Higher-order behavior of strings and the string-theory landscape

- How does the non-perturbative string landscape look like? -

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@ NCTS HEP Journal Club on Feb. 22, 2011

with

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Sheng-Yu Darren Shih (NTU -> UC Berkeley)
and Chi-Hsien Yeh (NTU)

References of the talk:

1. [CIY3 ’11], work in progress
2. [CIY2 ’10], **arXiv:1011.5745,** “*Stokes Phenomena and Non-perturbative completion in the multi-cut matrix models*”

3. [CIY1 ’10], **Nucl.Phys.B838:75-118,2010, [arXiv:1003.1626],** “*Fractional-superstring amplitudes, multi-cut matrix models and non-critical M theory*”
4. [CISY ’09], **Nucl.Phys.B828:536-580,2010 [arXiv:0909.1197],** “*macroscopic loop amplitudes in the multi-cut two-matrix models*”

5. [Irie ’09], **Nucl.Phys.B819:351-374,2009 [arXiv:0902.1676],** “*fractional supersymmetric Liouville theory and the multi-cut matrix models*”

Why string theory, and what's the Goal

1. *Consistent quantum theory of gravity:*
   ← almost the only one framework for Quantum Gravity,
   the most natural framework (cf. Gauge/String duality)

2. *String theory is a quite unique theory*
   ← various string theories are related
   by several string dualities.

3. *A promising candidate for the theory of everything*
   ← Unification of all the fundamental interactions

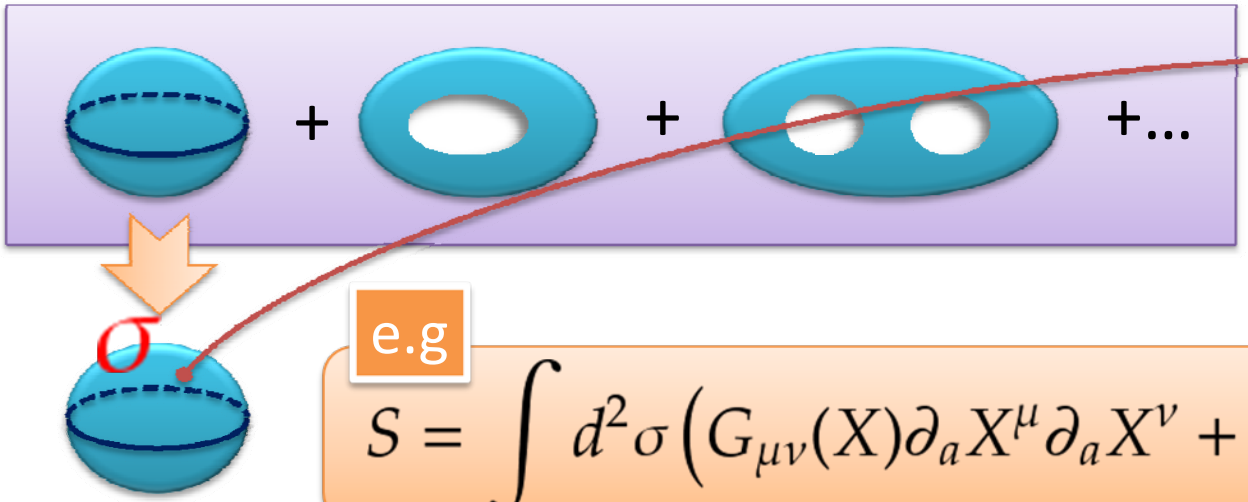
We wish to derive our universe
by some theoretical calculation
of the vacuum structure in string theory

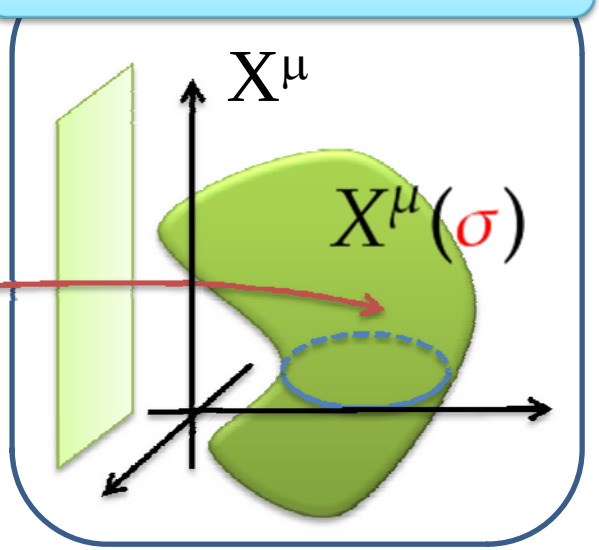
String Vacua

$G_{\mu\nu}(X),\ B_{\mu\nu}(X),\ \cdots$

Perturbative Vacua (Origin of "strings")

Feynman Graphs drawn by 2D surfaces:





e.g

$$S = \int d^2\sigma \left(G_{\mu\nu}(X)\partial_a X^\mu \partial_a X^\nu + \epsilon^{ab} B_{\mu\nu}(X)\partial_a X^\mu \partial_b X^\nu \right)$$

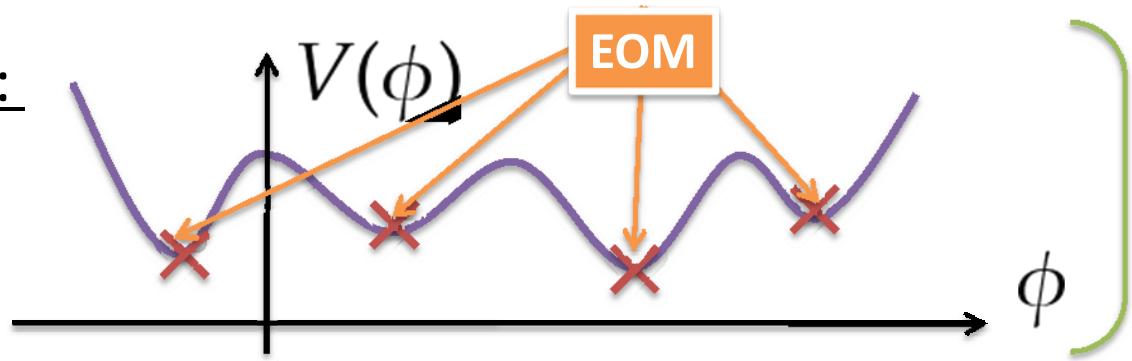
String Consistency ➔ This 2D QFT must be (anomaly free) CFT

1. Dimension of flat spacetime is 26/10D
2. *EOM of background fields ➔ Einstein eqn. for gravity, G,* and *Maxwell eqn. for 2-form gauge field, B.*
3. Allowed CFTs ⇔ Classical Solutions to the EOMs in spacetime

String Perturbative Vacua

Allowed CFTs = Classical Solutions to EOM = Perturbative Vacua

Cf) QFT:



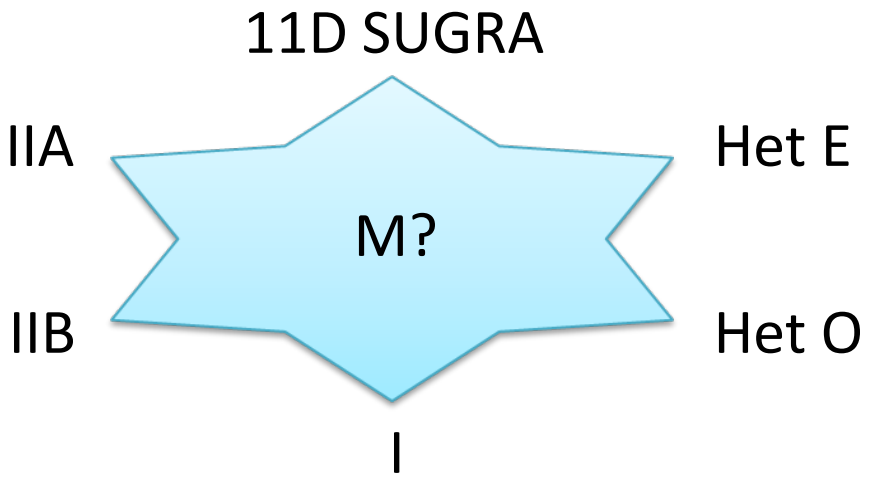
Deformation of CFT ⇔ background field configulations

String Perturbative Vacua

Allowed CFTs = Classical Solutions to EOM = Perturbative Vacua

Set of Allowed CFTs = Perturbative String-Theory Landscape

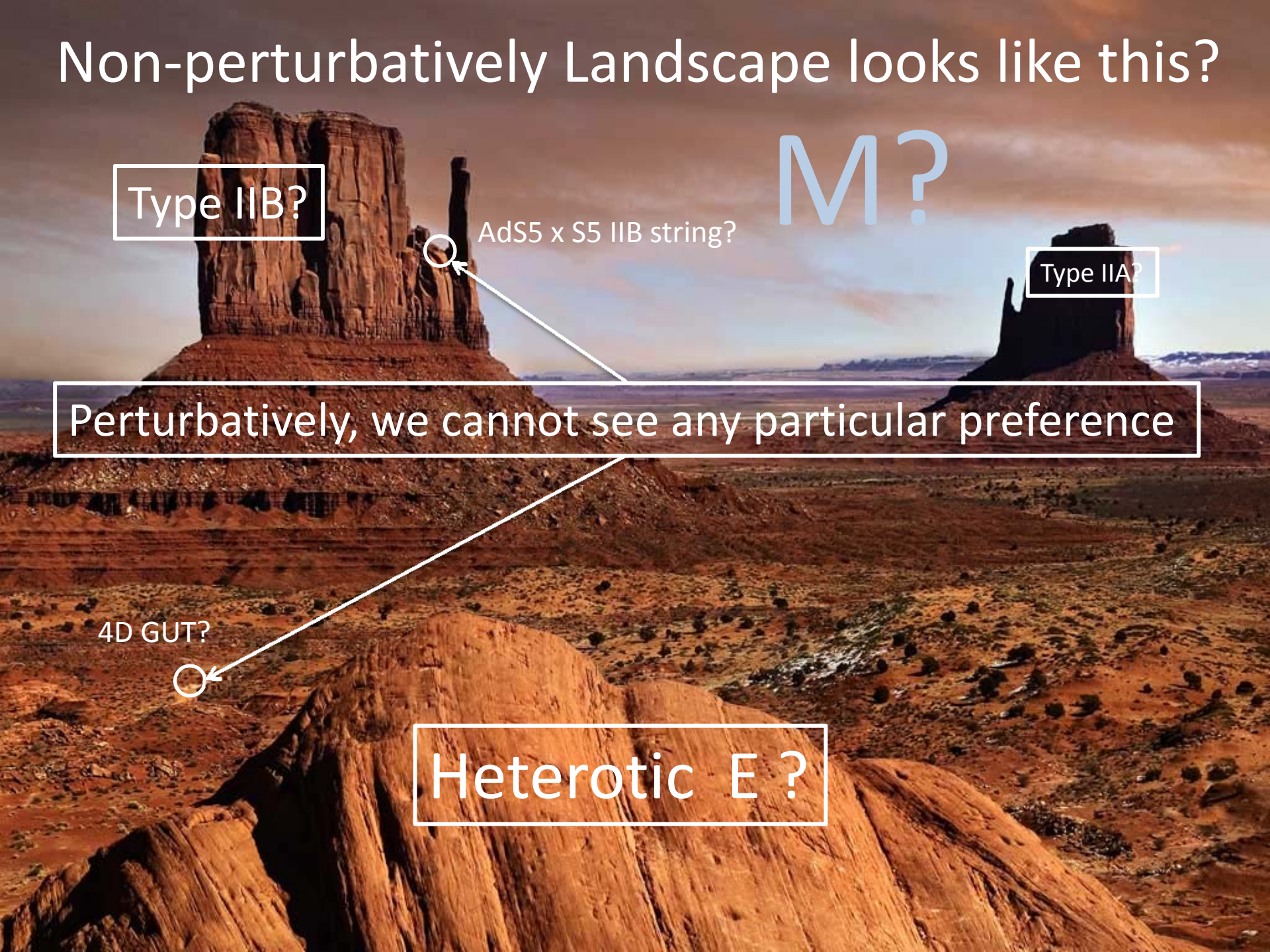
An IMAGE of Landscape of string-theory perturbative vacua



BUT, this procedure only guarantees Local Minimum (or Maximam)!!

The perturbative formulation is **not enough** to derive our universe

→ What is the Selection Principle for Our Universe??

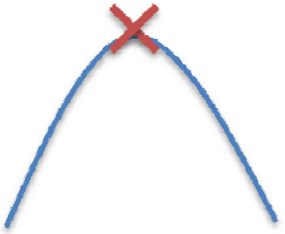

Non-perturbatively Landscape looks like this?
M?
Type IIB?
AdS5 x S5 IIB string?
Type IIA?
Perturbatively, we cannot see any particular preference
4D GUT?
Heterotic E ?

What we can get from perturbation theory

26D Bosonic string theory

There is a Tachyon mode

something like:



10D Superstring theory

Spacetime SUSY cancellation

something like:

People believe that non-perturbative effects
can give a non-trivial potential uplift

1. How non-trivial ??
2. What is the non-perturbative relationship among different perturbative vacua ??
3. Are there non-perturbative vacua ??

This is the long-standing and essential problem in string theory!

Non-perturbative string landscape and two-dimensional string theory

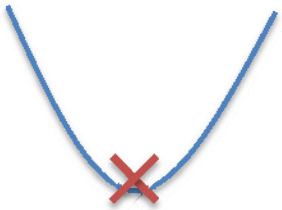
2D String Theory ('81-) Non-perturbatively exactly solvable with Matrix Models

One may wonder: *Why is it now ?*

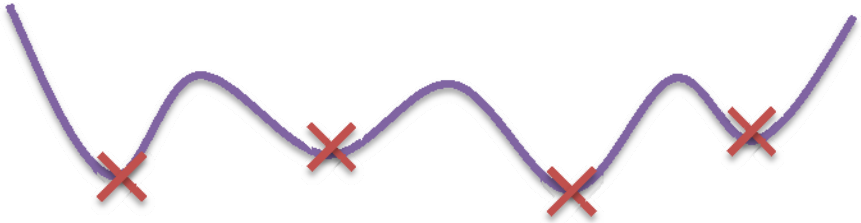
Why until now we could not derive the non-perturbative Landscape?

It is because 2D string theory was **TOO simple**

2D (bosonic) string is almost like:



On the other hand,
Our system is almost like:



Fractional Superstring Theory and Multi-Cut Matrix Models [Irie'09]

Plan of the talk

1. Introduction and Motivation

2. Multi-cut matrix models
   and fractional superstring theory

3. Non-perturbative String Landscape
   and non-critical M theory

4. Summary and future directions

2. Multi-cut matrix models and Fractional superstring theory

Brief look at matrix models

Matrix Models (N: size of Matrix)

$$\mathcal{Z} = \int dX dY \, e^{-N\mathrm{tr}[V_1(X)+V_2(Y)-XY]}$$

TW: Hermitian Matrix

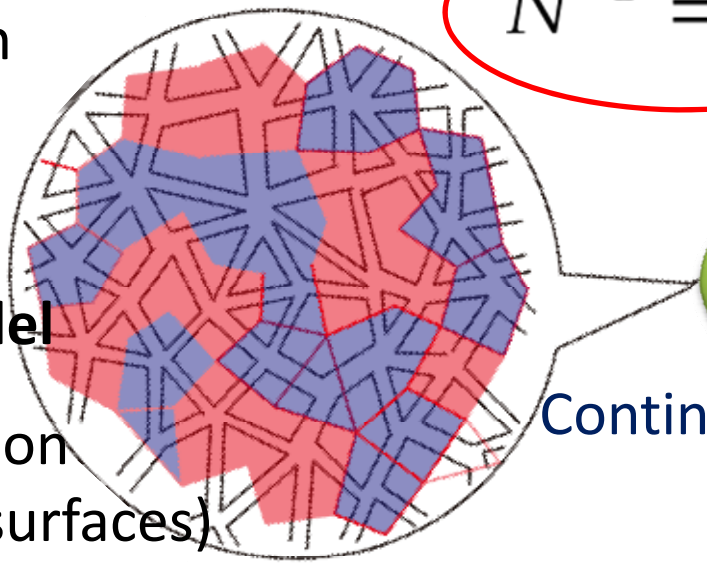
IW: NxN Hermitian Matrix

String Theory

t'Hooft's large N expansion of Yang-Mills theory

$$N^{-1} = g_{str}$$

Feynman Diagram



Worldsheet (2D surfaces)

+critical Ising model

Ising model

Triangulation (Random surfaces)

Continuum limit

Matrix Models → "**Multi-Cut**" Matrix Models

String Theory → **Fractional Superstring Theory [H.I'09]**

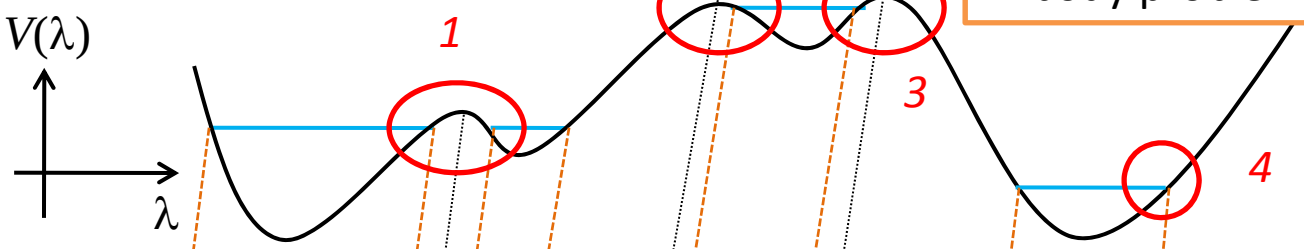
Let's see more details

Diagonalization: $U^\dagger M U = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_N)$

$$\mathcal{Z} = \int dM\, e^{-N \mathrm{tr} V(M)} \quad \Rightarrow \quad \mathcal{Z} = \int d^N\lambda \prod_{i>j} (\lambda_i - \lambda_j)^2 e^{-N \sum_i V(\lambda_i)}$$

In Large N limit (= semi-classical)

N-body problem in the potential V

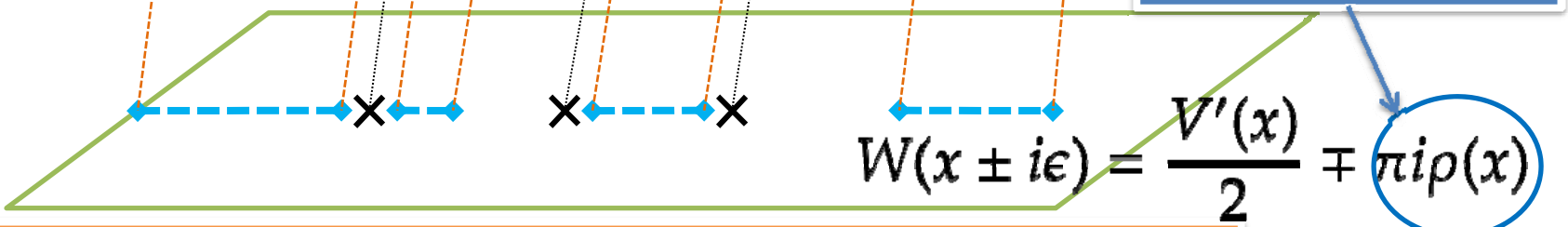


This can be seen by introducing the Resolvent
(Macroscopic Loop Amplitude)

$$W(x) = \frac{1}{N}\left\langle \mathrm{tr} \frac{1}{x - M} \right\rangle$$

which gives spectral curve (generally algebraic curve):

Eigenvalue density

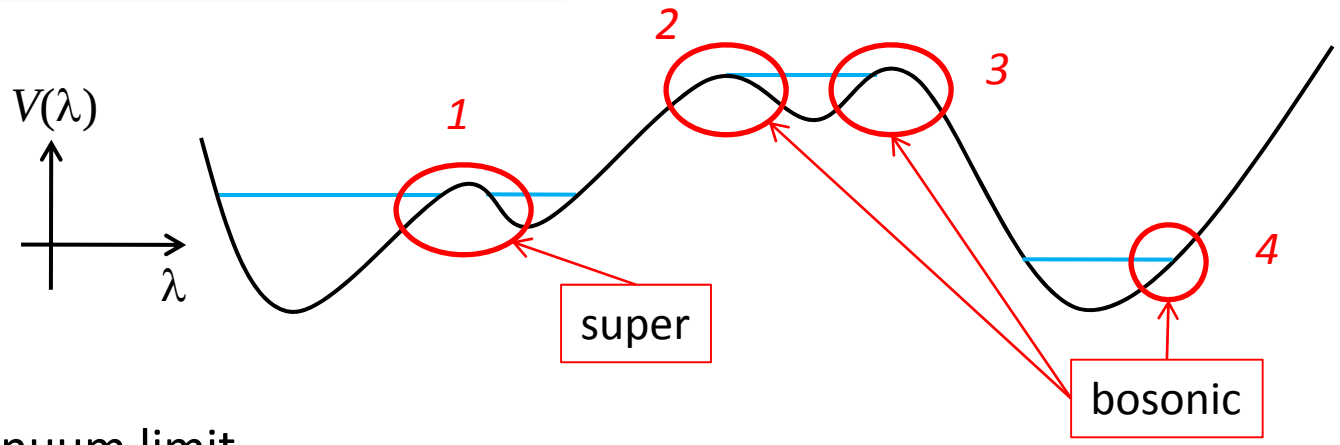


$$W(x \pm i\epsilon) = \frac{V'(x)}{2} \mp \pi i \rho(x)$$

Continuum limit = Blow up *some points of* x on the spectral curve

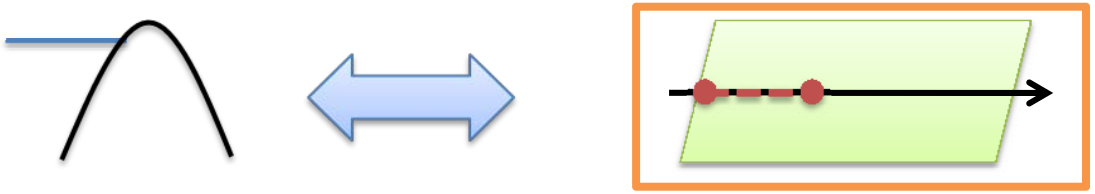
The nontrivial things occur only around the turning points

Correspondence with string theory



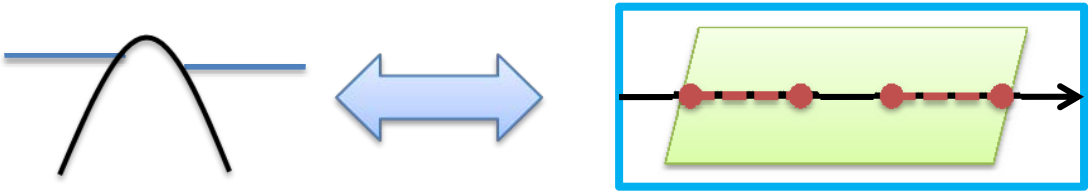
After continuum limit,

1-cut critical points (2, 3 and 4) give *(p,q) minimal (bosonic) string theory*



2-cut critical point (1) gives *(p,q) minimal superstring theory (SUSY on WS)*

[Takayanagi-Toumbas '03], [Douglas et.al. '03], [Klebanov et.al '03]

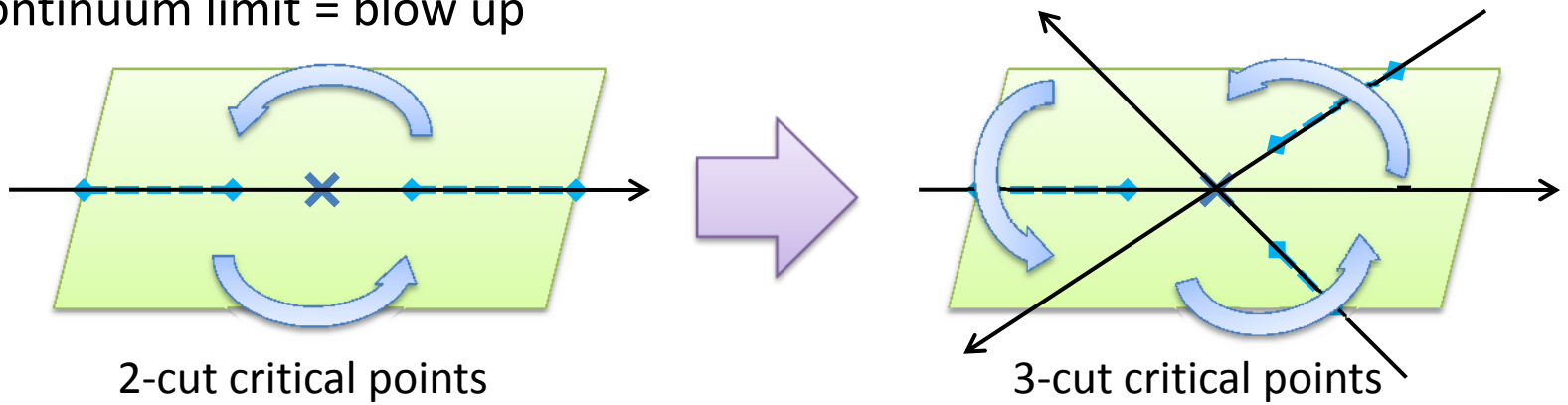


<u>TOO simple to claim string Landscape??</u>

Let's consider the *Multi-Cut Critical Points*:

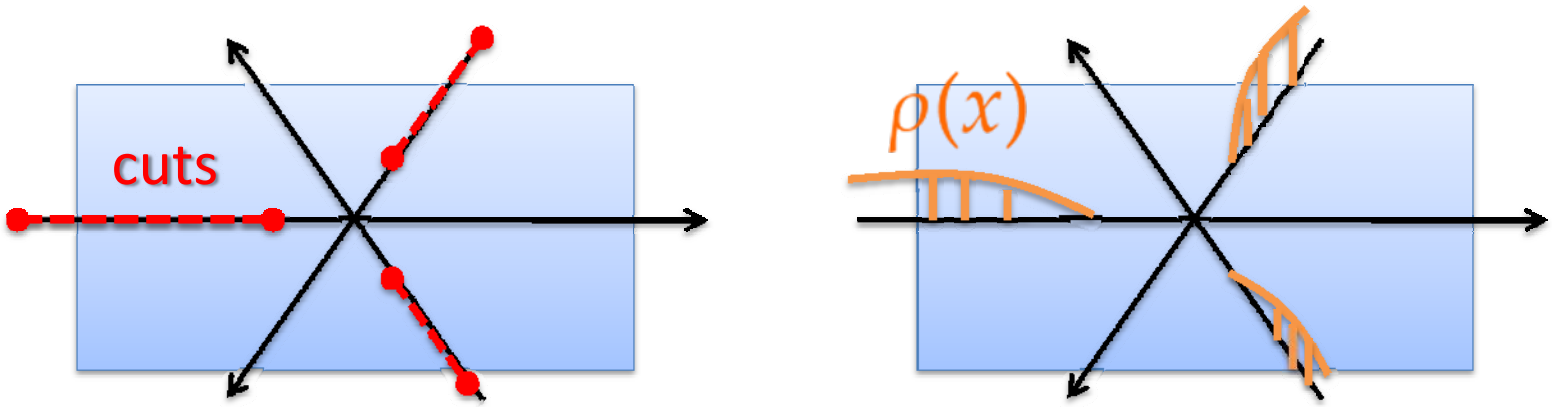
[Crinkovik-Moore '91]

Continuum limit = blow up



2-cut critical points

3-cut critical points



We can expect variety of vacua!

These consideration are only qualitative discussion.
Therefore, we show **quantitative** results of the system.

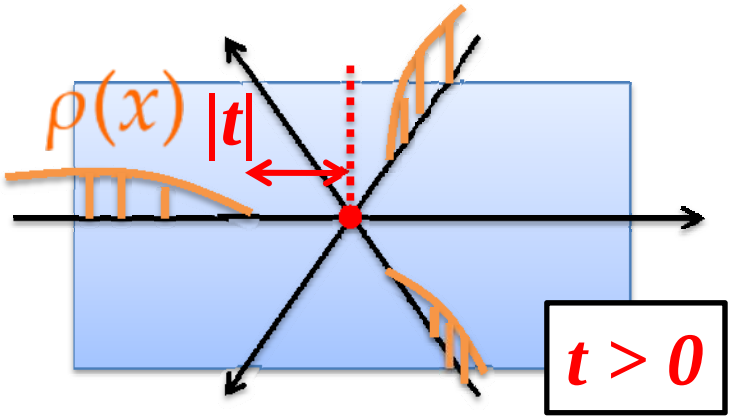
Actual solutions of the system [CISY'09, CIY'10]

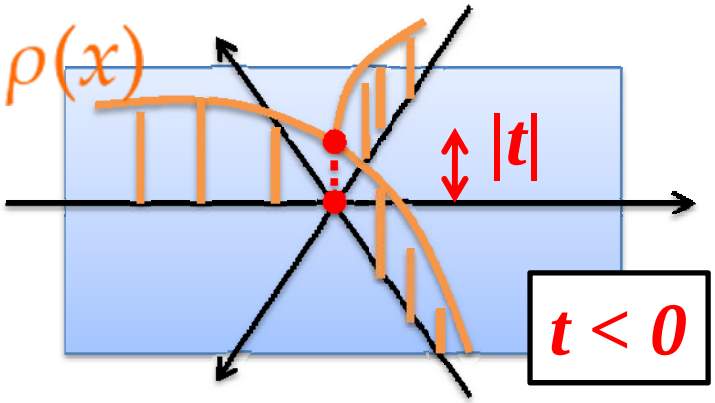
the Z_k symmetric case [CISY'09]: (p,q) critical points with k cuts

$$W(x):\quad \begin{cases} W = t^{\frac{q}{2p-1}}\, P_{q-1}^{(\frac{2l-k}{k},-\frac{2l-k}{k})}(z)\,\sqrt[k]{(z-1)^{k-l}(z+1)^{l}} \\ x = t^{\frac{p}{2p-1}}\, P_{p-1}^{(-\frac{2l-k}{k},\frac{2l-k}{k})}(z)\,\sqrt[k]{(z-1)^{l}(z+1)^{k-l}}, \end{cases}$$

(t: parameter)

e.g) the 3-cut cases are





the Z_k symmetric case [CISY'09]: (p,q) critical points with k cuts

$$W = t^{\frac{q}{2p-1}} P^{\left(\frac{2l-k}{k}, -\frac{2l-k}{k}\right)}_{q-1}(z) \sqrt[k]{(z-1)^{k-l}(z+1)^{l}}$$

$$x = t^{\frac{p}{2p-1}} P^{\left(-\frac{2l-k}{k}, \frac{2l-k}{k}\right)}_{p-1}(z) \sqrt[k]{(z-1)^{l}(z+1)^{k-l}},$$

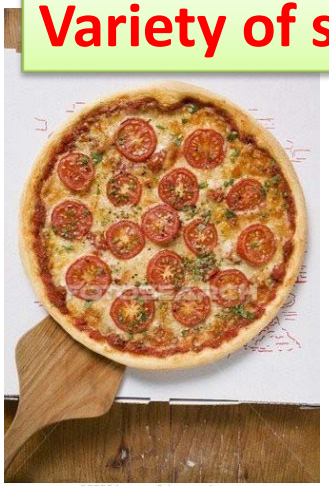
$l = 0, 1, \cdots, k-1, \quad \#l \sim k$ **Too many solutions!?**

$\#l = 2$ is natural because we have two choices

$$t > 0 \text{ or } t < 0$$

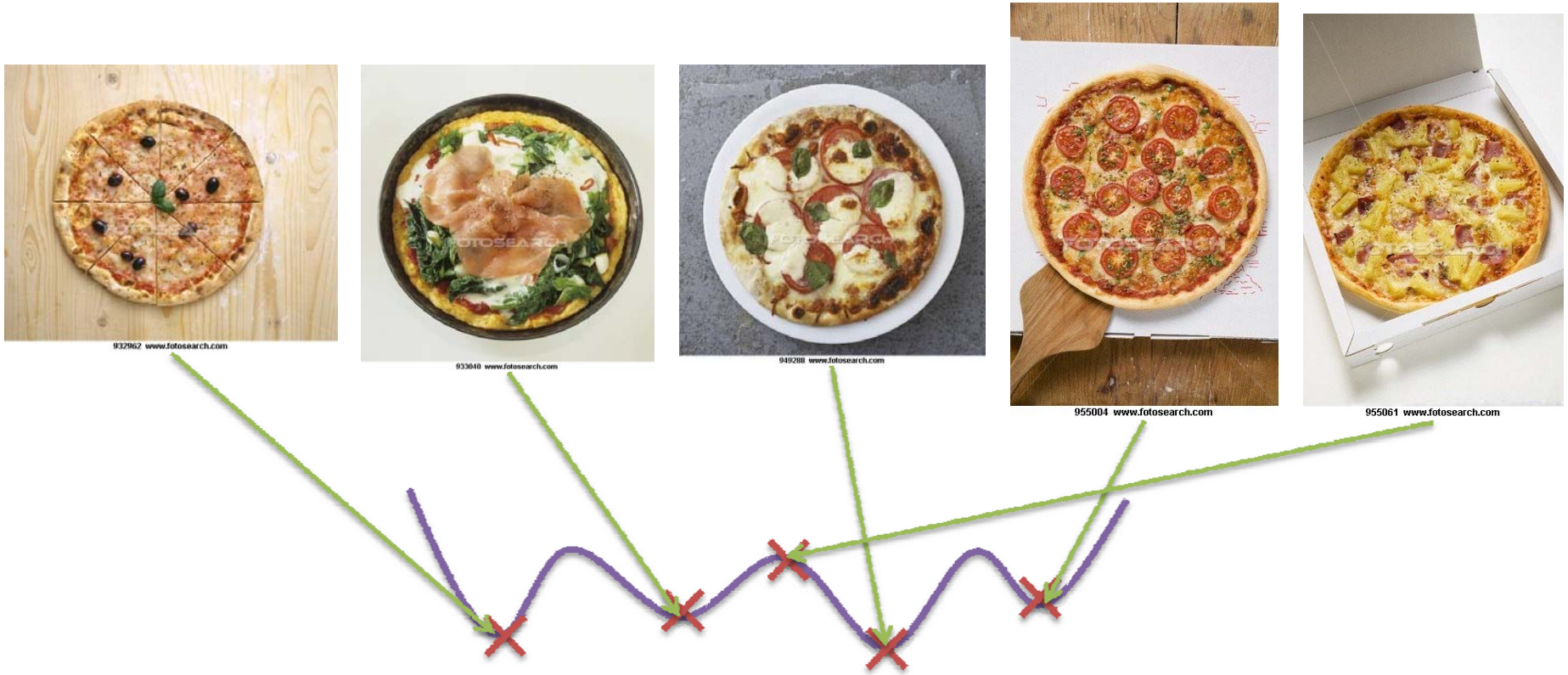
Each has different perturbative amplitudes

Variety of solutions



This implies that

the string Landscape of multi-cut matrix models is non-trivial



The multi-cut matrix models provide

non-trivial models for non-perturbative string landscape!

Fractional-superstrings provide more non-trivial situations!

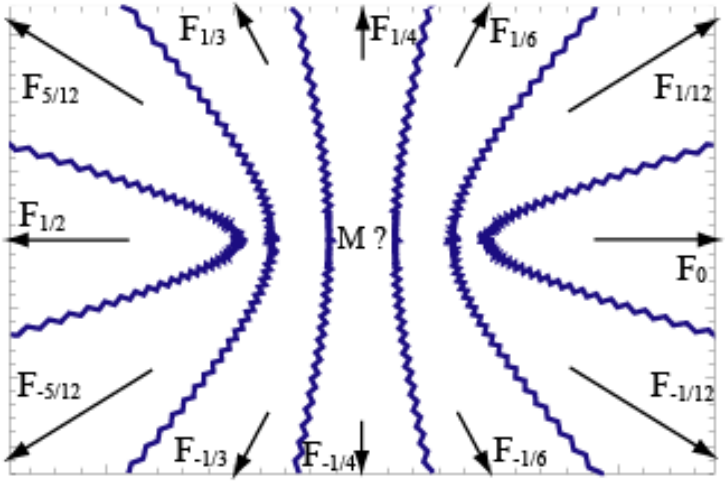
Actual solutions of the system [CISY'09, CIY'10]

the fractional superstring cases [CIY'10]:

(p,q) critical points with k cuts

$$\begin{cases} W = t^{\frac{q}{2p}} ch(q\tau + 2\pi i \frac{j-1}{k}) \\ x = t^{\frac{p}{2p}} ch(p\tau + 2\pi i \frac{j-1}{k}) \end{cases} \qquad \begin{cases} W = t^{\frac{q}{2p}} sh(q\tau + 2\pi i \frac{j-1}{k}) \\ x = t^{\frac{p}{2p}} sh(p\tau + 2\pi i \frac{j-1}{k}) \end{cases}$$

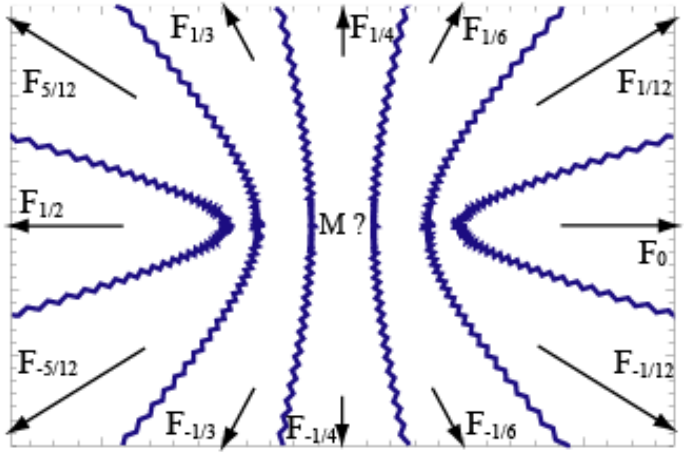
e.g) the 12-cut cases are

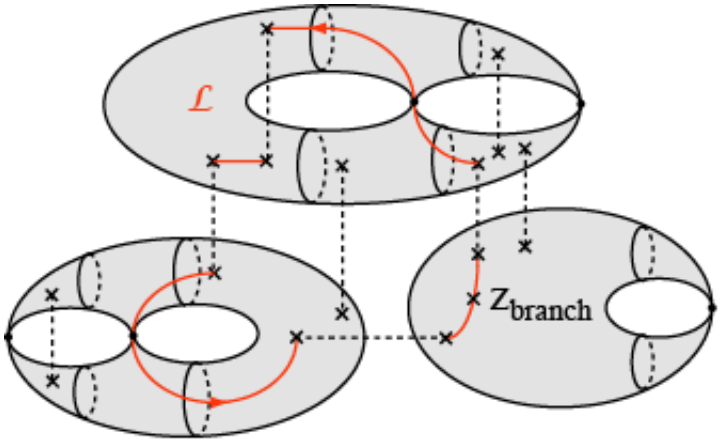


Next we will see the physical interpretation of this solution.

3. Non-Perturbative String Landscape and Non-Critical M theory

Factorization and Perturbative Isolation [CIY'10]





The algebraic equation of the solution
is **factorized** **into** ***irreducible curves***:

$$F(x,W) = F_0(x,W) \times F_{\pm 1/12}(x,W) \times \cdots \times F_{\pm 1/2}(x,W) = 0$$

EX:

reducible curves:

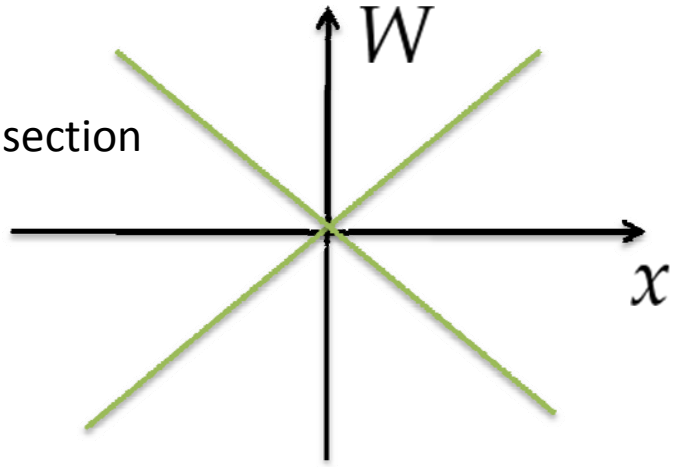
$$F(x,W) = W^2 - x^2 = (W-x)(W+x) = 0$$

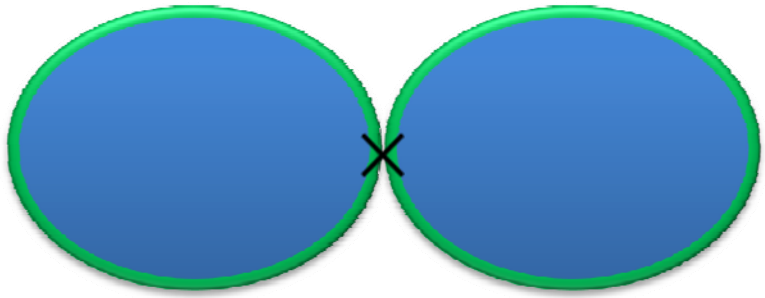
irreducible curves:

$$F(x,W) = W^2 + x^2 - 1 = 0$$

$$F(x, W) = W^2 - x^2 = (W - x)(W + x) = 0$$

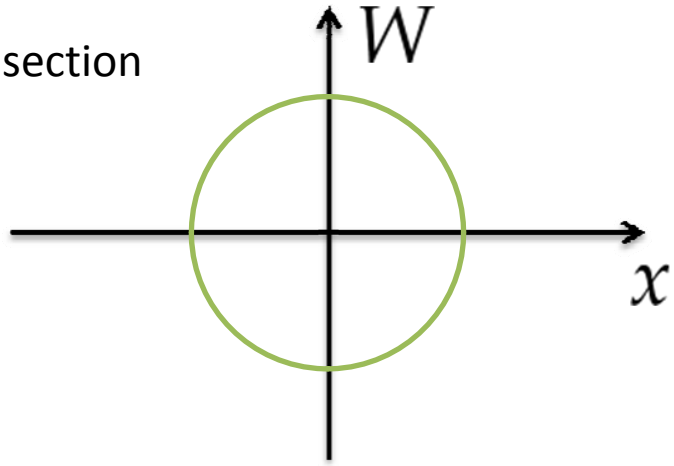
Real section

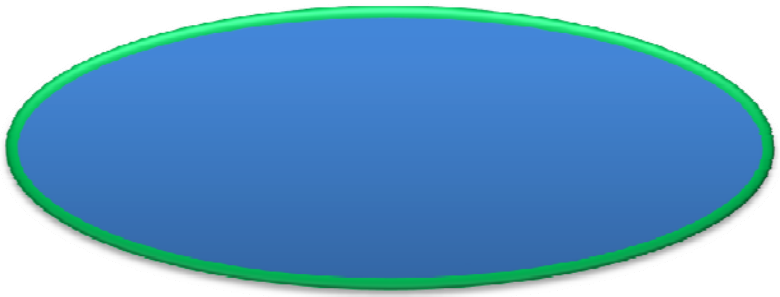




$$F(x, W) = W^2 + x^2 - 1 = 0$$

Real section





Factorization and Perturbative Isolation [CIY'10]

What is the physical meaning of these factrization?

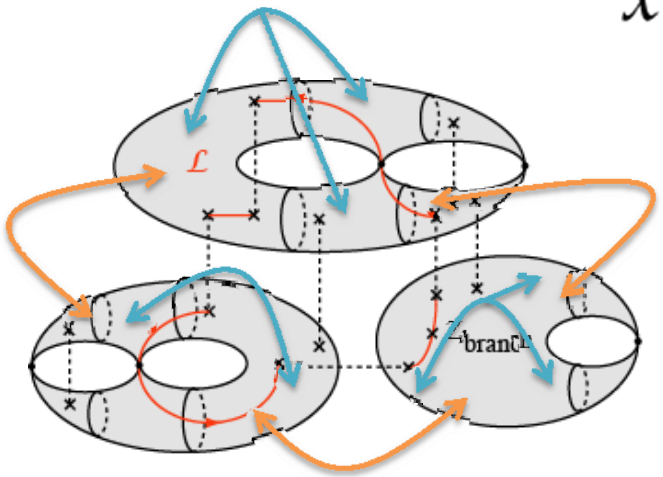
$$F(x,W) = F_0(x,W) \times F_{\pm 1/12}(x,W) \times \cdots \times F_{\pm 1/2}(x,W) = 0$$

Fact [Eynard-Orantin '07] **All order Perturbative correlators only depend on F(x,W)=0**

IF $F(x,W) = F_1(x,W) \times F_2(x,W) = 0$ (**All-order Perturbatively**)

$$\langle O_1(x_1)\cdots O_1(x_n) O_2(y_1)\cdots O_2(y_m)\rangle = \langle O_1(x_1)\cdots O_1(x_n)\rangle \langle O_2(y_1)\cdots O_2(y_m)\rangle$$

$$x_i \in C(F_1),\ y_j \in C(F_2)$$



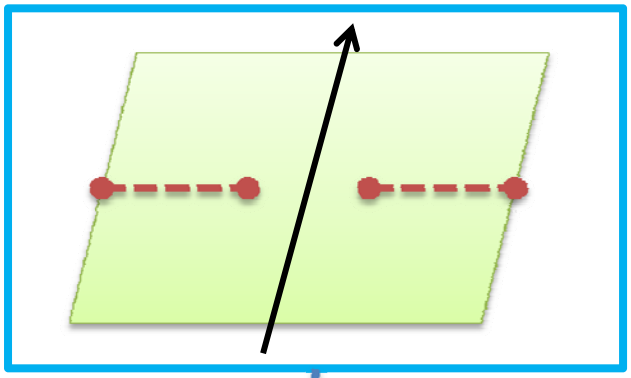
Perturbative interactions

Only non-perturbative interactions

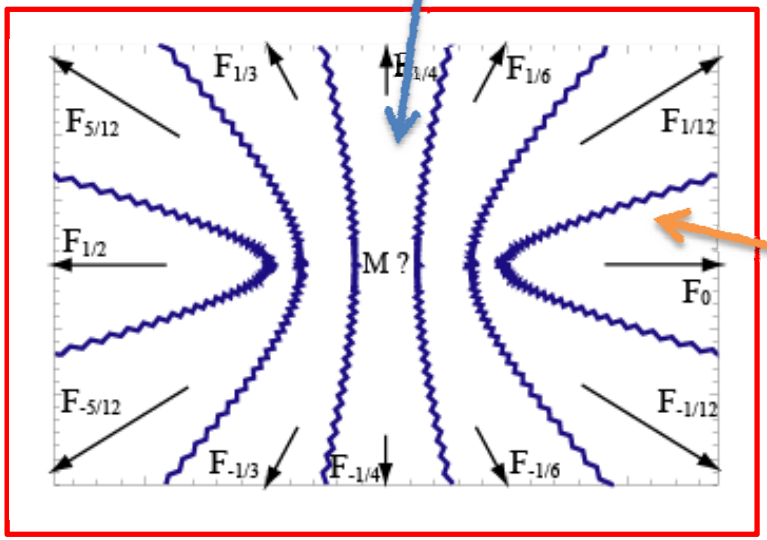
This system has many ***perturbatively isolated sectors***

Factorization and Perturbative Isolation [CIY'10]

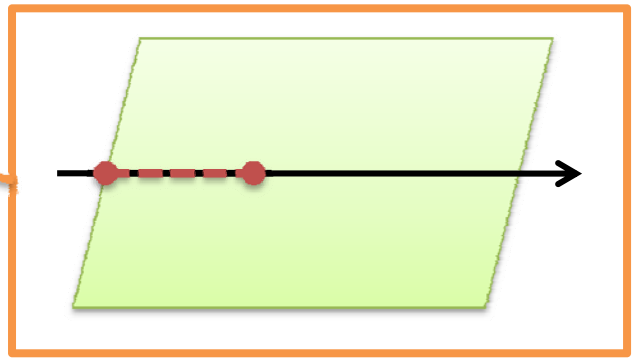
Perturbative vacua appear as perturbatively Isolated sectors!!



Perturbatively (all order)
type 0 Superstring



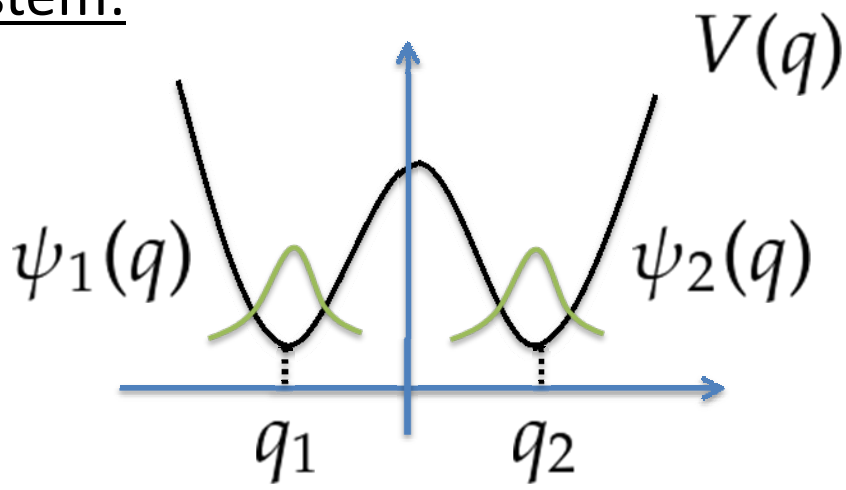
Perturbatively (all order) Bosonic string



Analogy to Quantum Mechanics System:

If we use perturbation theory,
we encounter two perturb. sectors:

$$\begin{cases} \psi_2(q) & q \sim q_1 \\ \psi_2(q) & q \sim q_2 \end{cases}$$



The true vacuum is *superposition* of these wave functions:

$$\psi_{vacuum}(q) = \frac{1}{\sqrt{2}}\left(\psi_1(q) + \psi_2(q)\right)$$

We know that the *coefficient* *is* *very important* *and* *non-perturbative*.

In our case, the perturbative sectors are like *superselection sectors*

Perturbative string theories are just ***segments*** *of* ***the whole system***

What is the whole system?

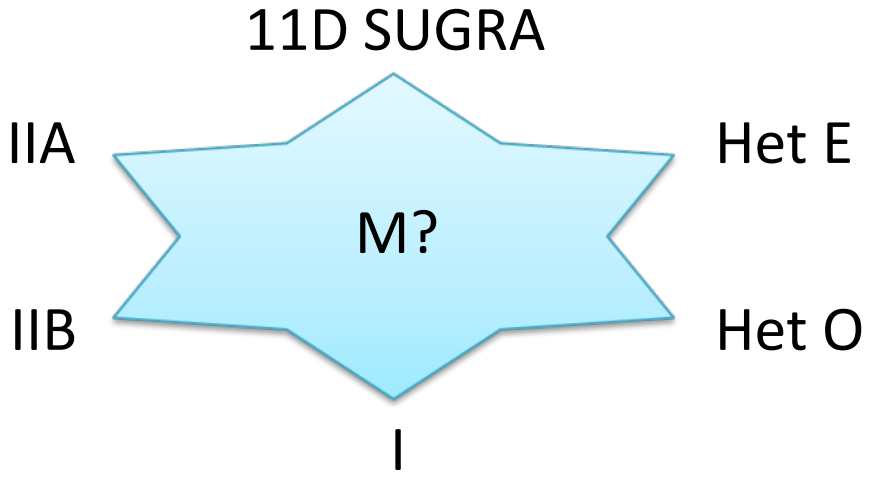
It's non-critical M theory!

As a non-critical **M** Theory [CIY'10]

M Theory is a Mysterious theory which unifies IIA BPS spectrum:

IIA(10Dim):	D0	F1	D2	D4	NS5
M(11Dim):	KK		M2		M5

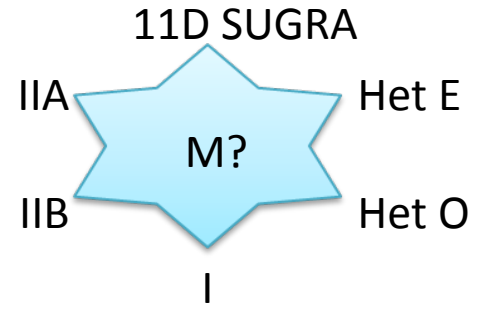
1. This lives in 11 dimensional spacetime (1Dim higher).
2. The fundamental DOF is Membrane (M2)
3. The low energy effective theory is 11D SUGRA.
4. Strong coupling dual theory of IIA string theory

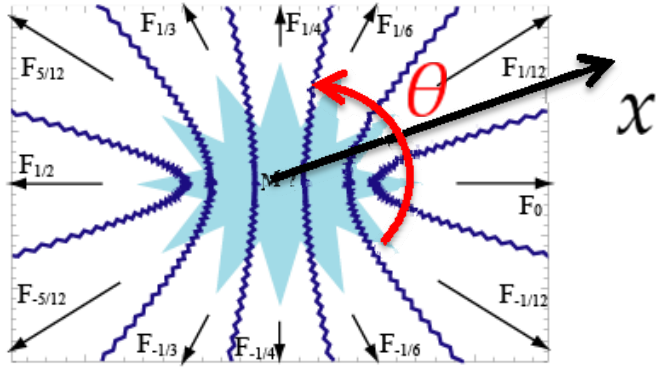


M theory should know the strong coupling dynamics of string theory

Evidence of Non-critical M-theory [CIY'10]

1. 3D Theory (as a Sequence of 2D pert. Strings)
2. The 3rd dim. is non-perturbative





3. [illegible]heory
4. [illegible]
5. [illegible]
6. Break down of the large N expansion – String is not good description in the strong coupling.

We found a complete and concrete definition of M theory, which is also solvable!!

Evidence of Non-critical M-theory [CIY'10]

1. 3D Theory (as a Sequence of 2D pert. Strings)
2. The 3rd dim. is non-perturbative
   (2D theory in all-order perturbation theory)
3. KK momentum = D0-brane charge of 0A string theory
4. Charge of D-brane = Position in the 3rd dim.
5. As a Mother theory
6. Break down of the large N expansion = string is not good description in the strong coupling.

We found a complete and concrete definition of M theory, which is also solvable!!

Summary

1. We showed that the multi-cut matrix models provide a nice laboratory to see new non-perturbative aspects of string theory:
   Non-perturbative string landscape and non-critical M theory
2. The quantitative analyses of the multi-cut matrix models have been carried out in various critical points of the models

future directions

1. ***Non-perturbative completion*** of the string theory
   → DONE!! [CIY2,10] [CIY3,11 in progress]
2. Integrable systems and ***description of non-critical M theory*** with the suitable degree of freedom
3. Dynamical principle of non-perturbative string landscape, and what is ***the selection principle of our universe***??