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Consistency of canonical formulation of Hořava Gravity

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(wth J. Yang, H. L. Yu ; J. Fernando Barbero G.)

*Einstein's General Relativity in 4-dimensions:

Not renormalizable as a perturbative QFT (Goroff, Sagnotti; t' Hooft Veltman; van der Ven …)

*GR with higher derivatives as perturbative QFTs :

Renormalizable; BUT not unitary (Stelle; Julve,Tonin; Fradkin,Tesytlin; Avramidi,Barvinsky;…)

$$\int d^4x\sqrt{g}\left[\frac{1}{16\pi G}(2\Lambda - R) + \frac{1}{2\lambda}C^2 - \frac{\omega}{3\lambda}R^2 + \frac{\theta}{\lambda}E\right]$$

$C^2 \equiv C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$ is the square of the Weyl tensor.

$E \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is the Gauss-Bonnet term.

taming of divergences due to higher derivatives

(General covariance => no. of time and space derivatives are equal)

=> problem with unitarity

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Quantum gravity at a Lifshitz point

Petr Hořava

*Horava's proposal:

improve convergence with higher spatial derivatives, but keep time derivatives to 2nd order only.

(=> Give up (!) spacetime covariance at the "fundamental" level)

Space and time are not on equal footing!

$$\frac{1}{\omega^2 - c^2\mathbf{k}^2 - G(\mathbf{k}^2)^z} = \frac{1}{\omega^2 - \mathbf{k}^2} + \frac{1}{\omega^2 - \mathbf{k}^2}G(\mathbf{k}^2)^z \times \frac{1}{\omega - \mathbf{k}^2} + \cdots.$$

*Reduce 4-dimensional diffeomorphism symmetry

-> 3-dimensional spatial diffeomorphism invariance

(?+? time reparametrization invariance)

$$ds^2 = -N^2(cdt)^2 + q_{ij}(dx^i + N^i cdt)(dx^j + N^j cdt)$$

Assume ADM decomposition of spacetime metric

*Horava's proposed action in canonical form:

$$S = \int \pi^{ij}\dot{q}_{ij}\, d^3x\, dt - \int (NH + N^i H_i)\, d^3x\, dt.$$

Guiding principle: maintain 3-dim. diffeomorphism invariance

»To eliminate many possible terms: Impose "detailed balance"

$$H = \frac{\kappa^2}{2}\frac{G_{ijkl}}{\sqrt{q}}\left[\pi^{ij}\pi^{kl} + \frac{\delta W_T}{\delta q_{ij}}\frac{\delta W_T}{\delta q_{kl}}\right] \qquad H_i = 2q_{ij}\nabla_k \pi^{kj}$$

Supermetric: $G_{ijkl} = \frac{1}{2}(q_{ik}q_{jl} + q_{il}q_{jk}) - \frac{\lambda}{3\lambda - 1} q_{ij}q_{kl}$

Deformation parameter λ

$W_T = W_{CS} + W_{EH\Lambda} = \frac{1}{4w^2}\int \tilde{\epsilon}^{ikj}(\Gamma^l_{im}\partial_j\Gamma^m_{kl} + \frac{2}{3}\Gamma^l_{im}\Gamma^m_{jn}\Gamma^n_{kl})\, d^3x + \frac{\mu}{2}\int \sqrt{q}(R - 2\Lambda_W)\, d^3x$. The Cotton tensor density can be expressed as $\tilde{C}^{ij} = w^2 \frac{\delta W_{CS}}{\delta q_{ij}} = w^2 \tilde{\epsilon}^{ikl}\nabla_k (R_l{}^j - \frac{1}{4}R\delta_l{}^j)$.

$$S = \int dt d^3\mathbf{x} \sqrt{g} N \Big\{ \frac{2}{\kappa^2} K_{ij} G^{ijk\ell} K_{k\ell}$$
$$- \frac{\kappa^2}{2} \Big[\frac{1}{w^2} C^{ij} - \frac{\mu}{2} \Big(R^{ij} - \frac{1}{2} R g^{ij} + \Lambda_W g^{ij} \Big) \Big]$$
$$\times \mathcal{G}_{ijk\ell} \Big[\frac{1}{w^2} C^{k\ell} - \frac{\mu}{2} \Big(R^{k\ell} - \frac{1}{2} R g^{k\ell} + \Lambda_W g^{k\ell} \Big) \Big] \Big\}.$$

$$S = \int dt d^3\mathbf{x} \sqrt{g} N \Big\{ \frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) - \frac{\kappa^2}{2w^4} C_{ij} C^{ij}$$
$$+ \frac{\kappa^2 \mu}{2w^2} \varepsilon^{ijk} R_{i\ell} \nabla_j R^{\ell}_k - \frac{\kappa^2 \mu^2}{8} R_{ij} R^{ij}$$
$$+ \frac{\kappa^2 \mu^2}{8(1-3\lambda)} \Big(\frac{1-4\lambda}{4} R^2 + \Lambda_W R - 3\Lambda_W^2 \Big) \Big\}.$$

$$G = \frac{\kappa^2 c^3}{32\pi}$$
$$\Lambda = \frac{3}{2} \Lambda_W$$
$$c = \frac{\kappa^2 \mu}{4} \sqrt{\frac{\Lambda_W}{1-3\lambda}}.$$

Short distance behavior: interacting fundamentally non-rel. gravitons
»Power-counting renormalizable in 3+1 dimensions.
=> If successful as perturbative QFT,
then coupling parameters obey renormalization group flow.
c, G emerge from non-relativistic fundamental theory.
Long distance behaviour : flows to Einstein's theory (hopefully(!)) $\lambda = 1$)
4-dim. spacetime covariance recovered at low energies/curvatures.

*Horava Gravity : *comes in 2 versions*

$$S = \int \pi^{ij}\dot{q}_{ij}\, d^3x\, dt - \int (NH + N^i H_i)\, d^3x\, dt$$

$$H = \frac{\kappa^2}{2\sqrt{q}}\left(\tilde{\pi}_{ij}\tilde{\pi}^{ij} - \frac{\lambda}{3\lambda - 1}\tilde{\pi}^2\right) + \sqrt{q}V(q)$$

1)*"Projectable" **(lapse function: N(t only))**

=> *Global (integrated) Hamiltonian constraint [$\int d^3x\, H(x)$] = 0

2) *"Non-projectable" **(lapse function N(t,x))**

=>*Local constraint H(x) = 0

*3) maybe "Projectable" is gauge-fixed version of "Non-projectable"

*Case 1) *Projectable version : with global (integrated) constraint

1 fewer local constraint than Gen. Rel. => (more than) 2 (local) d.o.f.

=> extra mode

Pathological behaviour of the scalar graviton in Hořava-Lifshitz gravity

arXiv 09101998

Kazuya Koyama* and Frederico Arroja†

$$S = \frac{M_{\rm pl}^2}{2}\int dt d^3x N\sqrt{-\gamma}\left(\left(K^{ij}K_{ij} - \lambda K^2\right) + {}^{(3)}R - 2\Lambda + \mathcal{L}_V\right)$$

$$ds^2 = -(N^2 - N_i N^i)dt^2 + 2N_i dx^i dt + \gamma_{ij}dx^i dx^j .$$

$$N = 1 + \alpha(t), \quad N_i = \partial_i \beta, \quad \gamma_{ij} = \delta_{ij} + 2\left(\delta_{ij} + k^{-2}\partial_i\partial_j\right)\zeta - 2k^{-2}\partial_i\partial_j\chi,$$

$$k^2 \equiv -\partial^2.$$

up to 2nd order in perturbations

$$\mathcal{L} = \pi_\zeta\dot{\zeta} + \pi_\chi\dot{\chi} - \mathcal{H} - \beta C_\beta - \alpha(t)C_\alpha,$$

$$C_\alpha = -2k^2\zeta, \quad C_\beta = -k^2\pi_\chi,$$

$$\mathcal{H} = -k^2\zeta^2 + \frac{1}{4(3\lambda - 1)}\left[2(2\lambda - 1)\pi_\chi^2 - 4\lambda\pi_\chi\pi_\zeta + (\lambda - 1)\pi_\zeta^2\right].$$

In GR, C_α and C_β are both constraints and they imply $k^2\zeta = 0$ and $\pi_\chi = 0$, then $\mathcal{H} = 0$ as $\lambda = 1$.

»Projectable version (integrated constraint)

=> NO restrictions.

$$0 = 2\alpha(t) \int d^3x \partial^2 \zeta.$$

$$\mathcal{L} = \pi_\zeta \dot{\zeta} - \mathcal{H}, \quad \mathcal{H} = -\frac{c_\zeta^2}{4}\pi_\zeta^2 - k^2\zeta^2,$$

$$c_\zeta^2 = \frac{1-\lambda}{3\lambda - 1},$$

$1/3 < \lambda < 1$, $c_\zeta^2 > 0$ but then the Hamiltonian is negative definite

$c_\zeta^2 < 0.$

EOM: $\ddot{\zeta} + c_\zeta^2 k^2 \zeta = 0.$ unstable

$\lambda \to 1$ limit

π_ζ disappears in the quadratic Hamiltonian in

$c_\zeta \to 0$ limit and the quantum fluctuations of π_ζ are unsuppressed

$$S_2 = \int dt d^3x \left[\dot{\zeta}\pi_\zeta - \left(-\frac{c_\zeta^2}{4a^3}\pi_\zeta^2 - a(\partial\zeta)^2\right)\right],$$

$$S_3 = \int dt d^3x \left[a\zeta\partial_i\zeta\partial^i\zeta - \frac{3c_\zeta^2}{4a^3}\zeta\pi_\zeta^2 + \frac{3}{2a}\zeta\left(\partial_i\partial_j\beta\partial^i\partial^j\beta - (\partial^2\beta)^2\right) - \frac{2}{a}\partial^2\beta\partial_i\zeta\partial^i\beta\right].$$

$$h_{ij} = a(t)^2 e^{2\zeta}\delta_{ij}$$

$$\partial^2\beta = \frac{\pi_\zeta}{2a}$$

$$N(t) = 1$$

Case 2) Non-Projectable N(t,x) with local super-Hamiltonian constraint

Einstein's General Relativity :

$$G_{ijkl} = (1/2) g^{-1/2} (g_{ik} g_{jl} + g_{il} g_{kj} - g_{ij} g_{kl})$$

$$\lambda = 1$$

$$\mathscr{H} = 2\kappa G_{ijkl} \pi^{ij} \pi^{kl} - (2\kappa)^{-1} g^{1/2} (R - \Lambda 2\lambda)$$

$$\mathscr{H}_i = -2\pi_i{}^j{}_{/j}$$

Constraints obeys the Dirac algebra :

$$[\mathscr{H}_i(x), \mathscr{H}_j(x')] = \mathscr{H}_i(x')\delta_{,j}(x,x') + \mathscr{H}_j(x)\delta_{,i}(x,x')$$

$$[\mathscr{H}_i(x), \mathscr{H}(x')] = \mathscr{H}(x)\delta_{,i}(x,x')$$

$$[\mathscr{H}(x), \mathscr{H}(x')] = (g^{ij}(x)\mathscr{H}_i(x) + g^{ij}(x')\mathscr{H}_i(x'))\,\delta_{,j}(x,x')$$

*Hallmark of spacetime covariance, and of the embeddability of hypersurface deformations (Hojman-Kuchar-Teitelboim)

Departures from General Relativity e.g. Horava gravity:

Q: What takes the place of the Dirac algebra?

*Conversely, Dirac Algebra :

$$[\mathscr{H}_\perp(x), \mathscr{H}_\perp(x')] = -\varepsilon[g^{rs}(x)\mathscr{H}_s(x) + g^{rs}(x')\mathscr{H}_s(x')]\delta_{,r}(x, x')$$

$$[\mathscr{H}_r(x), \mathscr{H}_\perp(x')] = \mathscr{H}_\perp(x)\delta_{,r}(x, x')$$

$$[\mathscr{H}_r(x), \mathscr{H}_s(x')] = \mathscr{H}_r(x')\delta_{,s}(x, x') + \mathscr{H}_s(x)\delta_{,r}(x, x')$$

with

$$\mathscr{H}_\perp^{\text{grav}} = \tfrac{1}{2}M_{ijkl}\pi^{ij}\pi^{kl} + V[g_{ij}]$$

=>

$$M_{ijkl} = 2\kappa g^{-1/2}\left(g_{ik}g_{jl} + g_{il}g_{jk} - \frac{2}{n-1}g_{ij}g_{kl}\right)$$

(=> $\lambda = 1$!)

AND

$$V = \varepsilon(2\kappa)^{-1}g^{1/2}(R - 2\Lambda)$$

$$G_{ijkl} = \tfrac{1}{2}(g_{ik}g_{jl} + g_{il}g_{jk} - g_{ij}g_{kl})$$

"Geometrodynamics Regained" program:

S. Hojman, K. Kuchar and C. Teitelboim, Nature Phys. Sci. **245**, 97 (1973); Ann. Phys.**96**, 88 (1976).
C. Teitelboim, *The Hamiltonian structure of spacetime*, in General Relativity and Gravitation Vol. 1, edited by A. Held (Plenum, New York, 1980).

$\lambda \neq 1$

*Case 2a) $H = \frac{2\kappa'}{\sqrt{q}} G_{ijkl}\pi^{ij}\pi^{kl}$ ultralocal theory (V=0)

$$\{\int NH\, d^3x, \int MH\, d^3y\}_{\mathrm{P.B.}} = 0$$

=> Modification of Dirac algebra;

but arbitrary hypersurface deformations (N, **N**) still allowed

*Case 2b) $H = (\frac{2\kappa'}{\sqrt{q}} G_{ijkl}\pi^{ij}\pi^{kl} - \frac{\sqrt{q}}{2\kappa'} R)$

$$G_{ijkl} = \frac{1}{2}(q_{ik}q_{jl} + q_{il}q_{jk}) - \frac{\lambda}{3\lambda-1} q_{ij}q_{kl}$$

$$\{\int NH\, d^3x, \int MH\, d^3y\}_{\mathrm{P.B.}}$$

$$= \int (N\nabla^i M - M\nabla^i N) H_i\, d^3x - \frac{2(1-\lambda)}{3\lambda-1} \int (N\nabla^i M - M\nabla^i N)\nabla_i \pi\, d^3x$$

$Z_i := \nabla_i \pi = 0$ Secondary constraint

$$\{\int \xi^i Z_i\, d^3x, \int \chi^j Z_j\, d^3y\}_{\mathrm{P.B.}} = \int \frac{3}{2}(\chi^i \nabla_j \xi^j - \xi^i \nabla_j \chi^j) Z_i\, d^3x$$

$$\{\int N^i H_i\, d^3x, \int \xi^i Z_i\, d^3y\}_{\mathrm{P.B.}} = \int (\mathcal{L}_{\vec{N}} \xi^i) Z_i\, d^3x$$

$$\{\int \xi^i Z_i \, d^3x, \int N H \, d^3y\}_{\mathrm{P.B.}} = \int [\frac{2\kappa'}{(3\lambda-1)\sqrt{q}} N \pi \xi^i Z_i + (\frac{1}{2} N \nabla_i \xi^i) H] \, d^3x + \frac{1}{\kappa'} \int d^3x \, \sqrt{q} \, (\nabla_j \xi^j) \nabla^i \partial_i N + \int d^3x \frac{2\kappa'}{(3\lambda-1)\sqrt{q}} \pi^2 \xi^i \partial_i N$$

=> $\partial_i N = 0$ restricted set of hypersurface deformations

Stability of constraints under evolution with

N (t) , N constant on Cauchy surface

constraints $\pi^i_{\vec{N}} = H_i = H = \nabla_i \pi = 0$

$$S = \int \pi^{ij}\dot{q}_{ij}\, d^3x\, dt - \int (NH + N^i H_i)\, d^3x\, dt,$$

Case 2c) Horava Gravity:

$$H = \frac{\kappa^2}{2}\frac{G_{ijkl}}{\sqrt{q}}\left[\pi^{ij}\pi^{kl} + \frac{\delta W_T}{\delta q_{ij}}\frac{\delta W_T}{\delta q_{kl}}\right]$$

$$H_i = 2q_{ij}\nabla_k \pi^{kj}$$

$$G_{ijkl} = \tfrac{1}{2}(q_{ik}q_{jl} + q_{il}q_{jk}) - \tfrac{\lambda}{3\lambda - 1} q_{ij}q_{kl}$$

$W_T = W_{CS} + W_{EH\Lambda} = \frac{1}{4w^2}\int \tilde{\epsilon}^{ikj}(\Gamma^l_{im}\partial_j \Gamma^m_{kl} + \frac{2}{3}\Gamma^l_{im}\Gamma^m_{jn}\Gamma^n_{kl})\, d^3x + \frac{\mu}{2}\int \sqrt{q}(R - 2\Lambda_W)\, d^3x$. The Cotton tensor density can be expressed as $\tilde{C}^{ij} = w^2 \frac{\delta W_{CS}}{\delta q_{ij}} = w^2 \tilde{\epsilon}^{ikl}\nabla_k (R_l{}^j - \frac{1}{4}R\delta_l{}^j)$.

Neither H nor is G_{ijkl} is of the form in “geometrodynamics regained”

*Non-Projectable Horava gravity with local super-hamiltonian constraint

Inconsistencies in the canonical formulation:

M. Li and Y. Pang, "A Trouble with Hořava-Lifshitz Gravity," JHEP **0908**, 015 (2009) [arXiv:0905.2751 [hep-th]].

"Troubles" in the constraint algebra of Horava Gravity:

$$\{\int d^3\mathrm{x}\zeta_1^i\mathcal{H}_i,\ \int d^3\mathrm{y}\zeta_2^j\mathcal{H}_j\}_{\mathrm{Pb}} = \int d^3\mathrm{x}(\zeta_1^i\partial_i\zeta_2^k - \zeta_2^i\partial_i\zeta_1^k)\mathcal{H}_k,$$

$$\{\int d^3\mathrm{x}\zeta^i\mathcal{H}_i,\ \int d^3\mathrm{y}\eta\mathcal{H}\}_{\mathrm{Pb}} = \int d^3\mathrm{x}\zeta^i\partial_i\eta\mathcal{H},$$

$$\{\mathcal{H}(\mathrm{x}),\ \int d^3\mathrm{y}\eta\mathcal{H}\}_{\mathrm{Pb}} = \underbrace{-2\sqrt{g}\frac{1}{k_W^4}(\alpha^{ijk}\nabla_k\nabla_j\nabla_i\eta + \beta^{ij}\nabla_j\nabla_i\eta + \gamma^i\nabla_i\eta + \omega\eta)}_{=:\ \Delta\eta}$$

Stability of local constraint under evolution

$$\triangle = -2\sqrt{g}\frac{1}{k_W^4}(\alpha^{ijk}\nabla_k\nabla_j\nabla_i + \beta^{ij}\nabla_j\nabla_i + \gamma^i\nabla_i + \omega)$$

$$\{\mathcal{H}(\mathrm{x}), \int d^3\mathrm{y}\eta\mathcal{H}\}_{\mathrm{Pb}} = -2\sqrt{g}\frac{1}{k_W^4}\underbrace{(\alpha^{ijk}\nabla_k\nabla_j\nabla_i\eta + \beta^{ij}\nabla_j\nabla_i\eta + \gamma^i\nabla_i\eta + \omega\eta)}_{=:\ \Delta\eta}$$

$$\alpha^{ijk} = (\widetilde{C}^{ilm}g^{jk} + \widetilde{C}^{klm}g^{ij} - \widetilde{C}^{ilk}g^{jm} - \widetilde{C}^{kli}g^{jm})K_{lm},$$

where $\widetilde{C}^{ijk}$ is defined as $\epsilon^{ijl}C_l{}^k$, in which $C_l{}^k = g_{lm}C^{mk}$

$$\beta^{ij}\nabla_j\nabla_i\eta = \nabla_{(j}\nabla_i\eta\nabla_{k)_c}(K_{lm}\widetilde{C}^{ilm}g^{jk} - K_{lm}\widetilde{C}^{ilk}g^{jm})$$

$$\begin{aligned}\gamma^i\nabla_i\eta = {} & t^{mlkji}\nabla_{(i}\eta\nabla_m\nabla_{k)_c}K_{jl} + K_{jl}\nabla_{(i}\eta\nabla_k\nabla_{m)_c}t^{mlkji} \\ & +2s^{lmijk}\nabla_i\eta\nabla_{[l}\nabla_{k]}K_{jm} + 2K_{jm}\nabla_i\eta\nabla_{[k}\nabla_{l]}s^{lmijk} \\ & +2(\widetilde{C}^{klj}R^i_l + \widetilde{C}^{lki}R^j_l + \widetilde{C}^{lij}R^k_l)K_{jk}\nabla_i\eta \\ & +(\frac{1}{2}R^i_{jkl}\widetilde{C}^{klj} - R^i_{jkl}\widetilde{C}^{jkl})K\nabla_i\eta\end{aligned}$$

$$\begin{aligned}\omega = {} & \nabla_i(\widetilde{C}^{jkl}R^i_kK_{jl} + \widetilde{C}^{jik}R^l_jK_{kl} + \widetilde{C}^{kji}R^l_kK_{jl}) \\ & +\widetilde{C}^{ijk}(\nabla_i\nabla_l\nabla_kK^l{}_j + \nabla_i\nabla_l\nabla_jK^l{}_k - \nabla_i\nabla^l\nabla_lK_{jk} - \nabla_i\nabla_k\nabla_jK) \\ & +(K^l{}_j\nabla_k\nabla_l\nabla_i + K^l{}_k\nabla_j\nabla_l\nabla_i - K_{jk}\nabla^l\nabla_l\nabla_i - K\nabla_j\nabla_k\nabla_i)\widetilde{C}^{ijk}.\end{aligned}$$

M. Henneaux, A. Kleinschmidt and G. L. Gómez, "A dynamical inconsistency of Hořava gravity," Phys. Rev. D 81, 064002 (2010) [arXiv:0912.0399 [hep-th]].

For Horava gravity with local Hamiltonian constraint :
Only consistent solution for stability of constraint under evolution is
N= 0

*Dirac algorithm resulting in N =0 suggests H constraint generates on-shell trivial time-reparametrization invariance ?

$\Rightarrow$? Only spatial diffeomorphisms are physically relevant gauge symmetries of the theory ?

*Conclusion:

Hamiltonian constraint of Horava Gravity:

1) Non-projectable version (with local constraint) $H(x)=0$:

Inconsistent constraint algebra (unless $N(x,t) = 0$) (Li-Pang, Henneaux,…)

2)Projectable version N(t only) (with global integrated constraint):

$[\int d^3x\, H(x)] = 0$ => Pathological extra d.o.f.

"Eating the cake and still having it":

Question:

Can the Hamiltonian constraint be local (=> removes extra d.o.f.)

AND

still be expressed as an integrated constraint (<=> projectable) ? !

*****Consistent Canonical Formulation:**

*Horava's "intended" theory:

$$S = \int dt \int d^3x \, [\tilde{\pi}^{ij} \dot{q}_{ij} + 2N_j \nabla_i \tilde{\pi}^{ij}] - \int dt \int d^3x \, N H$$

REPLACE** by ***Master Constraint Version:

$$S = \int dt \int d^3x \, [\tilde{\pi}^{ij} \dot{q}_{ij} + 2N_j \nabla_i \tilde{\pi}^{ij}] - \int dt \underbrace{N(t)}_{\varepsilon_0} \int d^3x \frac{H^2(x)}{\sqrt{q}}$$

=: M

$$H = \frac{\kappa^2}{2} \frac{G_{ijkl}}{\sqrt{q}} [\pi^{ij} \pi^{kl} + \frac{\delta W_T}{\delta q_{ij}} \frac{\delta W_T}{\delta q_{kl}}]$$

$$G_{ijkl} = \tfrac{1}{2}(q_{ik}q_{jl} + q_{il}q_{jk}) - \tfrac{\lambda}{3\lambda - 1} q_{ij} q_{kl}$$

$W_T = W_{CS} + W_{EH\Lambda} = \frac{1}{4w^2} \int \tilde{\epsilon}^{ikj} (\Gamma^l_{im} \partial_j \Gamma^m_{kl} + \frac{2}{3} \Gamma^l_{im} \Gamma^m_{jn} \Gamma^n_{kl}) \, d^3x + \frac{\mu}{2} \int \sqrt{q}(R - 2\Lambda_W) \, d^3x$. The Cotton tensor density can be expressed as $\tilde{C}^{ij} = w^2 \frac{\delta W_{CS}}{\delta q_{ij}} = w^2 \tilde{\epsilon}^{ikl} \nabla_k (R_l{}^j - \frac{1}{4} R \delta_l{}^j)$.

Dirac Algebra

$$\{H_i[N^i], H_j[M^j]\}_{P.B.} = \quad H_i[(\mathcal{L}_{\vec{N}} M)^i]$$
$$\{H_i[N^i], H[M]\}_{P.B.} = \quad H[(\mathcal{L}_{\vec{N}} M]$$
$$\{H[N], H[M]\}_{P.B.} = \quad H_i[(q^{ij}(N\partial_j M - M\partial_j N)]$$

Structure FUNCTIONS (not infinite dim. Lie Algebra)
Spatial diffeo. forms subgroup but not ideal.
Cannot solve constraint in 3-dim. diffeo. invariant subspace (superspace)
(H cannot be defined directly therein).

Recently, the master constraint programme for loop quantum gravity (LQG) was proposed as a classically equivalent way to impose the infinite number of Wheeler–DeWitt constraint equations in terms of a single master equation.

T. Thiemann, The Phoenix Project: master constraint programme for loop quantum gravity, Class. Quantum Grav. **23** (2006) 2211.

M-Theory: Master Constraint Program

Master Constraint Algebra:

$$\{\vec{H}(\vec{N}),\vec{H}(\vec{N}')\} = \vec{H}(\mathscr{L}_{\vec{N}}N')$$
$$\{\vec{H}(\vec{N}),\mathbf{M}\} = 0$$
$$\{\mathbf{M},\mathbf{M}\} = 0.$$

$$\mathbf{M} := \int_\Sigma \mathrm{d}^3x \frac{[H(x)]^2}{\sqrt{q(x)}}.$$

$\mathbf{M} = 0$ is then equivalent to $H(x) = 0, \forall x \in \Sigma$.

1st Class Constraints with structure constants

Tested with: finite-dimensional Abelian & non-Abelian algebras
with structure constants & also structure functions,
with contraints polynomial and non-polynomial in momenta,
with electrodynamics and Gauss Law, non-abelian gauge theories,
Free field QFT and interacting theories, linearized gravity.

References

[1] Thiemann T 2003 The Phoenix Project: master constraint programme for loop quantum gravity *Preprint* gr-qc/0305080

[2] Dittrich B and Thiemann T 2006 Testing the master constraint programme for loop quantum gravity: I. General framework *Class. Quantum Grav.* **23** 1025–65 (*Preprint* gr-qc/0411138)

[3] Dittrich B and Thiemann T 2006 Testing the master constraint programme for loop quantum gravity: II. Finite dimensional systems *Class. Quantum Grav.* **23** 1067–88 (*Preprint* gr-qc/0411139)

[4] Dittrich B and Thiemann T 2006 Testing the master constraint programme for loop quantum gravity: III. $SL(2, \mathbb{R})$ models *Class. Quantum Grav.* **23** 1089–120 (*Preprint* gr-qc/0411140)

[5] Dittrich B and Thiemann T 2006 Testing the master constraint programme for loop quantum gravity: V. Interacting field theories *Class. Quantum Grav.* **23** 1143–62 (*Preprint* gr-qc/0411142)

*Horava Gravity : explicit realization (representation) of the Master constraint algebra.

*Horava Gravity can't seem to be consistently formulated as a canonical theory otherwise.

$$\{q_{ij}, N(t)\frac{\mathrm{M}}{\epsilon_o} + \int N^k H_k \, d^3x\}|_{\mathrm{M}=0 \Leftrightarrow H=0} \approx$$

$$\{q_{ij}, \int N^k H_k \, d^3x\}_{\mathrm{P.B.}} = \mathcal{L}_{\vec{N}} q_{ij} \text{ (and similarly for } \pi^{ij})$$

Observables O:

$$\{O, \frac{N(t)}{\epsilon_0}\mathrm{M} + \int N^i H_i \, d^3x\}|_{\mathrm{M}=0 \Leftrightarrow H=0} \approx$$

$$\{O, \int N^i H_i \, d^3x\}_{\mathrm{P.B.}} = 0.$$

**Explicitly/concretely realizes

on-shell trivial time reparametrization generated by H;

physically relevant symmetry is 3-d (spatial) diffeomorphism invariance

**c.f. Einstein's theory

On-shell (modulo constraints +EOM),
constraints do generate 4-d diffeomorphisms
Eventhough Dirac algebra is NOT algebra of 4d diffeomorphisms

$$\delta_{\vec{N}} q_{ab} = \{H_i[N^i], q_{ab}\}_{P.B.} = \mathcal{L}_{\vec{N}} q_{ab}$$
$$\delta_N q_{ab} = \{H[N], q_{ab}\}_{P.B.} = 2NK_{ab} = \mathcal{L}_{N\vec{n}} q_{ab}$$

$$\delta_{\vec{N}} \pi^{ab} = \{H_i[N^i], \pi^{ab}\}_{P.B.} = \mathcal{L}_{\vec{N}} \pi^{ab}$$

$$\begin{aligned}\delta_N \pi^{ab} &= \{H[N], \pi^{ab}\}_{P.B.} \\ &= q^{ab}\frac{N}{2}H - N\sqrt{q}(q^{ca}q^{db} - q^{cd}q^{ab})R^{(4)}_{cd} + \mathcal{L}_{N\vec{n}}\pi^{ab}\end{aligned}$$

Master constraint program for Horava GR: requires N(t).

Existence of Black hole solution:

Painleve-Gulstrand form of metric

$$ds^2 = -dt^2 + \left(dr + \sqrt{\frac{M}{r} + \frac{\Lambda_W}{2} r^2 dt}\right)^2 + r^2 d\Omega^2.$$

Solution of Einstein's theory and of Horava GR ($\lambda = 1$ limit) with detailed balance [18] because of the spatially flat slicing (such slicings compatible with +ve cosmological constant[19]) . Gives same proper times as Schwarzschild solution and will pass empirical tests measuring proper times in Schwarzschild metric e.g. recent atomic interferometry GR redshift data[20]

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