A Geometric Approach to CP Violation: Applications to the MCPMFV SUSY Model

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* based on JHEP 1010:049,2010, arXiv:1006.3087 [hep-ph] and arXiv:1009.1151 [math.OC] with J. Ellis and A. Pilaftsis

♠ *Preliminary*

- The SUSY models such as the MSSM contain many possible sources of flavour and CP violation in the soft SUSY-breaking sector:

$30 \oplus 33 \oplus 46 = \mathbf{109}$!!!

  – Gaugino mass terms: $3 \oplus 3 = 6$

$$-\mathcal{L}_{\rm soft} \supset \frac{1}{2}(M_3\, \widetilde{g}\widetilde{g} + M_2\, \widetilde{W}\widetilde{W} + M_1\, \widetilde{B}\widetilde{B} + \text{h.c.})$$

  – Trilinear $\mathbf{a}$ terms $\mathbf{a}_{\mathbf{f}ij} \equiv \mathbf{h}_{\mathbf{f}ij} \cdot \mathbf{A}_{\mathbf{f}ij}$: $3 \times (3 \oplus 6 \oplus 9) = 54$

$$-\mathcal{L}_{\rm soft} \supset (\widetilde{u}^*_R\, \mathbf{a_u}\, \widetilde{Q} H_2 - \widetilde{d}^*_R\, \mathbf{a_d}\, \widetilde{Q} H_1 - \widetilde{e}^*_R\, \mathbf{a_e}\, \widetilde{L} H_1 + \text{h.c.})$$

  – Sfermion mass terms: $5 \times (3 \oplus 3 \oplus 3) = 45$

$$-\mathcal{L}_{\rm soft} \supset \widetilde{Q}^\dagger\, \mathbf{M}^2_{\widetilde{\mathbf{Q}}}\, \widetilde{Q} + \widetilde{L}^\dagger\, \mathbf{M}^2_{\widetilde{\mathbf{L}}}\, \widetilde{L} + \widetilde{u}^*_R\, \mathbf{M}^2_{\widetilde{\mathbf{u}}}\, \widetilde{u}_R + \widetilde{d}^*_R\, \mathbf{M}^2_{\widetilde{\mathbf{d}}}\, \widetilde{d}_R + \widetilde{e}^*_R\, \mathbf{M}^2_{\widetilde{\mathbf{e}}}\, \widetilde{e}_R$$

  – Higgs mass terms: $3 \oplus 1 = 4$

$$-\mathcal{L}_{\rm soft} \supset M^2_{H_u} H_2^\dagger H_2 + M^2_{H_d} H_1^\dagger H_1 - (m^2_{12} H_1 H_2 + \text{h.c.})$$

♠ *Preliminary*

- Recently, we have suggested **MCPMFV** framework with the maximal set of flavour-singlet mass scales: J. Ellis, JSL and A. Pilaftsis, Phys. Rev. D **76** (2007) 115011, [arXiv:0708.2079 [hep-ph]]

$$M_{1,2,3}\,, \quad M^2_{H_{u,d}}\,, \quad \widetilde{\mathbf{M}}^2_{Q,L,U,D,E} = \widetilde{M}^2_{Q,L,U,D,E}\,\mathbf{1}_3\,, \quad \mathbf{A}_{u,d,e} = A_{u,d,e}\,\mathbf{1}_3$$

$$3 \oplus 3 \qquad 2 \qquad 5 \qquad 3 \oplus 3$$

$13 \oplus 6 = 19$ Parameters !

For related approaches, see,

M. Argyrou, A. B. Lahanas and V. C. Spanos, JHEP **0805** (2008) 026; [arXiv:0804.2613 [hep-ph]]

G Colangelo, E. Nikolidakis and C. Smith, Eur. Phys. J. C **59** (2009) 75; [arXiv:0807.0801 [hep-ph]]

W. Altmannshofer, A. J. Buras and P. Paradisi, Phys. Lett. B **669** (2008) 239; [arXiv:0808.0707 [hep-ph]]

L. Mercolli and C. Smith, Nucl. Phys. B **817** (2009) 1; [arXiv:0902.1949 [hep-ph]]

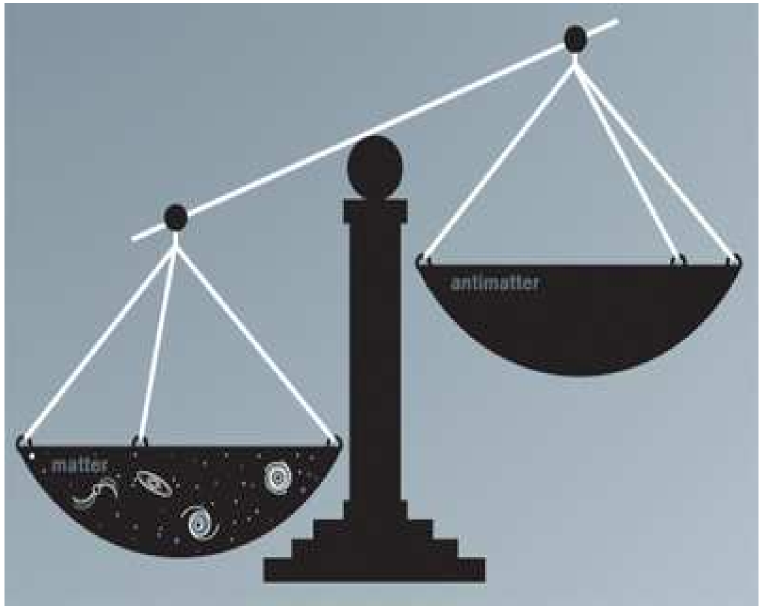
A. L. Kagan, G. Perez, T. Volansky and J. Zupan, Phys. Rev. D **80** (2009) 076002; [arXiv:0903.1794 [hep-ph]]

R. Zwicky and T. Fischbacher, Phys. Rev. D **80** (2009) 076009; [arXiv:0908.4182 [hep-ph]]

J. Ellis, R. N. Hodgkinson, JSL and A. Pilaftsis, JHEP **1002** (2010) 016; [arXiv:0911.3611 [hep-ph]]

♠ *Preliminary*

- Then, who ordered "more" CP violation beyond the SM CKM phase? A. D. Sakharov, JETP Letters **5**(1967)24

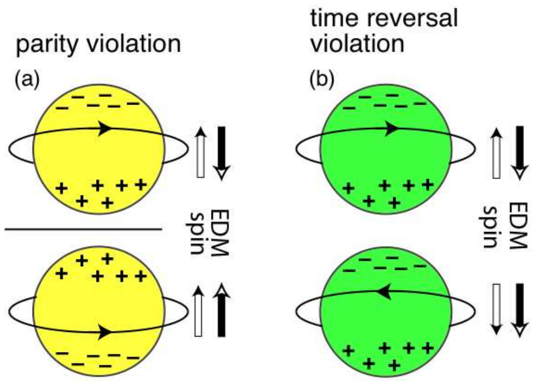


CP violation in the SM is too weak to explain the matter dominance of the Universe J. Cline, arXiv:hep-ph/0609145

The matter-dominated Universe did!

♠ *Preliminary*

- Electric Dipole Moments (EDMs): T violation ⇒ CP violation (under CPT)



$$\mathcal{H}^{\mathrm{EDM}} = -d\,\mathbf{E}\cdot\hat{\mathbf{s}}$$

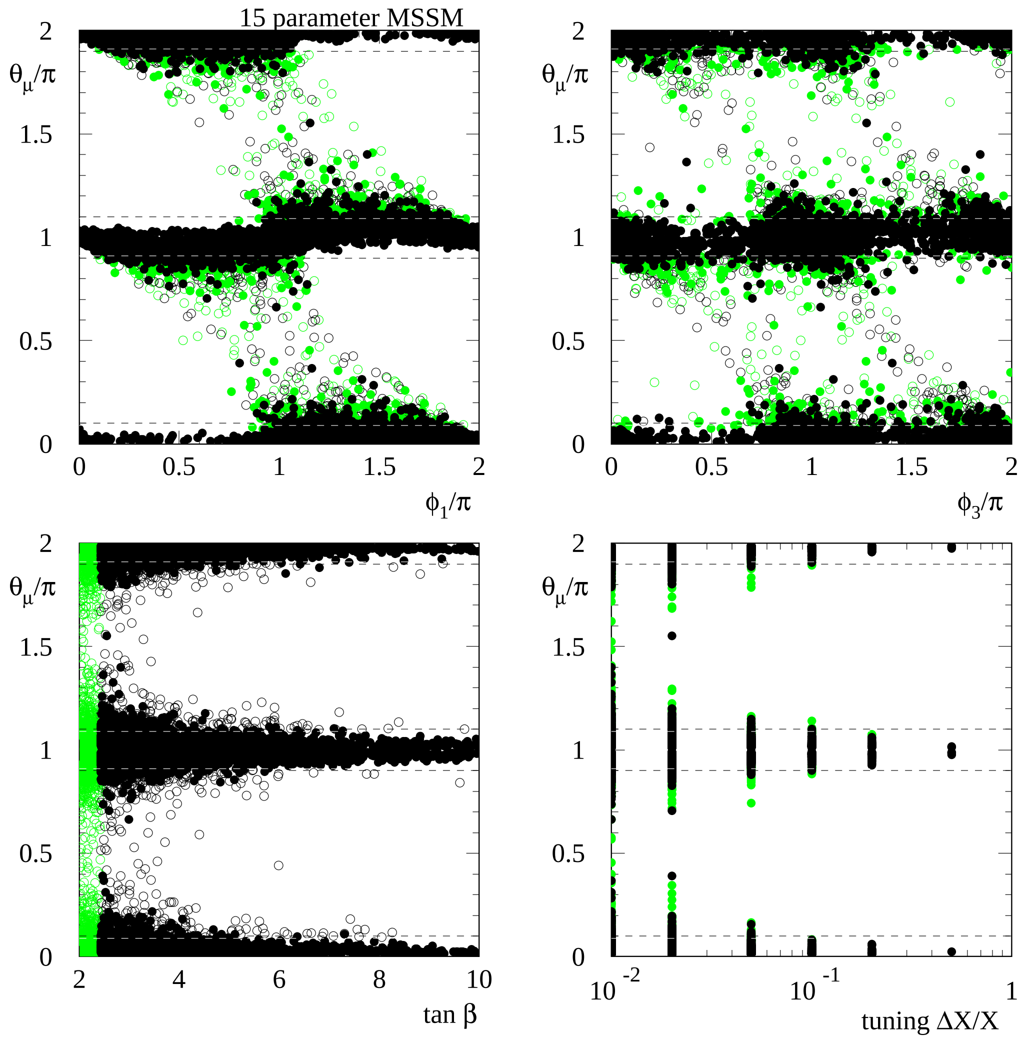
$$|d_{\mathrm{Tl}}| < 9\times 10^{-25}\,\mathrm{e\,cm}\,,\quad |d_{\mathrm{Hg}}| < 3.1\times 10^{-29}\,\mathrm{e\,cm}\,,\quad |d_{\mathrm{n}}| < 3\times 10^{-26}\,\mathrm{e\,cm}$$

B. C. Regan, E. D. Commins, C. J. Schmidt and D. DeMille, PRL**88** (2002) 071805; W. C. Griffith, M. D. Swallows, T. H. Loftus, M. V. Romalis, B. R. Heckel and E. N. Fortson, PRL**102** (2009) 101601; C. A. Baker *et al.*, PRL**97** (2006) 131801

Question: No large CP phases?

♠ *Preliminary*

- A scan method: See, for example, V. D. Barger, T. Falk, T. Han, J. Jiang, T. Li and T. Plehn, Phys. Rev. D **64** (2001) 056007 [arXiv:hep-ph/0101106].



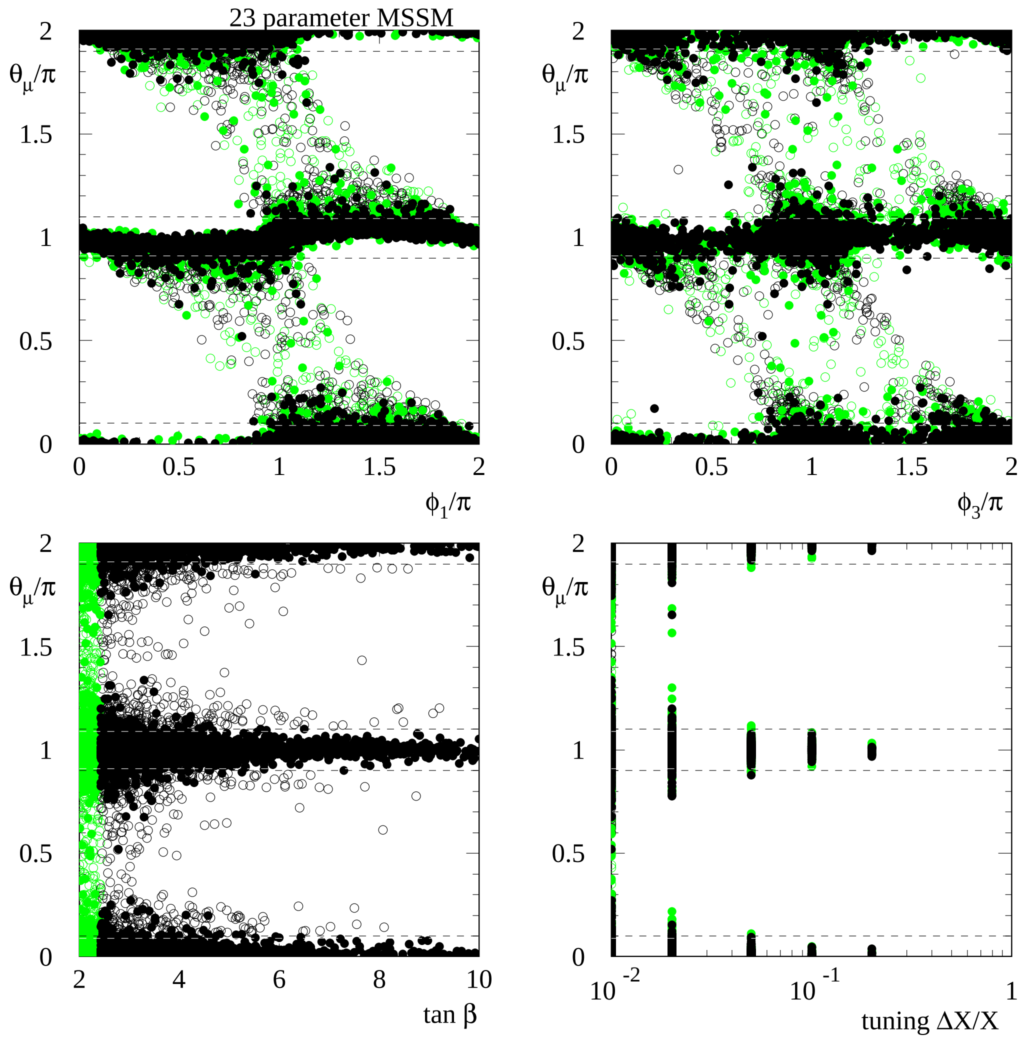
◇ 15 parameter MSSM:

$$
\begin{array}{rcccl}
0 & \leq & \theta_\mu, \phi_{1,3}, \theta_{Ad,e,u,t} & \leq & 2\pi \\
100\text{ GeV} & \leq & \mu & \leq & 1\text{ TeV} \\
100\text{ GeV} & \leq & 2M_1, M_2, M_3 & \leq & 1\text{ TeV} \\
0\text{ GeV} & \leq & |A| & \leq & 1\text{ TeV} \\
0\text{ GeV} & \leq & m_{\tilde{e}_R}, m_{\tilde{u}_R} & \leq & 1\text{ TeV} \\
2 & \leq & \tan\beta & \leq & 10
\end{array}
$$

Open circles suffer from parameter tuning $\Delta X/X$ worse than 1%. Green dots correspond to configurations with a light Higgs $m_h < 113$ GeV.

♠ *Preliminary*

- A scan method: See, for example, V. D. Barger, T. Falk, T. Han, J. Jiang, T. Li and T. Plehn, Phys. Rev. D **64** (2001) 056007 [arXiv:hep-ph/0101106].

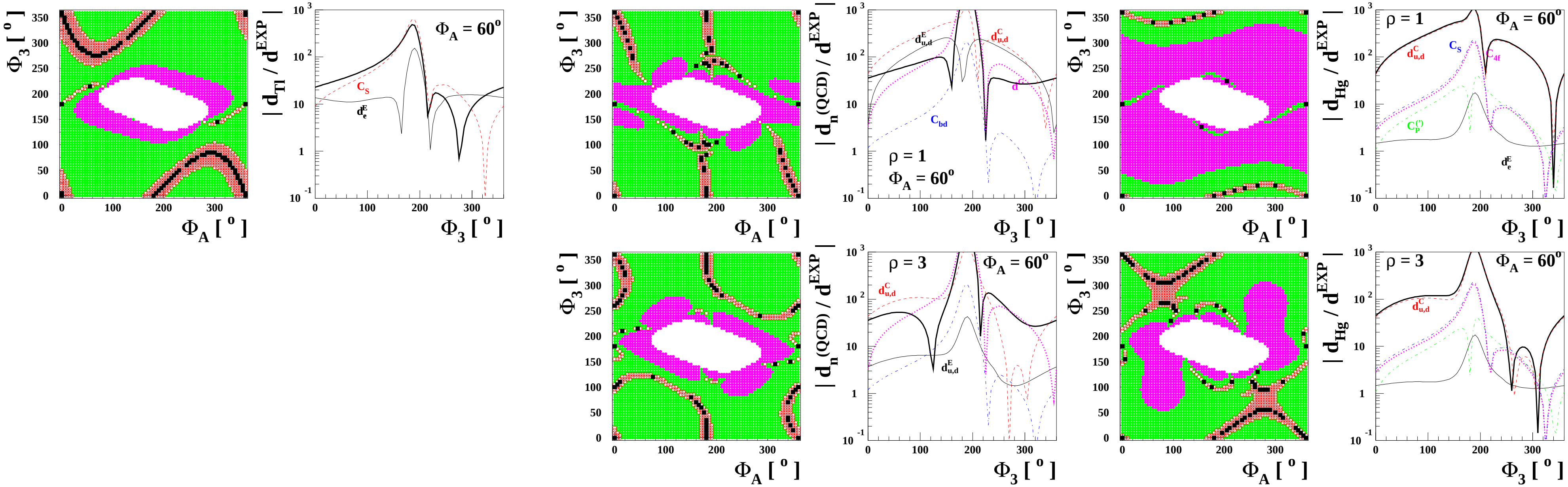


◇ 23 parameter MSSM:

0	$\leq$	$\theta_\mu, \theta_{M_i}, \theta_{A_i}$	$\leq$	2π
100 GeV	$\leq$	μ	$\leq$	1 TeV
100 GeV	$\leq$	$2M_1,\ M_2,\ M_3$	$\leq$	1 TeV
0 GeV	$\leq$	$\lvert A_{d,e,u,t}\rvert$	$\leq$	1 TeV
0 GeV	$\leq$	$m_{\tilde{\ell}_L}, m_{\tilde{e}_R}$	$\leq$	1 TeV
0 GeV	$\leq$	$m_{\tilde{q}_L}, m_{\tilde{u}_R}, m_{\tilde{d}_R}$	$\leq$	1 TeV
0 GeV	$\leq$	$m_{\tilde{q}_L^3}, m_{\tilde{t}_R}$	$\leq$	1 TeV
2	$\leq$	$\tan\beta$	$\leq$	10

♠ *Preliminary*

- <u>A scan method</u>: J. R. Ellis, JSL and A. Pilaftsis, JHEP **0810** (2008) 049 [arXiv:0808.1819 [hep-ph]]



d_{Tl} $d_{\mathrm{n}}^{\mathrm{QCD}}$ d_{Hg}

♠ *Preliminary*

- A scan method is like *"shooting in the dark"* ...

blind,

time consuming,

no guiding principle, etc

Any analytic, exact, and more effective method?

$\Longrightarrow$ **A Geometric Approach** to CP violation

♠ *Optimal Direction*

- A linear approximation: We consider the case with N CP-violating phases

In the N-dimensional CP-phase space, we define

$$N\text{−D phase vector}\quad \mathbf{\Phi} = (\Phi_1, \Phi_2, \cdots, \Phi_N)$$

and then any CP-odd observable O and any EDM E can be expanded as

$$O = \mathbf{\Phi} \cdot \mathbf{O} + \cdots\,; \qquad E = \mathbf{\Phi} \cdot \mathbf{E} + \cdots$$

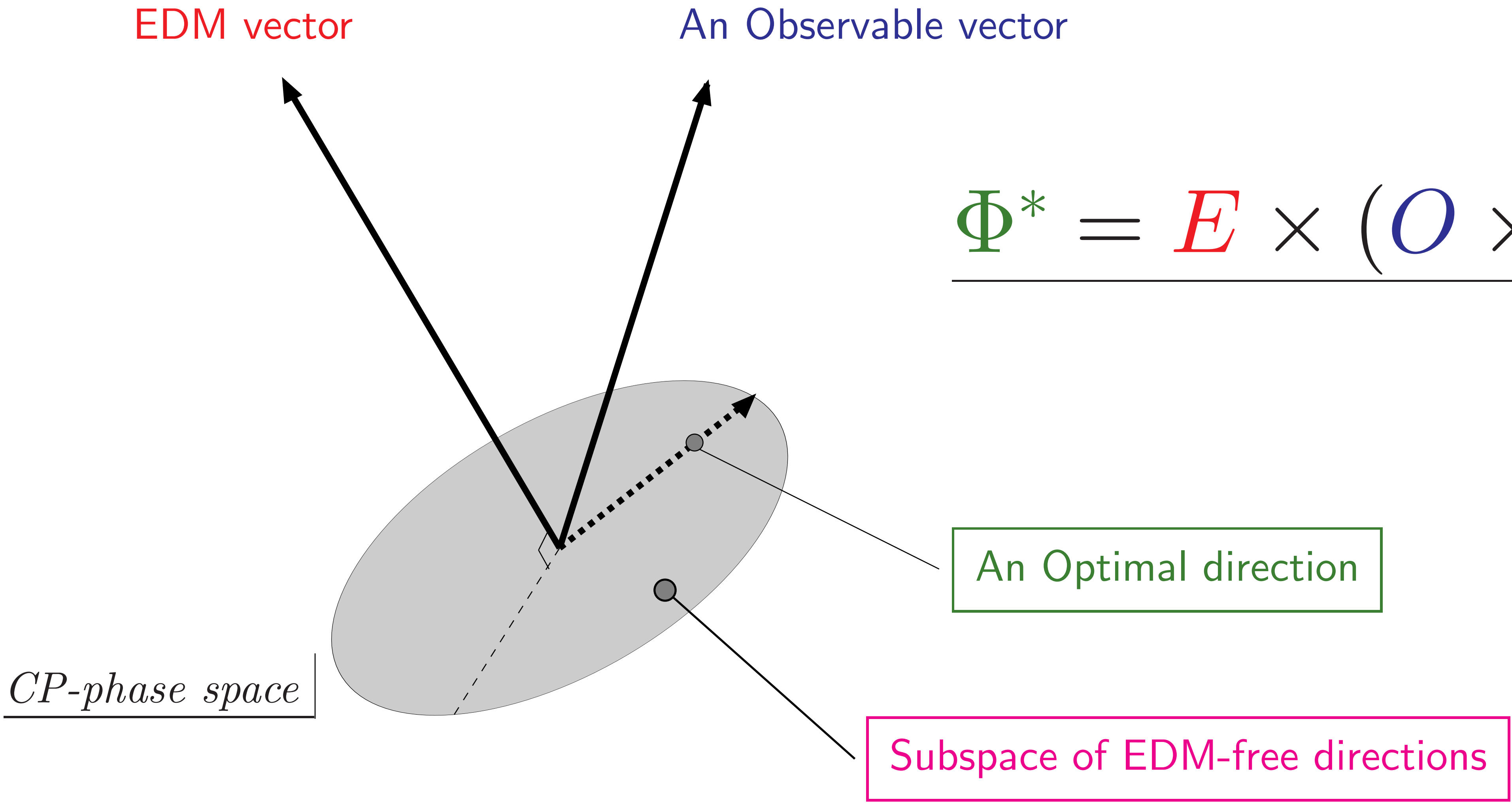
Formally, we define

$$\mathbf{O} \equiv \nabla O\,; \qquad\qquad \mathbf{E} \equiv \nabla E$$

with $\nabla \equiv (\partial/\partial\phi_1, \partial/\partial\phi_2, \cdots, \partial/\partial\phi_N)$

♠ *Optimal Direction*

- [Simple 3D example] with 3 CP phases and 1 EDM constraints: EDM-free subspace and Optimal direction in the linear approximation



$$\Phi^* = E \times (O \times E)$$

♠ *Optimal Direction*

- THE HIGHER-DIMENSIONAL GENERALIZATION with

N CP phases and n EDM constraints

The N-dimensional vector of the optimal direction

$$\Phi^*{}_\alpha = \epsilon_{\alpha\beta_1\cdots\beta_n\gamma_1\cdots\gamma_{N-n-1}} E^{(1)}_{\beta_1} \cdots E^{(n)}_{\beta_n} B_{\gamma_1\cdots\gamma_{N-n-1}}$$

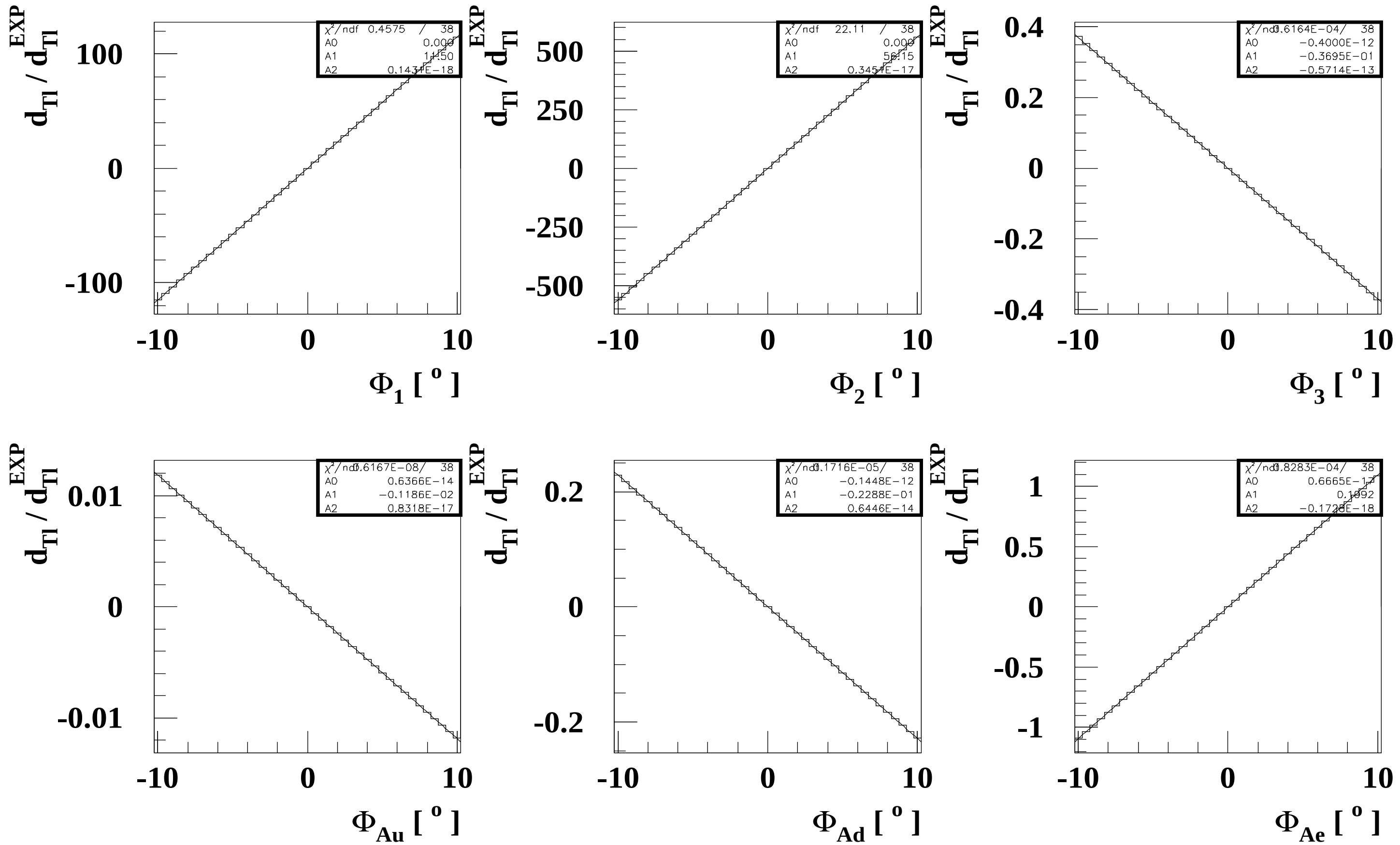
where (N-n-1)-dimensional B form is

$$B_{\gamma_1\cdots\gamma_{N-n-1}} = \epsilon_{\gamma_1\cdots\gamma_{N-n-1}\sigma\beta_1\cdots\beta_n} O_\sigma E^{(1)}_{\beta_1} \cdots E^{(n)}_{\beta_n}$$

The maximum allowed value of O is: $O^{\max} = (\phi^*)^{\max}\, \widehat{\mathbf{\Phi}}^* \cdot \mathbf{O}$ with the normalized optimal-direction vector $\widehat{\mathbf{\Phi}}^*$ and ϕ^* which may be practically determined by the validity of the small-phase approximation of the EDMs.

♠ *Optimal Direction*

- *How good is the linear approximation?* The quadratic fit to the Thallium EDM that is used to obtain the 6D vector $\mathbf{E}^{d_{\rm Tl}} \equiv \nabla(d_{\rm Tl}/d_{\rm Tl}^{\rm EXP})$ in an expansion around $\widetilde{\varphi}_\alpha = 0^\circ$ for the MCPMFV scenario: $|M_{1,2,3}| = 250 \;\; {\rm GeV}\,, M^2_{H_u} = M^2_{H_d} = \widetilde{M}^2_Q = \widetilde{M}^2_U = \widetilde{M}^2_D = \widetilde{M}^2_L = \widetilde{M}^2_E = (100 \;\; {\rm GeV})^2\,, |A_u| = |A_d| = |A_e| = 100 \;\; {\rm GeV}$, and $\tan\beta = 40$.



♠ *Numerical Illustration*

- We are considering J. Ellis, JSL, and A. Pilaftsis, JHEP 0810:049,2008, arXiv:0808.1819 [hep-ph] ; K. Cheung, O. C. W. Kong, and JSL, JHEP 0906:020,2009, arXiv:0904.4352 [hep-ph]; J. Ellis, JSL and A. Pilaftsis, Phys. Rev. D **76** (2007) 115011, [arXiv:0708.2079 [hep-ph]]
  - Thallium EDM
  - Neutron EDM
  - Mercury EDM
  - Deuteron EDM
  - muon EDM
  - $A_{\rm CP}(b \to s\gamma)$

These are all implemented in `CPsuperH2.2`

♠ *Numerical Illustration*

- Thallium EDM; I.B. Khriplovich and S.K. Lamoreaux, *CP Violation Without Strangeness* (Springer, New York, 1997); M. Pospelov and A. Ritz, Annals Phys. **318** (2005) 119,[arXiv:hep-ph/0504231]

$$d_{\rm Tl}\,[e\,{\rm cm}] \;=\; -585\cdot d_e^E\,[e\,{\rm cm}] \;-\; 8.5\times 10^{-19}\,[e\,{\rm cm}]\cdot(C_S\,{\rm TeV}^2) \;+\;\cdots$$

$$\begin{aligned} d_e^E &= (d_e^E)^{\tilde\chi^\pm} + (d_e^E)^{\tilde\chi^0} + (d_e^E)^H \\ C_S &= (C_S)^{4f} + (C_S)^g \end{aligned}$$

where $(C_S)^{4f} \;=\; C_{de}\frac{29\,{\rm MeV}}{m_d} + C_{se}\frac{\kappa\times 220\,{\rm MeV}}{m_s}$ with $\kappa\equiv\langle N|m_s\bar s s|N\rangle/220\ {\rm MeV}\simeq 0.50\pm 0.25$ and

$$(C_S)^g = (0.1\,{\rm GeV})\,\frac{m_e}{v^2}\sum_{i=1}^{3}\frac{g^S_{H_igg}g^P_{H_i\bar e e}}{M^2_{H_i}}$$

♠ *Numerical Illustration*

- Neutron EDM [QCD sum rule techniques (QCD)]; M. Pospelov and A. Ritz, Phys. Rev. Lett. **83** (1999) 2526, [arXiv:hep-ph/9904483]; M. Pospelov and A. Ritz, Nucl. Phys. B **573** (2000) 177, [arXiv:hep-ph/9908508]; M. Pospelov and A. Ritz, Phys. Rev. D **63** (2001) 073015, [arXiv:hep-ph/0010037]; D. A. Demir, M. Pospelov and A. Ritz, Phys. Rev. D **67** (2003) 015007,[arXiv:hep-ph/0208257]; D. A. Demir, O. Lebedev, K. A. Olive, M. Pospelov and A. Ritz, Nucl. Phys. B **680** (2004) 339, [arXiv:hep-ph/0311314]; K. A. Olive, M. Pospelov, A. Ritz and Y. Santoso, Phys. Rev. D **72** (2005) 075001, [arXiv:hep-ph/0506106]

$$d_n = d_n(d_q^E, d_q^C) + d_n(d^G) + d_n(C_{bd}) + \cdots ,$$

$$\begin{aligned} d_n(d_q^E, d_q^C) &= (1.4 \pm 0.6)\,(d_d^E - 0.25\,d_u^E) + (1.1 \pm 0.5)\,e\,(d_d^C + 0.5\,d_u^C)/g_s\,, \\ d_n(d^G) &\sim \pm\, e\,(20 \pm 10)\ \mathrm{MeV}\,d^G\,, \\ d_n(C_{bd}) &\sim \pm\, e\,2.6 \times 10^{-3}\ \mathrm{GeV}^2 \left[\frac{C_{bd}}{m_b} + 0.75\frac{C_{db}}{m_b}\right] \end{aligned}$$

where $d^G = d^G(1\ \mathrm{GeV}) \simeq 8.5\,d^G(\mathrm{EW})$

♠ *Numerical Illustration*

- Mercury EDM; D. A. Demir, O. Lebedev, K. A. Olive, M. Pospelov and A. Ritz, Nucl. Phys. B **680** (2004) 339, [arXiv:hep-ph/0311314]; K. A. Olive, M. Pospelov, A. Ritz and Y. Santoso, Phys. Rev. D **72** (2005) 075001, [arXiv:hep-ph/0506106]

$$
\begin{aligned}
d_{\rm Hg} \;=\; & 7\times 10^{-3}\, e\,(d_u^C - d_d^C)/g_s \;+\; 10^{-2}\, d_e^E \\
& - 1.4\times 10^{-5}\, e\,{\rm GeV}^2\left[\frac{0.5 C_{dd}}{m_d} + 3.3\kappa\frac{C_{sd}}{m_s} + (1-0.25\kappa)\frac{C_{bd}}{m_b}\right] \\
& + (3.5\times 10^{-3}\ {\rm GeV})\, e\, C_S \\
& + (4\times 10^{-4}\ {\rm GeV})\, e\left[C_P + \left(\frac{Z-N}{A}\right)_{\rm Hg} C_P'\right]
\end{aligned}
$$

where $\mathcal{L}_{C_P} \;=\; C_P\,\bar{e}e\,\bar{N}i\gamma_5 N \;+\; C_P'\,\bar{e}e\,\bar{N}i\gamma_5\tau_3 N$ with

$$
\begin{aligned}
C_P \;&=\; (C_P)^{4f} \;\simeq\; -375\ {\rm MeV}\sum_{q=c,s,t,b}\frac{C_{eq}}{m_q}\,, \\
C_P' \;&=\; (C_P')^{4f} \;\simeq\; -806\ {\rm MeV}\,\frac{C_{ed}}{m_d} - 181\ {\rm MeV}\sum_{q=c,s,t,b}\frac{C_{eq}}{m_q}
\end{aligned}
$$

♠ *Numerical Illustration*

- Deuteron EDM; O. Lebedev, K. A. Olive, M. Pospelov and A. Ritz, Phys. Rev. D **70** (2004) 016003,[arXiv:hep-ph/0402023]

$$
\begin{aligned}
d_D \;\simeq\; & -\left[5^{+11}_{-3} + (0.6 \pm 0.3)\right] \, e\,(d_u^C - d_d^C)/g_s \\
& -(0.2 \pm 0.1)\, e\,(d_u^C + d_d^C)/g_s \; + \; (0.5 \pm 0.3)(d_u^E + d_d^E) \\
& +(1 \pm 0.2) \times 10^{-2}\, e\,\mathrm{GeV}^2 \left[\frac{0.5 C_{dd}}{m_d} + 3.3\kappa \frac{C_{sd}}{m_s} + (1 - 0.25\kappa)\frac{C_{bd}}{m_b}\right] \\
& \pm\; e\,(20 \pm 10)\; \mathrm{MeV}\, d^G
\end{aligned}
$$

The projective sensitivity: Y. K. Semertzidis *et al.* [EDM Collaboration], AIP Conf. Proc. **698** (2004) 200, [arXiv:hep-ex/0308063]; Y. F. Orlov, W. M. Morse and Y. K. Semertzidis, Phys. Rev. Lett. **96** (2006) 214802, [arXiv:hep-ex/0605022]; and also see `http://www.bnl.gov/edm/deuteron_proposal_080423_final.pdf`

$$
|d_D| < 3 \times 10^{-27} - 10^{-29} \; e\,\mathrm{cm}
$$

For our numerical study, we take $3 \times 10^{-27}\; e\,\mathrm{cm}$ as a representative expected value

♠ *Numerical Illustration*

- CP-violating QCD θ-term: O. Lebedev, K. A. Olive, M. Pospelov and A. Ritz, Phys. Rev. D **70** (2004) 016003, [arXiv:hep-ph/0402023]; M. Pospelov and A. Ritz, Annals Phys. **318** (2005) 119, [arXiv:hep-ph/0504231]; J. Ellis, JSL, and A. Pilaftsis, arXiv:1006.3087 [hep-ph]

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\,\bar\theta\,G^a_{\mu\nu}\tilde G^{\mu\nu,a} \quad \text{with} \quad \bar\theta = \theta_{\rm QCD} + {\rm Arg\,Det}\,M_q$$

$$\begin{aligned} d_n(\bar\theta) &= (1\pm 0.5)\frac{|\langle\bar q q\rangle|}{(225\ {\rm MeV})^3}\,\bar\theta\,\times 2.5\times 10^{-16}\,e\,{\rm cm} \\ d_{\rm Hg}(\bar\theta) &\simeq +2.0\times 10^{-6}\,\bar\theta\,e\,{\rm GeV}^{-1} \simeq 3.9\times 10^{-20}\,\bar\theta\,e\,{\rm cm} \\ d_D(\bar\theta) &\simeq -e\,[(3.5\pm 1.4)+(1.4\pm 0.4)]\times 10^{-3}\,\bar\theta\,{\rm GeV}^{-1} \simeq -9.7\times 10^{-17}\,\bar\theta\,e\,{\rm cm} \end{aligned}$$

♠ *Numerical Illustration*

- [Summary] EDMs and Observables under consideration

$$d_{\rm Tl}/d_{\rm Tl}^{\rm EXP}\,, \quad d_{\rm n}/d_{\rm n}^{\rm EXP}\,, \quad d_{\rm Hg}/d_{\rm Hg}^{\rm EXP}\,,$$
$$d_{\rm D}/d_{\rm D}^{\rm EXP}\,, \quad d_{\mu}/d_{\mu}^{\rm EXP}\,, \quad A_{\rm CP}(b\to s\gamma)[\%]\,,$$

where we choose the following normalization factors

$$d_{\rm Tl}^{\rm EXP} = 9\times 10^{-25}\,e\,{\rm cm}\,, \quad d_{\rm n}^{\rm EXP} = 3\times 10^{-26}\,e\,{\rm cm}\,, \quad d_{\rm Hg}^{\rm EXP} = 3.1\times 10^{-29}\,e\,{\rm cm}\,,$$
$$d_{\rm D}^{\rm EXP} = 3\times 10^{-27}\,e\,{\rm cm}\,, \quad d_{\mu}^{\rm EXP} = 1\times 10^{-24}\,e\,{\rm cm}$$

♠ *Numerical Illustration*

- The EDMs and Observables under consideration are functions of

7 parameters :

$$\Phi_1\,,\quad \Phi_2\,,\quad \Phi_3\,,\quad \Phi_{A_u}\,,\quad \Phi_{A_d}\,,\quad \Phi_{A_e}\,,\quad \bar{\theta}$$

and then

$$\nabla_\alpha \equiv \left(\frac{\partial}{\partial\Phi_1},\ \frac{\partial}{\partial\Phi_2},\ \frac{\partial}{\partial\Phi_3},\ \frac{\partial}{\partial\Phi_{A_u}},\ \frac{\partial}{\partial\Phi_{A_d}},\ \frac{\partial}{\partial\Phi_{A_e}},\ \frac{\partial}{\partial\widehat{\theta}}\right)$$

The CP-violating phases $\Phi_{1,2,3}$ and Φ_{A_u,A_d,A_e} are specified in degrees and we normalize $\bar{\theta}$ in units of 10^{-10}: $\widehat{\theta} \equiv \bar{\theta} \times 10^{10}$

♠ *Numerical Illustration*

- A scenario: We consider CP-violating variants of a typical CMSSM scenario with

$$|M_{1,2,3}| = 250 \ \ \mathrm{GeV}\,,$$
$$M^2_{H_u} = M^2_{H_d} = \widetilde{M}^2_Q = \widetilde{M}^2_U = \widetilde{M}^2_D = \widetilde{M}^2_L = \widetilde{M}^2_E = (100 \ \ \mathrm{GeV})^2\,,$$
$$|A_u| = |A_d| = |A_e| = 100 \ \ \mathrm{GeV}\,,$$

at the GUT scale, varying $\tan\beta\,(M_{\mathrm{SUSY}})$

We adopt the convention that $\Phi_\mu = 0^\circ$, and we vary independently the following six MCPMFV phases at the GUT scale:

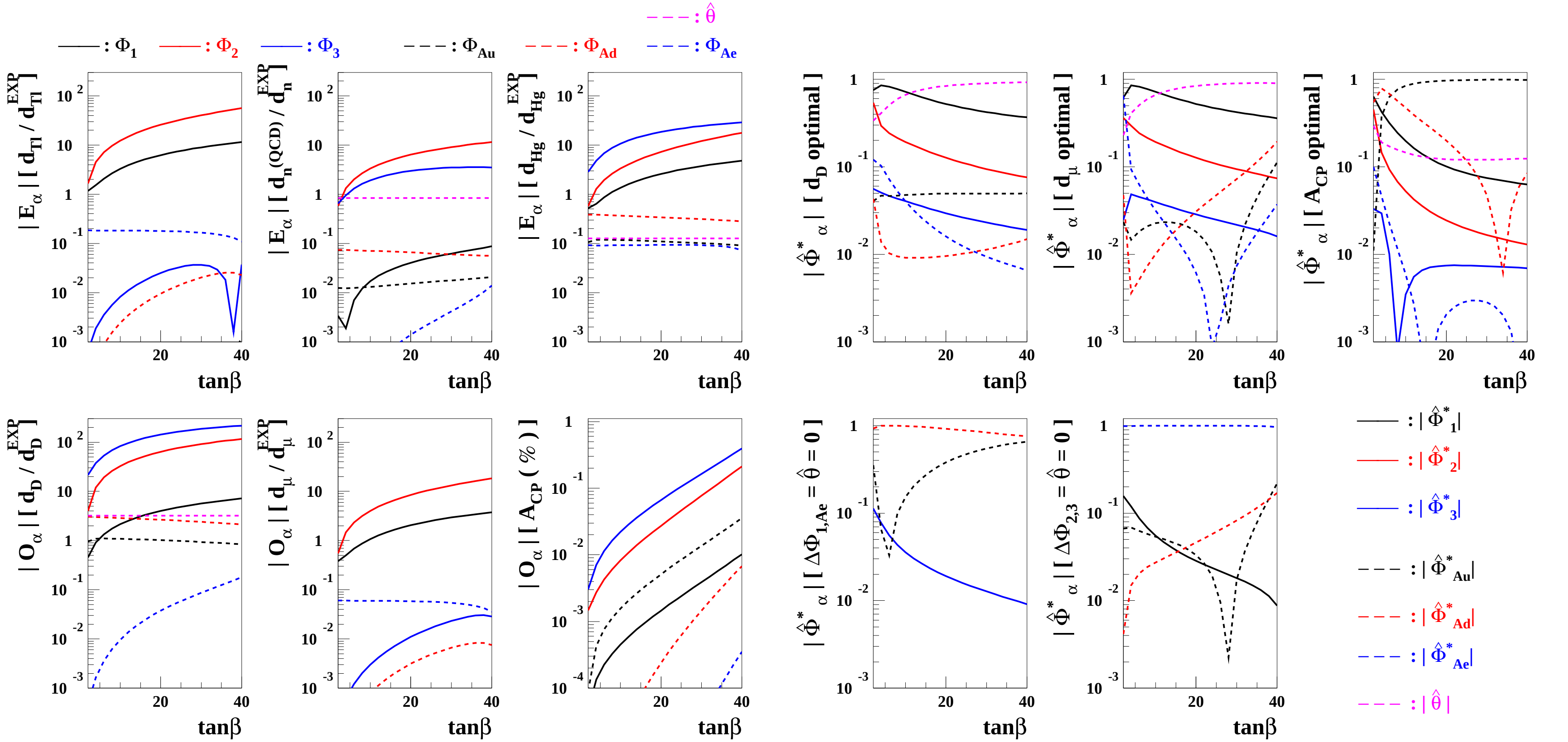
$$\Phi_1\,, \quad \Phi_2\,, \quad \Phi_3\,, \quad \Phi_{A_u}\,, \quad \Phi_{A_d}\,, \quad \Phi_{A_e}$$

in addition to the QCD θ term: $\bar{\theta}$

This scenario becomes the SPS1a point when $\tan\beta = 10$, $\Phi_{1,2,3} = 0^\circ$ and $\Phi_{A_u,A_d,A_e} = 180^\circ$

♠ *Numerical Illustration*

- Components of the vectors $\mathbf{E} \equiv \nabla E\,,\ \mathbf{O} \equiv \nabla O\,;\ \widehat{\Phi}^*$

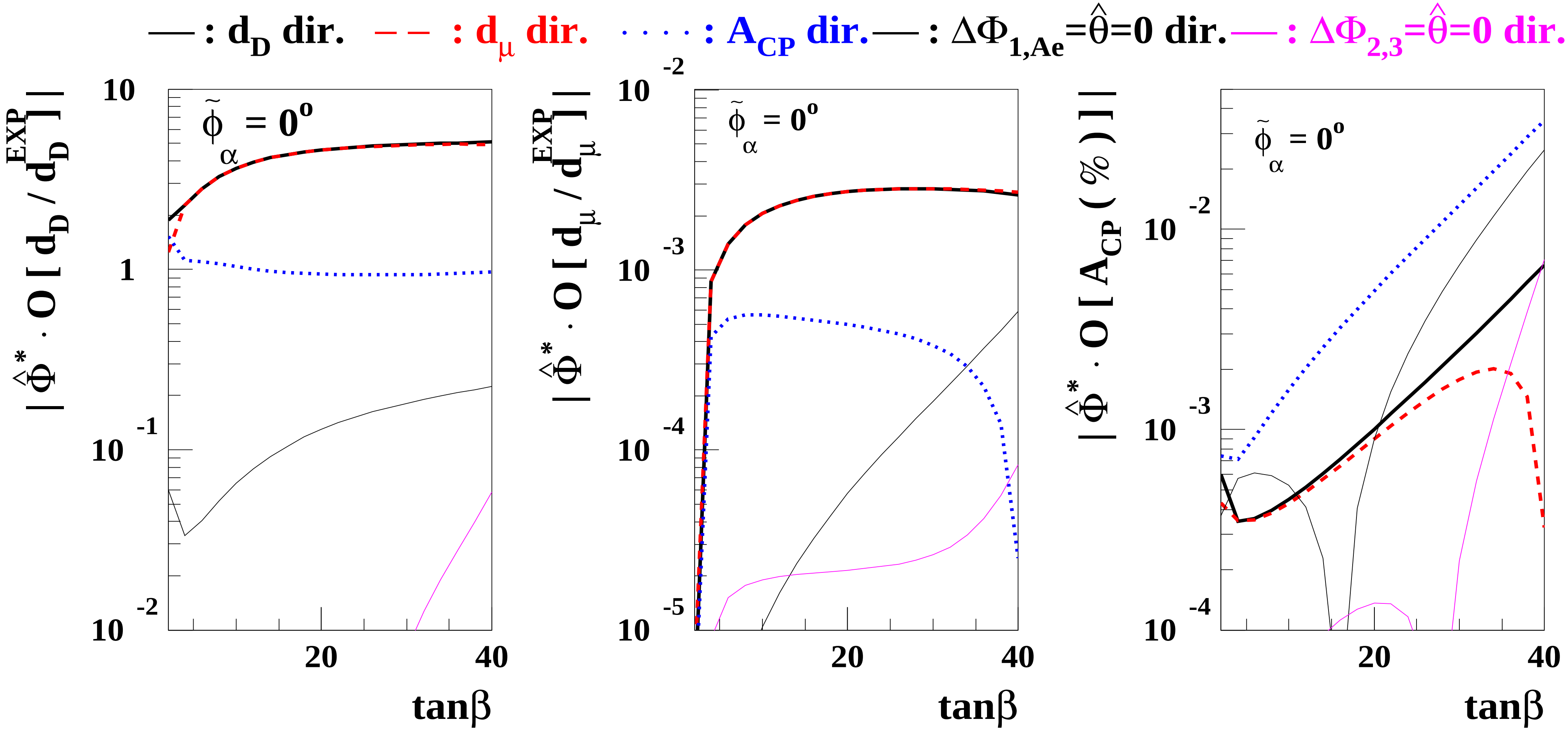


$$\widehat{\Phi}^*{}_\alpha \;=\; \mathcal{N}\, \varepsilon_{\alpha\beta\gamma\delta\mu\nu\rho}\, E_\beta^{d_{\rm Tl}}\, E_\gamma^{d_{\rm n}}\, E_\delta^{d_{\rm Hg}}\, B_{\mu\nu\rho}$$

with the 3-form $B_{\mu\nu\rho} \;=\; \varepsilon_{\mu\nu\rho\lambda\sigma\tau\omega}\, O_\lambda\, E_\sigma^{d_{\rm Tl}}\, E_\tau^{d_{\rm n}}\, E_\omega^{d_{\rm Hg}}$ or $N_\mu^{(1)} N_\nu^{(2)} N_\rho^{(3)}$ for some reference directions

♠ *Numerical Illustration*

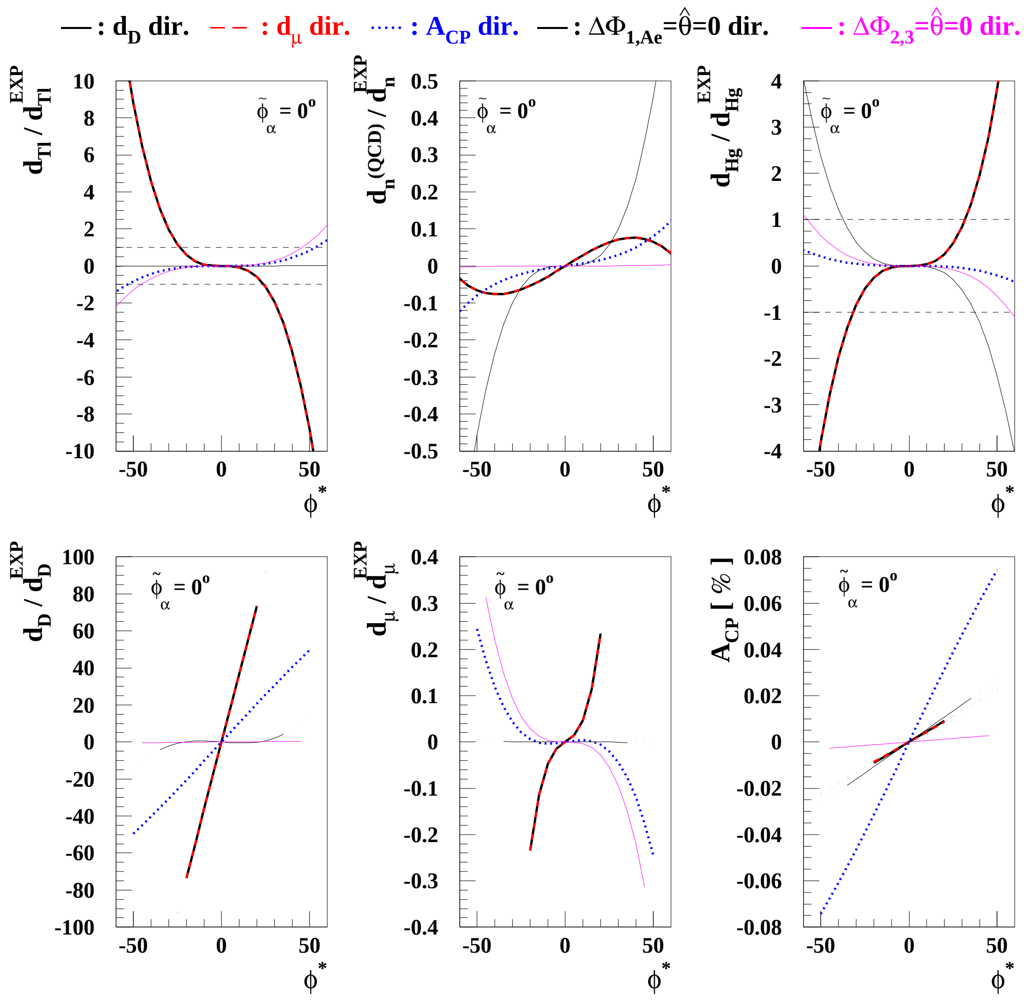
- The products $\widehat{\Phi}^* \cdot \mathbf{O}$ Recall the relation $O^{\max} = (\phi^*)^{\max}\, \widehat{\Phi}^* \cdot \mathbf{O}$



♠ *Numerical Illustration*

- The maximum values of the observables along the optimal directions $\tan\beta = 10$

  Again, recall the relation $O^{\max} = (\phi^*)^{\max}\, \widehat{\Phi}^* \cdot \mathbf{O}$

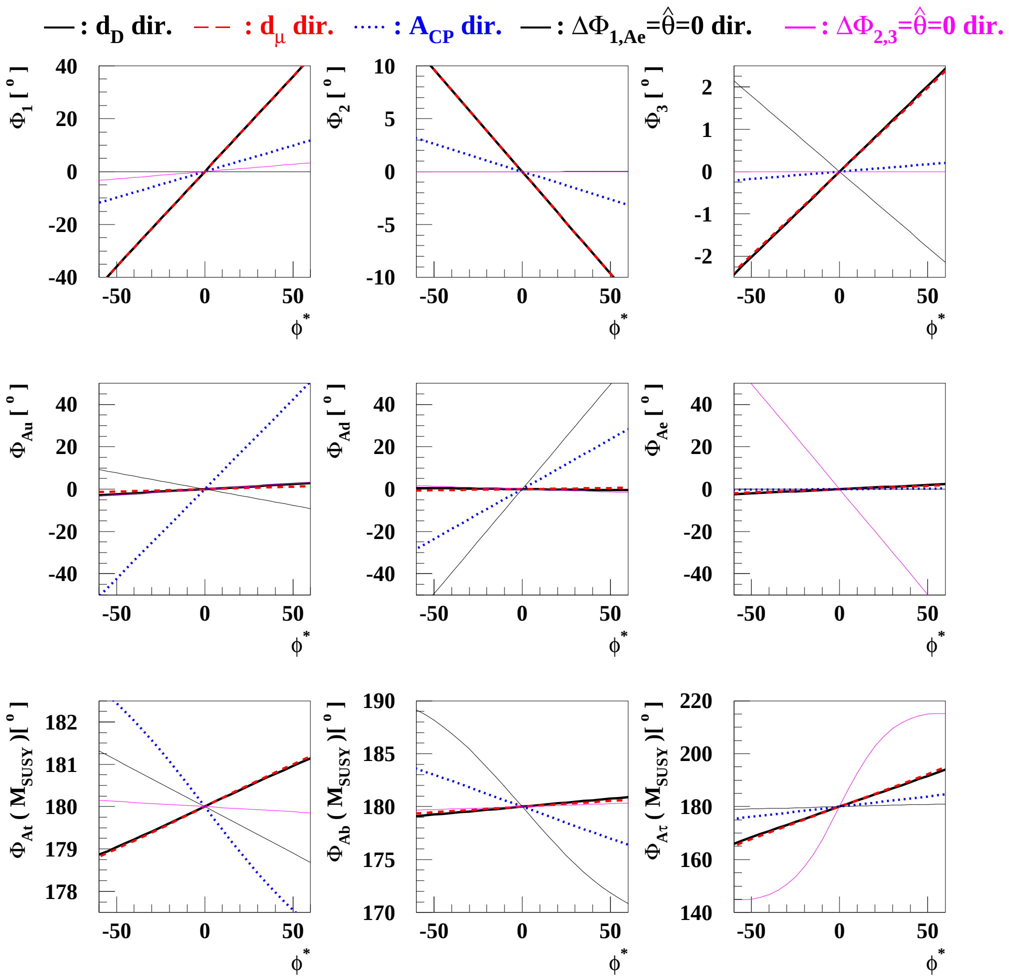


◇ The maximum values of ϕ^* for each EDM-free direction from the figure: $(\phi^*)^{\max} \sim$ 25 (d_D-optimal), 25 (d_μ-optimal), 50 (A_{CP}-optimal), 40 ($\Delta\Phi_{1,A_e} = \widehat{\theta} = 0$), and 45 ($\Delta\Phi_{2,3} = \widehat{\theta} = 0$), which are mainly constrained by d_{Tl}, d_{Tl}, d_{Tl}, d_{Hg}, and d_{Tl}, respectively.

◇ The maximum values of the observables

♠ Numerical Illustration

- The maximum values of the CP phases along the optimal directions $\tan\beta = 10$



In the top panels we see that Φ_1 and Φ_2 can be as large as $\sim 20^\circ$ and $\sim 5^\circ$, respectively, for $(\phi^*)^{\rm max} \sim 25$ along the $d_{\rm D}$- and d_μ-optimal directions denoted by the thick solid and dashed lines.

We also note in the middle and bottom panels that the phases of $A_{d,u,e}$ could be are large, in general, though they are suppressed at the $M_{\rm SUSY}$ scale.

Finally, we note (not shown) that $\bar{\theta}$ could be as large as $\sim 2 \times 10^{-9}$ along the $d_{\rm D}$- and d_μ-optimal directions with $(\phi^*)^{\rm max} \sim 25$.

♠ *Summary*

- We are proposing a geometric method which provides an accurate parametric determination of the optimal cancellation regions where any given physical observable is maximized in the linear approximation

- Our geometric approach is exact, efficient and less computationally-intensive than a naive scan of a multi-dimensional space

- This constitutes an *analytic* solution to the so-called *linear programming problem*

- You may want to apply this method to your problem if you are trying to achieve the best outcome in a given requirements expressed in linear equations