



---

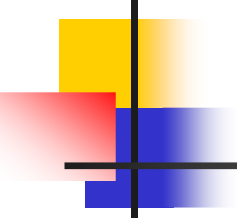
# Bound-State Effects on Top-Quark Pair Production at the LHC

Hiroshi YOKOYA (NCTS<sub>n</sub>/NTU)

K.Hagiwara,Y.Sumino,HY,Phys.Lett.B666,71(2008)

Y.Sumino,HY,JHEP09,034(2010)

[K.Hagiwara,HY,JHEP10,049(2009)]

- 
- 
- Outline :
1. Introduction : Top-Quark at hadron colliders
  2. Bound-state effects on  $t\bar{t}$  production  
at hadron colliders
  3. Differential cross-section / Event Generation
  4. Summary



---

# 1. Introduction

# 1. Introduction

## □ Properties of the top-quark

- **Mass :**

(CDF and D0 combined, [arXiv:0903.2503](https://arxiv.org/abs/0903.2503))

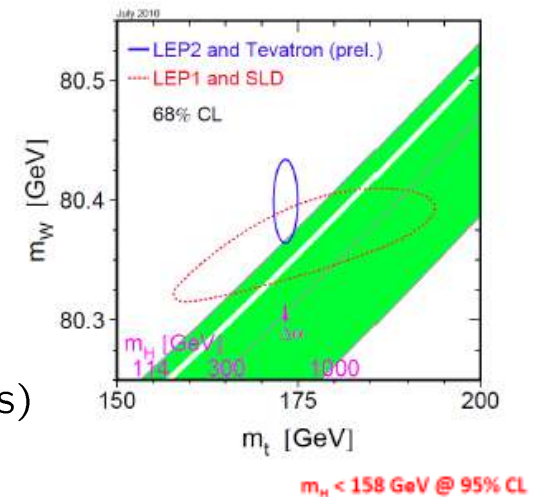
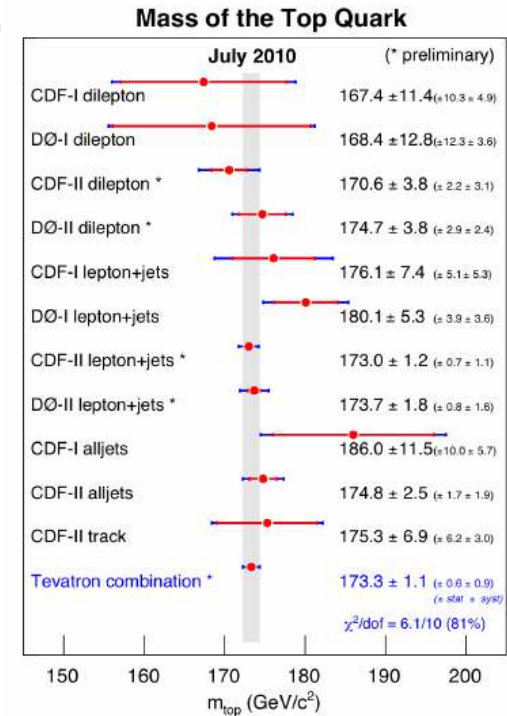
$$m_t = 173.1 \pm 0.6(\text{stat.}) \pm 1.1(\text{syst.}) [\text{GeV}]$$

- Heaviest fundamental particle ever found
- Important input for EW precision test
- $y_t \sim 1 \Rightarrow$  may be related to the BSM physics

- **Width :** SM prediction;  $\Gamma_t \simeq \frac{G_F m_t^3}{8\sqrt{2}\pi} |V_{tb}|^2 \sim 1.5 [\text{GeV}]$

D0 measurement ('10);  $\Gamma_t = 1.99^{+0.69}_{-0.55} \text{ GeV}$

- decay before hadronization,  $\Gamma_t \gg \Lambda_{\text{QCD}}$  ( $\tau \sim 10^{-25}\text{s}$ )  
thus spin information is preserved in decay products

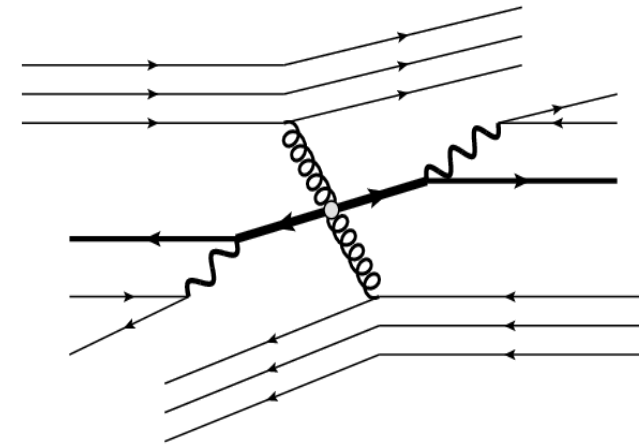


# 1. Introduction

- Top-quark production at Hadron Colliders

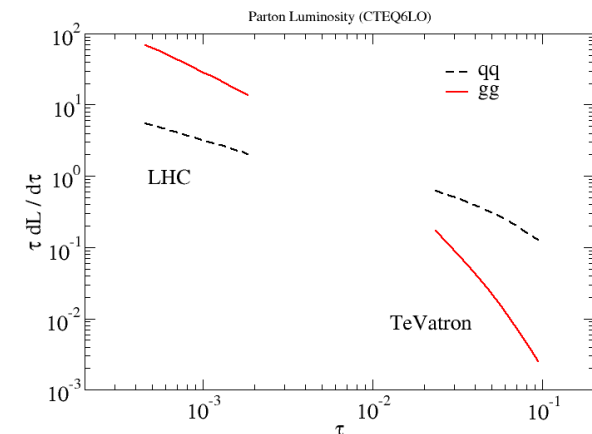
$$\sigma_{t\bar{t}}(s) = \sum_i \int d\tau \frac{dL_i}{d\tau}(\tau) \hat{\sigma}_i(\hat{s} = \tau s)$$

partonic luminosity      partonic cross-section



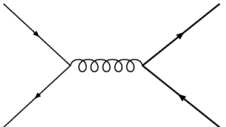
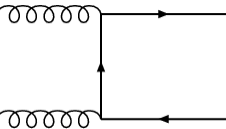
- We can calculate the partonic cross-section perturbatively.
- Partonic luminosity is a convolution of the parton distribution functions (PDFs) of the two protons, which are measured by experiments.

$$\frac{dL_i}{d\tau}(\tau, \mu_F) = \int dx_1 dx_2 \delta(\tau - x_1 x_2) f_a(x_1, \mu_F) f_b(x_2, \mu_F)$$



# 1. Introduction

- Partonic subprocess at Hadron Colliders

			Tevatron	LHC
$q\bar{q} \rightarrow t\bar{t}$		Color: <b>Octet</b> $ J =1$	85%	10%
			..	..
$gg \rightarrow t\bar{t}$		Color: <b>Singlet, Octet</b> $ J =0,1,2,...$	15%	90%

- Color decomposition in gg process is obtained from the color-matrix in the amplitude

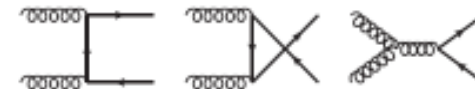
$$\begin{aligned} \mathcal{M}_{gg \rightarrow t\bar{t}} &= (T^a T^b)_{ij} \mathcal{M}_1 + (T^b T^a)_{ij} \mathcal{M}_2 \\ &= \frac{1}{2} \{T^a, T^b\}_{ij} \mathcal{M}_S + \frac{1}{2} [T^a, T^b]_{ij} \mathcal{M}_A \end{aligned}$$

$$\frac{1}{2} \{T^a, T^b\} = \frac{1}{2N_c} \delta^{ab} \delta_{ij} + \frac{1}{2} d^{abc} T_{ij}^c$$

color-singlet

$$\left| \frac{1}{2N_c} \delta^{ab} \delta_{ij} \right|^2 / \left| \frac{1}{2} d^{abc} T_{ij}^c \right|^2 = \frac{2}{N_c^2 - 4}$$

~20-30% of t-tbar are color-singlet in gg





# 1. Introduction

---

- Perturbative calculations for the **partonic cross-section** :

$$\hat{\sigma}_i(\hat{s}) = \hat{\sigma}_i^{(0)}(\hat{s}) + \frac{\alpha_s}{\pi} \hat{\sigma}_{ij}^{(1)}(\hat{s}; \mu) + \left(\frac{\alpha_s}{\pi}\right)^2 \hat{\sigma}_i^{(2)}(\hat{s}; \mu) + \dots$$

- Fixed-Order Calculation** :
- NLO : Dawson,Ellis,Nason('88), Beenakker etal.('90)  
(Analytic): Cazkon,Mitov('08)
  - Building blocks for **full NNLO** correction :  
Korner etal.('06),Dittmaier etal.('07),Cazkon etal.('07),,,
  - (appx.) NNLO : Kidonakis,Vogt('08),,
  - 1-loop electroweak correction : Bernreuther,Fuecker,Si('06),  
Kuhn,Scharf,Uwer('06),Moretti,Nolten,Ross('06)
- Resummation** :
- Threshold Resummation  
NLL : Bonciani etal.('98),,, ; NNLL : Moch,Uwer('08),
  - **Coulomb Summation** : Catani,Mangano,Nason,Trentadue('96),  
Hagiwara,Sumino,HY('08), Kiyono etal('08), Sumino,HY('10)

# 1. Introduction

- Total cross-section and its uncertainties :

$$\sigma_{t\bar{t}}(\text{TeV}) \sim 8 \text{ [pb]}$$

$$\sigma_{t\bar{t}}(\text{LHC } 7\text{TeV}) \sim 150 \text{ [pb]} \quad \sigma_{t\bar{t}}(\text{LHC } 14\text{TeV}) \sim 900 \text{ [pb]}$$

Theoretical uncertainties = (Ren & Fac scales) + (PDF)  $\sim 5\%$  (TeV)  
 $\sim 10\%$   $\sim 3\%$  (LHC)

**MOCH&UWER VS CACCIARI ET AL: TEV**

$m_{\text{top}} = 171 \text{ GeV}$

**CTEQ6.5**

M&U	$\sigma = 7.93$	$^{+0.06(1.0\%)}_{-0.28(3.5\%)}$	(scales)	$^{+0.44(5.5\%)}_{-0.45(5.5\%)}$	(PDFs) pb
C&cal	$\sigma = 7.61$	$^{+0.38(5.1\%)}_{-0.80(10.9\%)}$	(scales)	$^{+0.49(6.6\%)}_{-0.34(4.6\%)}$	(PDFs) pb

**MRSTW-06**

M&U	$\sigma = 8.23$	$^{+0.08(1.0\%)}_{-0.33(4.0\%)}$	(scales)	$^{+0.21(2.6\%)}_{-0.23(2.8\%)}$	(PDFs) pb
C&cal	$\sigma = 7.93$	$^{+0.34(4.3\%)}_{-0.56(7.1\%)}$	(scales)	$^{+0.24(3.1\%)}_{-0.20(2.5\%)}$	(PDFs) pb.

**MOCH&UWER VS CACCIARI ET AL: LHC**

$m_{\text{top}} = 171 \text{ GeV}$

**CTEQ6.5**

M&U	$\sigma = 918$	$^{-9(1.0\%)}_{-39(4.2\%)}$	(scales)	$^{+30(3.3\%)}_{-30(3.3\%)}$	(PDFs) pb
C&cal	$\sigma = 908$	$^{+82(9.0\%)}_{-85(9.3\%)}$	(scales)	$^{+30(3.3\%)}_{-29(3.2\%)}$	(PDFs) pb

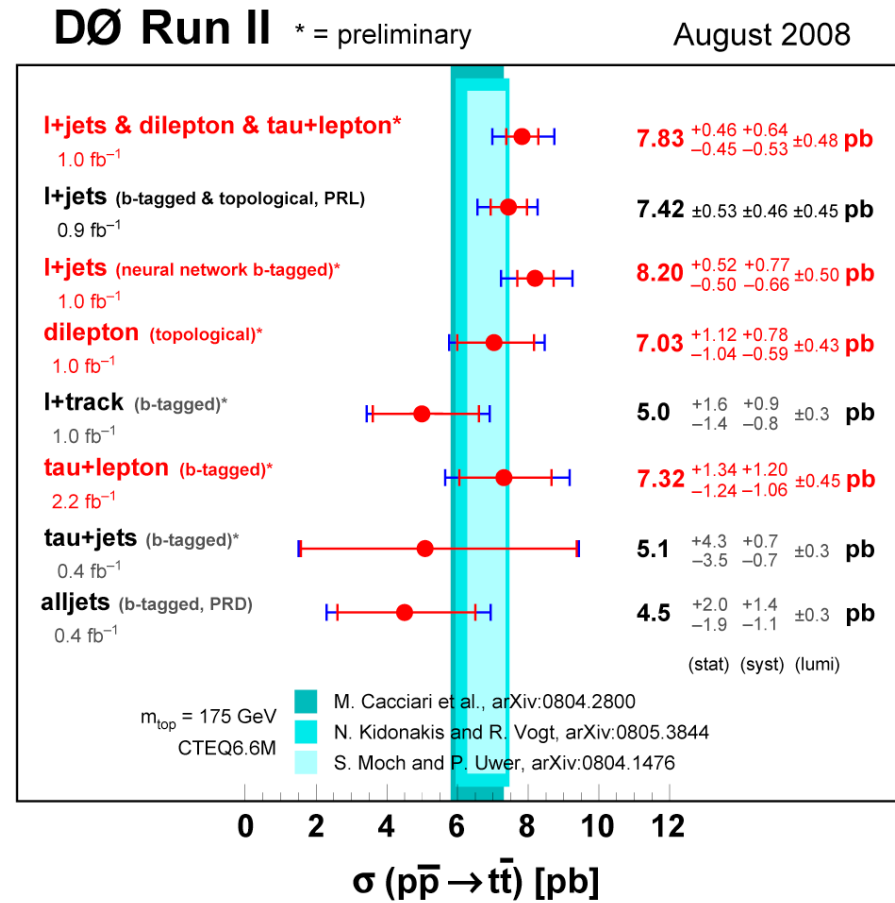
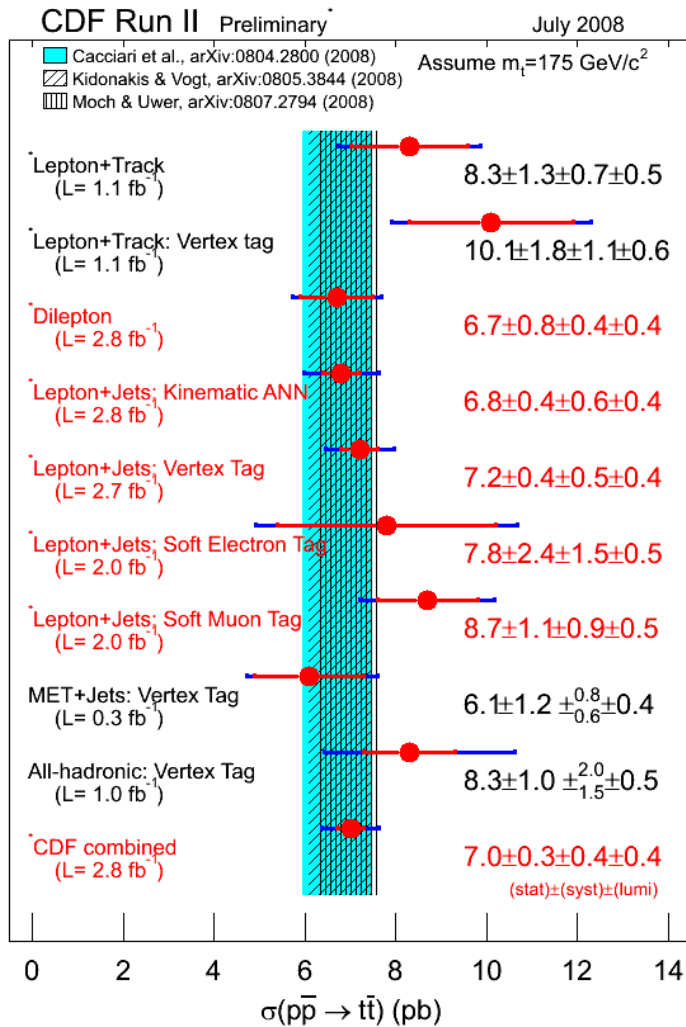
**MRSTW-06**

M&U	$\sigma = 969$	$^{-13(1.3\%)}_{-39(4.0\%)}$	(scales)	$^{+11(1.1\%)}_{-11(1.1\%)}$	(PDFs) pb
C&cal	$\sigma = 961$	$^{+89(9.2\%)}_{-91(9.4\%)}$	(scales)	$^{+11(1.1\%)}_{-12(1.2\%)}$	(PDFs) pb

- Central values within 1%
- PDF uncertainty results agree, and confirm that  $\delta_{\text{PDF}}$  is underestimated
- NNLL scale uncertainty smaller than NLL?

# 1. Introduction

## Cross-Section Measurements at CDF and D0



# 1. Introduction

## LHC has already found Top-Quarks



CMS-TOP-10-001

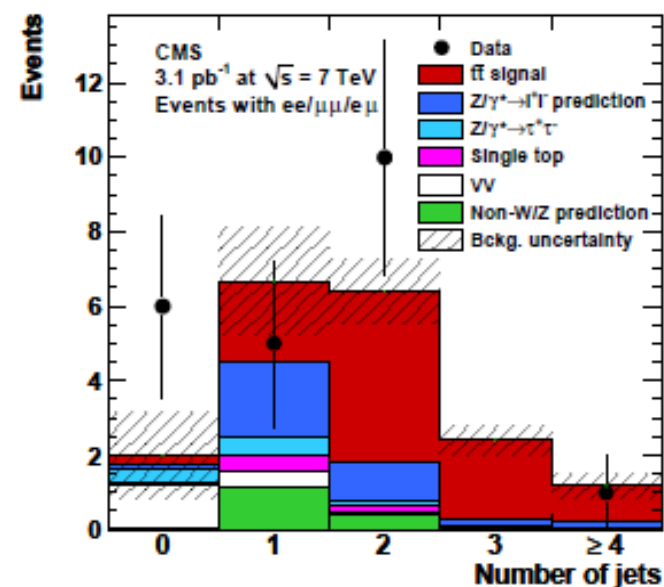
CERN-PH-EP/2010-039  
2010/11/12

### First Measurement of the Cross Section for Top-Quark Pair Production in Proton-Proton Collisions at $\sqrt{s} = 7$ TeV

The CMS Collaboration\*

#### Abstract

The first measurement of the cross section for top-quark pair production in pp collisions at the LHC at center-of-mass energy  $\sqrt{s} = 7$  TeV has been performed using  $3.1 \pm 0.3 \text{ pb}^{-1}$  of data recorded by the CMS detector. This result utilizes the final state with two isolated, highly energetic charged leptons, large missing transverse energy, and two or more jets. Backgrounds from Drell-Yan and non-W/Z boson production are estimated from data. Eleven events are observed in the data with  $2.1 \pm 1.0$  events expected from background. The measured cross section is  $194 \pm 72(\text{stat.}) \pm 24(\text{syst.}) \pm 21(\text{lumi.}) \text{ pb}$ , consistent with next-to-leading order predictions.



# 1. Introduction



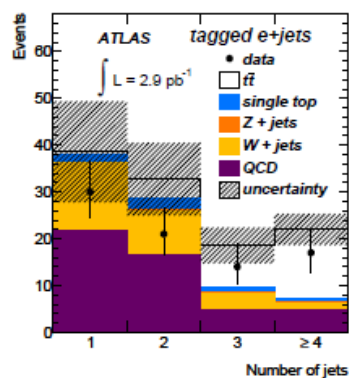
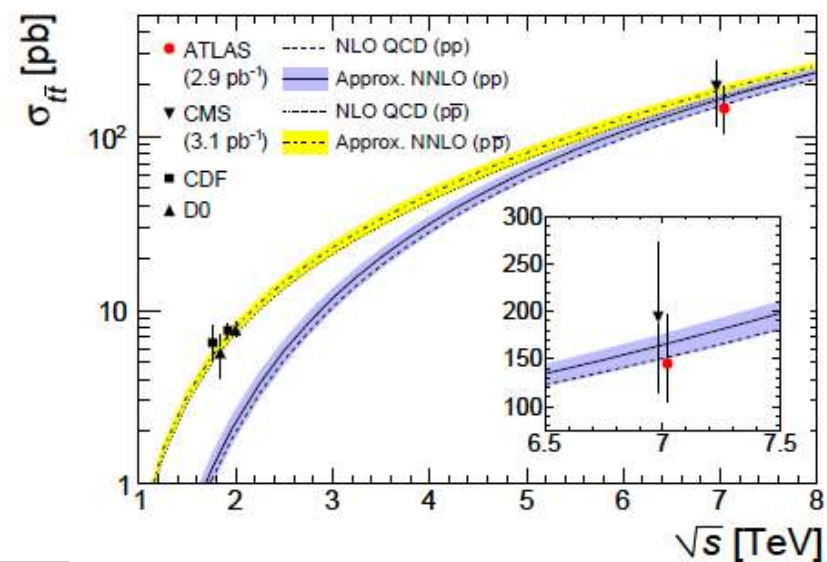
CERN-PH-EP-2010-064  
(Submitted to EPJC)

December 8, 2010

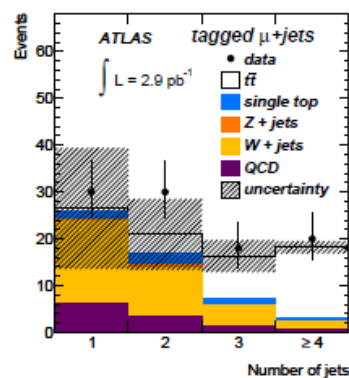


Measurement of the top quark-pair production cross section  
with ATLAS in pp collisions at  $\sqrt{s} = 7$  TeV

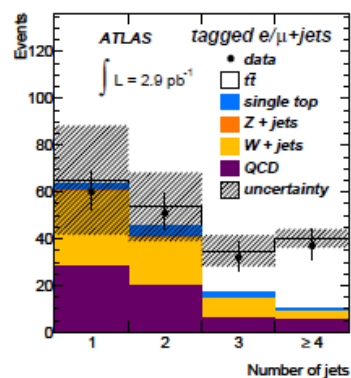
The ATLAS Collaboration



(d)



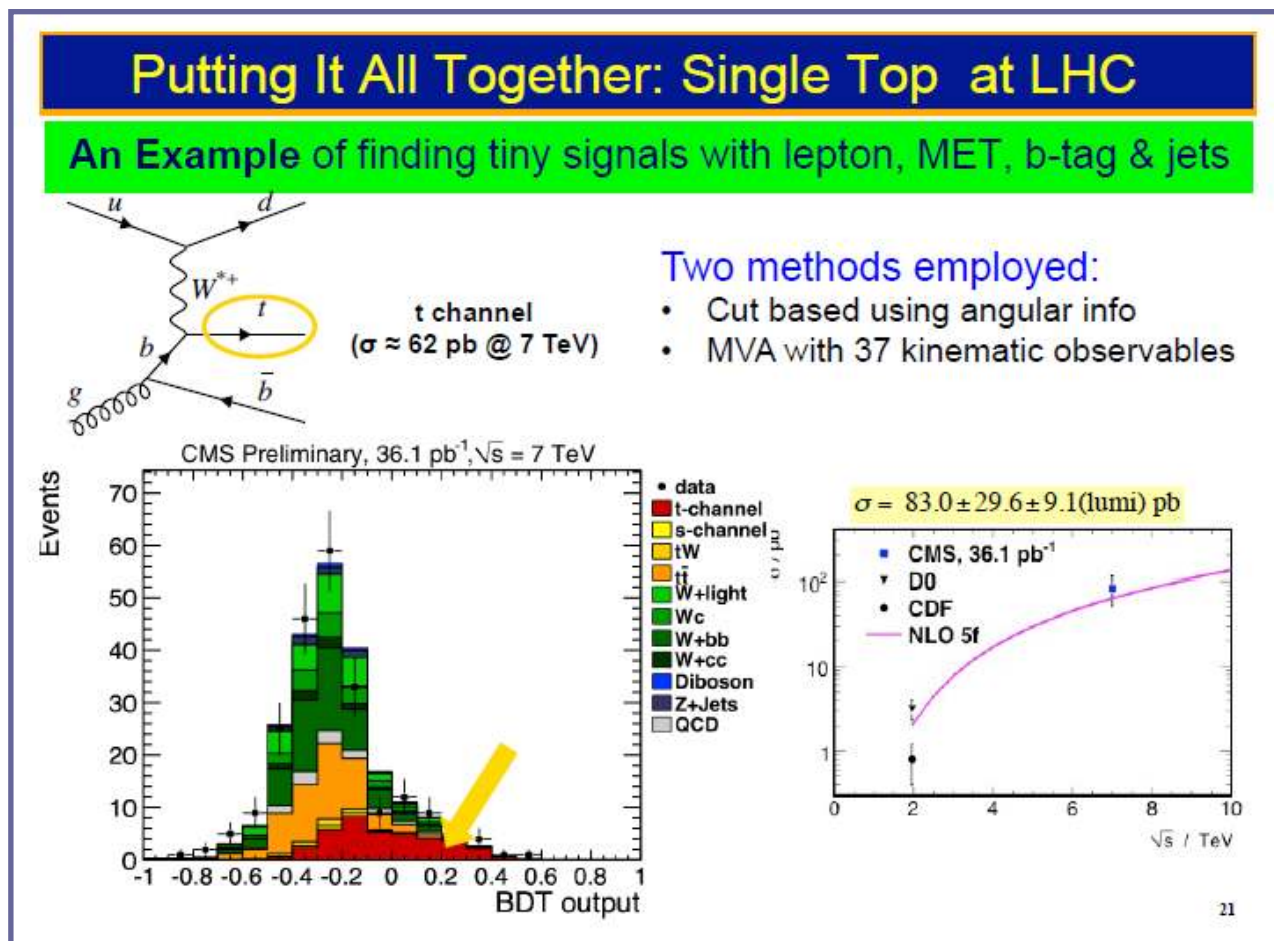
(e)



(f)

# 1. Introduction

Single Top event is also rediscovered.





# 1. Introduction

---

## LHC = Top Factory

So top-quark physics can be more precisely at the LHC

- Cross-Section at the LHC (14 TeV):
  - Pair production cross-section  $\sim 900\text{pb}$
  - Single production cross-section  $\sim 300\text{pb}$  (t-ch.  $>$  tW  $>$  s-ch.)
- QCD physics (standard candle process):
 

Luminosity monitor, PDF monitor, Jet energy scale, B-tagging,,,
- Electroweak physics :
 

Decay (width, distribution,  $V_{tb}$ ), Interaction with Long. weak-bosons,,,

$$\Gamma_t = \underbrace{\Gamma_L}_{70\%} + \underbrace{\Gamma_T}_{30\%}$$

- And Beyond Standard Model physics :

# 1. Introduction

- Precise mass (and width) determination :

What is the top-quark mass observed so far?

pole mass? a parameter in the Monte Carlo?

**Fact** : Pole mass is not a well-defined.

Theoretical prediction using pole mass has bad pert. convergency

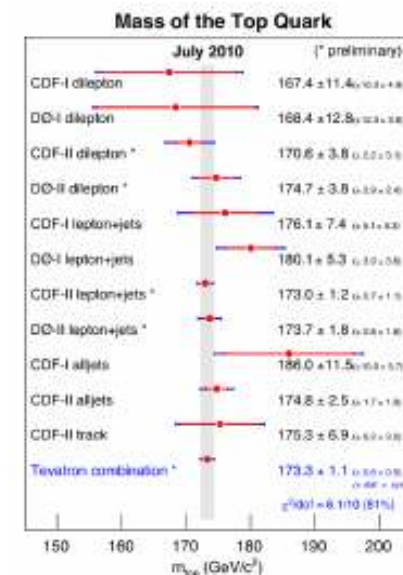
**Important** : definition of the mass in an IR-safe manner  
(short-distance mass)

Langenfeld,Moch,Uwer('09) determination of the  $\overline{\text{MS}}$  mass from the total cross-section at Tevatron.

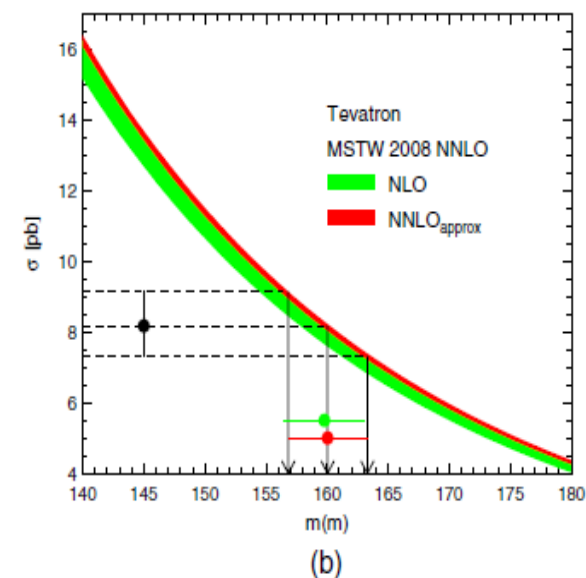
$$\bar{m}(\bar{m}) = 160.0^{+3.3}_{-3.2} [\text{GeV}] \quad (\text{NNLO})$$

$$\rightarrow \text{pole mass } m_t = 168.9 \pm 3.5 \text{ GeV}$$

Another choice may be the "threshold mass".



PHYSICAL REVIEW D **80**, 054009 (2009)



# 1. Introduction

- At  $e^+e^-$  colliders, “Threshold Scan” can be performed

precise determinations of the top-quark mass, width and strong coupling constant are possible

$$\delta m_t \sim 100 \text{ [MeV]} \quad \delta \Gamma_t / \Gamma_t \sim 20 \text{ [%]}$$

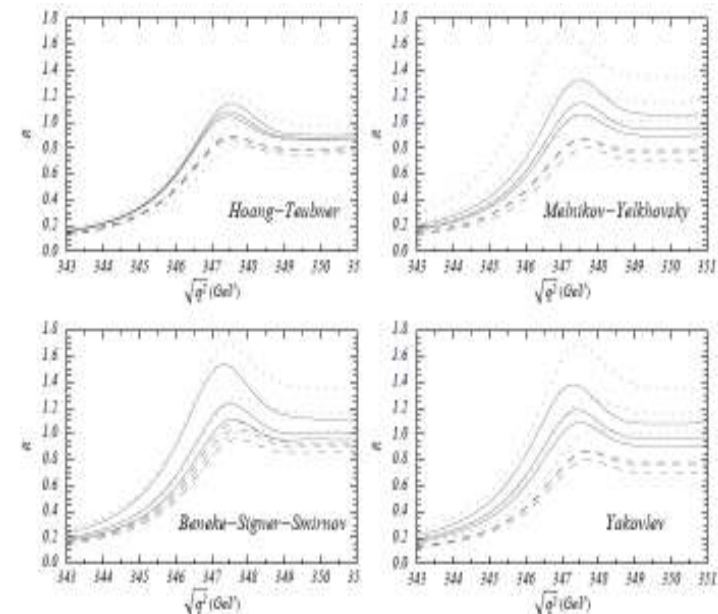
- Peak of the 1S resonance  $\Leftrightarrow$  Threshold mass (1S mass, PS mass,,,) )

Relation between the threshold mass,  $\overline{\text{MS}}$  mass and pole mass is well-known to higher-orders

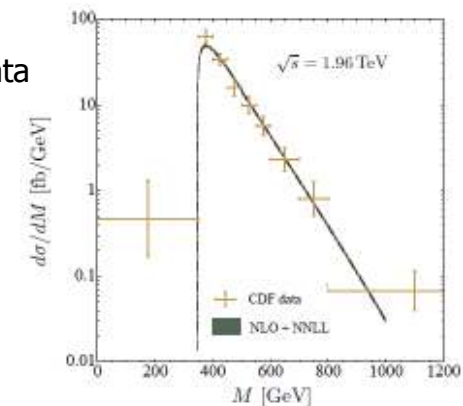
- However, at hadron colliders, (partonic) collision energy is not fixed, so one has to reconstruct top-pair invariant mass.

It would be a very challenging task to have good precision

hep-ph/0001286

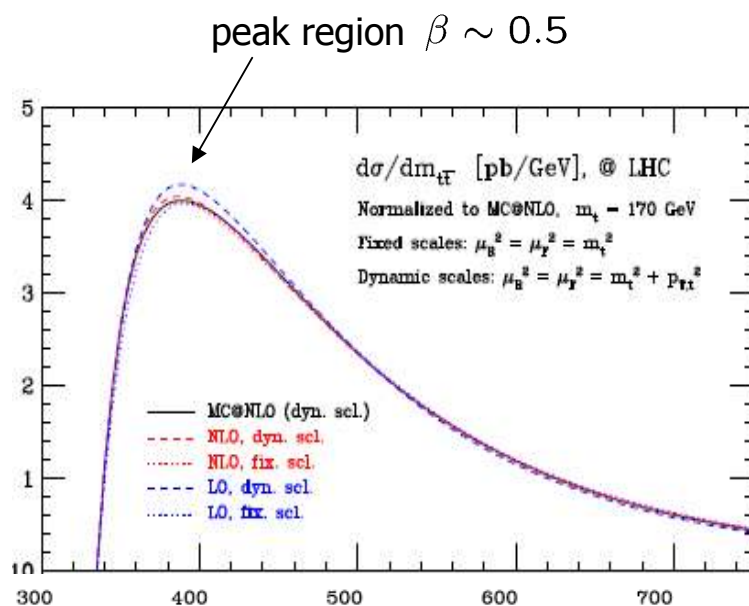


CDF data



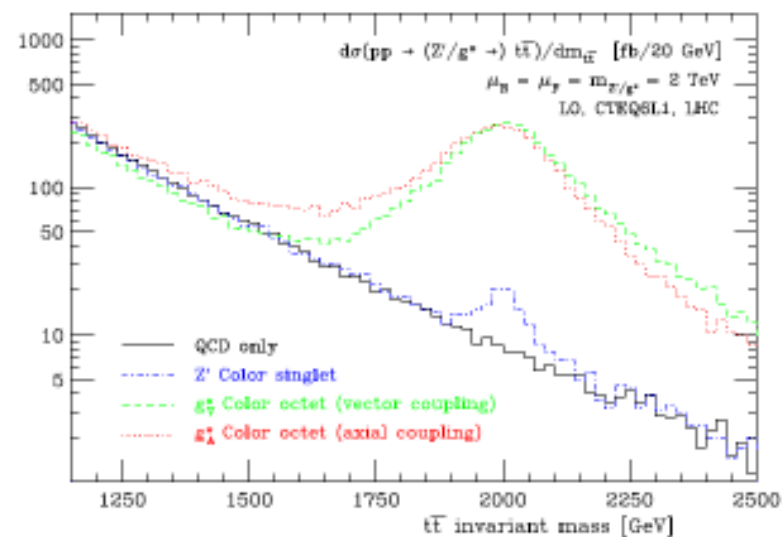
# 1. Introduction

- ttbar Invariant-mass distribution :  $m_{tt} = (p_t + p_{\bar{t}})^2$



threshold region  $\beta \leq 0.3$  ( $m_{tt} < 360$  GeV)

Frederix, Maltoni ('07)



High-energy region  $\beta \sim 1$

(Boosted top, resonances, FB asymmetries?,,,)

$$\beta = \sqrt{1 - \frac{4m_t^2}{m_{tt}}} \quad : \text{velocity of top-quarks in } t\bar{t}\text{bar CM frame}$$



---

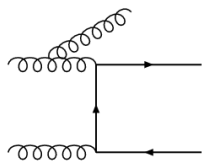
## 2. Bound-state effects on $t\bar{t}$ production at Hadron Colliders

## 2. Bound-state effects at Hadron colliders

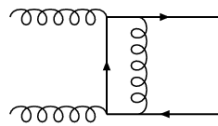
- NLO correction near partonic threshold :  $\left( \beta = \sqrt{1 - \frac{4m_t^2}{\hat{s}}} \rightarrow 0 \right)$   
velocity of top-quarks in  $t\bar{t}$  CM frame

$$\hat{\sigma}_i^{e,(1)} \sim \alpha_s \hat{\sigma}_i^{e,(0)} \left[ \underbrace{A_i \ln^2(8\beta^2)}_{\text{red}} + \underbrace{B_i^{(c)} \ln(8\beta^2)}_{\text{green}} + \underbrace{C_i^{(c)} \frac{\pi^2}{\beta}}_{\text{green}} + \underbrace{D_i^{(e)}}_{\text{blue}} + \mathcal{O}(\beta) \right] \quad i=qq,gg$$

**Threshold logs:** emission of soft and/or collinear gluon in initial-state and final-state



**Coulomb singularity:** Coulomb gluon exchange between  $t$  and  $t$ -bar



**Hard correction:** process dependent



## 2. Bound-state effects at Hadron colliders

---

To all orders, we expect,

$$\begin{aligned}\hat{\sigma}_i &= \hat{\sigma}_i^0 \times \left[ 1 + \alpha_s \ln^2 \beta + \alpha_s \ln \beta + \alpha_s^2 \ln^4 \beta + \dots \right] \\ &\times \left[ 1 + \frac{\alpha_s}{\beta} + \frac{\alpha_s^2}{\beta^2} + \dots \right] \times \left[ 1 + \alpha_s + \alpha_s^2 + \dots \right] \\ &+ \text{(non - decoupling terms)}\end{aligned}$$

- C.F., factorization of each contribution is shown by

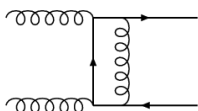
Beneke,Falgari,Schwinn ('09)

Beneke,Czakov,Falgari,Mitov,Schwinn('10)

## 2. Bound-state effects at Hadron colliders

### □ Coulomb corrections to all-orders

- Coulomb singularity  $\propto C^{(c)} \frac{\alpha_s}{\beta}$   $\mathcal{O}(1)$  for  $\beta \simeq \alpha_s$

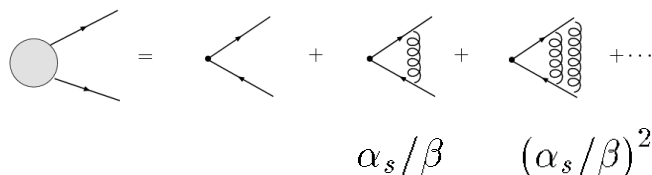


$$\text{color-factor} \begin{cases} \text{singlet} & C^{(1)} = -C_F \\ \text{octet} & C^{(8)} = C_A/2 - C_F \end{cases}$$

- Summation of ladder diagrams = Sommerfeld factor

Sommerfeld, Sakharov (QED)

$$S(z) = \frac{z}{1 - \exp[-z]}, \quad z = C^{(c)} \pi \alpha_s / \beta$$



## 2. Bound-state effects at Hadron colliders

### □ Coulomb corrections to all-orders

- **Green's function formalism (NRQCD)** Fadin, Khoze('87),...

Schrodinger's Eq. 
$$\left[ (E + i\Gamma_t) - \left\{ -\frac{\nabla^2}{m_t} + V_{QCD}^{(c)}(r) \right\} \right] G^{(c)}(E, \vec{x}) = \delta^3(\vec{x})$$

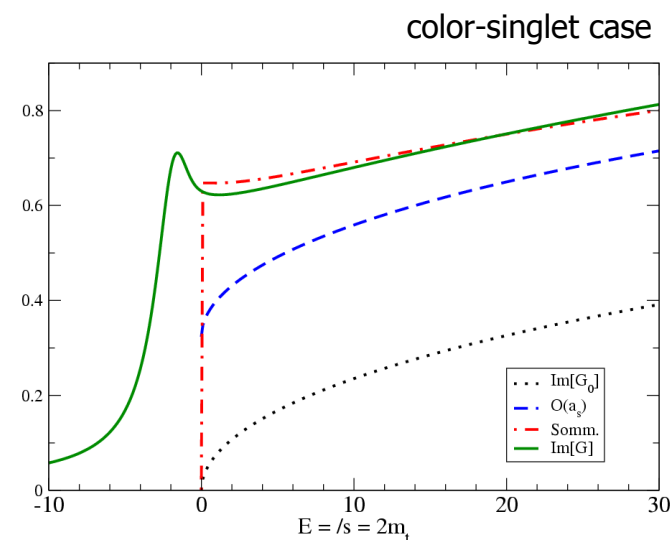


where  $E = m_{tt} - 2m_t$

**finite width effect** is incorporated by complex energy  
 $\Rightarrow$  Off-shellness of top-quarks

$$G(E, \vec{x}) = \sum_n \frac{\Psi_n(\vec{x}) \Psi_n^*(0)}{E - E_n + i\Gamma_n/2} + \text{continuum}$$

- Large width smears the multiple resonance structure, but only one broad peak can be seen as a remnant.



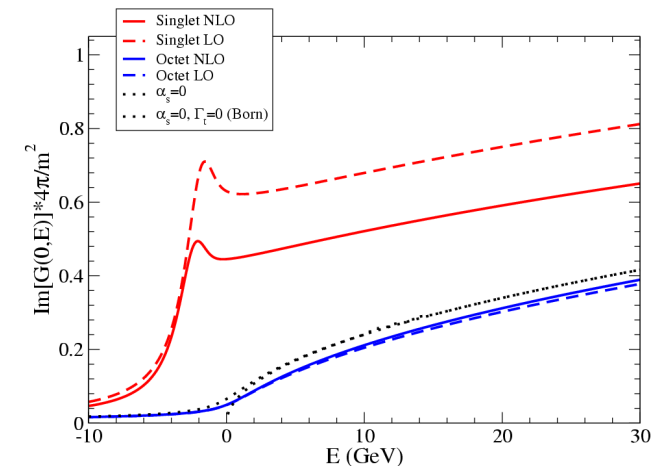
## 2. Bound-state effects at Hadron colliders

- Perturbative QCD potential (NLO), since an IR cut-off by  $r \lesssim \frac{1}{\Gamma_t}$

$$V_{\text{QCD}}^{(c)}(r) = C^{(c)} \frac{\alpha_s(\mu_B)}{r} \times \left[ 1 + \frac{\alpha_s}{\pi} v_1^{(c)}(r) + \dots \right]$$

$$\begin{cases} \text{singlet} & C^{(1)} = -4/3 \\ \text{octet} & C^{(8)} = 1/6 \end{cases}$$

Singlet is attractive,  
but octet is repulsive and small correction.



- Toponium system :  $m_t \gg \mu_B > E_B \simeq \Gamma_t \gg \Lambda_{\text{QCD}}$

- Binding energy :  $E_B \simeq m_t \alpha_s^2 \simeq 2\text{GeV}$

If  $\Gamma_t > E_B$ , top-quark decays  
before bound-state formation

- Bohr radius :  $\mu_B \simeq m_t \alpha_s \simeq 20 - 30\text{GeV}$

(typical momentum of the Coulomb gluon)

## 2. Bound-state effects at Hadron colliders

- Cross-section is proportional to the Imaginary part of the Green function by Optical theorem.

$$\hat{\sigma}_{tt}^{(c)} \rightarrow \hat{\sigma}_{tt,\text{Born}}^{(c)} \cdot \text{Im}[G^{(c)}(E, \vec{0})]$$

- Combining Initial-state/Final-state radiation effects,

$$\frac{d\sigma}{dm_{tt}}(s, m_{tt}^2) = \hat{\sigma}_i^{(c)}(m_{tt}^2) \cdot K_i^{(c)} \int_{\tau_0}^1 \frac{dz}{z} F_i^{(c)}(z) \frac{d\mathcal{L}_i(\tau_0/z)}{d\tau}$$

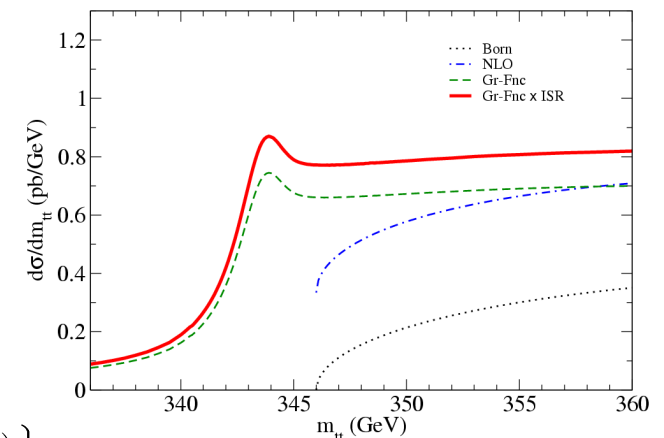
$$F_i^{(c)}(z) = \delta(1-z) + \frac{\alpha_s}{\pi} \left[ A_i \left\{ \left( \frac{\ln(1-z)}{1-z} \right)_+ - \left( \frac{1}{1-z} \right)_+ \ln \left( \frac{\mu_F}{2m_t} \right) \right\} + D_{tt}^{(c)} \left( \frac{1}{1-z} \right)_+ + k_i^{(c)} \delta(1-z) \right]$$

- Color-dependent **hard correction** :  $K_i^{(c)} = 1 + \frac{\alpha_s}{\pi} h_i^{(c)}$   
can be extracted from the NLO Quarkonium production

Petrelli,Cacciari,Greco,Maltoni,Mangano ('98)

+Non-decoupling term HSY('08), Czakon,Mitov('08)

gg->tt, color-singlet at the LHC



NLL : Kiyo etal.('08)

$$h_{gg}^{(1)}\left(\frac{\mu_R}{m_t}\right) = C_A \left(1 + \frac{\pi^2}{12}\right) + C_F \left(-5 + \frac{\pi^2}{4}\right) + \beta_0 \ln\left(\frac{\mu_R}{2m_t}\right),$$

$$h_{gg}^{(8)}\left(\frac{\mu_R}{m_t}\right) = C_A \left(3 - \frac{\pi^2}{24}\right) + C_F \left(-5 + \frac{\pi^2}{4}\right) + \beta_0 \ln\left(\frac{\mu_R}{2m_t}\right),$$

$$h_{q\bar{q}}^{(8)}\left(\frac{\mu_R}{m_t}\right) = C_A \left(\frac{59}{9} - \frac{\pi^2}{4} + \frac{2\ln 2}{3}\right) + C_F \left(-8 + \frac{\pi^2}{3}\right) - \frac{5}{9}n_q - \frac{8}{9} + \beta_0 \ln\left(\frac{\mu_R}{2m_t}\right).$$

## 2. Bound-state effects at Hadron colliders

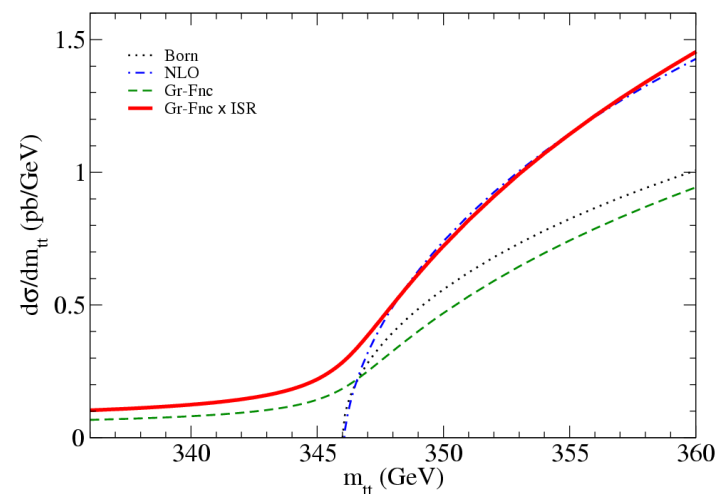
### □ $t\bar{t}$ invariant-mass distributions

Black : Born  
 Blue :  $O(\alpha_s)$  corr. (NLO)  
 Green : Gr-Fnc. without ISR  
 Red : Gr-Fnc. with ISR

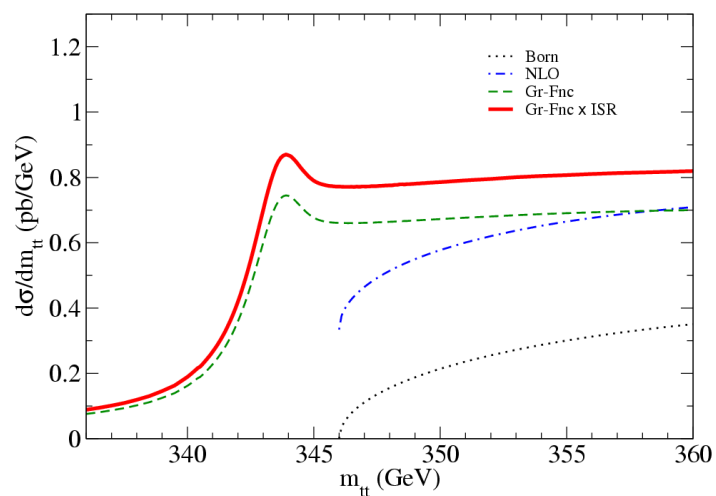
ISR : up to  $O(\alpha_s)$  (soft/collinear)

$m_t=173$  GeV,  $\Gamma_t=1.5$  GeV, CTEQ6M

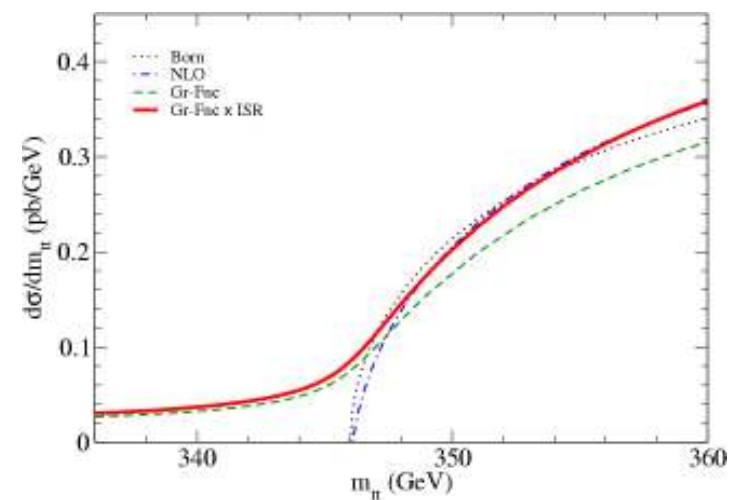
gg->tt, color-octet



gg->tt, color-singlet



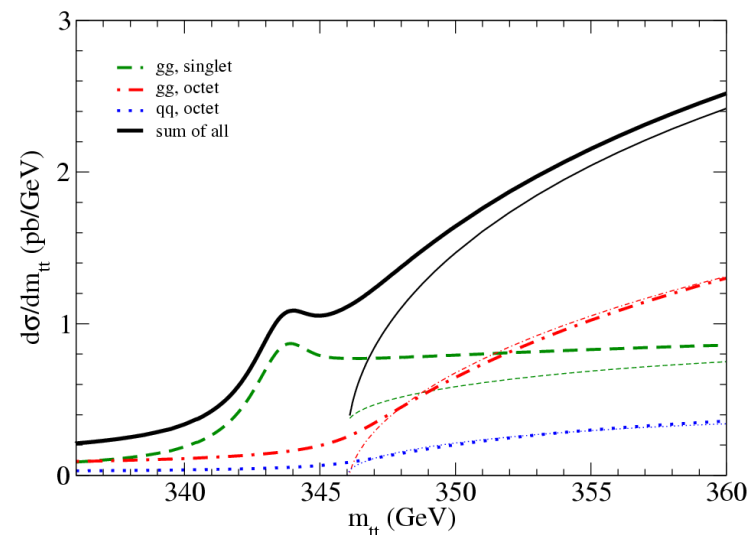
qq->tt, color-octet



## 2. Bound-state effects at Hadron colliders

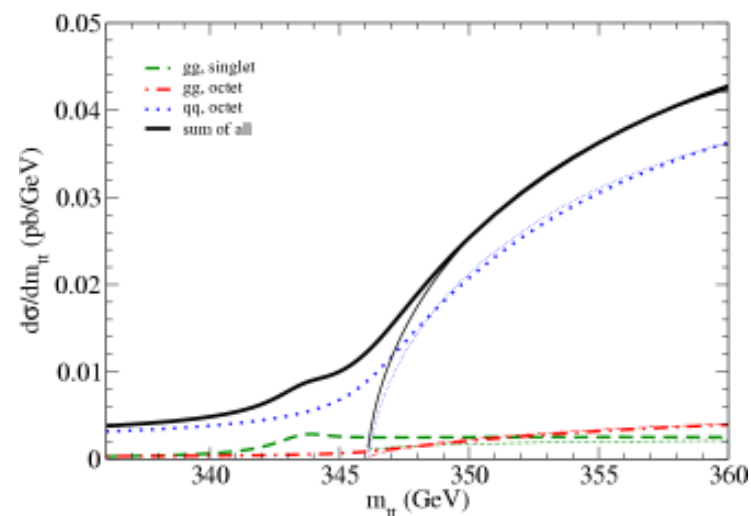
### ◆ In total at the LHC :

- BS effects **deform the invariant-mass distribution** near threshold
- Form **a broad resonance peak** below threshold (observable in principal)
- Enhance the total cross-section by **10 pb  $\sim O(1\%)$**



### ◆ On the other hand at the Tevatron :

Resonance can't be seen,  
due to the color-octet dominance.

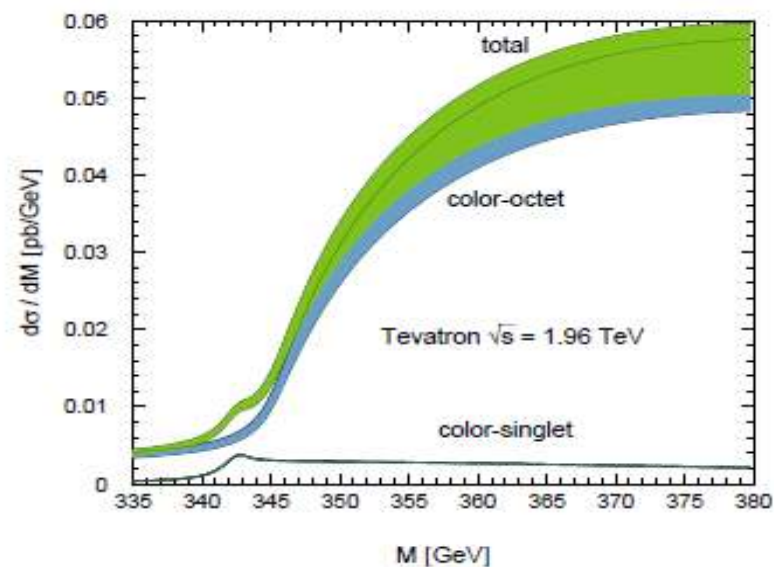
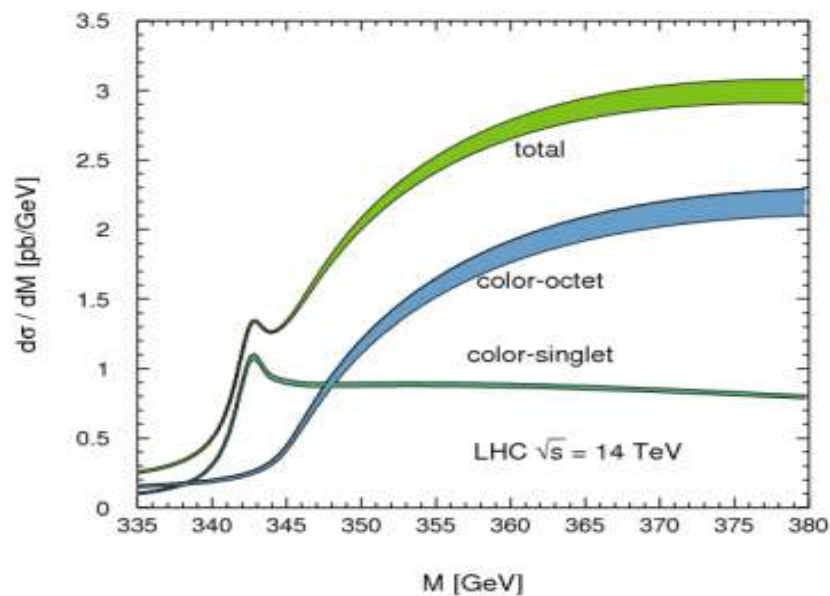


## 2. Bound-state effects at Hadron colliders

- Kiyo,Kuhn,Moch,Steinhauser,Uwer ('08)

- + full  $O(a_s)$  ISR (non-singular term)
- + Resummed ISR (NLL)

Small difference on the overall normalization.  
The conclusions agree with each other.





---

### 3. Differential Cross-section / Event Generation with BS effects

### 3. Differential Cross-section

#### □ Coulomb correction to differential cross-sections

- Differential distributions are useful for the analysis with kinematical cuts.
- Differential distribution is needed to generate events in MC simulation.
- Method to include BS effects is developed in  $e^+e^-$  collider study

Jezabek,Kuhn,Teubner('92)

Sumino,Fujii,Hagiwara,Murayama,Ng('93),,,

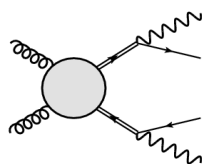
- Take into account the “**Leading-order**” contribution in both region :
  - **Threshold region** :  $(\alpha_s/\beta)^n$  but not  $\alpha_s^n, \beta^n$
  - **High-energy region** :  $\beta^n$  but not  $\alpha_s^n$
  - plus some NLO effects (K-factor, width, QCD potential,,)

note,  $\Gamma_t/m_t \sim \alpha_W \sim \alpha_s^2$

### 3. Differential Cross-section

#### □ Coulomb correction in differential cross-section

- **gg/qq to bWbW** process to take into account the off-shellness of top-quarks
- Divide **resonant part** and **non-resonant part** of the amplitude
- Double resonant part of the amplitude :



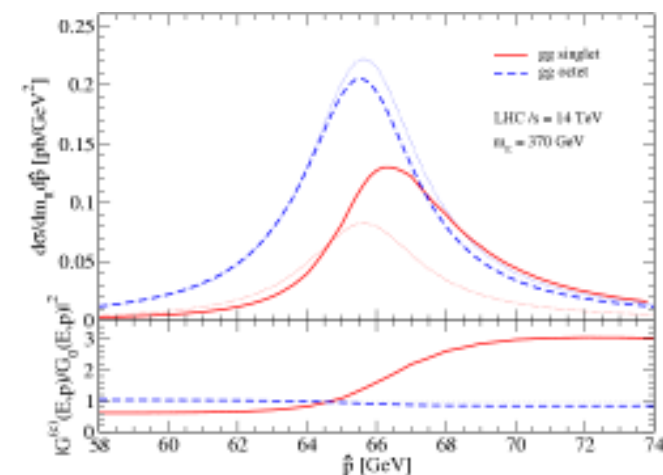
$$\mathcal{M}_{t\bar{t}} = \bar{D}_{t \rightarrow bW} \cdot \frac{i}{\not{p}_t - m_t + i\Gamma_t} \cdot P_{i \rightarrow t\bar{t}} \cdot \frac{i}{-\not{p}_{\bar{t}} - m_t + i\Gamma_t} \cdot D_{\bar{t} \rightarrow \bar{b}W}$$

Coulomb correction :  $P_i \rightarrow P_i \times \tilde{G}^{(c)}(E, \vec{p})$

Green function in momentum-space

- Top-quark momentum distribution (of the  $t\bar{t}$  cm frame) is affected by the Coulomb correction.

$$\frac{d\sigma}{d|\vec{p}|} \propto |\tilde{G}(E, \vec{p})|^2 \quad \begin{array}{l} \text{color-singlet : } \delta p > 0 \\ \text{color-octet : } \delta p < 0 \end{array}$$



### 3. Event Generation

Sumino, HY ('10)

<http://madgraph.kek.jp/~yokoya/TopBS>

#### □ Event Generator (LO + all-order Coulomb) :

- Full  $gg/qq \rightarrow bWbW$  plus  $W$ -decays Matrix-Elements (6-bodys)
- Color-decomposition in  $gg \rightarrow bWbW$  process
- Bound-state correction to the double-resonant amplitudes
- Color-dependent  $K$ -factors to reproduce NLO  $m_{tt}$  dist. near threshold

- Difference from General-purpose Monte-Carlo's :

MadGraph/MadEvent, Sherpa,,, (PYTHIA, HERWIG,,,) )

LO(Tree-level), ○ non-resonant effects, off-shell effect,,,

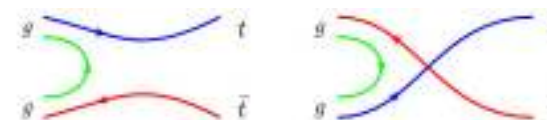
MCFM, MC@NLO,,,

NLO ( $\alpha_s/\beta$  term), × non-resonant effects, Breit-Wigner,,,

### 3. Event Generation

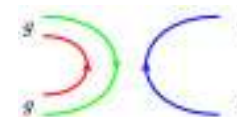
#### □ Color-flow assignment

in gluon-fusion process,



$$\begin{aligned}\mathcal{M}_{gg \rightarrow t\bar{t}} &= (T^a T^b)_{ij} \mathcal{M}_1 + (T^b T^a)_{ij} \mathcal{M}_2 \\ &= \frac{1}{2} \{T^a, T^b\}_{ij} \mathcal{M}_S + \frac{1}{2} [T^a, T^b]_{ij} \mathcal{M}_A\end{aligned}$$

symmetric part :  $\frac{1}{2} \{T^a, T^b\} = \frac{1}{2N_c} \delta^{ab} \delta_{ij} + \frac{1}{2} d^{abc} T_{ij}^c$



color-singlet

ratio of the amplitudes squared :  $\left| \frac{1}{2N_c} \delta_{ab} \delta_{ij} \right|^2 / \left| \frac{1}{2} d^{abc} T_{ij}^c \right|^2 = \frac{2}{N_c^2 - 4}$

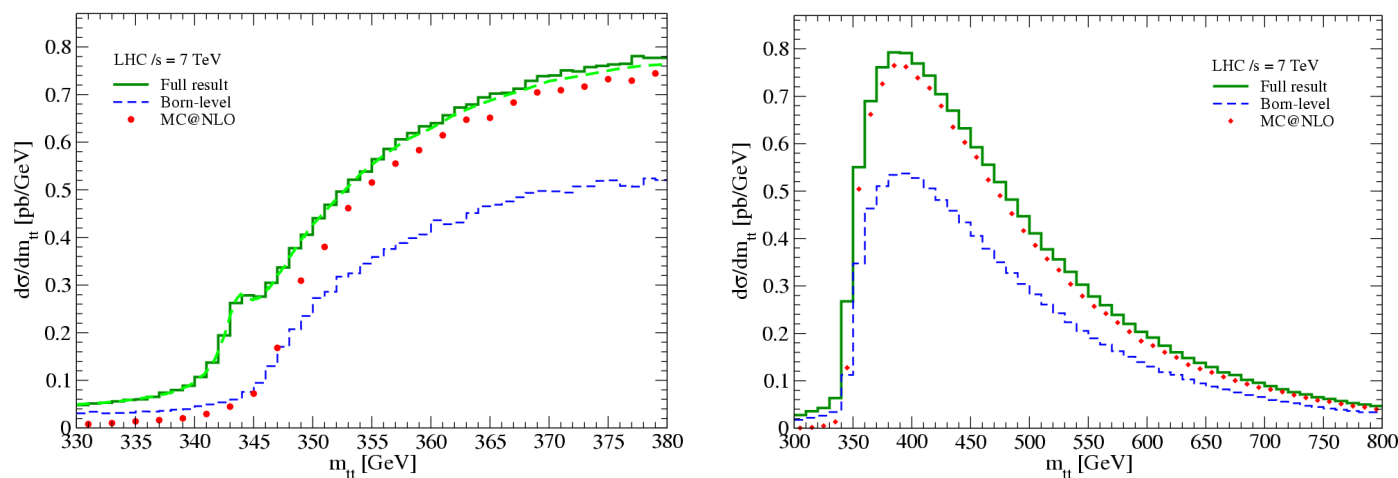
this is zero in large-N limit, but not in QCD

our color-singlet events have correct color-flow assignment in the LHEF record

### 3. Event Generation

#### □ Some Examples (at partonic-level)

(1)  $t\bar{t}$  invariant-mass ( $m_{t\bar{t}}$ ) distribution :



- The only generator which describes the threshold enhancement and resonance
- Effectively, well reproduce MC@NLO results at large  $m_{t\bar{t}}$  by taking the scales as

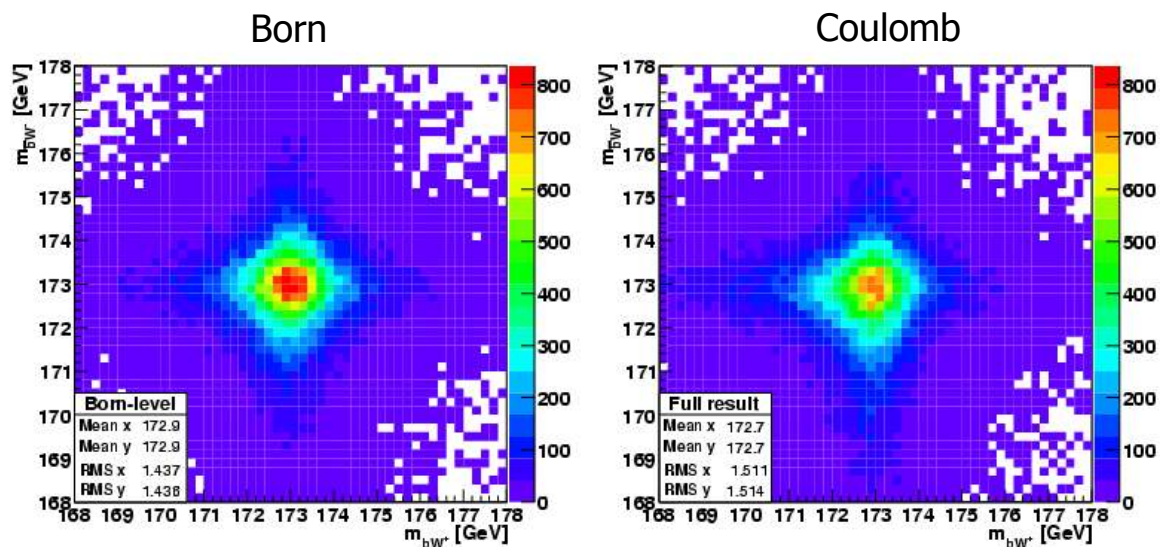
$$\mu = m_t \quad (\mu = \sqrt{m_t^2 + p_T^2} \quad \text{in MC@NLO})$$

### 3. Event Generation

#### □ Some Examples (at partonic-level)

(2) (bW)-(bW) double invariant-mass distribution of top-quarks ;  $m_{bW} = (p_b + p_W)^2$

limiting for the events with  $m_{tt} < 370$  GeV (10% of the total event)



$$\delta m \simeq -200 \text{ [MeV]}$$

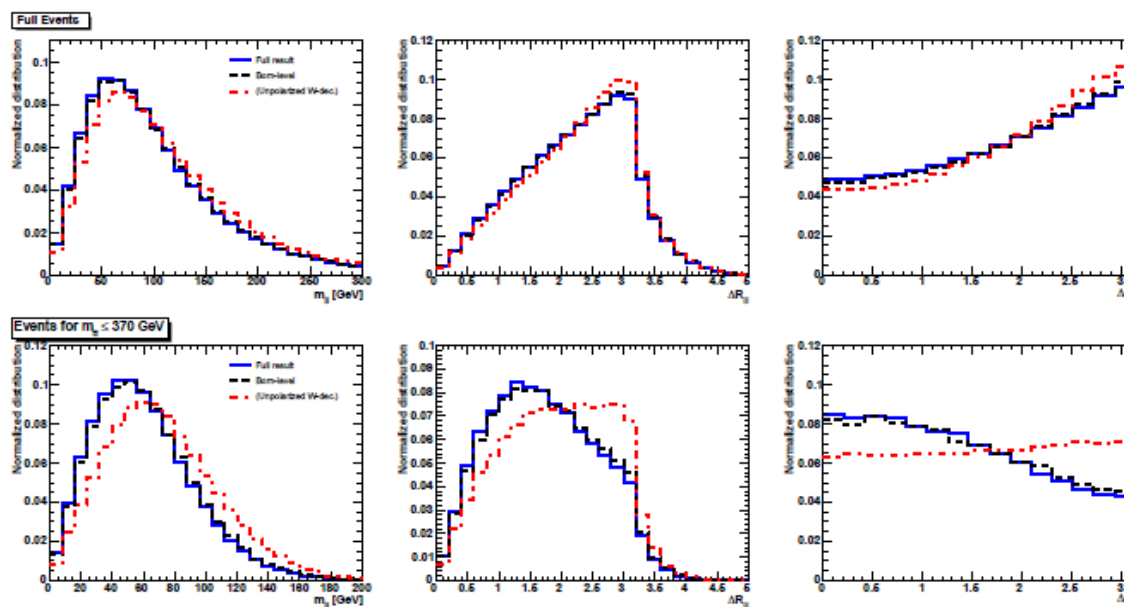
**correlated deviation** : one top-quark is still on-shell,  
but the other invariant-mass is reduced

## 3. Event Generation

### □ Some Examples (at partonic-level)

#### (3) lepton angular distributions (di-lepton case)

- Unfortunately, small bound-state effects in final lepton angular distributions
- bWbW MEs plus W-decay by parton-shower give wrong distribution (no W polarization)  
⇒ use 6-body MEs





## 4. Summary

---

We have studied the bound-state effect to the top-quark production at hadron colliders

- At the LHC, **gluon-fusion process** dominates and substantial amount of  $t\bar{t}$  pair is **color-singlet**.
  - The bound-state effects are calculated for the  $m_{t\bar{t}}$  distribution at Hadron Colliders up to NLO (Green's func., gluon radiation, hard-correction).
  - Large corrections in  $m_{t\bar{t}}$  dist. near threshold is predicted, and there appears a broad resonance below the threshold.
- **Differential cross-sections** are also calculated including **BS effects**, non-resonant amp's as well as decays of  $W$ 's
  - incorporate momentum-space Green functions for color-singlet and octet
  - smooth interpolation to the high-energy region
  - non-resonant diagrams are taken into account



## 4. Summary

---

- **Event Generator** including Bound-state effects
  - MC simulation including BS effects is now possible
  - Deviation from the Breit-Wigner distribution
  - Color-connection between  $t\bar{t}$  in color-singlet channel
- **Future** :
  - Application to the production of heavy colored particles in BSM  
gluino (Hagiwara, HY '09, ), squark (Beneke, Falgari, Schwinn '09, '10), KK gluon (Kahawala, Kats '11),,  
Coulomb corrections become relatively important for heavier particles production.
  - Detail phenomenological studies for the LHC measurements :  
how to extract threshold events? useful for the mass/width determination?