Phenomenology in HTM with A4 Symmetry

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NCTS seminar

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Phenomenology in the Higgs Triplet Model (HTM) with the A_4 Symmetry

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Outline

- Introduction
- Higgs Triplet Model (HTM)
- A4 symmetry
- HTM with A4
- Phenomenology
- Summary

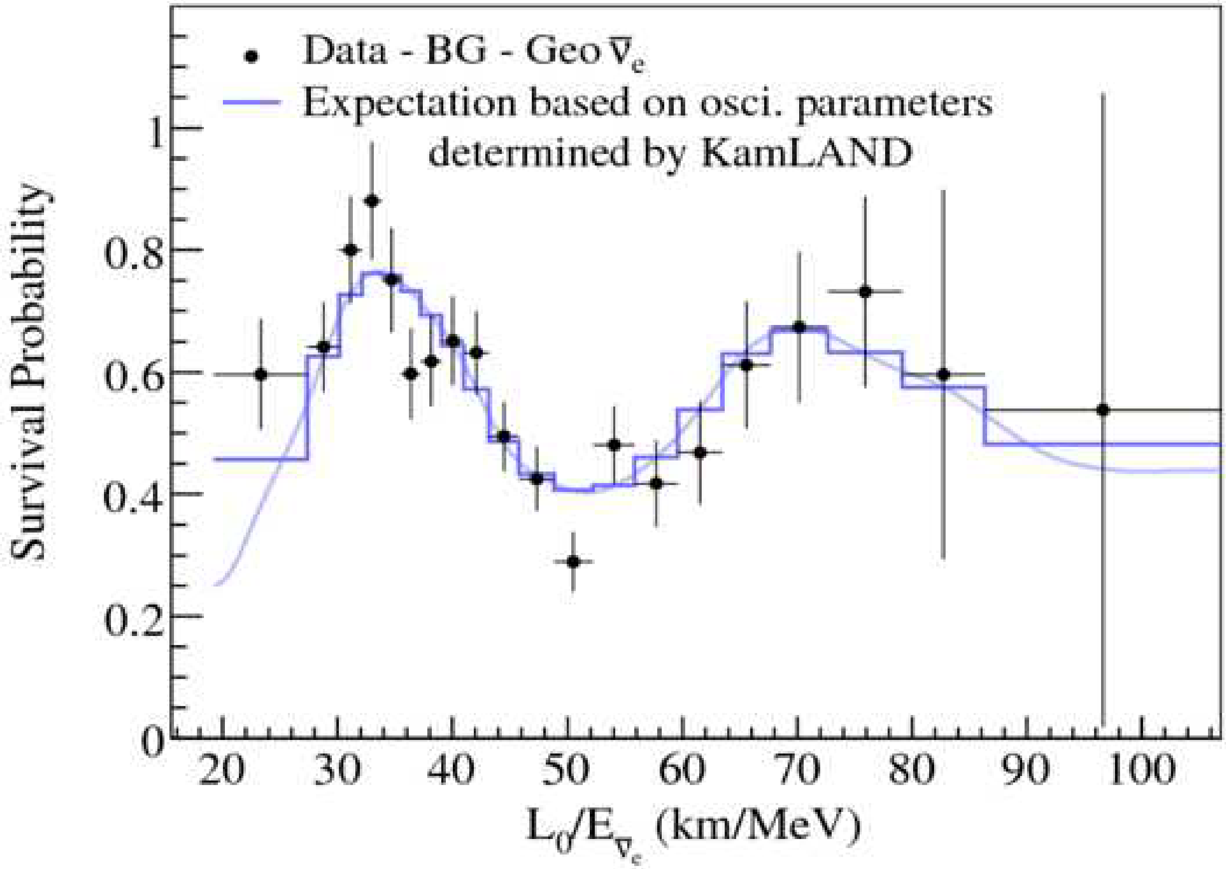
Neutrino

Neutrinos are massless in the SM

But, solar/atmospheric neutrino deficits are observed.
→ … Massive neutrino?

Neutrino oscillation

- **Manifestly oscillating**



→ **Massive Neutrino (a clear evidence of BSM)**

Massive neutrino can be mixed

Flavor eigenstates

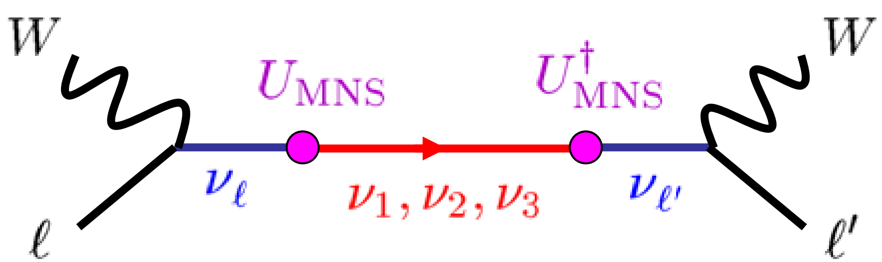
Mass eigenstates

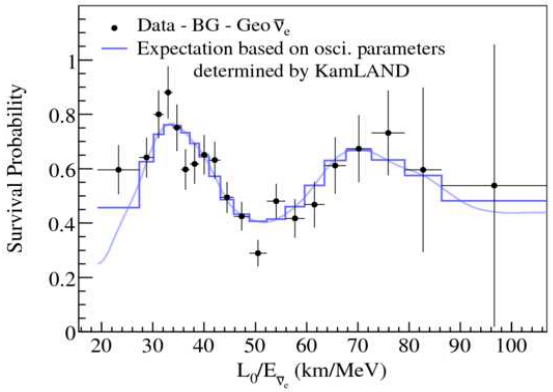
$$\nu_\ell = \sum_i (U_{\mathrm{MNS}})_{\ell i}\, \nu_i$$

Neutrino (MNS) mixing matrix

Neutrino flavor oscillation

$$P(\nu_\ell \to \nu_{\ell'}) = \left| \sum_i (U_{\mathrm{MNS}})_{\ell i}\ \exp\left(i \frac{m_i^2 L}{2E} \right) (U^\dagger_{\mathrm{MNS}})_{i\ell'} \right|^2$$





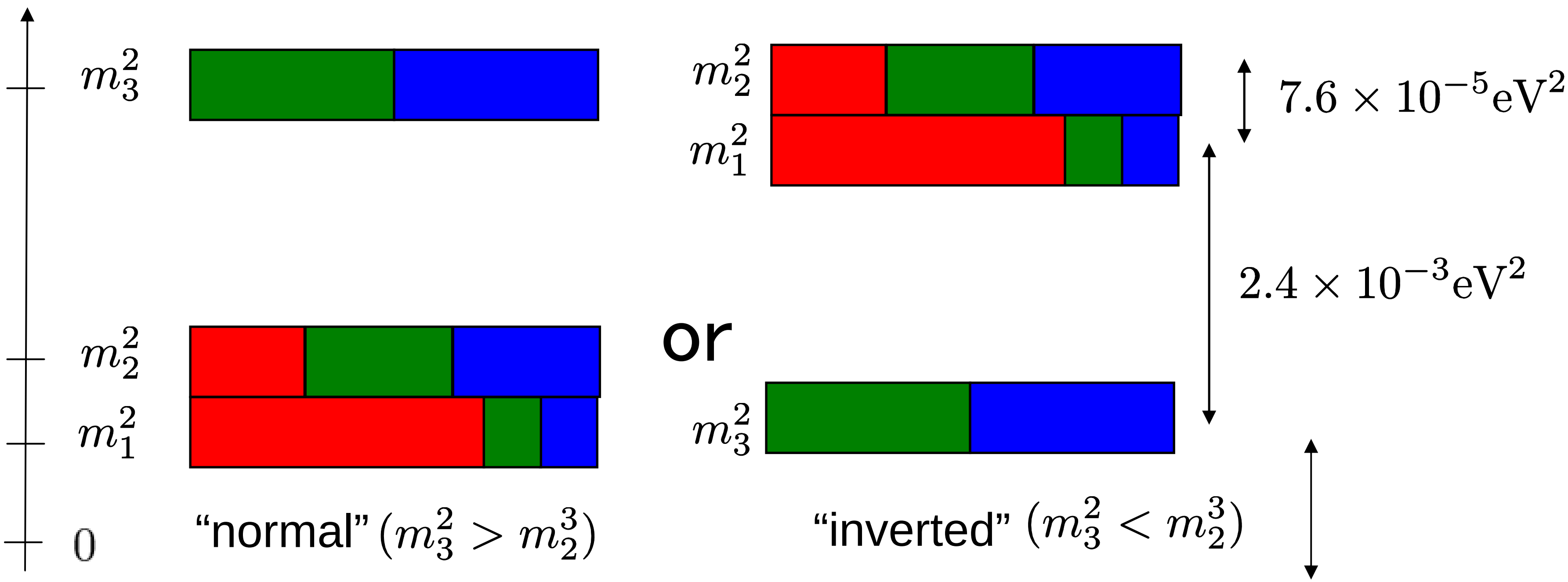
Tiny neutrino masses

- **Oscillation data:**

$$\Delta m^2_{21} = 7.6 \times 10^{-5}\ \mathrm{eV}^2$$ The sign can be determined by matter effects.

$$\left|\Delta m^2_{31}\right| = 2.4 \times 10^{-3}\ \mathrm{eV}^2 \longrightarrow m_\nu \geq \sqrt{|\Delta m^2_{31}|} \approx 0.05\mathrm{eV}$$

Absolute mass scale



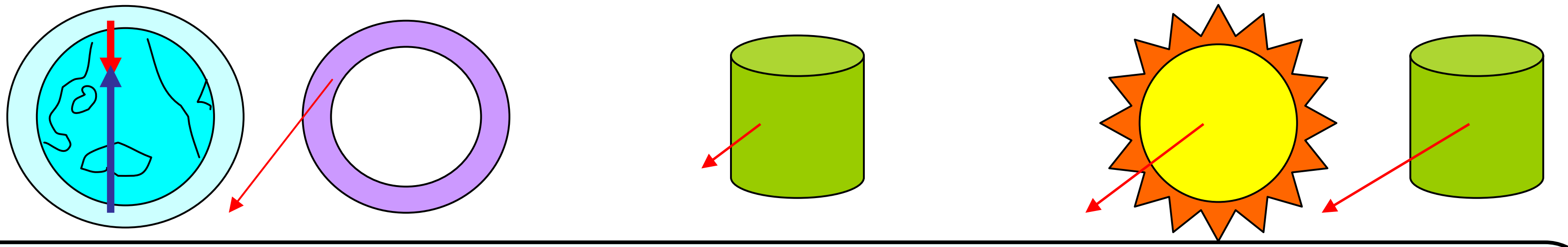
Large neutrino mixing

- **Oscillation data:**

Atmospheric (SK)
Accelerator (K2K,T2K,MINOS)

Reactor (CHOOZ)

Solar (SK,SNO)
Reactor (KamLAND)



$$U_{\mathrm{MNS}} \simeq \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{0.5} & \sqrt{0.5} \\ 0 & -\sqrt{0.5} & \sqrt{0.5} \end{pmatrix}}_{45^\circ} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{< 11^\circ \,\&\, \delta?} \underbrace{\begin{pmatrix} \sqrt{0.68} & \sqrt{0.32} & 0 \\ -\sqrt{0.32} & \sqrt{0.68} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{34^\circ}$$

Maximal mixing

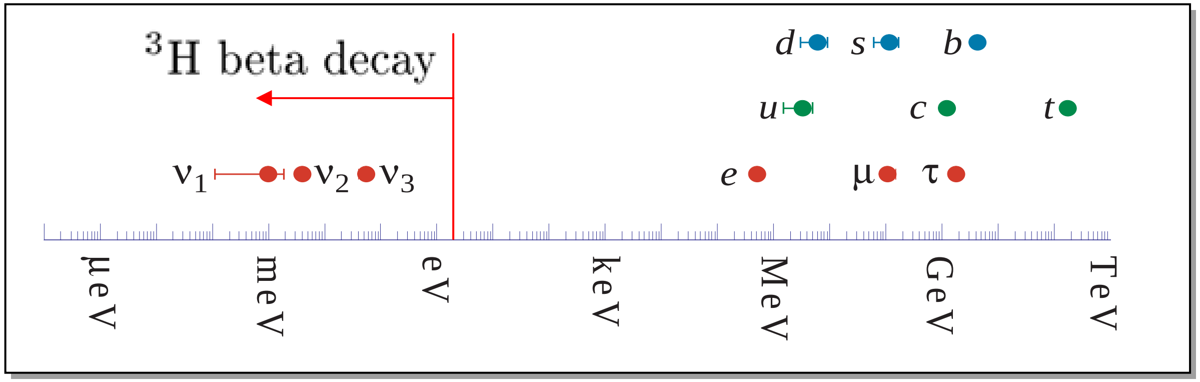
Small mixing
(not yet measured)

Large mixing

Why are neutrino masses so important?

Comparison with other fermions

- **Extremely small mass** → **suggest new phys. Scale?**



- **Large mixing** → **new phys. in lepton sector?**

$$U_{\rm MNS} \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{0.5} & \sqrt{0.5} \\ 0 & -\sqrt{0.5} & \sqrt{0.5} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{0.68} & \sqrt{0.32} & 0 \\ -\sqrt{0.32} & \sqrt{0.68} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$V_{\rm CKM} \simeq \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\,\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\,\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Fundamental fermion?

- **Majorana nature for neutral fermions**
  - Mass term can be written by left-handed field.

$$\boxed{\frac{1}{2} m \overline{(\nu_L)^c} \nu_L + \mathrm{H.c.}}$$

cf. Charged fermion mass term: $m \overline{f_L} f_R + \mathrm{H.c.}$

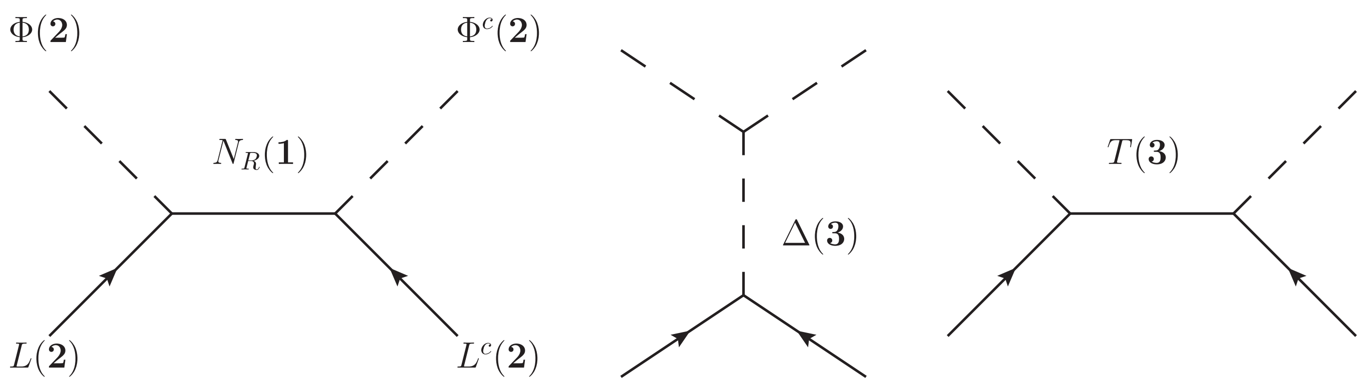
Connections to new physics

- **Dim.5 Weinberg op.**

$$\boxed{\mathcal{O}_W = (L\Phi)^\dagger (L\Phi)} \quad \rightarrow \frac{1}{2} m \overline{(\nu_L)^c} \nu_L + \mathrm{H.c.}$$

  - Possible origin of neutrino Majorana mass in the eff. SM

→ **Seesaw I, II and III** (tree-level decomposition)

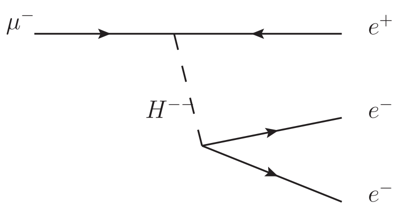


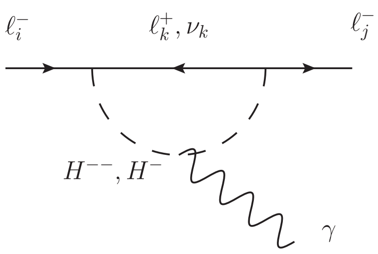
→ New source of mass scale other than EW vev

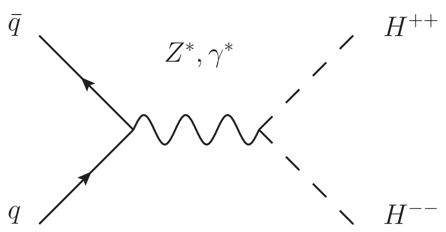
Higgs Triplet Model:

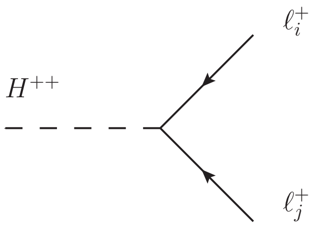
A model for Majorana neutrino mass

- **Motivations:**
  - Rich LFV phenomenology
  - Interesting collider phenomenology









Higgs Triplet Model (HTM)

- **Adding a complex SU(2) triplet scalar with Y=2**

$$\Delta \equiv \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$$

Doubly charged Higgs boson

- **Neutrino mass generation in HTM**

L# = -2

$$h_{\ell\ell'}\left(-\overline{(\ell_L)^c}, \overline{(\nu_{\ell L})^c}\right)\begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}\begin{pmatrix} \nu_{\ell' L} \\ \ell'_L \end{pmatrix} + \text{h.c.}$$

Triplet scalar develops vev: $v_\Delta \equiv \sqrt{2}\langle \Delta^0 \rangle \simeq \frac{\mu\, v^2}{2M^2}$

$$\frac{1}{2}\sqrt{2}\, v_\Delta h_{\ell\ell'}\,\overline{(\nu_{\ell L})^c}\nu_{\ell' L} + \text{h.c.} + \cdots$$

L# violation generates NGB?

HTM potential

- **Explicit L# breaking to avoid NGB (Majoron)**

$$
\begin{aligned}
V_{\mathrm{HTM}} =& -m^2(\Phi^\dagger\Phi) + \lambda_1(\Phi^\dagger\Phi)^2 \\
& + M^2 \mathrm{Tr}(\Delta^\dagger\Delta) + \lambda_2[\mathrm{Tr}(\Delta^\dagger\Delta)]^2 + \lambda_3 \mathrm{Det}(\Delta^\dagger\Delta) \\
& + \lambda_4(\Phi^\dagger\Phi)\mathrm{Tr}(\Delta^\dagger\Delta) + \lambda_5(\Phi^\dagger\sigma^i\Phi)\mathrm{Tr}(\Delta^\dagger\sigma^i\Delta) \\
& + \left(\frac{1}{\sqrt{2}}\mu(\Phi^T i\sigma^2\Delta^\dagger\Phi) + \mathrm{h.c.}\right)
\end{aligned}
$$

Triplet scalar develops vev: $v_\Delta \equiv \sqrt{2}\langle\Delta^0\rangle \simeq \dfrac{\mu\, v^2}{2M^2}$

Soft L# breaking parameter

Possible realizations for tiny neutrino mass

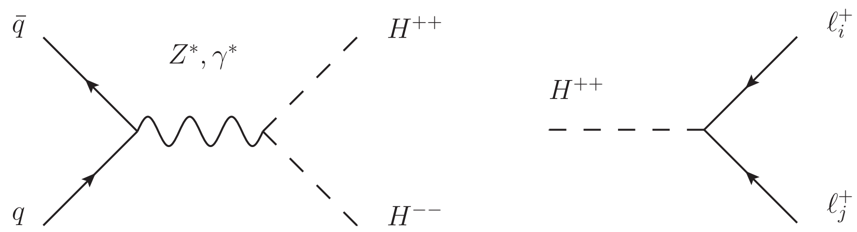
Possible realizations

$$(M_\nu)_{\ell\ell'} = \sqrt{2}\, v_\Delta\, h_{\ell\ell'} \simeq \frac{\mu v^2}{\sqrt{2} M^2} h_{\ell\ell'}$$

- Heavy triplet scalar (M) : often called type2-seesaw
- Small Yukawa (h_ll)
- Small L# breaking (μ): moderate (M & h_ll)

→ h_ll can affect low energy LFV,
and triplet scalar can be discovered at the LHC

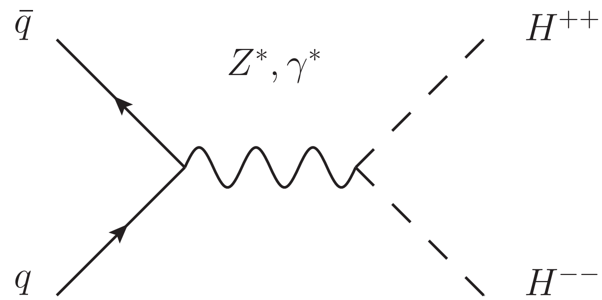
HTM at the LHC

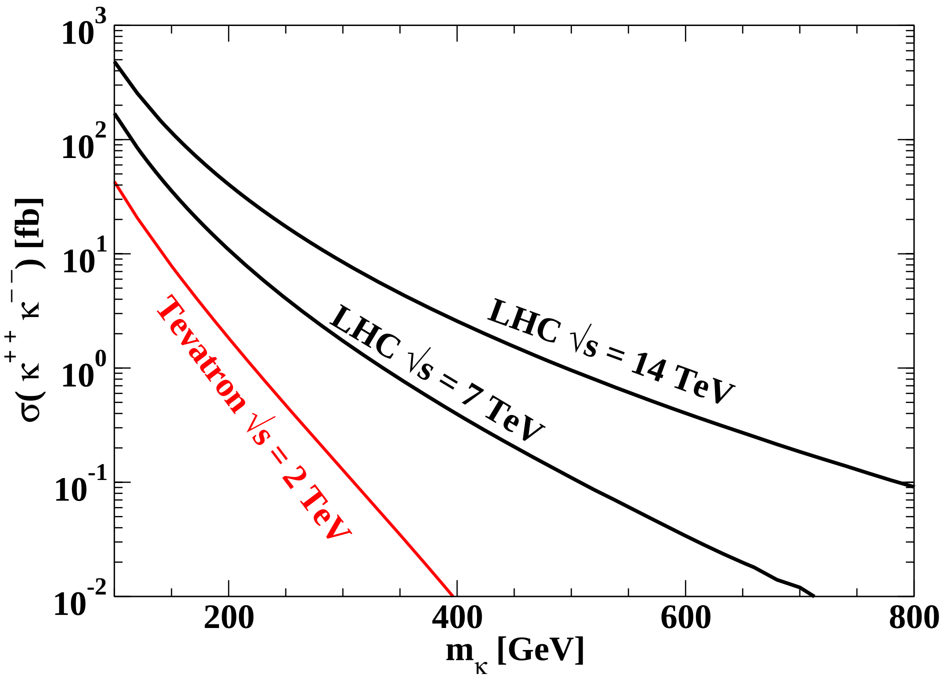


Phenomenology of double charged Higgs bosons

- Pair produced by gauç

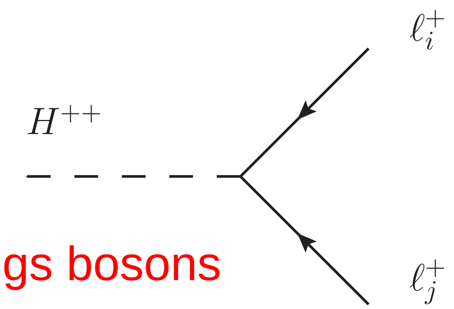
$$q\bar{q} \to \gamma^*, Z^* \to H_i^{++} H_i^{--}$$





- Clear leptonic decay signal

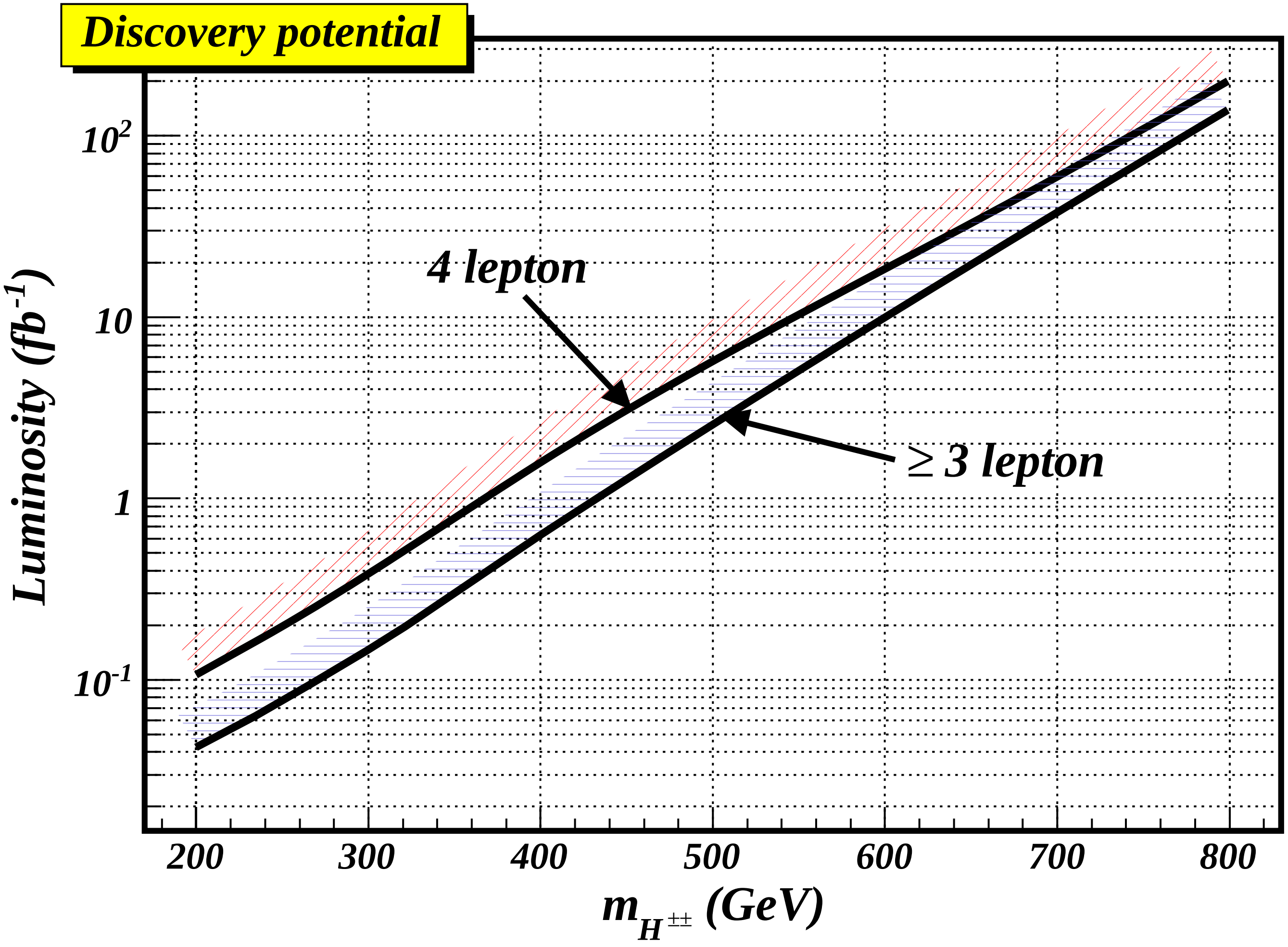
$$H_i^{++} \to \ell_j^+ \ell_k^+$$



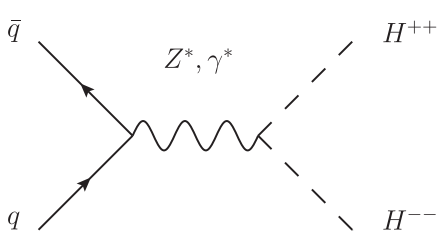
Decays of doubly charged Higgs bosons

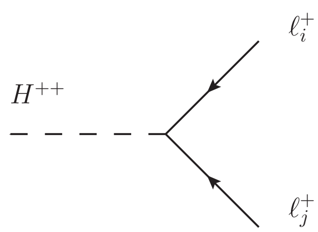
Discovery pot. of triplet Higgs boson @ LHC

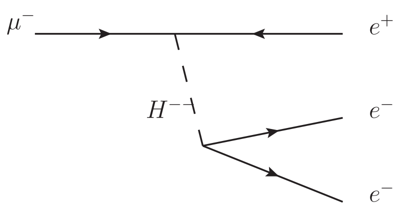
- **Akeroyd, Chiang, JHEP11(2010)005**

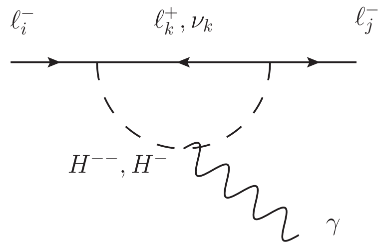


LHC vs Low energy data









Rich Higgs phenomenology in HTM

- **Yukawa (h_ll) prop. to Neutrino mass**

$$(M_\nu)_{\ell\ell'} = \sqrt{2}\, v_\Delta\, h_{\ell\ell'} \simeq \frac{\mu v^2}{\sqrt{2} M^2} h_{\ell\ell'}$$

  - **Rich Higgs phenomenology**
    - H++ can be produced at LHC; M < 1 TeV
    - H++ decays (Testable!!)

      vs neutrino oscillation data

      vs low energy LFV (Lepton flavor violation)

      vs low energy L#V (Lepton number violation)

A4 flavor symmetry

A4 group: alternating group for 4 letters

flavor sym.: Origin of fermion masses and mixings

Why are we focusing on A4 symmetry?

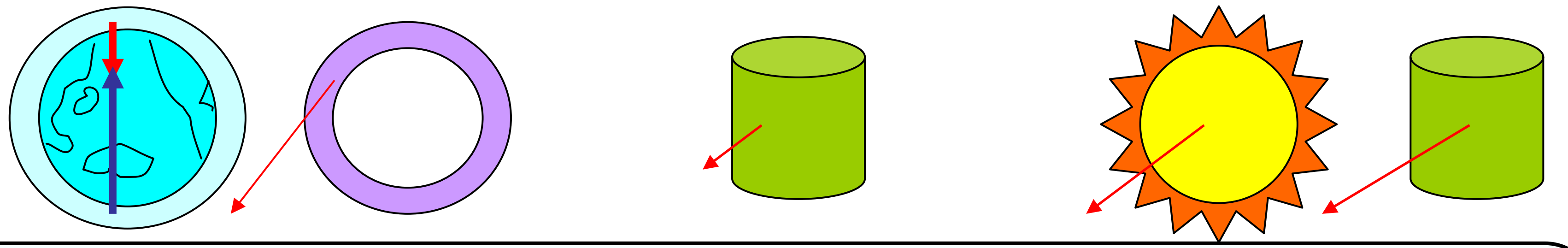
Large neutrino mixing

Oscillation data:

Atmospheric (SK)
Accelerator (K2K,T2K,MINOS)

Reactor (CHOOZ)

Solar (SK,SNO)
Reactor (KamLAND)



$$U_{\mathrm{MNS}} \simeq \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{0.5} & \sqrt{0.5} \\ 0 & -\sqrt{0.5} & \sqrt{0.5} \end{pmatrix}}_{45^\circ} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{< 11^\circ \;\&\; \delta ?} \underbrace{\begin{pmatrix} \sqrt{0.68} & \sqrt{0.32} & 0 \\ -\sqrt{0.32} & \sqrt{0.68} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{34^\circ}$$

Maximal mixing

Small mixing
(not yet measured)

Large mixing

Neutrino mixing

- Tri-Bi-Maximal mixing: good agreement with experiments.

$$U_{\mathrm{MNS}} = U_{\mathrm{TB}} \equiv \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sim \underbrace{\frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}} \times \underbrace{\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}}$$

$$\omega = e^{2\, i\, \pi/3}$$

Z3 symmetry
in charged lepton sector

Z2 symmetry
in neutrino sector

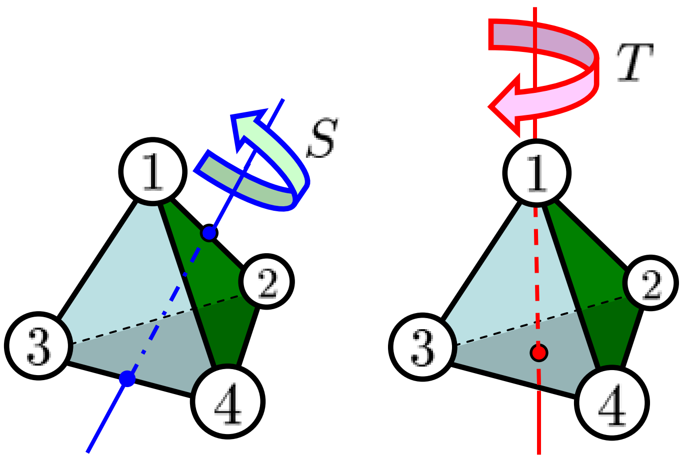
Z6, S3 also contain Z2 and Z3, but there is no irr. 3-rep. → **A4**

A4 group (alternating group for 4 letters)

- **even-permutation of 4 letters** (12 elements)
  - Elemental transposition S: $S^2 = I \rightarrow$ **Z2**
  - Elemental transposition T: $T^3 = I \rightarrow$ **Z3**

  others can be obtained from products of S and T, ex., ST, STS, …

$$(1,2,3,4) \left\{ \begin{array}{l} \xrightarrow{\quad e \quad} (1,2,3,4) \\ \xrightarrow{a_1 \equiv S} (\underline{2,1},\underline{4,3}) \\ \xrightarrow{a_2 \equiv T} (1,\underline{3,4,2}) \\ \qquad\qquad \vdots \\ \xrightarrow{\quad a_{11} \quad} (2,3,1,4) \end{array} \right.$$



Irreducible representations of A4

- **Transformations under A4**

$$S^2 = T^3 = (ST)^3 = 1$$

1-dim. rep. : $\begin{cases} \underline{\mathbf{1}} : \; S\,\underline{\mathbf{1}} = \underline{\mathbf{1}}, \quad T\,\underline{\mathbf{1}} = \underline{\mathbf{1}} \\ \underline{\mathbf{1}}' : \; S\,\underline{\mathbf{1}}' = \underline{\mathbf{1}}', \quad T\,\underline{\mathbf{1}}' = \omega\underline{\mathbf{1}}' \\ \underline{\mathbf{1}}'' : \; S\,\underline{\mathbf{1}}'' = \underline{\mathbf{1}}'', \quad T\,\underline{\mathbf{1}}'' = \omega^2\underline{\mathbf{1}}'' \end{cases}$ $\qquad \omega \equiv \exp\left(\frac{2\pi i}{3}\right)$

3-dim. rep. : $\underline{\mathbf{3}} = \begin{pmatrix} \mathbf{3}_x \\ \mathbf{3}_y \\ \mathbf{3}_z \end{pmatrix} : \; S\,\underline{\mathbf{3}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \underline{\mathbf{3}}, \quad T\,\underline{\mathbf{3}} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \underline{\mathbf{3}}$

3-dim. rep. may be related for 3 generation of fermion family

Computation rules

$$\boxed{\underline{\mathbf{3}} \otimes \underline{\mathbf{3}} = \underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'' \oplus \underline{\mathbf{3}}_s \oplus \underline{\mathbf{3}}_a}$$

$$\underline{\mathbf{3}} \otimes \underline{\mathbf{3}} \to \underline{\mathbf{1}} : \ (ab)_{\mathbf{1}} \equiv a_x b_x + a_y b_y + a_z b_z$$

$$\underline{\mathbf{3}} \otimes \underline{\mathbf{3}} \to \underline{\mathbf{1}}' : \ (ab)_{\mathbf{1}'} \equiv a_x b_x + \omega^2 a_y b_y + \omega a_z b_z$$

$$\left\{ \begin{aligned} &\xrightarrow{S} a_x b_x + \omega^2(-a_y)(-b_y) + \omega(-a_z)(-b_z) \\ &\qquad\qquad = a_x b_x + \omega^2 a_y b_y + \omega a_z b_z \\ &\xrightarrow{T} a_y b_y + \omega^2 a_z b_z + \omega a_x b_x \\ &\qquad\qquad = \omega(a_x b_x + \omega^2 a_y b_y + \omega a_z b_z) \end{aligned} \right.$$

$$\underline{\mathbf{3}} \otimes \underline{\mathbf{3}} \to \underline{\mathbf{1}}'' : \ (ab)_{\mathbf{1}''} \equiv a_x b_x + \omega a_y b_y + \omega^2 a_z b_z$$

$$\underline{\mathbf{3}} \otimes \underline{\mathbf{3}} \to \underline{\mathbf{3}}_s : \ (ab)_{\mathbf{3}_s} \equiv \begin{pmatrix} a_y b_z + a_z b_y \\ a_z b_x + a_x b_z \\ a_x b_y + a_y b_x \end{pmatrix} \qquad (ab)_{\mathbf{3}_s} = (ba)_{\mathbf{3}_s}$$

$$\underline{\mathbf{3}} \otimes \underline{\mathbf{3}} \to \underline{\mathbf{3}}_a : \ (ab)_{\mathbf{3}_a} \equiv \begin{pmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{pmatrix} \qquad (ab)_{\mathbf{3}_a} = -(ba)_{\mathbf{3}_a}$$

A4HTM:

The minimal A4 symmetric extension of the Higgs triplet model with soft A4 breaking terms

A4HTM (particle contents)

	ψ^-_{1R}	ψ^-_{2R}	ψ^-_{3R}	$\Psi_{AL} = \begin{pmatrix} \psi^0_{AL} \\ \psi^-_{AL} \end{pmatrix}$
A_4	$\underline{\mathbf{1}}$	$\underline{\mathbf{1}}'$	$\underline{\mathbf{1}}''$	$\underline{\mathbf{3}}$
$SU(2)_L$	singlet	singlet	singlet	doublet
$U(1)_Y$	-2	-2	-2	-1

	$\Phi_A = \begin{pmatrix} \phi^+_A \\ \phi^0_A \end{pmatrix}$	$\delta = \begin{pmatrix} \frac{\delta^+}{\sqrt{2}} & \delta^{++} \\ \delta^0 & -\frac{\delta^+}{\sqrt{2}} \end{pmatrix}$	$\Delta_A = \begin{pmatrix} \frac{\Delta^+_A}{\sqrt{2}} & \Delta^{++}_A \\ \Delta^0_A & -\frac{\Delta^+_A}{\sqrt{2}} \end{pmatrix}$
	$\underline{\mathbf{3}}$	$\underline{\mathbf{1}}$	$\underline{\mathbf{3}}$
	doublet	triplet	triplet
	1	2	2

A4 Yukawa interaction for charged fermions

- **Mass generation**

$$\left(\overline{\Psi_{xL}}\,\Phi_x,\ \overline{\Psi_{yL}}\,\Phi_y,\ \overline{\Psi_{zL}}\,\Phi_z \right) \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} y_1 \Psi^-_{1R} \\ y_2 \Psi^-_{2R} \\ y_3 \Psi^-_{3R} \end{pmatrix} + \text{h.c.}$$

  - Developing aligned vev: $\langle \phi^0_x \rangle = \langle \phi^0_y \rangle = \langle \phi^0_z \rangle = v/\sqrt{6}$

  - Mass eigenvalues:

$$m_e \equiv \frac{y_1}{\sqrt{2}}, \quad m_\mu \equiv \frac{y_2}{\sqrt{2}}, \quad m_\tau \equiv \frac{y_3}{\sqrt{2}}$$

Structures are same for up and down quarks

A4HTM potetial 1

$$
\begin{aligned}
V_{\mathrm{A4HTM}} \equiv & V_m + V_1 + V_2 + V_3 + V_4 + V_5 + V_\mu, \\
V_m \equiv & - m_\Phi^2 \, (\Phi^\dagger \Phi)_{\mathbf{1}} + M_\delta^2 \, \mathrm{Tr}(\delta^\dagger \delta) + M_\Delta^2 \, \mathrm{Tr}(\Delta^\dagger \Delta)_{\mathbf{1}}, \\
V_4 \equiv & \lambda_{4\delta} \, (\Phi^\dagger \Phi)_{\mathbf{1}} \, \mathrm{Tr}\big(\delta^\dagger \delta\big) + \lambda_{4\Delta} \, (\Phi^\dagger \Phi)_{\mathbf{1}} \, \mathrm{Tr}(\Delta^\dagger \Delta)_{\mathbf{1}} \\
& + \left\{ \lambda'_{4\Delta p} \, (\Phi^\dagger \Phi)_{\mathbf{1}''} \, \mathrm{Tr}(\Delta^\dagger \Delta)_{\mathbf{1}'} + \mathrm{h.c.} \right\} \\
& + \lambda_{4\Delta ss} \, (\Phi^\dagger \Phi)_{\mathbf{3}_s} \, \mathrm{Tr}(\Delta^\dagger \Delta)_{\mathbf{3}_s} + \lambda_{4\Delta aa} \, (\Phi^\dagger \Phi)_{\mathbf{3}_a} \, \mathrm{Tr}(\Delta^\dagger \Delta)_{\mathbf{3}_a} \\
& + i\lambda_{4\Delta sa} \, (\Phi^\dagger \Phi)_{\mathbf{3}_s} \, \mathrm{Tr}(\Delta^\dagger \Delta)_{\mathbf{3}_a} + i\lambda_{4\Delta as} \, (\Phi^\dagger \Phi)_{\mathbf{3}_a} \, \mathrm{Tr}(\Delta^\dagger \Delta)_{\mathbf{3}_s} \\
& + \left\{ \lambda'_{4s} \, \delta^*_{\beta\alpha} \left[\Delta_{\beta\alpha} \, (\Phi^\dagger \Phi)_{\mathbf{3}_s} \right]_{\mathbf{1}} + \lambda'_{4a} \, \delta^*_{\beta\alpha} \left[\Delta_{\beta\alpha} \, (\Phi^\dagger \Phi)_{\mathbf{3}_a} \right]_{\mathbf{1}} + \mathrm{h.c.} \right\}, \\
V_5 \equiv & \lambda_{5\delta} \, (\Phi^\dagger \sigma^i \Phi)_{\mathbf{1}} \, \mathrm{Tr}(\delta^\dagger \sigma^i \delta) + \lambda_{5\Delta} \, (\Phi^\dagger \sigma^i \Phi)_{\mathbf{1}} \, \mathrm{Tr}(\Delta^\dagger \sigma^i \Delta)_{\mathbf{1}} \\
& + \left\{ \lambda'_{5\Delta p} \, (\Phi^\dagger \sigma^i \Phi)_{\mathbf{1}''} \, \mathrm{Tr}(\Delta^\dagger \sigma^i \Delta)_{\mathbf{1}'} + \mathrm{h.c.} \right\} \\
& + \lambda_{5\Delta ss} \, (\Phi^\dagger \sigma^i \Phi)_{\mathbf{3}_s} \, \mathrm{Tr}(\Delta^\dagger \sigma^i \Delta)_{\mathbf{3}_s} + \lambda_{5\Delta aa} \, (\Phi^\dagger \sigma^i \Phi)_{\mathbf{3}_a} \, \mathrm{Tr}(\Delta^\dagger \sigma^i \Delta)_{\mathbf{3}_a} \\
& + i\lambda_{5\Delta sa} \, (\Phi^\dagger \sigma^i \Phi)_{\mathbf{3}_s} \, \mathrm{Tr}(\Delta^\dagger \sigma^i \Delta)_{\mathbf{3}_a} + i\lambda_{5\Delta as} \, (\Phi^\dagger \sigma^i \Phi)_{\mathbf{3}_a} \, \mathrm{Tr}(\Delta^\dagger \sigma^i \Delta)_{\mathbf{3}_s} \\
& + \left\{ \lambda'_{5s} \, (\delta^\dagger \sigma^i)_{\alpha\beta} \left[\Delta_{\beta\alpha} \, (\Phi^\dagger \sigma^i \Phi)_{\mathbf{3}_s} \right]_{\mathbf{1}} \right. \\
& \qquad \qquad \left. + \lambda'_{5a} \, (\delta^\dagger \sigma^i)_{\alpha\beta} \left[\Delta_{\beta\alpha} \, (\Phi^\dagger \sigma^i \Phi)_{\mathbf{3}_a} \right]_{\mathbf{1}} + \mathrm{h.c.} \right\}.
\end{aligned}
$$

A4HTM potential 2

$$V_{\rm A4HTM} \equiv V_m + V_1 + V_2 + V_3 + V_4 + V_5 + V_\mu,$$

$$\begin{aligned}
V_1 = {} & \lambda_1 \left[(\Phi^\dagger\Phi)_{\mathbf{1}}\right]^2 + \lambda_{1p}(\Phi^\dagger\Phi)_{\mathbf{1}'}(\Phi^\dagger\Phi)_{\mathbf{1}''} \\
& + \lambda_{1ss}\Big((\Phi^\dagger\Phi)_{\mathbf{3}_s}(\Phi^\dagger\Phi)_{\mathbf{3}_s}\Big)_{\mathbf{1}} + \lambda_{1aa}\Big((\Phi^\dagger\Phi)_{\mathbf{3}_a}(\Phi^\dagger\Phi)_{\mathbf{3}_a}\Big)_{\mathbf{1}} \\
& + i\lambda_{1sa}(\Phi^\dagger\Phi)_{\mathbf{3}_s}(\Phi^\dagger\Phi)_{\mathbf{3}_a},
\end{aligned}$$

$$\begin{aligned}
V_2 = {} & \lambda_{2\delta}\left[\mathrm{Tr}(\delta^\dagger\delta)\right]^2 \\
& + \lambda_{2\Delta}\left[\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{1}}\right]^2 + \lambda_{2\Delta p}\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{1}'}\,\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{1}''} \\
& + \lambda_{2\Delta ss}\left(\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_s}\,\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_s}\right)_{\mathbf{1}} + \lambda_{2\Delta aa}\left(\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_a}\,\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_a}\right)_{\mathbf{1}} \\
& + i\lambda_{2\Delta sa}\left(\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_s}\,\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_a}\right)_{\mathbf{1}} \\
& + \lambda_{2\delta\Delta 1}\,\mathrm{Tr}(\delta^\dagger\delta)\,\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{1}} + \lambda_{2\delta\Delta 2}\,(\delta^*_{\beta\alpha}\delta_{\omega\gamma})\,(\Delta_{\beta\alpha}\Delta^*_{\omega\gamma})_{\mathbf{1}} \\
& + \left\{\lambda'_{2\delta\Delta 3}\,(\delta^*_{\beta\alpha}\delta^*_{\omega\gamma})\,\left[\Delta_{\beta\alpha}\Delta_{\omega\gamma}\right]_{\mathbf{1}} + \mathrm{h.c.}\right\} \\
& + \left\{\lambda'_{2\delta\Delta s}\,\delta^*_{\beta\alpha}\,\left[\Delta_{\beta\alpha}(\Delta^*_{\omega\gamma}\Delta_{\omega\gamma})_{\mathbf{3}_s}\right]_{\mathbf{1}} + \mathrm{h.c.}\right\} \\
& + \left\{\lambda'_{2\delta\Delta a}\,\delta^*_{\beta\alpha}\,\left[\Delta_{\beta\alpha}(\Delta^*_{\omega\gamma}\Delta_{\omega\gamma})_{\mathbf{3}_a}\right]_{\mathbf{1}} + \mathrm{h.c.}\right\}.
\end{aligned}$$

A4HTM potential 3

$$V_{\rm A4HTM} \equiv V_m + V_1 + V_2 + V_3 + V_4 + V_5 + V_\mu,$$

$$\begin{aligned}
V_3 = & \frac{1}{2}\lambda_{3\delta}\left\{\left[\mathrm{Tr}(\delta^\dagger\delta)\right]^2 - \mathrm{Tr}\left(\left[\delta^\dagger\delta\right]^2\right)\right\} \\
& + \frac{1}{2}\lambda_{3\Delta}\left\{\left[\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{1}}\right]^2 - \mathrm{Tr}\left(\left[(\Delta^\dagger\Delta)_{\mathbf{1}}\right]^2\right)\right\} \\
& + \frac{1}{2}\lambda_{3\Delta p}\left\{\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{1}'}\,\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{1}''} - \mathrm{Tr}\left((\Delta^\dagger\Delta)_{\mathbf{1}'}\,(\Delta^\dagger\Delta)_{\mathbf{1}''}\right)\right\} \\
& + \frac{1}{2}\lambda_{3\Delta ss}\left\{\left(\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_s}\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_s}\right)_{\mathbf{1}} - \mathrm{Tr}\left((\Delta^\dagger\Delta)_{\mathbf{3}_s}(\Delta^\dagger\Delta)_{\mathbf{3}_s}\right)_{\mathbf{1}}\right\} \\
& + \frac{1}{2}\lambda_{3\Delta aa}\left\{\left(\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_a}\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_a}\right)_{\mathbf{1}} - \mathrm{Tr}\left((\Delta^\dagger\Delta)_{\mathbf{3}_a}(\Delta^\dagger\Delta)_{\mathbf{3}_a}\right)_{\mathbf{1}}\right\} \\
& + \frac{1}{2}i\lambda_{3\Delta sa}\left\{\left(\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_s}\,\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_a}\right)_{\mathbf{1}} - \mathrm{Tr}\left((\Delta^\dagger\Delta)_{\mathbf{3}_s}\,(\Delta^\dagger\Delta)_{\mathbf{3}_a}\right)_{\mathbf{1}}\right\} \\
& + \frac{1}{2}\lambda_{3\delta\Delta 1}\left\{\mathrm{Tr}(\delta^\dagger\delta)\,\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{1}} - \mathrm{Tr}\left((\delta^\dagger\delta)\,(\Delta^\dagger\Delta)_{\mathbf{1}}\right)\right\} \\
& + \frac{1}{2}\lambda_{3\delta\Delta 2}\left\{\delta^*_{\beta\alpha}(\Delta_{\beta\alpha}\Delta^*_{\omega\gamma})_{\mathbf{1}}\delta_{\omega\gamma} - \mathrm{Tr}\left(\delta^\dagger(\Delta\Delta^\dagger)_{\mathbf{1}}\delta\right)\right\} \\
& + \ldots
\end{aligned}$$

A4HTM potential 4

$$
\begin{aligned}
V_{\mathrm{A4HTM}} &\equiv V_m + V_1 + V_2 + V_3 + V_4 + V_5 + V_\mu, \\
V_3 &= \dots \\
&+ \left\{ \frac{1}{2} \lambda'_{3\delta\Delta 3} \left((\delta^*_{\beta\alpha} \delta^*_{\omega\gamma}) \left[\Delta_{\beta\alpha} \Delta_{\omega\gamma} \right]_{\mathbf{1}} - \delta^*_{\beta\alpha} \delta^*_{\omega\gamma} \left[\Delta_{\beta\gamma} \Delta_{\omega\alpha} \right]_{\mathbf{1}} \right) + \mathrm{h.c.} \right\} \\
&+ \left\{ \frac{1}{2} \lambda'_{3\delta\Delta s} \left(\delta^*_{\beta\alpha} \left[\Delta_{\beta\alpha} (\Delta^*_{\omega\gamma} \Delta_{\omega\gamma})_{\mathbf{3}_s} \right]_{\mathbf{1}} - \delta^*_{\beta\alpha} \left[\Delta_{\beta\gamma} (\Delta^*_{\omega\gamma} \Delta_{\omega\alpha})_{\mathbf{3}_s} \right]_{\mathbf{1}} \right) + \mathrm{h.c.} \right\} \\
&+ \left\{ \frac{1}{2} \lambda'_{3\delta\Delta a} \delta^*_{\beta\alpha} \left[\Delta_{\beta\alpha} (\Delta^*_{\omega\gamma} \Delta_{\omega\gamma})_{\mathbf{3}_a} \right]_{\mathbf{1}} + \mathrm{h.c.} \right\}, \\
V_\mu &= \frac{1}{\sqrt{2}} \mu_\delta \left[\Phi_\alpha \Phi_\beta \right]_{\mathbf{1}} (i\sigma^2 \delta^\dagger)_{\alpha\beta} + \frac{1}{\sqrt{2}} \mu_\Delta \Big((\Phi_\alpha \Phi_\beta)_{\mathbf{3}_s} (i\sigma^2 \Delta^\dagger)_{\alpha\beta} \Big)_{\mathbf{1}} + \mathrm{h.c.}
\end{aligned}
$$

- **Soft A4 breaking terms**

$$
\boxed{\tilde{V}_\mu = \frac{1}{\sqrt{2}} \mu_\delta \left[\Phi_\alpha \Phi_\beta \right]_{\mathbf{1}} (i\sigma^2 \delta^\dagger)_{\alpha\beta} + \frac{1}{\sqrt{2}} \mu_{\Delta_x} (2\Phi_{y\alpha} \Phi_{z\beta}) (i\sigma^2 \Delta^\dagger_x)_{\alpha\beta} + \mathrm{h.c.}}
$$

Quark mixing

- Unitary transf.

$$\begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} \equiv U_L^\dagger \begin{pmatrix} \Psi_{xL} \\ \Psi_{yL} \\ \Psi_{zL} \end{pmatrix} \qquad \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \psi_{1R}^- \\ \psi_{2R}^- \\ \psi_{3R}^- \end{pmatrix}$$

$$L_\ell \equiv \begin{pmatrix} \nu_{\ell L} \\ \ell_L \end{pmatrix} \qquad \boxed{U_L \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}}$$

Z3 sym. in quark and charged lepton mixing

- **Quark CKM mixing**

$$\boxed{U_{\rm CKM} = (U_L^d)^\dagger U_L^u = U_L^\dagger U_L = I}$$

Unit matrix at LO.
(or Quarks can be coupled with other $\Phi[1]$, next slide)

Possible quark sector

- **Effectively Type-X 2HDM**

  Quarks and leptons couple to other Higgs doublet

$$\Phi_1 = \frac{1}{\sqrt{3}}(\Phi_x + \Phi_y + \Phi_z) : \quad \text{for leptons}$$

Φ_2 : for quarks

	$\psi_{iR}^{\frac{2}{3}}$	$\psi_{iR}^{-\frac{1}{3}}$	$\Psi_{iQ} = \begin{pmatrix} \psi_{iL}^{\frac{2}{3}} \\ \psi_{iL}^{-\frac{1}{3}} \end{pmatrix}$	Φ_2
A_4	$\underline{\mathbf{1}}$	$\underline{\mathbf{1}}$	$\underline{\mathbf{1}}$	$\underline{\mathbf{1}}$
$SU(2)_L$	singlet	singlet	doublet	doublet
$U(1)_Y$	$4/3$	$-2/3$	$1/3$	1

Neutrino mixing

- Tri-Bi-Maximal mixing: good agreement with experiments.

$$U_{\rm MNS} = U_{\rm TB} \equiv \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sim \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \times \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Z3 symmetry
in charged lepton sector

Z2 symmetry
in neutrino sector

$$\langle \phi_x^0 \rangle = \langle \phi_y^0 \rangle = \langle \phi_z^0 \rangle = v/\sqrt{6}$$

Triplet Yukawa interaction for neutrino masses

$$\left(\overline{(\Psi_{xL})^c},\ \overline{(\Psi_{yL})^c},\ \overline{(\Psi_{zL})^c} \right) \begin{pmatrix} h_\delta i\sigma^2\delta & h_\triangle i\sigma^2\Delta_z & h_\triangle i\sigma^2\Delta_y \\ h_\triangle i\sigma^2\Delta_z & h_\delta i\sigma^2\delta & h_\triangle i\sigma^2\Delta_x \\ h_\triangle i\sigma^2\Delta_y & h_\triangle i\sigma^2\Delta_x & h_\delta i\sigma^2\delta \end{pmatrix} \begin{pmatrix} \Psi_{xL} \\ \Psi_{yL} \\ \Psi_{zL} \end{pmatrix} + \text{h.c.}$$

- 2-3 maximal mixing is preferred in this basis

$$\langle \delta^0 \rangle = \frac{v_\delta}{\sqrt{2}}, \quad \langle \Delta_x^0 \rangle = \frac{v_\triangle}{\sqrt{2}}, \quad \langle \Delta_y^0 \rangle = \langle \Delta_z^0 \rangle = 0$$

$$\frac{1}{\sqrt{2}} \left(\overline{(\psi^0_{xL})^c},\ \overline{(\psi^0_{yL})^c},\ \overline{(\psi^0_{zL})^c} \right) \begin{pmatrix} h_\delta v_\delta & 0 & 0 \\ 0 & h_\delta v_\delta & h_\triangle v_\triangle \\ 0 & h_\triangle v_\triangle & h_\delta v_\delta \end{pmatrix} \begin{pmatrix} \psi^0_{xL} \\ \psi^0_{yL} \\ \psi^0_{zL} \end{pmatrix} + \cdots + \text{h.c.}$$

$$M_\nu = \sqrt{2}\, U_L^T \begin{pmatrix} h_\delta v_\delta & 0 & 0 \\ 0 & h_\delta v_\delta & h_\triangle v_\triangle \\ 0 & h_\triangle v_\triangle & h_\delta v_\delta \end{pmatrix} U_L$$

Neutrino mixing

- Tri-Bi-Maximal mixing: good agreement with experiments.

$$U_{\rm MNS} = U_{\rm TB} \equiv \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sim \underbrace{\frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}} \times \underbrace{\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}}$$

Z3 symmetry
in charged lepton sector

$$\langle \phi_x^0 \rangle = \langle \phi_y^0 \rangle = \langle \phi_z^0 \rangle = v/\sqrt{6}$$

Z2 symmetry
in neutrino sector

$$\langle \Delta_x^0 \rangle = \frac{v_\Delta}{\sqrt{2}}, \langle \Delta_y^0 \rangle = \langle \Delta_z^0 \rangle = 0$$

Neutrino masses and mixings under A4

- **Diagonalize:** $\mathrm{diag}(m_1 e^{i\alpha_{12}},\ m_2,\ m_3 e^{i\alpha_{32}}) = U_{\mathrm{MNS}}^T M_\nu U_{\mathrm{MNS}}$

$$m_1 e^{i\alpha_{12}} = h_\delta v_\delta + h_\triangle v_\triangle$$

$$m_2 = h_\delta v_\delta \in \mathbf{R}$$

$$m_3 e^{i\alpha_{32}} = -h_\delta v_\delta + h_\triangle v_\triangle$$

$$U_{\mathrm{MNS}} = U_{\mathrm{TB}} \equiv \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

- **A4 sym. + vev alignment → TriBiMaximal mixing**

Good agreement with experiments.

- Note: TB-mixing can be obtained in model without δ[1], but it is required to solve mass degeneracy of m1 & m3.

$$M_\nu = \sqrt{2}\, U_L^T \begin{pmatrix} h_\delta v_\delta & 0 & 0 \\ 0 & h_\delta v_\delta & h_\triangle v_\triangle \\ 0 & h_\triangle v_\triangle & h_\delta v_\delta \end{pmatrix} U_L$$

Approximate symmetry in the broken phase

- Doublet vev is symmetric under T

$$\begin{pmatrix} v/\sqrt{6} \\ v/\sqrt{6} \\ v/\sqrt{6} \end{pmatrix} \xrightarrow{T} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} v/\sqrt{6} \\ v/\sqrt{6} \\ v/\sqrt{6} \end{pmatrix} = \begin{pmatrix} v/\sqrt{6} \\ v/\sqrt{6} \\ v/\sqrt{6} \end{pmatrix}$$

A4 → Z3 sym

- Tiny Triplet vev is symmetric under S

$$\begin{pmatrix} v_\Delta/\sqrt{2} \\ 0 \\ 0 \end{pmatrix} \xrightarrow{S} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} v_\Delta/\sqrt{2} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} v_\Delta/\sqrt{2} \\ 0 \\ 0 \end{pmatrix}$$

A4 → Z2 sym

- **EW precision obs. ρ:** $v \sim 246\mathrm{GeV}, \sqrt{v_\delta^2 + v_\Delta^2} \lesssim 1\mathrm{GeV} \ll v$

→ **approx. Z3 symmetry**
(slightly broken by triplet vev)

All the particle in A4HTM can be classified by approx. Z3 charge !!

Z3 classification

- **Singlets: by default**

$$\underline{\mathbf{1}} : T\,\underline{\mathbf{1}} = \underline{\mathbf{1}}$$
$$\underline{\mathbf{1}}' : T\,\underline{\mathbf{1}}' = \omega\underline{\mathbf{1}}'$$
$$\underline{\mathbf{1}}'' : T\,\underline{\mathbf{1}}'' = \omega^2\underline{\mathbf{1}}''$$

- **Triplets**

$$\underline{\mathbf{3}} : \begin{pmatrix} a_\xi \\ a_\eta \\ a_\zeta \end{pmatrix} \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$$

$$\sqrt{3}a_\xi = a_x + a_y + a_z \xrightarrow{T} a_y + a_z + a_x = \sqrt{3}a_\xi$$
$$\sqrt{3}a_\eta = a_x + \omega^2 a_y + \omega a_z \xrightarrow{T} a_y + \omega^2 a_z + \omega a_x = \omega\sqrt{3}a_\eta$$
$$\sqrt{3}a_\zeta = a_x + \omega a_y + \omega^2 a_z \xrightarrow{T} a_y + \omega a_z + \omega^2 a_x = \omega^2\sqrt{3}a_\zeta$$

	$\underline{\mathbf{1}}$, a_ξ	$\underline{\mathbf{1}}'$, a_η	$\underline{\mathbf{1}}''$, a_ζ
Z_3–charge	1	ω	ω^2

Z3 charges for leptons

	ψ^-_{1R}	ψ^-_{2R}	ψ^-_{3R}	$\Psi_{AL} = \begin{pmatrix} \psi^0_{AL} \\ \psi^-_{AL} \end{pmatrix}$
A_4	$\underline{\mathbf{1}}$	$\underline{\mathbf{1}'}$	$\underline{\mathbf{1}''}$	$\underline{\mathbf{3}}$
$SU(2)_L$	singlet	singlet	singlet	doublet
$U(1)_Y$	-2	-2	-2	-1

$$\begin{array}{c} \text{Z3} \\ 1 \\ \omega \\ \omega^2 \end{array} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \psi^-_{1R} \\ \psi^-_{2R} \\ \psi^-_{3R} \end{pmatrix} \begin{array}{c} \text{A4} \\ \underline{\mathbf{1}} \\ \underline{\mathbf{1}'} \\ \underline{\mathbf{1}''} \end{array}$$

$$\begin{array}{c} \text{Z3} \\ 1 \\ \omega \\ \omega^2 \end{array} \begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} \equiv U_L^\dagger \begin{pmatrix} \Psi_{xL} \\ \Psi_{yL} \\ \Psi_{zL} \end{pmatrix} \begin{array}{c} \text{A4} \\ \\ \underline{\mathbf{3}} \\ \\ \end{array}$$

	$L_e,\ e_R$	$L_\mu,\ \mu_R$	$L_\tau,\ \tau_R$
Z_3–charge	1	ω	ω^2

Doubly charged Higgs bosons

- Mass eigenstates can be determined approximately by neglecting tiny effects from triplet vev

$$\begin{pmatrix} H_1^{++} \\ H_2^{++} \\ H_3^{++} \\ H_4^{++} \end{pmatrix} \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{\pm\pm} & s_{\pm\pm} \\ 0 & 0 & -s_{\pm\pm} & c_{\pm\pm} \end{pmatrix} \begin{pmatrix} 1 & \omega & \omega^2 & 0 \\ 1 & \omega^2 & \omega & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & \sqrt{3}\, e^{-i\alpha_{\pm\pm}} \end{pmatrix} \begin{pmatrix} \Delta_x^{++} \\ \Delta_y^{++} \\ \Delta_z^{++} \\ \delta^{++} \end{pmatrix}$$

	$H_3^{++},\ H_4^{++}$	H_2^{++}	H_1^{++}
Z_3–charge	1	ω	ω^2

Doubly charged Higgs Yukawa interaction

$$(h_{i\pm\pm})_{\ell\ell'}\,\overline{(\ell_L)^c}\,\ell'_L\,H_i^{++} + \text{h.c.}$$

$$\frac{2}{\sqrt{3}}h_\Delta\left\{-\overline{(e_L)^c}\,\mu_L + \overline{(\tau_L)^c}\,\tau_L\right\}H_1^{++}$$

$\mathbf{1}$ ω ω^2 ω^2 ω^2

$$h_{1\pm\pm} = \frac{1}{\sqrt{3}}h_\Delta\begin{pmatrix}0 & -1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 2\end{pmatrix}$$

Zeros are consequence of Z3 sym.

$$h_{2\pm\pm} = \frac{1}{\sqrt{3}}h_\Delta\begin{pmatrix}0 & 0 & 1\\ 0 & 2 & 0\\ 1 & 0 & 0\end{pmatrix}$$

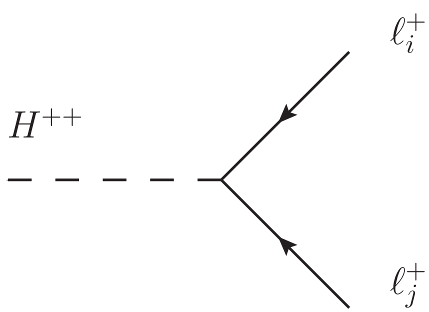
$$h_{3\pm\pm} = \frac{1}{\sqrt{3}}h_\Delta c_{\pm\pm}\begin{pmatrix}2 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix} + h_\delta s_{\pm\pm}e^{i\alpha_{\pm\pm}}\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & -1\\ 0 & -1 & 0\end{pmatrix}$$

$$h_{4\pm\pm} = -\frac{1}{\sqrt{3}}h_\Delta s_{\pm\pm}\begin{pmatrix}2 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix} + h_\delta c_{\pm\pm}e^{i\alpha_{\pm\pm}}\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & -1\\ 0 & -1 & 0\end{pmatrix}$$

BRs of doubly charged Higgs bosons

- **A4HTM predicts unique Ratios of BRs.**

	$\mathcal{B}(H^{--} \to \ell\ell')$ $ee(11) : \mu\mu(\omega\omega) : \tau\tau(\omega^2\omega^2) : e\mu(1\omega) : e\tau(\omega^2) : \mu\tau(\omega\omega^2)$
$H_1^{\pm\pm}(\omega)$	$0 : 0 : 2/3 : 1/3 : 0 : 0$
$H_2^{\pm\pm}(\omega^2)$	$0 : 2/3 : 0 : 0 : 1/3 : 0$
$H_3^{\pm\pm}(1)$	$R_3^{\pm\pm}/(1+R_3^{\pm\pm}) : 0 : 0 : 0 : 0 : 1/(1+R_3^{\pm\pm})$
$H_4^{\pm\pm}(1)$	$R_4^{\pm\pm}/(1+R_4^{\pm\pm}) : 0 : 0 : 0 : 0 : 1/(1+R_4^{\pm\pm})$

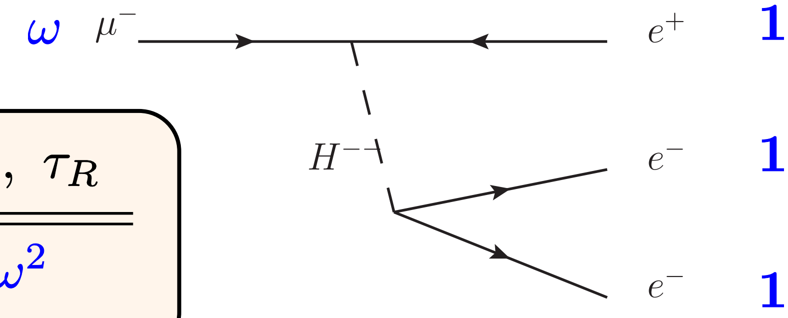


$$R_3^{\pm\pm} \equiv \frac{|2h_\Delta c_{\pm\pm} + \sqrt{3}\, h_\delta s_{\pm\pm} e^{i\alpha_{\pm\pm}}|^2}{2|h_\Delta c_{\pm\pm} - \sqrt{3}\, h_\delta s_{\pm\pm} e^{i\alpha_{\pm\pm}}|^2}$$

$$R_4^{\pm\pm} \equiv \frac{|2h_\Delta s_{\pm\pm} - \sqrt{3}\, h_\delta c_{\pm\pm} e^{i\alpha_{\pm\pm}}|^2}{2|h_\Delta s_{\pm\pm} + \sqrt{3}\, h_\delta c_{\pm\pm} e^{i\alpha_{\pm\pm}}|^2}$$

- **A4HTM (approx. Z3 sym.) forbids specific LFV modes**

In particular, $\mu^- \not\to e^- e^- e^+$



	$L_e,\ e_R$	$L_\mu,\ \mu_R$	$L_\tau,\ \tau_R$
Z_3–charge	1	ω	ω^2

Same for

$$\begin{array}{ccccccccccccccccccc} \tau^- & \not\to & e^+ & e^- & e^-, & \mu^+ & \mu^- & \mu^-, & e^+ & e^- & \mu^-, & \mu^+ & \mu^- & e^- \\ \omega^2 & & 1 & 1 & 1 & \omega^2 & \omega & \omega & 1 & 1 & \omega & \omega^2 & \omega & 1 \end{array}$$

- **A4HTM predicts specific LFV tau decays**

$$\begin{array}{cccccccc} \tau^- & \to & e^+ & \mu^- & \mu^-, & \mu^+ & e^- & e^- \\ \omega^2 & & 1 & \omega & \omega & \omega^2 & 1 & 1 \end{array}$$

(Triplet-like) Singly charged Higgs Yukawa interaction

$$\sqrt{2}(h_{i\pm})_{\ell\ell'}\,\overline{(\nu_L)^c}\,\ell'_L\,H_i^+ + \mathrm{h.c.}$$

$$h_{1\pm} = h_{1\pm\pm} = \frac{1}{\sqrt{3}} h_\Delta \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$h_{2\pm} = h_{2\pm\pm} = \frac{1}{\sqrt{3}} h_\Delta \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$h_{3\pm} = \frac{1}{\sqrt{3}} h_\Delta \underline{c_\pm} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + h_\delta \underline{s_\pm e^{i\alpha_\pm}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

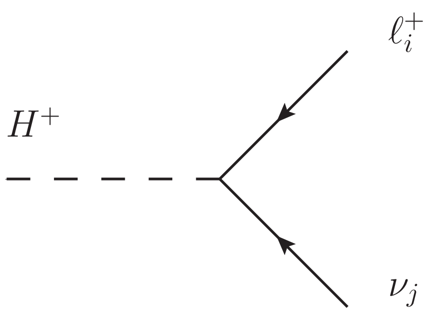
$$h_{4\pm} = -\frac{1}{\sqrt{3}} h_\Delta \underline{s_\pm} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + h_\delta \underline{c_\pm e^{i\alpha_\pm}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

Decays of singly charged Higgs bosons

- **Ratios of BRs**

$$\mathcal{B}(H^- \to \ell\nu) \equiv \sum_i \mathcal{B}(H^- \to \ell\nu_i)$$

	$\mathcal{B}(H^- \to \ell\nu)$ $e\nu : \mu\nu : \tau\nu$
$H_1^\pm$	$1/6 : 1/6 : 2/3$
$H_2^\pm$	$1/6 : 1/6 : 2/3$
$H_3^\pm$	$R_3^\pm/(1+R_3^\pm) : \frac{1}{2}/(1+R_3^\pm) : \frac{1}{2}/(1+R_3^\pm)$
$H_4^\pm$	$R_4^\pm/(1+R_4^\pm) : \frac{1}{2}/(1+R_4^\pm) : \frac{1}{2}/(1+R_4^\pm)$



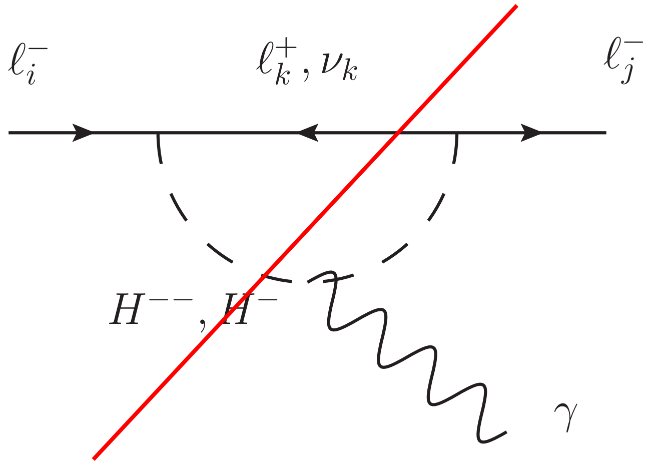
$$R_3^\pm \equiv \frac{|2h_\Delta c_\pm + \sqrt{3}\, h_\delta s_\pm e^{i\alpha_\pm}|^2}{2|h_\Delta c_\pm - \sqrt{3}\, h_\delta s_\pm e^{i\alpha_\pm}|^2}$$

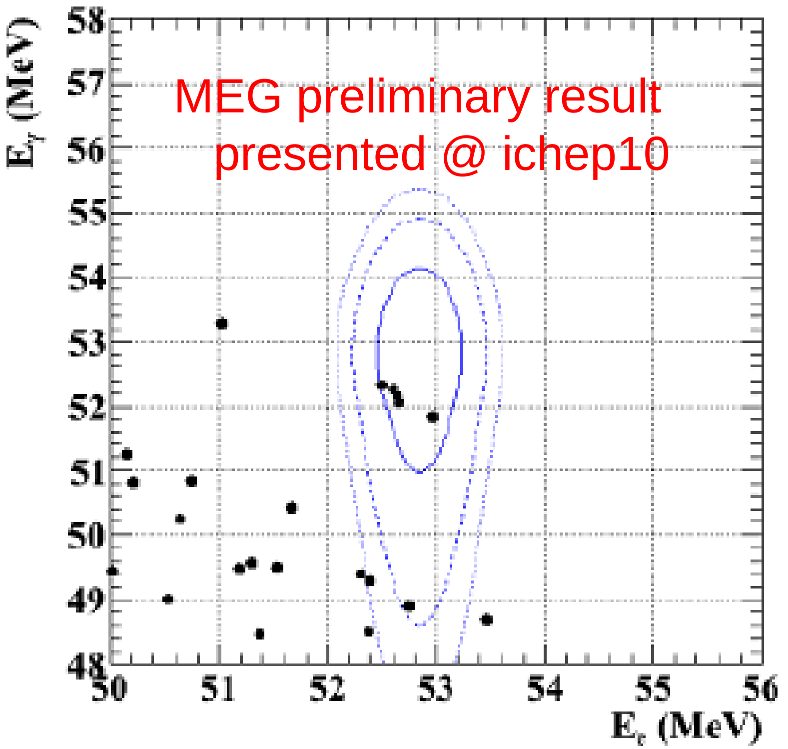
$$R_4^{\pm\pm} \equiv \frac{|2h_\Delta s_\pm - \sqrt{3}\, h_\delta c_\pm e^{i\alpha_\pm}|^2}{2|h_\Delta s_\pm + \sqrt{3}\, h_\delta c_\pm e^{i\alpha_\pm}|^2}$$

Lepton flavor violation $\ell_i^- \not\to \ell_j^- \gamma$

- **Natural suppression**

	$L_e,\ e_R$	$L_\mu,\ \mu_R$	$L_\tau,\ \tau_R$
Z_3–charge	1	ω	ω^2



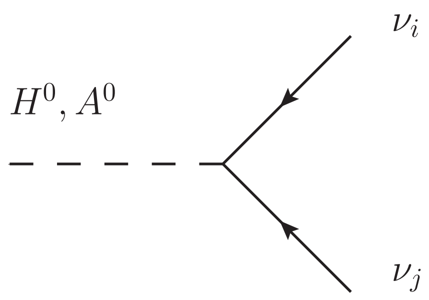


Discovery of $\mu \rightarrow e\,\gamma$ excludes A4HTM

It can be excluded by coming MEG data

Neutral Higgs phenomenology

- **Only couple to neutrinos**
  - H0, A0 phenomenology may be pooooooooor



Summary

- **HTM provides new source for neutrino mass.**
- **A4 sym. can give large neutrino mixing and small quark mixing even in HTM.**
- **Remaining Z3 sym. plays an important role in A4HTM.**
  - Unique predictions of triplet Higgs decays
  - Natural suppression of muon LFV

Thank you very much for your attention.

Back up

Lepton universality

- **Charged Higgs contributes to leptonic decay**

$$\boxed{2\sqrt{2}(G_W + G_{\mu e\ell\ell'})(\bar{\nu}_\ell \gamma_\mu P_L \mu)(\bar{e}\gamma^\mu P_L \nu_{\ell'})}$$

$$G_{\mu e\ell\ell'} = \sum_i \frac{(h_{i\pm})_{\ell'\mu}(h^*_{i\pm})_{\ell e}}{2\sqrt{2}m^2_{H_i^+}}$$

Non-standard neutrino interactions

- **Charged Higgs contributes to NSI**

$$\boxed{2\sqrt{2}G_F\epsilon_{\ell\ell'}^{fX}(\bar{f}\gamma_\mu P_X f)(\bar{\nu}_\ell\gamma^\mu P_L\nu_{\ell'})}$$

$$\epsilon_{\ell\ell}^{eL} = \sum_i \frac{(h_{i\pm})_{\ell'\mu}(h_{i\pm}^*)_{\ell e}}{2\sqrt{2}G_F m_{H_i^+}^2}$$

Doublet like charged Higgs bosons

- **Doublet-triplet mixing is suppressed by vev ratio**

$$\begin{pmatrix} H^+_{1D} \\ H^+_{2D} \\ H^+_{NG} \end{pmatrix} \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \phi^+_x \\ \phi^+_y \\ \phi^+_z \end{pmatrix}$$