Higgs mass and effects of brane kinetic terms on phenomenologically viable Gauge-Higgs unification models

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Hierarchy problem

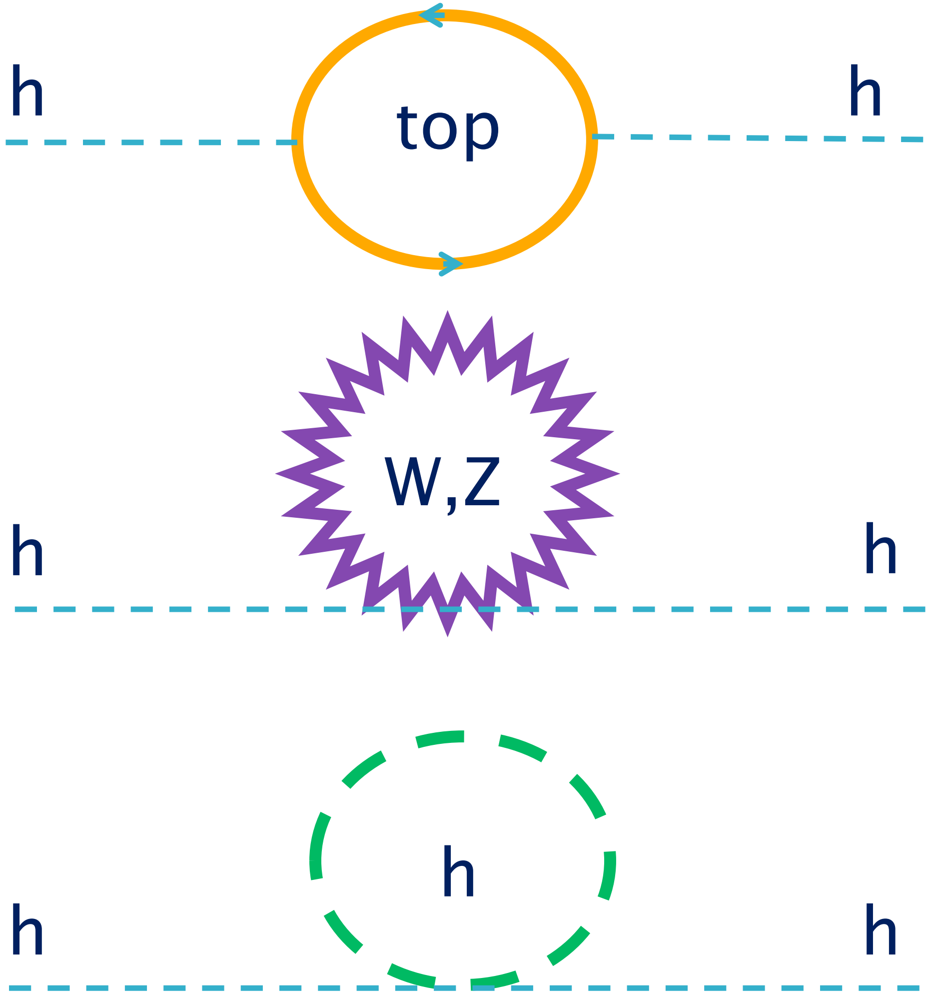
Mass hierarchy, Gauge hierarchy …………..

a **hierarchy problem** occurs when the fundamental parameters (couplings or masses) of some Lagrangian are vastly different (usually larger) from the parameters measured by experiment. This can happen because measured parameters are related to the fundamental parameters by a prescription known as renormalization.

Typically the renormalization parameters are closely related to the fundamental parameters, but in some cases, it appears that there has been a delicate cancellation between the fundamental quantity and the quantum corrections to it. Hierarchy problems are related to fine-tuning problems and problems of naturalness.

From Wikipedia, the free encyclopedia

Significant Higgs loop corrections in the standard Model



$$= -\frac{3}{4\pi}\frac{m_t}{v^2}\Lambda^2 = -\frac{3}{8\pi^2}\lambda_t^2\Lambda^2$$

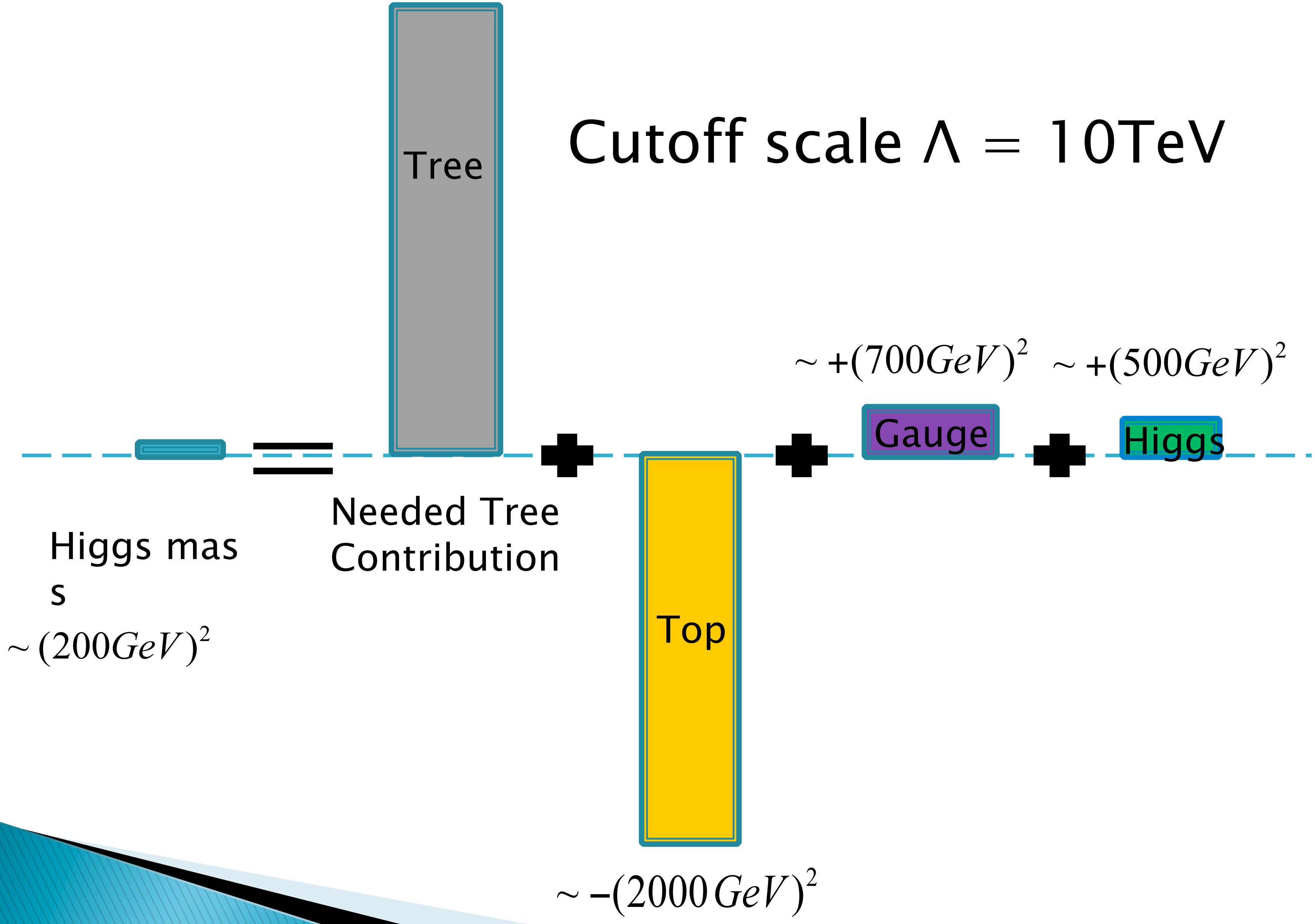
$$\frac{\lambda_t}{\sqrt{2}}v = m_t$$

$$= \frac{1}{16\pi^2}g^2\Lambda^2$$

$$= \frac{1}{16\pi^2}\lambda\Lambda^2$$

'Little'(low mass) Higgs and Fine Tuning

Cutoff scale Λ = 10TeV



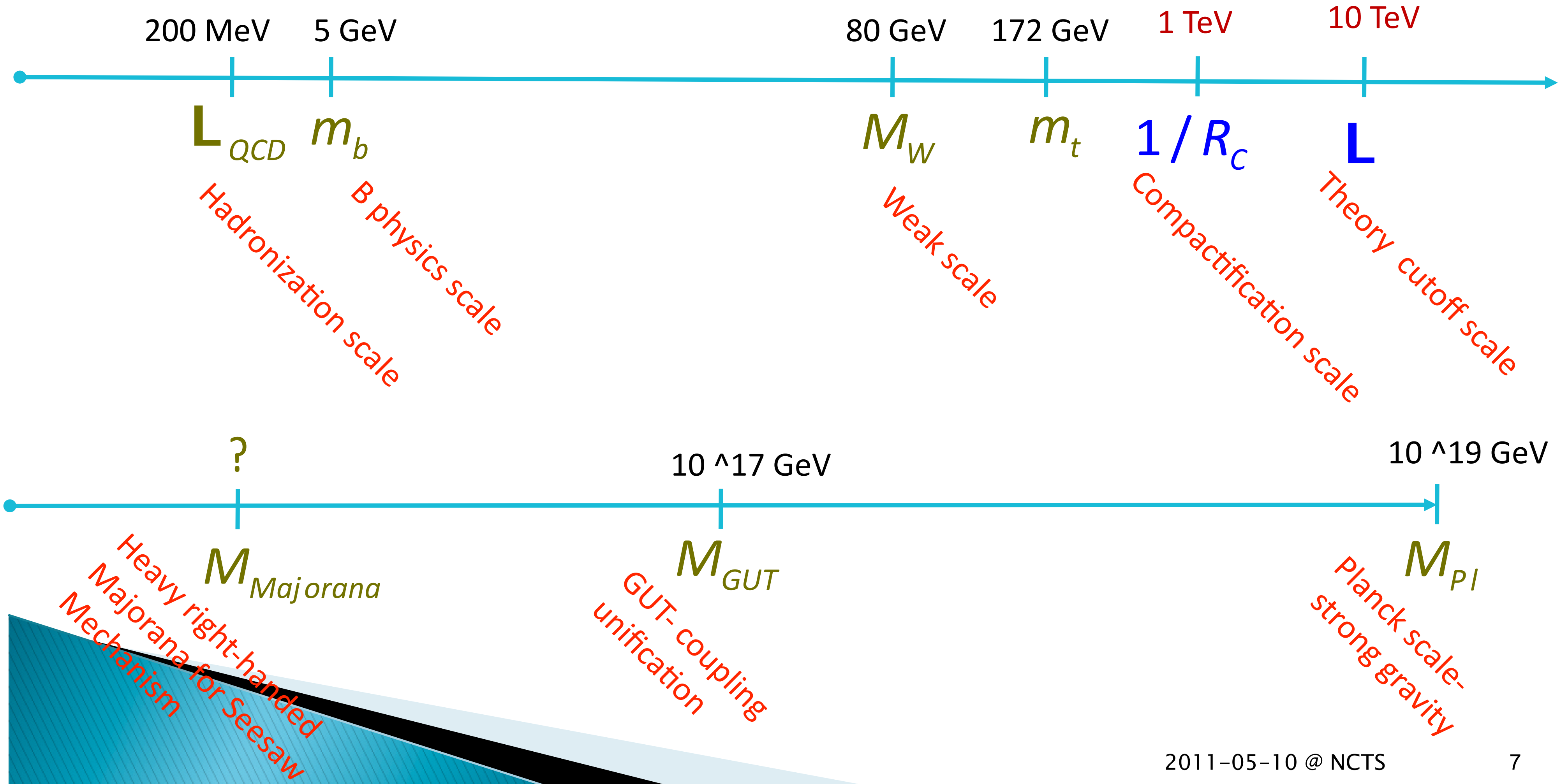
So we need **incredible fine tuning** to explain why the Higgs mass (~ Weak scale order) is so much lighter than other mass parameter sc ales (Planck, GUT or Heavy Majorana scale) when we take the Cuto ff scale Λ as P or G or H.

This is not **NATURAL**. (NATURALNESS problem)

In order to solve the hierarchy problem naturally (**without fine tuning**), we can expect that there exist at least the new p hysics beyond the Standard Model if we accept the big-desert betwe en weak energy scale and P or G or H. .

LEP and Tevatron have probed directly up to a few hundred GeV, an d indirectly between 1 and 10 TeV through the precision measure ments.

Energy scales



Goals

- **Stability of the electroweak scale (from the quadratic divergences)**
- **Higgs potential**
  - to trigger the electroweak symmetry breakin g

Other models

- Composite Higgs
  - Little Higgs (from UV completion)
  - Tecnicolor (new Strong-type interation)

- Extra dimension
  - Large extra dimension (ADD)
    - Universal extra dimension (UED)
  - Small extra dimension
    - With the warped spacetime (RS)
  - Higgsless

Toy example – 5D S_1/Z_2 SU(3)

ORBIFOLD BOUNDARY CONDITIONS

$$A_\mu(x,y) = P^{-1}A_\mu(x,-y)P, \qquad A_5(x,y) = -P^{-1}A_5(x,-y)P \qquad P = \mathrm{diag}(-1,-1,+1)$$

PURE HIGHER-DIMENSIONAL GAUGE THEORY

$$\mathcal{L}_{5D} = \int d^4x \int dy - \frac{1}{4}(F^a_{MN})^2 \qquad A_M = A^a_M \frac{\lambda^a}{2}$$

$$A_\mu = \begin{pmatrix} + & + & - \\ + & + & - \\ - & - & + \end{pmatrix} \qquad A_5 = \begin{pmatrix} - & - & + \\ - & - & + \\ + & + & - \end{pmatrix}$$

$$A^{(0)}_\mu = \frac{1}{2}\begin{pmatrix} A^3_\mu + \frac{1}{\sqrt{3}}A^8_\mu & A^1 - iA^2_\mu & 0 \\ A^1 + iA^2_\mu & -A^3_\mu + \frac{1}{\sqrt{3}}A^8_\mu & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}}A^8_\mu \end{pmatrix} \qquad A^{(0)}_5 = \frac{1}{2}\begin{pmatrix} 0 & 0 & A^4_5 + iA^5_5 \\ 0 & 0 & A^6_5 + iA^7_5 \\ A^4_5 - iA^5_5 & A^6_5 - iA^7_5 & 0 \end{pmatrix}$$

$$A^{(0)}_\mu + A^{(0)}_5 = \frac{1}{2}\begin{pmatrix} W^3_\mu + \frac{1}{\sqrt{3}}B^8_\mu & \sqrt{2}\,W^+_\mu & \sqrt{2}\,H^*_5 \\ \sqrt{2}\,W^-_\mu & -W^3_\mu + \frac{1}{\sqrt{3}}B^8_\mu & \sqrt{2}\,H^0_5 \\ \sqrt{2}\,H^-_5 & \sqrt{2}\,H^{*0}_5 & -\frac{2}{\sqrt{3}}B^8_\mu \end{pmatrix}$$

We only focus on the zero modes,

$$\mathcal{L}_{5D} = \int d^4x \int_0^{\pi R} dy - \frac{1}{4}(F_{\mu\nu}^{a(0)})^2 + \cdots$$

After we integrate out fifth dimension,

$$\int d^4x - \frac{1}{4}(F_{\mu\nu}^{a(0)})^2 \cdot Z_0^2 \,;$$

Volume factor

$$Z_0^2 \equiv \pi R,$$

And rescale the gauge field,

$$A_\mu^{(0)a} \to Z_0 A_\mu^{(0)a} \,;$$

$$F_{\mu\nu}{}^a = (\partial_\mu Z_0 A_\nu - \partial_\nu Z_0 A_\mu + \frac{g_{5D}}{Z_0} f^{abc} Z_0 A_\mu{}^b Z_0 A_\nu{}^c)$$

RELATION BETWEEN 4D AND 5D GAUGE COUPLINGS

$$g_{4D} = \frac{g_{5D}}{Z_0} = \frac{g_{5D}}{\sqrt{\pi R}}$$

Adding to brane kinetic terms

$$\mathcal{L}_{B.K} = \int d^4x \int dy \; -\frac{1}{4}\delta(y)\left[c_1(F^a_{\mu\nu})^2 + c_2(F^b_{\mu\nu})^2\right]$$

SU(2)

U(1)

We can easily understand that these terms can give a modification to the gauge couplings without any change of given models.

$$\mathcal{L}_{eff.} = (Z_0^2 + c_1)(-\frac{1}{4}(F^{a(0)}_{\mu\nu})^2)$$

From the effective Lagrangian, we can expect this relation

$$g'_{4D,\ SU(2)} = \frac{g_{5D}}{\sqrt{Z_0^2 + c_1}} = \frac{g_{5D}}{Z_0}\frac{1}{\sqrt{1 + \frac{c_1}{Z_0^2}}} = \frac{g_{4D}}{\sqrt{Z_1}},$$

$$Z_i = 1 + \frac{c_i}{Z_0^2}, \quad (i = 1 \text{ or } 2)$$

Similarly, for the U(1) coupling

$$g'_{4D,\ U(1)} = \sqrt{3}\frac{g_4}{\sqrt{Z_2}},$$

Final 4D effective Lagrangian

$$\mathcal{L}_{4D} = -\frac{1}{4}(F^{a(0)}_{\mu\nu})^2 + -\frac{1}{4}(F^{a(8)}_{\mu\nu})^2 + \left|\left(\partial_\mu - i\frac{g_4}{\sqrt{Z_1}}W^a_\mu T^a - i\frac{g_4}{\sqrt{Z_2}}\sqrt{3}B_\mu Y\right)H\right|^2$$

$\frac{g_4}{\sqrt{Z_1}}$ → g $\frac{g_4}{\sqrt{Z_2}}\sqrt{3}$ → g'

NO MASS TERM OF THE HIGGS BECAUSE OF HIGHER DIMENSIONAL GAUGE SYMMETRY

Weak mixing angle

$$\tan\theta_W = \frac{g'}{g} = \sqrt{3}\sqrt{\frac{Z_1}{Z_2}}.$$

* Note that the value of tangent angle for weak mixing angle is $\sqrt{3}$ *when* $c_1 = c_2 = 0$.

This number is completely fixed by the analysis of structure constants of given Lie group (or Lie algebra) regardless of volume factor Z if there are no brane kinetic terms in given models.

Problems in the toy model

- **Wrong weak mixing angle**
  ($\sin^2\theta_{exp} = 0.22292$ **,** $\tan\theta_{\exp} \approx \frac{1}{\sqrt{3}}$ **)**

- **No Higgs potential (to trigger the EWSB).**
  - may generate too low Higgs mass (or top quark)
  even if we use quantum corrections to make its potential.

- **Realistic construction of Yukawa couplings**

Possible answers for these problems

- **Wrong weak mixing angle**
  - Brane kinetic terms

$$\mathcal{L}_{B.K} = \int d^4x \int dy \ -\frac{1}{4}\delta(y)\left[c_1(F^a_{\mu\nu})^2 + c_2(F^b_{\mu\nu})^2\right]$$

- Violation of Lorentz symmetry (SO(1,4) -> SO(1,3))

$$L(a) = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{a}{4}F_{\mu 5}F^{\mu 5}$$

- Graded Lie algebra
(ex. $SU(3) \rightarrow SU(2|1)$ **)**

- Using a non-simple group.
an anomalous additional U(1) (or U(1)s)

→ Abandon the gauge coupling unification scheme .

- **No Higgs potential (to trigger the EWSB).**

- Using a non-simply connected extra-dimension (the fluctuation of the AB type phase – loop quantum correction)

- Using a 6D (or more) pure gauge theory.

$$L \sim tr(F_{56}{}^2)$$

- Using a background field like a monopole in extra dimensional space.

$$L \sim [A_5, A_B]^2$$

Phenomenologically viable models

Alfredo Aranda and Jose Wudka, PRD 82, 096005

- To find phenomenologically viable models the y demand following 4 constraints:

  (0) simple group ~ the gauge coupling unification.

  (1) three massive gauge bosons W+,W-,Z0 at t he electroweak scale

  (2) rho =1 at tree level

  (3) existence of representations that can cont ain all Standard Model(SM) particle, especially hyper charge 1/6.

  (4) correct weak mixing angle.

POSSIBLE ALL GROUPS THAT SATISFY (0), (1), (2), (3) CONSTRAINTS EXCEPT (4) – WEAK MIXING ANGLE

group	$s_{\rm W}^2$	α	y	$\tan\theta_W$
$SU(3l)$	$3l/(6l-2)$	α^1	$\tilde{\mu}_2/2$	$\sqrt{3l/(3l-2)}$
$SO(2n+1)$	$3/4$	α^1	$\tilde{\mu}_2/6$	$\sqrt{3}$
G_2	$3/4$	α^1	$\tilde{\mu}_2/6$	$\sqrt{3}$
F_4	$3/4$	α^1	$\tilde{\mu}_2/6$	$\sqrt{3}$
E_6	$3/8$	$\alpha^{1,5}$	$\tilde{\mu}_{2,3}/2$	$\sqrt{3/5}$
E_7	$3/4,\ 3/5$	$\alpha^{1,7}$	$\tilde{\mu}_{2,3}/6$	$\sqrt{3}, \sqrt{3/2}$
E_8	$9/16,\ 3/8$	$\alpha^{1,8}$	$\tilde{\mu}_{2,3}/6$	$\sqrt{9/7}, \sqrt{3/5}$

Any GHU model can not expl
ain correct weak mixing ang
le.

Simple roots cor. to SU(2)

One cartan generator c
or. to U(1)

Higgs potential in 6D

NOTATIONS OF LIE ALGEBRA SU(2) AND U(1)

$$J_0 = \frac{1}{|\alpha|^2}\alpha\cdot\mathbf{C}, \quad J_+ = \frac{\sqrt{2}}{|\alpha|}E_\alpha, \quad J_- = (J_+)^\dagger$$ ➡ SU(2) generators

$$Y = y\cdot\mathbf{C}\,,$$ ➡ U(1) generator

COMMUTATION RELATIONS

$$[\mathbf{C},\beta] = \beta E_\beta, \quad [E_\beta, E_{-\beta}] = \beta\cdot\mathbf{C}$$

$$[E_\beta, E_\gamma] = N_{\beta,\gamma}E_{\beta+\gamma} \;\; \text{if } \beta+\gamma\neq 0$$

ORTHONORMAL BASIS

$$\mathrm{tr}C_iC_j = \delta_{i,j}, \quad \mathrm{tr}E_\alpha E_\beta = \delta_{\alpha+\beta,0}, \quad \mathrm{tr}E_\alpha C_i = 0\,.$$

A GENERAL FORM OF ZERO MODES IN TERMS OF GENERATORS OF LIE ALGEBRA

$$A_\mu = W_\mu^+ E_\alpha + W_\mu^- E_{-\alpha} + W_\mu^0 \hat{\alpha} \cdot \mathbf{C} + B_\mu \hat{y} \cdot \mathbf{C} + \cdots$$

$$A_n = \sum_{\beta > 0} (\phi_{n,\beta} E_\beta + \phi_{n,\beta}^* E_\beta) + \cdots$$

We focus on the mass term,

$$-\mathrm{tr}\,[A_\mu, A_n]^2 \supset \sum_{\beta>0\ ;\ isodoublets} |\phi_{n,\beta}|^2 \left\{ \frac{1}{2}\alpha^2 W_\mu^+ W^{-\mu} + \frac{1}{4}\alpha^2 \left(W_\mu^0 - \frac{1}{|\alpha||y|} B_\mu \right)^2 \right\}$$

and the mixing angle,

$$\tan\theta_W = \frac{1}{|\alpha||y|}$$

From previous toy example, we can easily expect that our brane kinetic ter ms can modify the coupling constants, that is, the mixing angle,

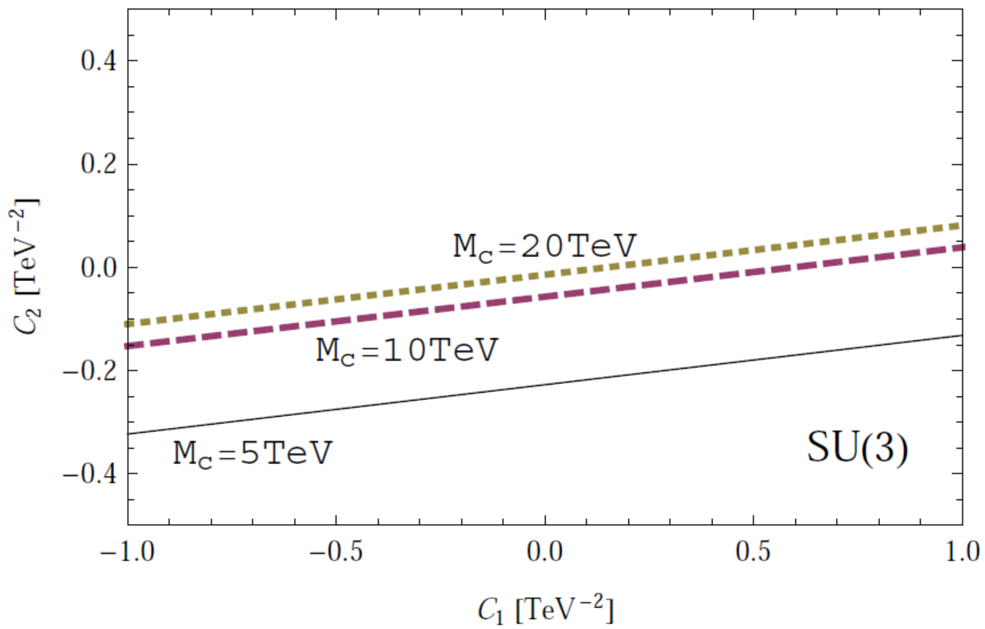
$$\tan\theta_W' = \frac{1}{|\alpha||y|}\sqrt{\frac{Z_1}{Z_2}}$$

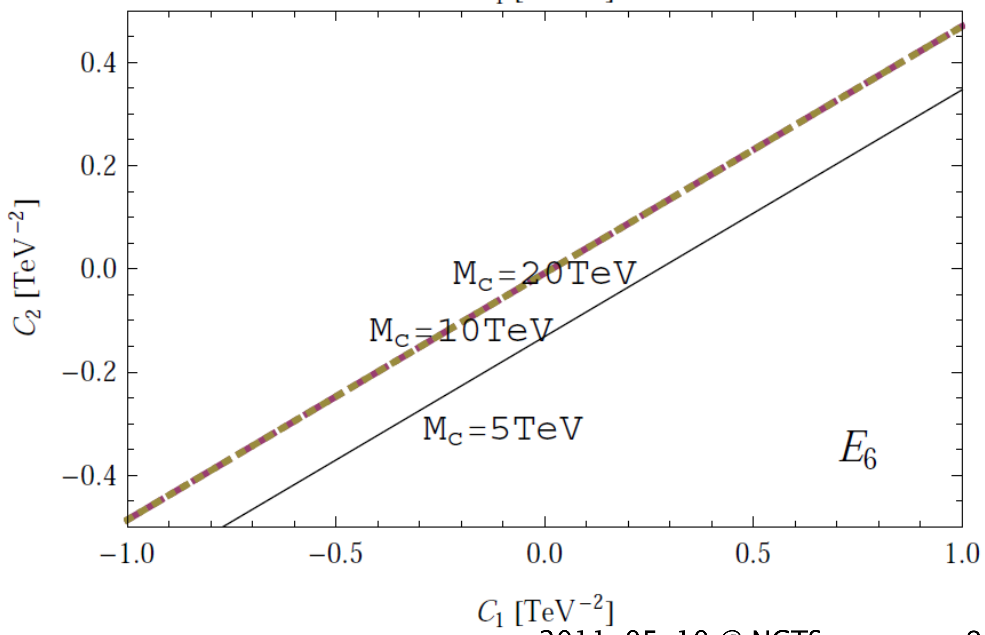
FROM EXPERIMENTAL VALUE OF WEAK MIXING ANGLE,

$$Z_1 = \frac{\tan^2\theta_{exp}}{\tan^2\theta_W} Z_2$$

⇕

$$c_1 = Z_0^2\left(\frac{\tan^2\theta'_W}{\tan^2\theta_W} - 1\right) + \frac{\tan^2\theta'_W}{\tan^2\theta_W} c_2$$



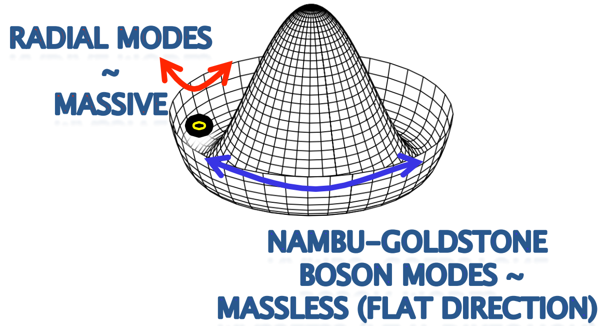


VALUES OF C1 AND C2
WHICH ARE CONSISTENT WITH THE PRESENT EXPERIMENT VALUE OF THE WEAK MIXING A NGLE
AND
EACH GROUP THEORETIC NUMERICAL FACTO R
IN 6 DIMENSIONAL SU(3) AND E6 GAUGE H IGGS UNIFICATION MODELS ON S^2/Z_2.

STRAIGHT, DASHED AND DOTTED LINES COR RESPOND TO THE COMPACTIFICATION SCAL ES 5, 10 AND 20 TEV, RESPECTIVELY.

HIGGS POTENTIAL,

$$V(H) = -\mu^2|H|^2 + \lambda|H|^4, \quad \boxed{\lambda = \frac{g^2}{2}}$$



After the Higgs obtains $\langle |H| \rangle = v$, $\quad M_H = \sqrt{2}\mu = \sqrt{2\lambda}\, v, \quad M_W = \frac{gv}{2}$

Finally, we can get this relation,

$$\frac{M_H}{M_W} = 2\sqrt{Z_1}$$

We can rewrite the equation with previous relation,

$$\frac{M_H}{M_W} = 2\frac{\tan\theta_{exp}}{\tan\theta_W}\sqrt{\left(1 + \frac{c_2}{Z_0^2}\right)}$$

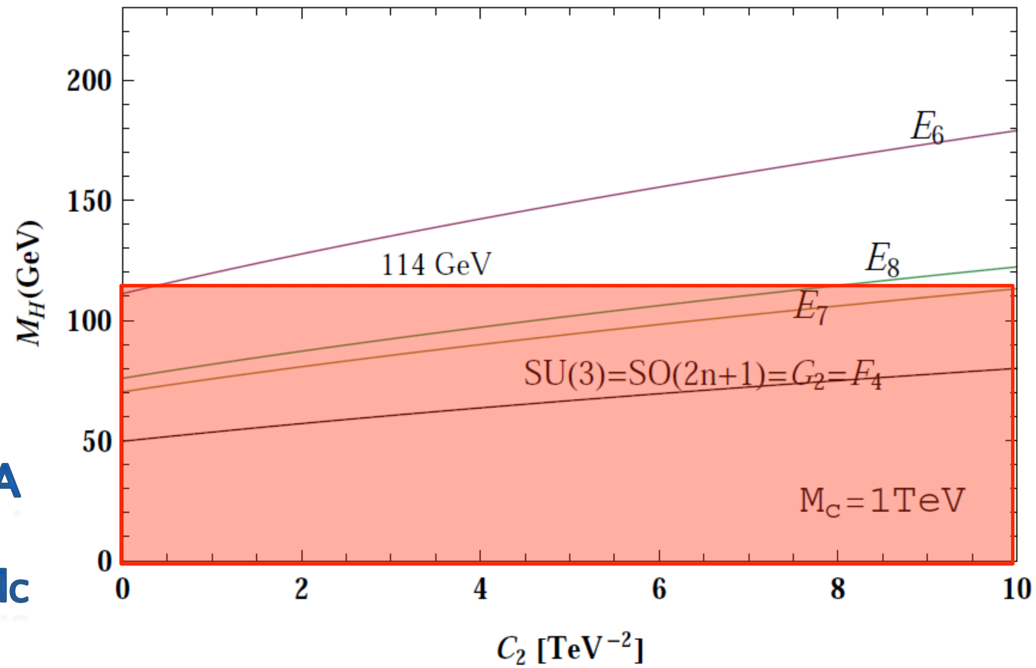
Numerical results.

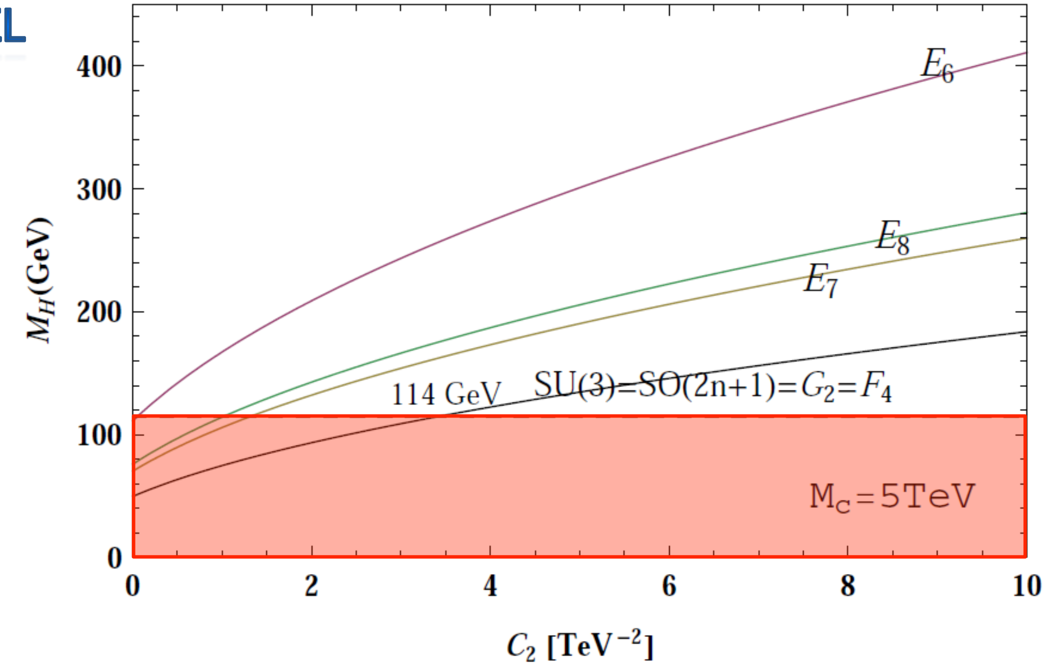
1. POSSIBLE GAUGE GROUPS AND HIGGS MASS UNDER PRESENCE OF GAUGE KINETIC TERMS

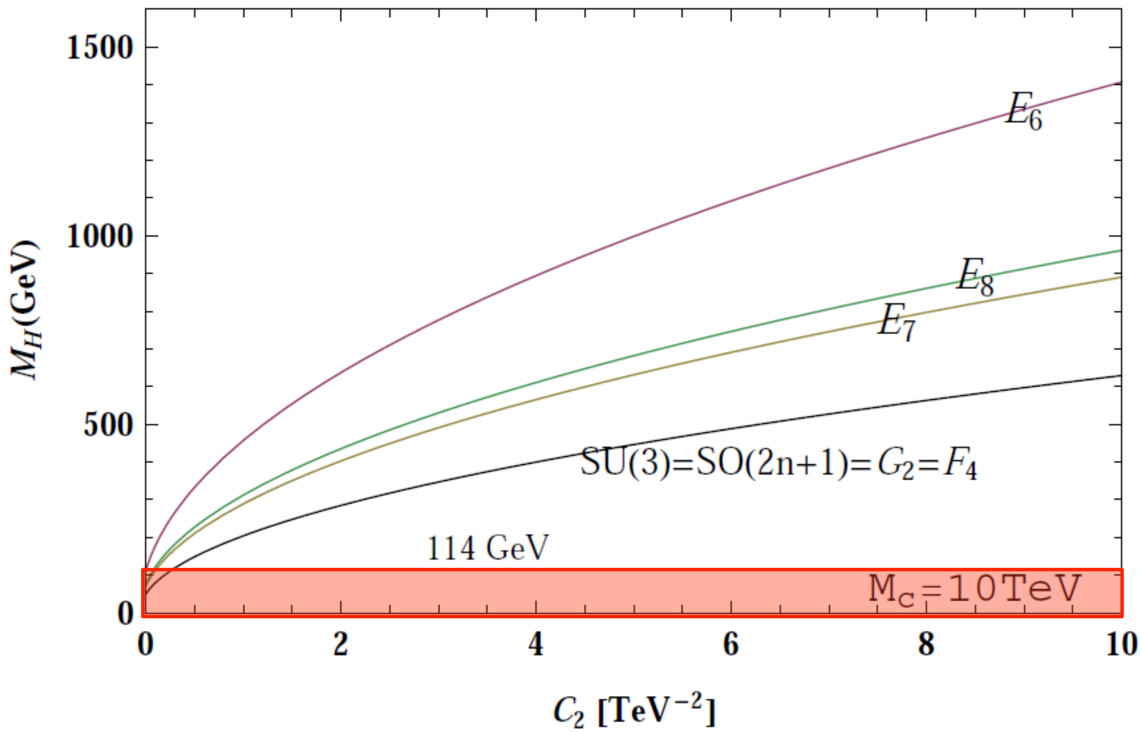
Group	α	y	$\tan\theta_W / \sqrt{\frac{Z_1}{Z_2}}$	Higgs mass [GeV], $c_2 = 0$
$SU(3l)$	α^1	$\tilde{\mu}_2/2$	$\sqrt{3l/(3l-2)}$	$49.7235 \times \sqrt{(3l-2)/l}$
$SO(2n+1)$	α^1	$\tilde{\mu}_2/6$	$\sqrt{3}$	49.7235
G_2	α^1	$\tilde{\mu}_2/6$	$\sqrt{3}$	49.7235
F_4	α^1	$\tilde{\mu}_2/6$	$\sqrt{3}$	49.7235
E_6	$\alpha^{1,5}$	$\tilde{\mu}_{2,3}/2$	$\sqrt{3/5}$	111.185
E_7	$\alpha^{1,7}$	$\tilde{\mu}_{2,3}/6$	$\sqrt{3}, \sqrt{3/2}$	49.7235, 70.3196
E_8	$\alpha^{1,8}$	$\tilde{\mu}_{2,3}/6$	$\sqrt{9/7}, \sqrt{3/5}$	75.9539, 111.185

✡ All masses are smaller than 114.4 GeV.

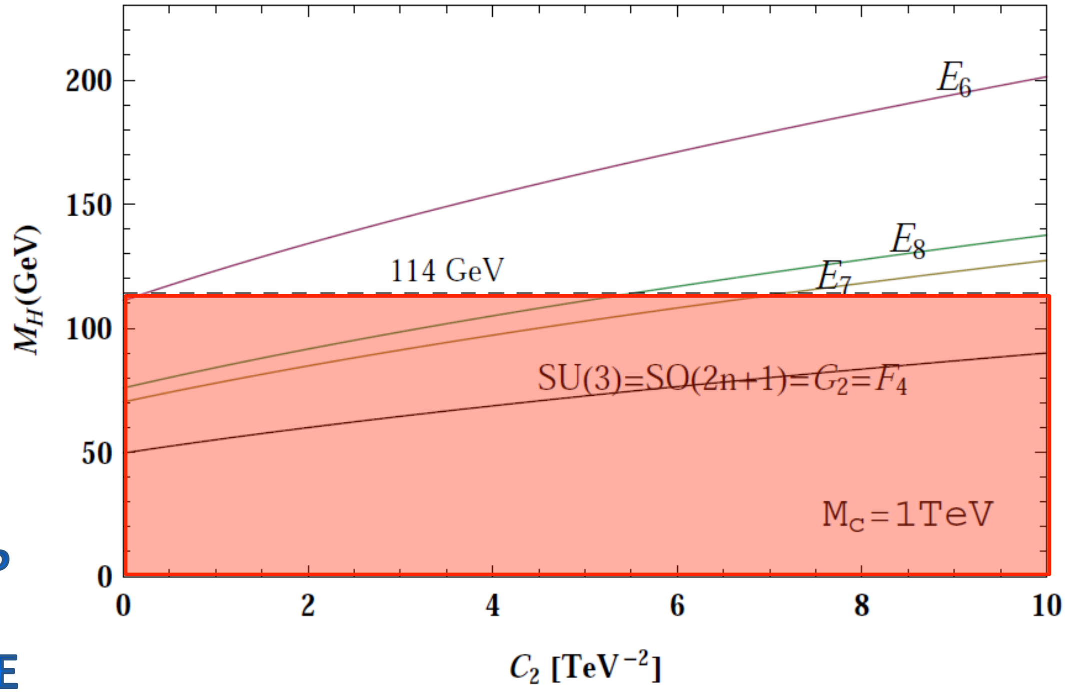
2. THE HIGGS MASS OF EACH GROUP AS A FUNCTION OF C_2 ON THE COMPACTIFICATION SCALE M_C =1, 5 AND 10 TEV IN THE 6 DIMENSIONAL GHU MODEL ON S^2/Z_2.

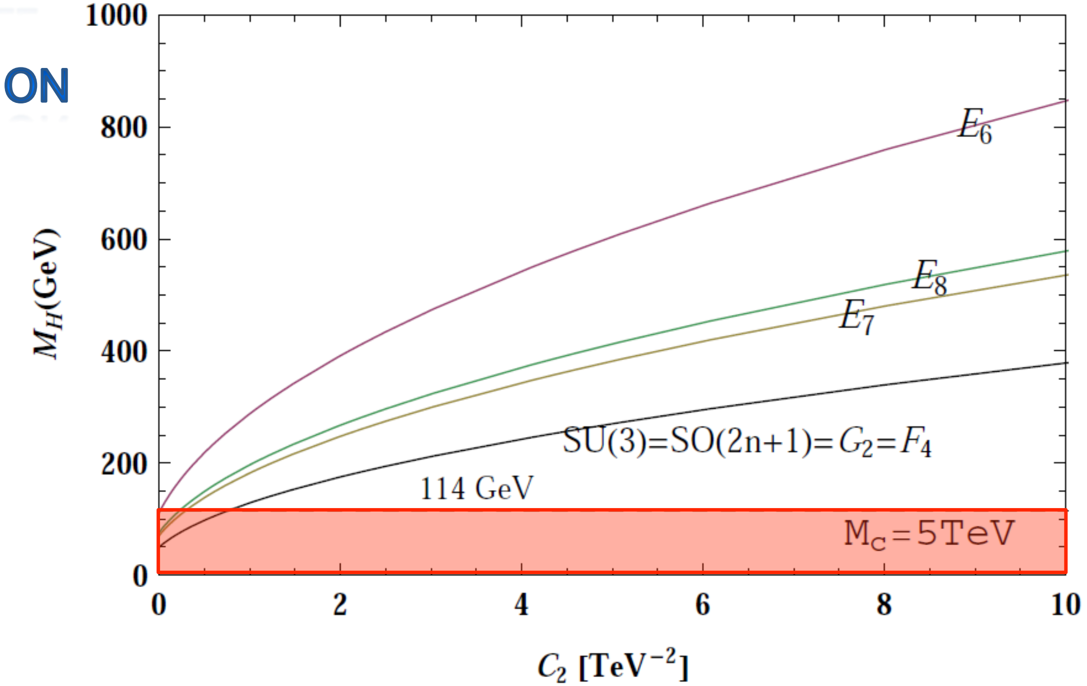


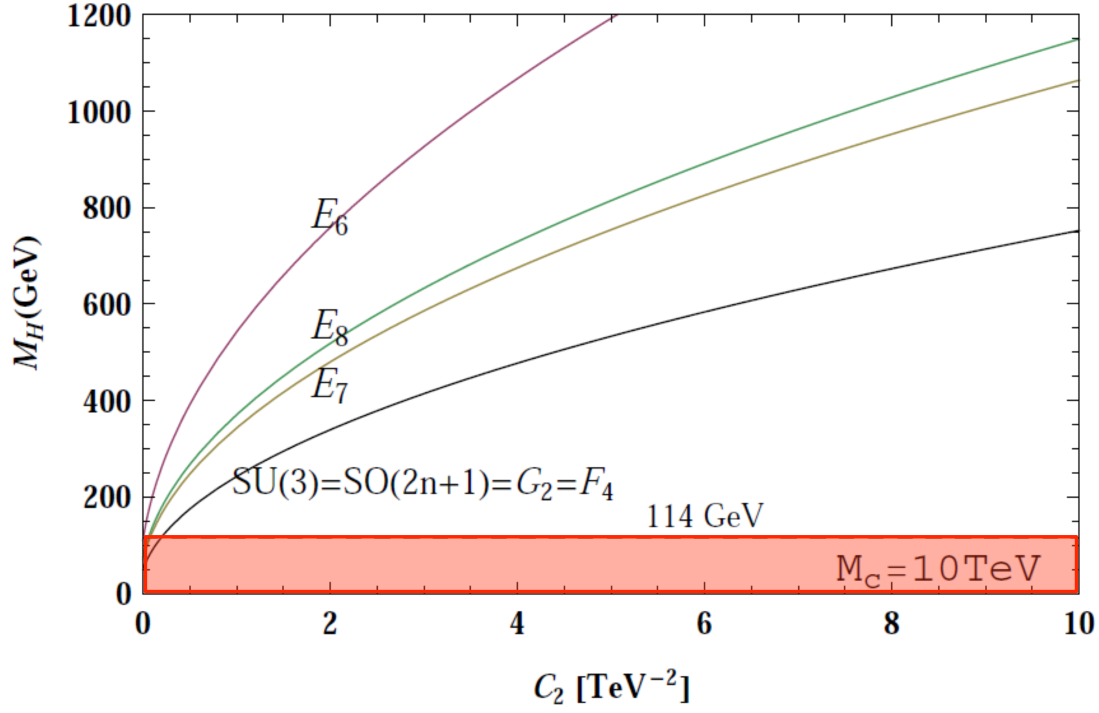



1500
1000
500
0
M_H(GeV)
E_6
E_8
E_7
SU(3)=SO(2n+1)=G_2=F_4
114 GeV
M_c=10TeV
0
2
4
6
8
10
C_2 [TeV^{-2}]

3. THE HIGGS MASS OF EACH GROUP AS A FUNCTION OF C_2 ON THE COMPACTIFICATION SCALE M_C=1, 5 AND 10 TEV IN THE 6 DIMENSIONAL GHU MODEL ON T_2/Z_3.



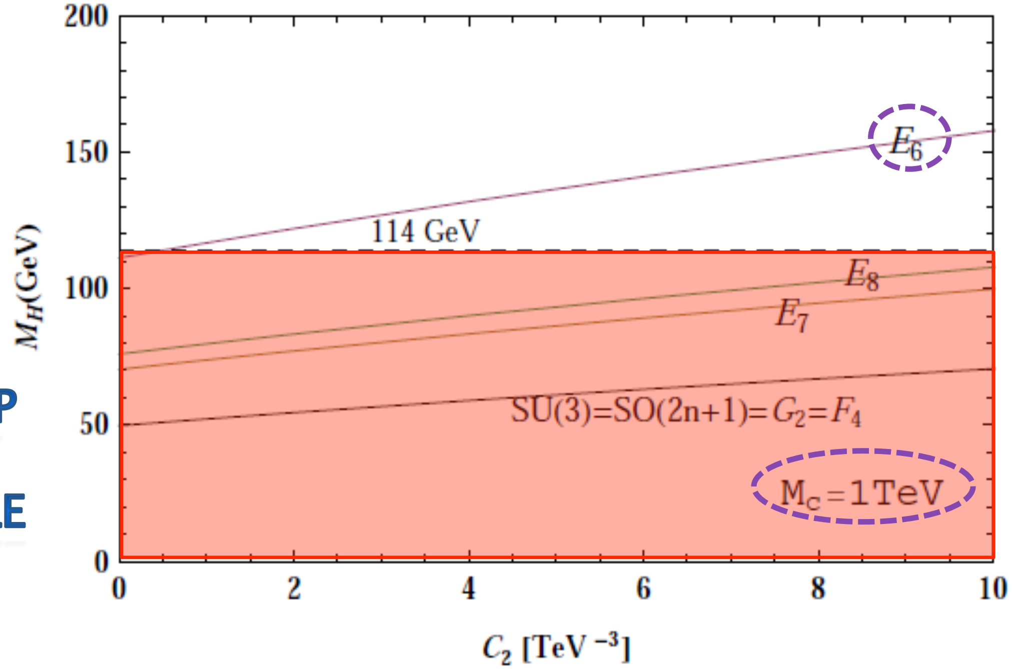


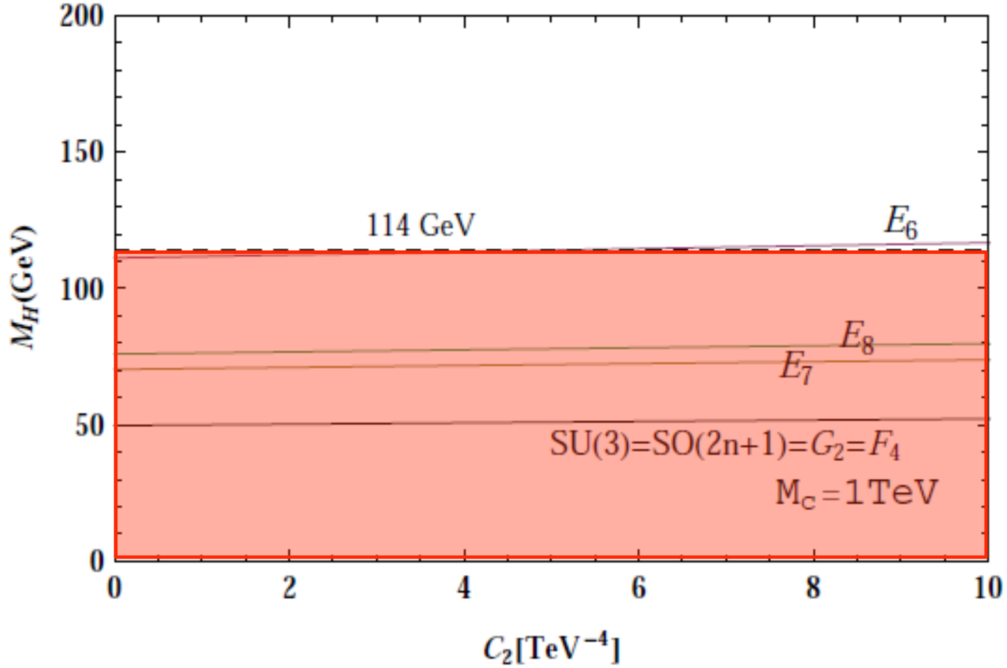

1200
1000
800
600
400
200
0
M_H(GeV)
E_6
E_8
E_7
SU(3)=SO(2n+1)=G_2=F_4
114 GeV
M_c=10TeV
0
2
4
6
8
10
C_2 [TeV^{-2}]

4. VOLUME FACTORS AND SLOPES OF SEVERAL EXAMPLES IN 6D, 7D AND 8D

Dimension	Space	Volume	M_C	slope at $c_2 = 0$	slope at $c_2 = 10$	Remark
6D	$S^2/\mathbb{Z}_N$	$4\pi R^2/N$	0.5 TeV	2.212	2.195	$N = 2,\ E_6$
			1 TeV	3.957	3.534	$N = 2,\ SU(3)$
			1 TeV	8.848	7.903	$N = 2,\ E_6$
			1 TeV	17.70	12.47	$N = 4,\ E_6$
			5 TeV	221.2	17.52	$N = 2,\ E_6$
	$T^2/\mathbb{Z}_N$	$(2\pi R/N)^2$	0.5 TeV	3.168	3.118	$N = 3,\ E_6$
			1 TeV	5.668	4.598	$N = 3,\ SU(3)$
			1 TeV	12.67	10.28	$N = 3,\ E_6$
			1 TeV	50.69	16.61	$N = 6,\ E_6$
			5 TeV	316.8	17.55	$N = 3,\ E_6$
7D	$S^3/\mathbb{Z}_N$	$2\pi^2 R^3/N$	0.5 TeV	0.7041	0.7035	$N = 2,\ E_6$
			1 TeV	5.633	5.364	$N = 2,\ E_6$
			5 TeV	704.1	17.57	$N = 2,\ E_6$
8D	$T^4/\mathbb{Z}_N$	$(2\pi R/N)^4$	0.5 TeV	0.0357	0.0357	$N = 2,\ E_6$
			1 TeV	0.5707	0.5704	$N = 2,\ E_6$
			5 TeV	356.7	17.56	$N = 2,\ E_6$

3. THE HIGGS MASS OF EACH GROUP AS A FUNCTION OF C2 ON THE COMPACTIFICATION SCALE Mc=1, 5 AND 10 TEV IN THE 7 DIMENSIONAL MODEL ON S_3/Z_2 (TOP) AND 8 DIMENSIONAL T_4/Z_2 (BOTTOM) .





Summary

- 1. A present exclusion bound of the Higgs mass always tends to favor exceptional group E6, 7, 8 than other SU(3l), SO(2n+1), G2, and F4 groups independently of the compactification scales. Particularly the E6 can always have the largest Higgs mass above the bound except the very tiny range at the beginning of c2 value compared to all other groups.

- 2. As the compactification scale lowers below 1 TeV, SU(3l), SO(2n+1), G2, and F4 group mo dels can not easily escape from present boun d without big hierarchical c2 number, this me ans that the introduction of brane kinetic term s just replace the original hierarchy problem b y the new c2 hierarchy problem, and so it does not work correctly in these models.

- 3. As we go to more higher dimensional case, both two example cases, 7-dimensional(7D) S 3/Z2 and 8-dimensional(8D) T4/Z2, show that e xcept E6 they can not absolutely escape from present bound at lower compactification scale below 1 TeV due to their volume factor in the slope without huge c2 number. However beca use as the compactification scale get larger th an 1 TeV, the slope can get larger dramatically , they can avoid the constraint more easier. Th erefore we can expect that these higher dime nsional GHU models need more larger compa ctification scales above 1 TeV to survive from t he low energy constraints.