Phenomenology in HTM with A4 Symmetry

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Phenomenology in the Higgs Triplet Model (HTM) with the A_4 Symmetry
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Outline

- Introduction
- Higgs Triplet Model (HTM)
- A4 symmetry
- HTM with A4
- Phenomenology
- Summary

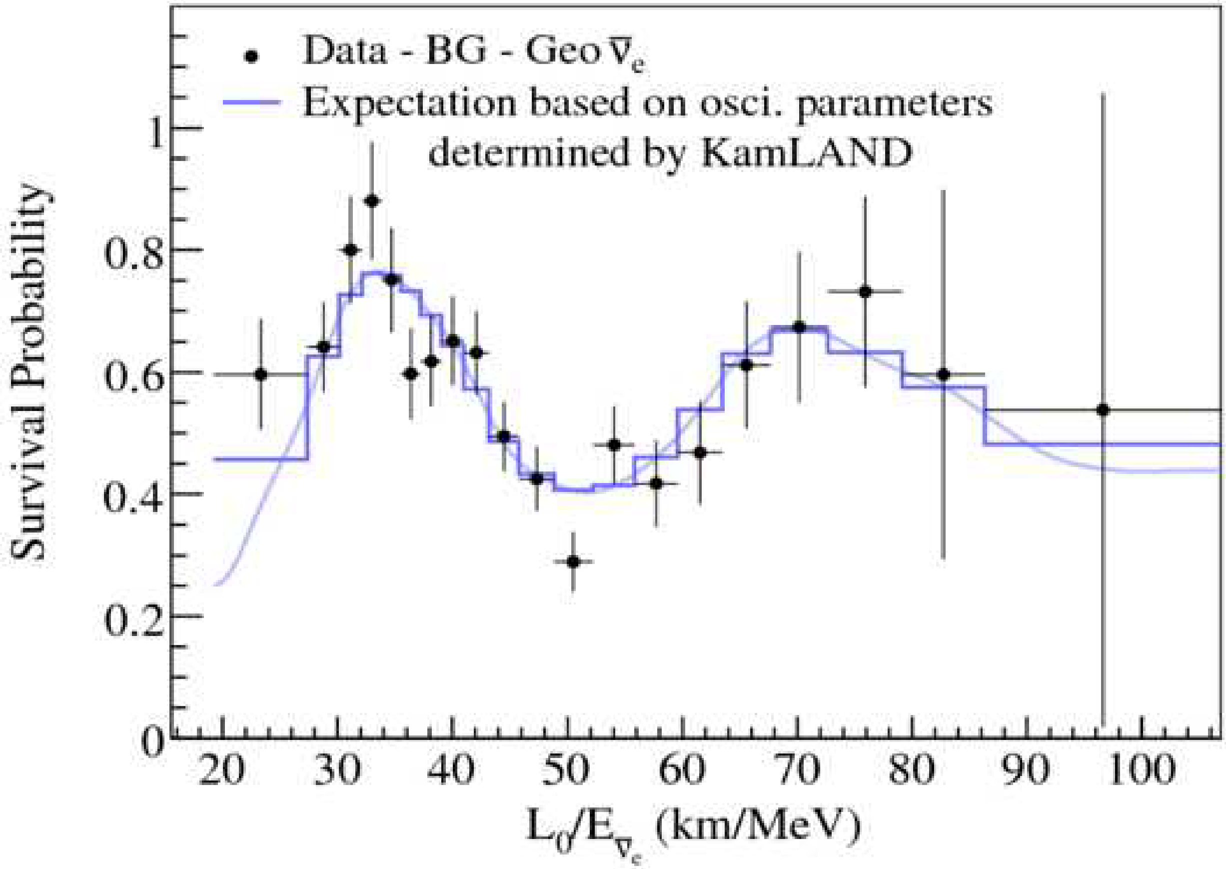
Neutrino

Neutrinos are massless in the SM

But, solar/atmospheric neutrino deficits are observed.
→ … Massive neutrino?

Neutrino oscillation

- Manifestly oscillating



→ **Massive Neutrino (a clear evidence of BSM)**

Introduction

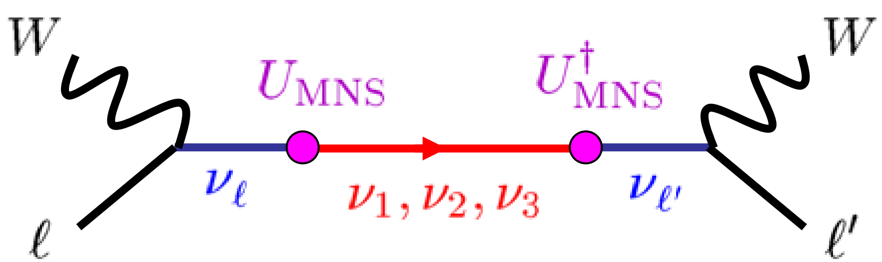
▫ Massive neutrino can be mixed

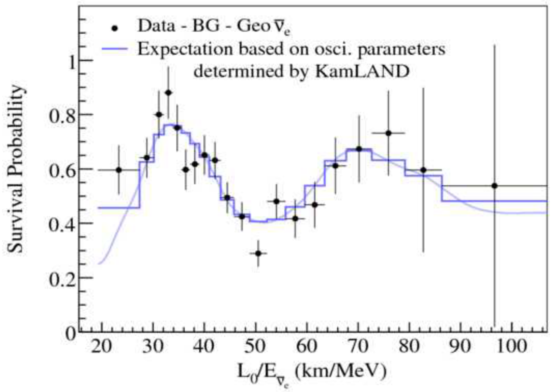
Flavor eigenstates → ν_ℓ; Mass eigenstates → ν_i; Neutrino (MNS) mixing matrix → $(U_{\rm MNS})_{\ell i}$

$$\nu_\ell = \sum_i (U_{\rm MNS})_{\ell i}\, \nu_i$$

▫ Neutrino flavor oscillation

$$P(\nu_\ell \to \nu_{\ell'}) = \left| \sum_i (U_{\rm MNS})_{\ell i} \ \exp\left(i \frac{m_i^2 L}{2E} \right) (U^\dagger_{\rm MNS})_{i\ell'} \right|^2$$





Tiny neutrino masses

- **Oscillation data:**

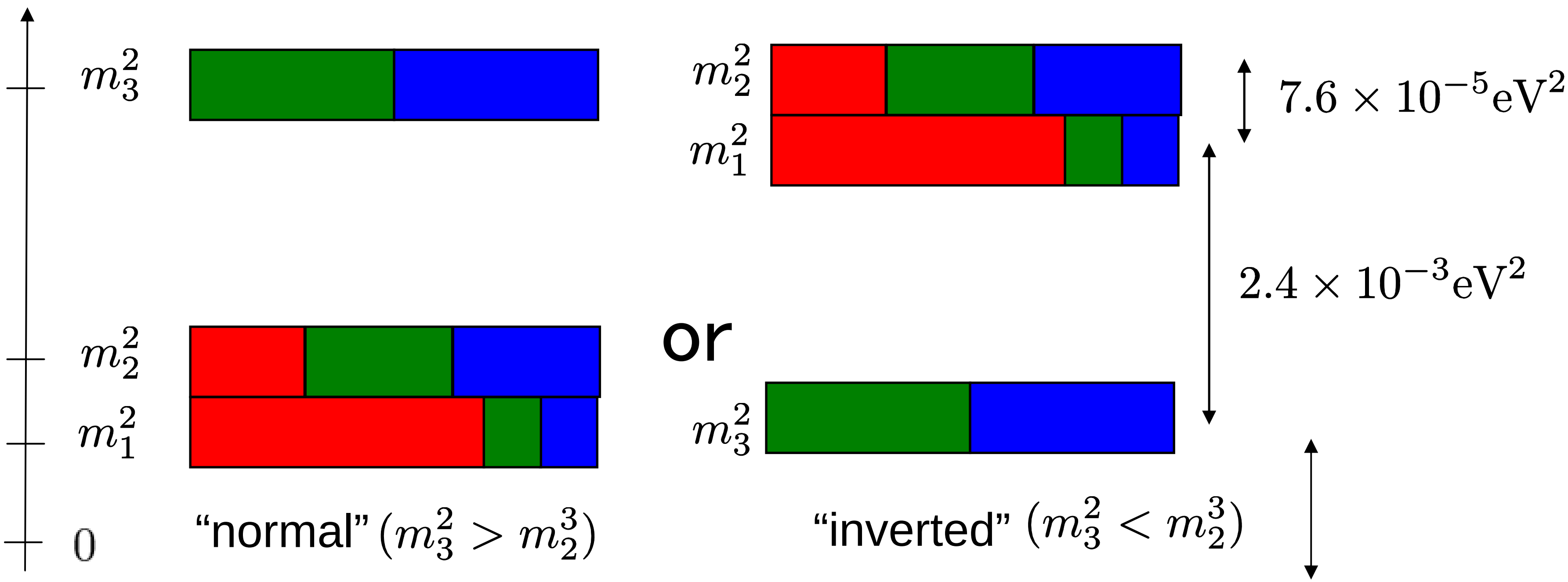
$$\Delta m^2_{21} = 7.6 \times 10^{-5}\ \mathrm{eV}^2$$

← The sign can be determined by matter effects.

$$\left|\Delta m^2_{31}\right| = 2.4 \times 10^{-3}\ \mathrm{eV}^2$$

→ $m_\nu \geq \sqrt{|\Delta m^2_{31}|} \approx 0.05\mathrm{eV}$

Absolute mass scale



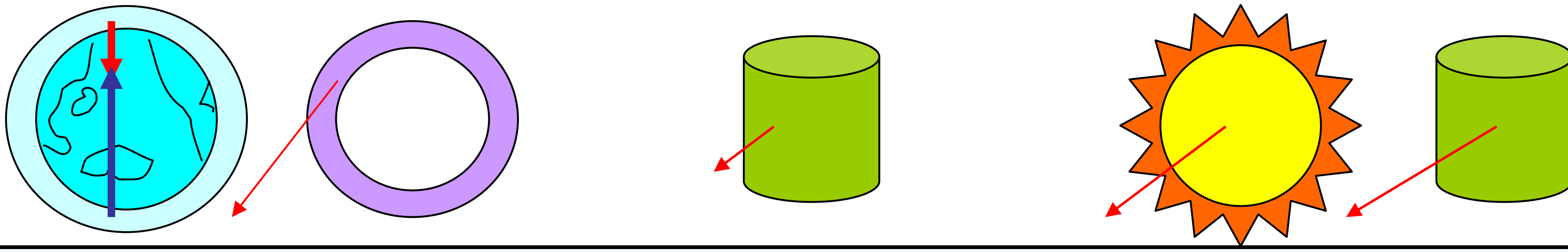
Large neutrino mixing

- **Oscillation data:**

Atmospheric (SK)
Accelerator (K2K,T2K,MINOS)

Reactor (CHOOZ)

Solar (SK,SNO)
Reactor (KamLAND)



$$U_{\mathrm{MNS}} \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{0.5} & \sqrt{0.5} \\ 0 & -\sqrt{0.5} & \sqrt{0.5} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{0.68} & \sqrt{0.32} & 0 \\ -\sqrt{0.32} & \sqrt{0.68} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

45° $\qquad$ $< 11^\circ \,\&\, \delta?$ $\qquad$ 34°

Maximal mixing

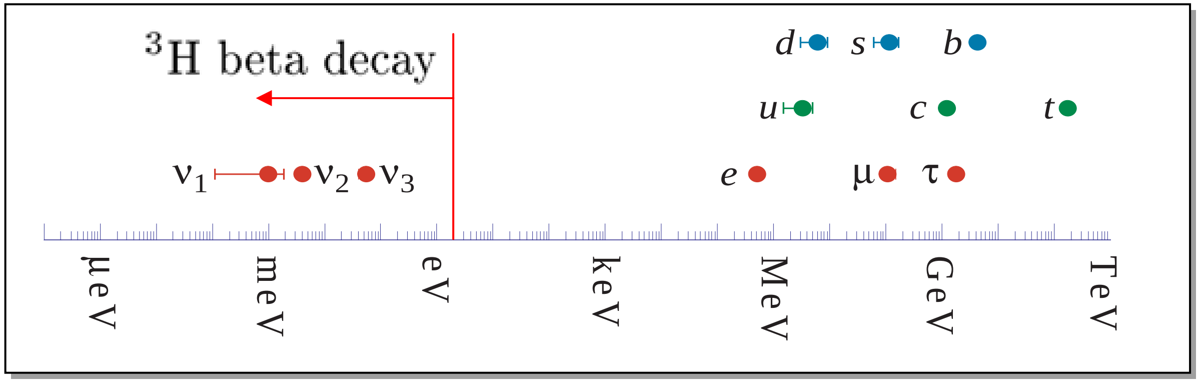
Small mixing
(not yet measured)

Large mixing

Why are neutrino masses so important?

Comparison with other fermions

- **Extremely small mass** → **suggest new phys. Scale?**



- **Large mixing** → **new phys. in lepton sector?**

$$U_{\rm MNS} \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{0.5} & \sqrt{0.5} \\ 0 & -\sqrt{0.5} & \sqrt{0.5} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{0.68} & \sqrt{0.32} & 0 \\ -\sqrt{0.32} & \sqrt{0.68} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$V_{\rm CKM} \simeq \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\,\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\,\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Fundamental fermion?

- **Majorana nature for neutral fermions**
  - Mass term can be written by left-handed field.

$$\frac{1}{2} m \overline{(\nu_L)^c} \nu_L + \mathrm{H.c.}$$

cf. Charged fermion mass term: $m \overline{f_L} f_R + \mathrm{H.c.}$

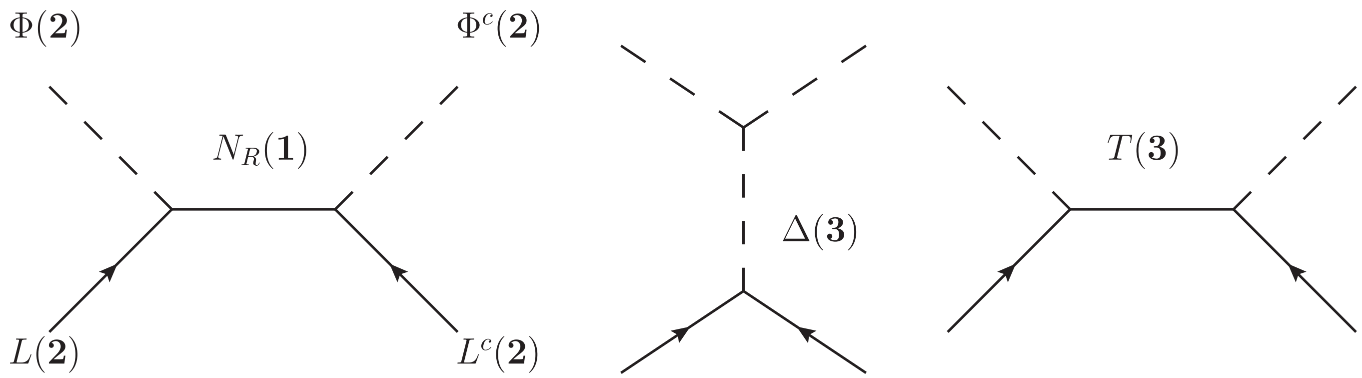
Connections to new physics

- **Dim.5 Weinberg op.**

$$\boxed{\mathcal{O}_W = (L\Phi)^\dagger (L\Phi)} \quad \to \frac{1}{2} m \overline{(\nu_L)^c} \nu_L + \text{H.c.}$$

  - Possible origin of neutrino Majorana mass in the eff. SM

→ **Seesaw I, II and III** (tree-level decomposition)

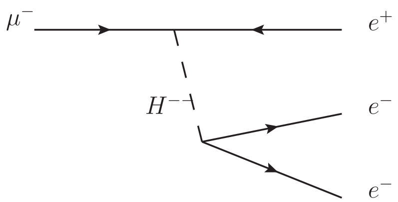


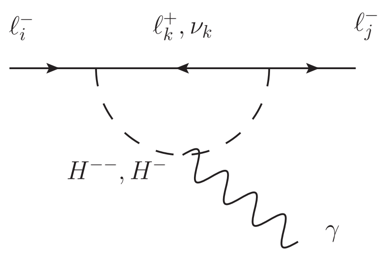
→ New source of mass scale other than EW vev

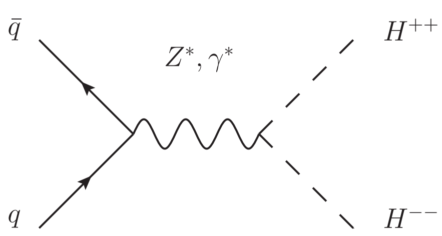
Higgs Triplet Model:

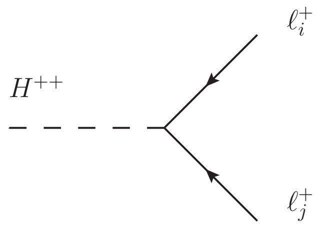
A model for Majorana neutrino mass

- **Motivations:**
  - Rich LFV phenomenology
  - Interesting collider phenomenology









Higgs Triplet Model (HTM)

- **Adding a complex SU(2) triplet scalar with Y=2**

$$\Delta \equiv \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$$

Doubly charged Higgs boson

- **Neutrino mass generation in HTM**

L# = -2

$$h_{\ell\ell'} \left(-\overline{(\ell_L)^c}, \overline{(\nu_{\ell L})^c} \right) \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix} \begin{pmatrix} \nu_{\ell' L} \\ \ell'_L \end{pmatrix} + \text{h.c.}$$

Triplet scalar develops vev: $v_\Delta \equiv \sqrt{2}\langle \Delta^0 \rangle \simeq \frac{\mu v^2}{2M^2}$

$$\frac{1}{2}\sqrt{2}\, v_\Delta h_{\ell\ell'} \overline{(\nu_{\ell L})^c} \nu_{\ell' L} + \text{h.c.} + \cdots$$

L# violation generates NGB?

HTM potential

- Explicit L# breaking to avoid NGB (Majoron)

$$V_{\rm HTM} = -m^2(\Phi^\dagger\Phi) + \lambda_1(\Phi^\dagger\Phi)^2 + M^2{\rm Tr}(\Delta^\dagger\Delta) + \lambda_2[{\rm Tr}(\Delta^\dagger\Delta)]^2 + \lambda_3{\rm Det}(\Delta^\dagger\Delta) + \lambda_4(\Phi^\dagger\Phi){\rm Tr}(\Delta^\dagger\Delta) + \lambda_5(\Phi^\dagger\sigma^i\Phi){\rm Tr}(\Delta^\dagger\sigma^i\Delta) + \left(\frac{1}{\sqrt{2}}\mu(\Phi^T i\sigma^2\Delta^\dagger\Phi) + {\rm h.c.}\right)$$

Triplet scalar develops vev: $v_\Delta \equiv \sqrt{2}\langle\Delta^0\rangle \simeq \frac{\mu v^2}{2M^2}$

Soft L# breaking parameter

Possible realizations for tiny neutrino mass

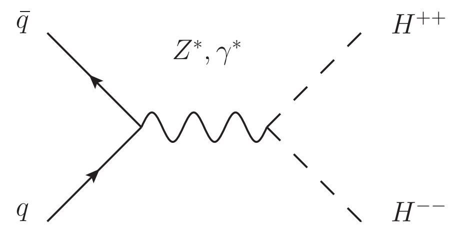
◻ Possible realizations

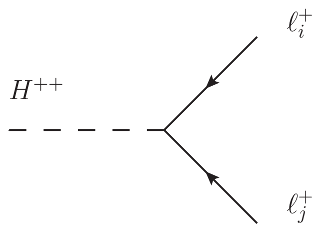
$$(M_\nu)_{\ell\ell'} = \sqrt{2}\, v_\Delta\, h_{\ell\ell'} \simeq \frac{\mu v^2}{\sqrt{2} M^2} h_{\ell\ell'}$$

- ◻ Heavy triplet scalar (M) : often called type2-seesaw
- ◻ Small Yukawa (h_ll)
- ◻ Small L# breaking (μ): moderate (M & h_ll)

→ h_ll can affect low energy LFV,
and triplet scalar can be discovered at the LHC

HTM at the LHC

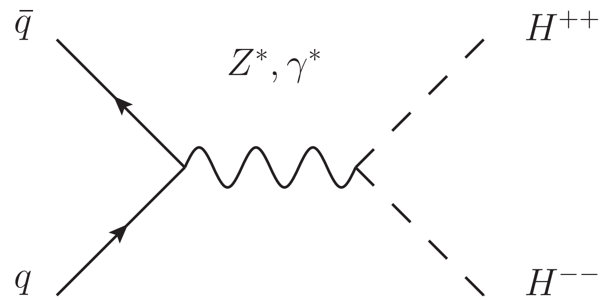


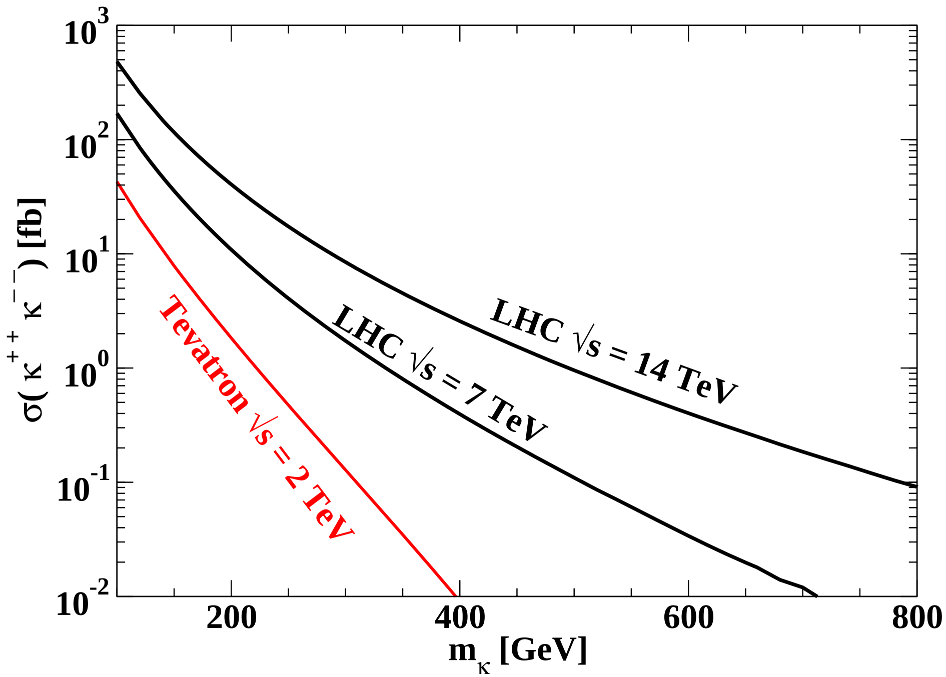


Phenomenology of double charged Higgs bosons

- Pair produced by gaug

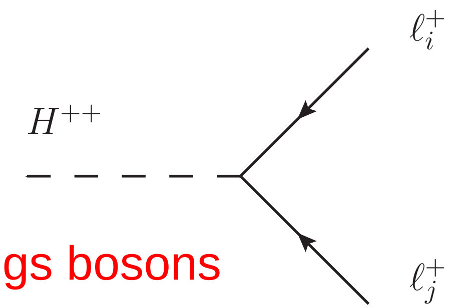
$$q\bar{q} \to \gamma^*, Z^* \to H_i^{++} H_i^{--}$$





- Clear leptonic decay signal

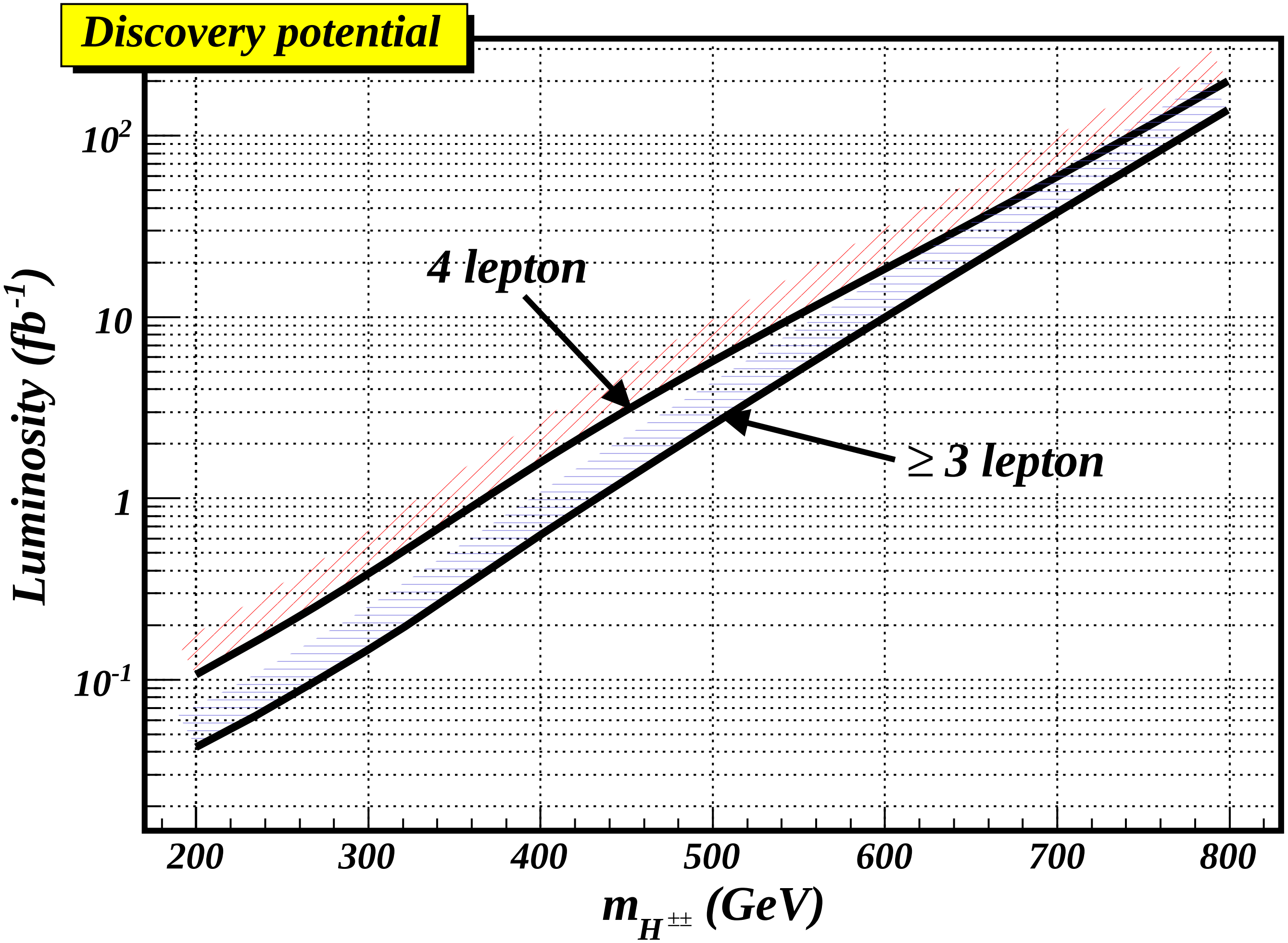
$$H_i^{++} \to \ell_j^+ \ell_k^+$$



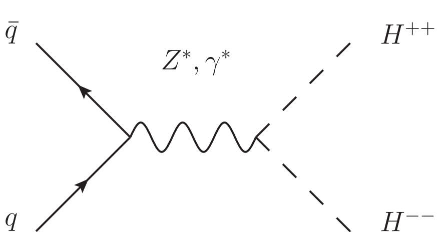
Decays of doubly charged Higgs bosons

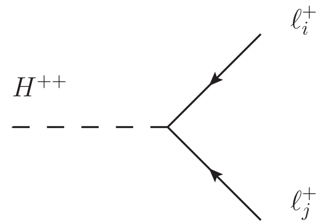
Discovery pot. of triplet Higgs boson @ LHC

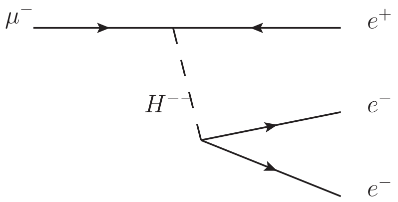
- **Akeroyd, Chiang, JHEP11(2010)005**

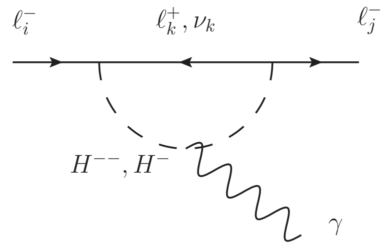


LHC vs Low energy data









Rich Higgs phenomenology in HTM

- Yukawa (h_ll) prop. to Neutrino mass

$$(M_\nu)_{\ell\ell'} = \sqrt{2}\, v_\Delta\, h_{\ell\ell'} \simeq \frac{\mu v^2}{\sqrt{2} M^2} h_{\ell\ell'}$$

- **Rich Higgs phenomenology**
  - H++ can be produced at LHC; M < 1 TeV
  - H++ decays (Testable!!)

    vs neutrino oscillation data

    vs low energy LFV (Lepton flavor violation)

    vs low energy L#V (Lepton number violation)

A4 flavor symmetry

A4 group: alternating group for 4 letters

flavor sym.: Origin of fermion masses and mixings

Why are we focusing on A4 symmetry?

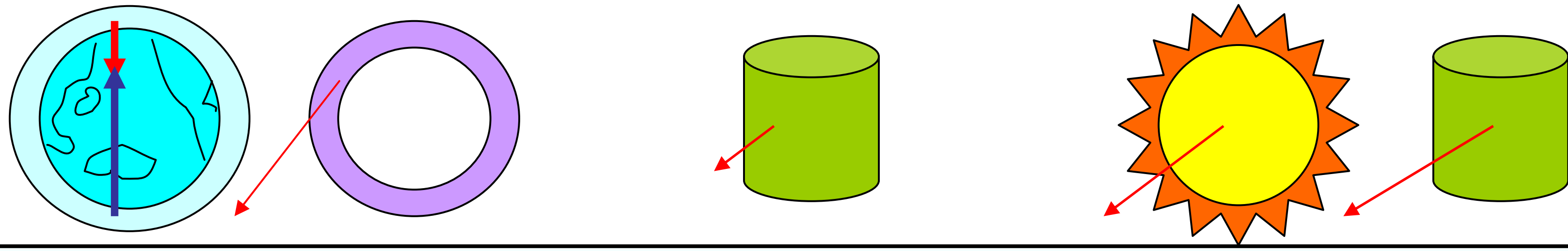
Large neutrino mixing

- **Oscillation data:**

Atmospheric (SK)
Accelerator (K2K,T2K,MINOS)

Reactor (CHOOZ)

Solar (SK,SNO)
Reactor (KamLAND)



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45° $\quad$ $< 11^\circ \ \& \ \delta?$ $\quad$ 34°

Maximal mixing

Small mixing
(not yet measured)

Large mixing

Neutrino mixing

- Tri-Bi-Maximal mixing: good agreement with experiments.

$$U_{\rm MNS} = U_{\rm TB} \equiv \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sim \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \times \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\omega = e^{2\,i\,\pi/3}$$

Z3 symmetry
in charged lepton sector

Z2 symmetry
in neutrino sector

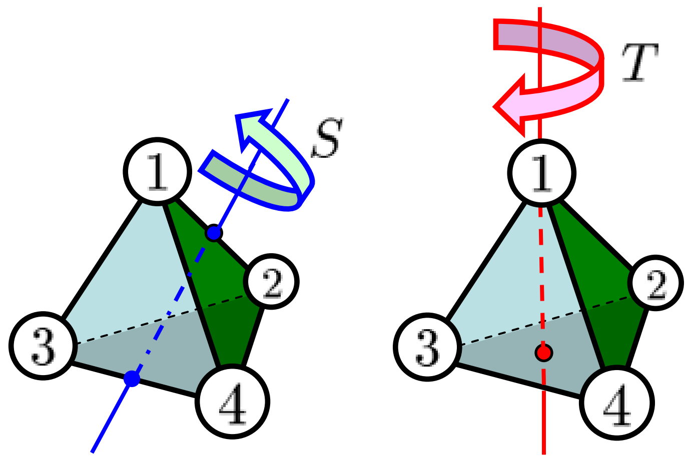
Z6, S3 also contain Z2 and Z3, but there is no irr. 3-rep. → **A4**

A4 group (alternating group for 4 letters)

- **even-permutation of 4 letters** (12 elements)
  - Elemental transposition S: $S^2 = I$ → **Z2**
  - Elemental transposition T: $T^3 = I$ → **Z3**

  others can be obtained from products of S and T, ex., ST, STS, …

$$(1,2,3,4) \left\{ \begin{array}{l} \xrightarrow{\quad e \quad} (1,2,3,4) \\ \xrightarrow{a_1 \equiv S} (\underline{2,1},\underline{4,3}) \\ \xrightarrow{a_2 \equiv T} (1,\underline{3,4,2}) \\ \qquad\quad \vdots \\ \xrightarrow{\quad a_{11} \quad} (2,3,1,4) \end{array} \right.$$



Irreducible representations of A4

- **Transformations under A4**

$$S^2 = T^3 = (ST)^3 = 1$$

1-dim. rep. : $\begin{cases} \underline{\mathbf{1}} : \; S\,\underline{\mathbf{1}} = \underline{\mathbf{1}}, \quad T\,\underline{\mathbf{1}} = \underline{\mathbf{1}} \\ \underline{\mathbf{1}}' : \; S\,\underline{\mathbf{1}}' = \underline{\mathbf{1}}', \quad T\,\underline{\mathbf{1}}' = \omega\underline{\mathbf{1}}' \\ \underline{\mathbf{1}}'' : \; S\,\underline{\mathbf{1}}'' = \underline{\mathbf{1}}'', \quad T\,\underline{\mathbf{1}}'' = \omega^2\underline{\mathbf{1}}'' \end{cases}$

$$\omega \equiv \exp\left(\frac{2\pi i}{3}\right)$$

3-dim. rep. : $\underline{\mathbf{3}} = \begin{pmatrix} \mathbf{3}_x \\ \mathbf{3}_y \\ \mathbf{3}_z \end{pmatrix} : \; S\,\underline{\mathbf{3}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \underline{\mathbf{3}}, \quad T\,\underline{\mathbf{3}} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \underline{\mathbf{3}}$

3-dim. rep. may be related for 3 generation of fermion family

Computation rules

$$\underline{\mathbf{3}} \otimes \underline{\mathbf{3}} = \underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'' \oplus \underline{\mathbf{3}}_s \oplus \underline{\mathbf{3}}_a$$

$$\underline{\mathbf{3}} \otimes \underline{\mathbf{3}} \to \underline{\mathbf{1}} : \ (ab)_{\mathbf{1}} \equiv a_x b_x + a_y b_y + a_z b_z$$

$$\underline{\mathbf{3}} \otimes \underline{\mathbf{3}} \to \underline{\mathbf{1}}' : \ (ab)_{\mathbf{1}'} \equiv a_x b_x + \omega^2 a_y b_y + \omega a_z b_z$$

$$\begin{cases} \xrightarrow{S} a_x b_x + \omega^2(-a_y)(-b_y) + \omega(-a_z)(-b_z) \\ \qquad = a_x b_x + \omega^2 a_y b_y + \omega a_z b_z \\ \xrightarrow{T} a_y b_y + \omega^2 a_z b_z + \omega a_x b_x \\ \qquad = \omega(a_x b_x + \omega^2 a_y b_y + \omega a_z b_z) \end{cases}$$

$$\underline{\mathbf{3}} \otimes \underline{\mathbf{3}} \to \underline{\mathbf{1}}'' : \ (ab)_{\mathbf{1}''} \equiv a_x b_x + \omega a_y b_y + \omega^2 a_z b_z$$

$$\underline{\mathbf{3}} \otimes \underline{\mathbf{3}} \to \underline{\mathbf{3}}_s : \ (ab)_{\mathbf{3}_s} \equiv \begin{pmatrix} a_y b_z + a_z b_y \\ a_z b_x + a_x b_z \\ a_x b_y + a_y b_x \end{pmatrix} \qquad (ab)_{\mathbf{3}_s} = (ba)_{\mathbf{3}_s}$$

$$\underline{\mathbf{3}} \otimes \underline{\mathbf{3}} \to \underline{\mathbf{3}}_a : \ (ab)_{\mathbf{3}_a} \equiv \begin{pmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{pmatrix} \qquad (ab)_{\mathbf{3}_a} = -(ba)_{\mathbf{3}_a}$$

A4HTM:

The minimal A4 symmetric extension of the Higgs triplet model with soft A4 breaking terms

A4HTM (particle contents)

	ψ_{1R}^-	ψ_{2R}^-	ψ_{3R}^-	$\Psi_{AL} = \begin{pmatrix} \psi_{AL}^0 \\ \psi_{AL}^- \end{pmatrix}$
A_4	$\underline{\mathbf{1}}$	$\underline{\mathbf{1}}'$	$\underline{\mathbf{1}}''$	$\underline{\mathbf{3}}$
$SU(2)_L$	singlet	singlet	singlet	doublet
$U(1)_Y$	-2	-2	-2	-1

	$\Phi_A = \begin{pmatrix} \phi_A^+ \\ \phi_A^0 \end{pmatrix}$	$\delta = \begin{pmatrix} \frac{\delta^+}{\sqrt{2}} & \delta^{++} \\ \delta^0 & -\frac{\delta^+}{\sqrt{2}} \end{pmatrix}$	$\Delta_A = \begin{pmatrix} \frac{\Delta_A^+}{\sqrt{2}} & \Delta_A^{++} \\ \Delta_A^0 & -\frac{\Delta_A^+}{\sqrt{2}} \end{pmatrix}$
	$\underline{\mathbf{3}}$	$\underline{\mathbf{1}}$	$\underline{\mathbf{3}}$
	doublet	triplet	triplet
	1	2	2

A4 Yukawa interaction for charged fermions

- **Mass generation**

$$\left(\overline{\Psi_{xL}}\,\Phi_x,\ \overline{\Psi_{yL}}\,\Phi_y,\ \overline{\Psi_{zL}}\,\Phi_z \right) \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} y_1 \Psi^-_{1R} \\ y_2 \Psi^-_{2R} \\ y_3 \Psi^-_{3R} \end{pmatrix} + \text{h.c.}$$

  - Developing aligned vev: $\langle \phi^0_x \rangle = \langle \phi^0_y \rangle = \langle \phi^0_z \rangle = v/\sqrt{6}$
  - Mass eigenvalues:

$$m_e \equiv \frac{y_1}{\sqrt{2}}, \quad m_\mu \equiv \frac{y_2}{\sqrt{2}}, \quad m_\tau \equiv \frac{y_3}{\sqrt{2}}$$

Structures are same for up and down quarks

A4HTM potetial 1

$$
\begin{aligned}
V_{\rm A4HTM} \equiv & V_m + V_1 + V_2 + V_3 + V_4 + V_5 + V_\mu, \\
V_m \equiv & - m_\Phi^2\, (\Phi^\dagger\Phi)_{\mathbf{1}} + M_\delta^2\, {\rm Tr}(\delta^\dagger\delta) + M_\Delta^2\, {\rm Tr}(\Delta^\dagger\Delta)_{\mathbf{1}}, \\
V_4 \equiv & \lambda_{4\delta}\, (\Phi^\dagger\Phi)_{\mathbf{1}}\, {\rm Tr}\big(\delta^\dagger\delta\big) + \lambda_{4\Delta}\, (\Phi^\dagger\Phi)_{\mathbf{1}}\, {\rm Tr}(\Delta^\dagger\Delta)_{\mathbf{1}} \\
& + \left\{\lambda'_{4\Delta p}\, (\Phi^\dagger\Phi)_{\mathbf{1}''}\, {\rm Tr}(\Delta^\dagger\Delta)_{\mathbf{1}'} + {\rm h.c.}\right\} \\
& + \lambda_{4\Delta ss}\, (\Phi^\dagger\Phi)_{\mathbf{3}_s}\, {\rm Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_s} + \lambda_{4\Delta aa}\, (\Phi^\dagger\Phi)_{\mathbf{3}_a}\, {\rm Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_a} \\
& + i\lambda_{4\Delta sa}\, (\Phi^\dagger\Phi)_{\mathbf{3}_s}\, {\rm Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_a} + i\lambda_{4\Delta as}\, (\Phi^\dagger\Phi)_{\mathbf{3}_a}\, {\rm Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_s} \\
& + \left\{\lambda'_{4s}\, \delta^*_{\beta\alpha}\, \left[\Delta_{\beta\alpha}\, (\Phi^\dagger\Phi)_{\mathbf{3}_s}\right]_{\mathbf{1}} + \lambda'_{4a}\, \delta^*_{\beta\alpha}\, \left[\Delta_{\beta\alpha}\, (\Phi^\dagger\Phi)_{\mathbf{3}_a}\right]_{\mathbf{1}} + {\rm h.c.}\right\}, \\
V_5 \equiv & \lambda_{5\delta}\, (\Phi^\dagger\sigma^i\Phi)_{\mathbf{1}}\, {\rm Tr}(\delta^\dagger\sigma^i\delta) + \lambda_{5\Delta}\, (\Phi^\dagger\sigma^i\Phi)_{\mathbf{1}}\, {\rm Tr}(\Delta^\dagger\sigma^i\Delta)_{\mathbf{1}} \\
& + \left\{\lambda'_{5\Delta p}\, (\Phi^\dagger\sigma^i\Phi)_{\mathbf{1}''}\, {\rm Tr}(\Delta^\dagger\sigma^i\Delta)_{\mathbf{1}'} + {\rm h.c.}\right\} \\
& + \lambda_{5\Delta ss}\, (\Phi^\dagger\sigma^i\Phi)_{\mathbf{3}_s}\, {\rm Tr}(\Delta^\dagger\sigma^i\Delta)_{\mathbf{3}_s} + \lambda_{5\Delta aa}\, (\Phi^\dagger\sigma^i\Phi)_{\mathbf{3}_a}\, {\rm Tr}(\Delta^\dagger\sigma^i\Delta)_{\mathbf{3}_a} \\
& + i\lambda_{5\Delta sa}\, (\Phi^\dagger\sigma^i\Phi)_{\mathbf{3}_s}\, {\rm Tr}(\Delta^\dagger\sigma^i\Delta)_{\mathbf{3}_a} + i\lambda_{5\Delta as}\, (\Phi^\dagger\sigma^i\Phi)_{\mathbf{3}_a}\, {\rm Tr}(\Delta^\dagger\sigma^i\Delta)_{\mathbf{3}_s} \\
& + \left\{\lambda'_{5s}\, (\delta^\dagger\sigma^i)_{\alpha\beta}\, \left[\Delta_{\beta\alpha}\, (\Phi^\dagger\sigma^i\Phi)_{\mathbf{3}_s}\right]_{\mathbf{1}} \right. \\
& \qquad\qquad \left. + \lambda'_{5a}\, (\delta^\dagger\sigma^i)_{\alpha\beta}\, \left[\Delta_{\beta\alpha}\, (\Phi^\dagger\sigma^i\Phi)_{\mathbf{3}_a}\right]_{\mathbf{1}} + {\rm h.c.}\right\}.
\end{aligned}
$$

A4HTM potential 2

$$V_{\rm A4HTM} \equiv V_m + V_1 + V_2 + V_3 + V_4 + V_5 + V_\mu,$$

$$\begin{aligned}
V_1 = {} & \lambda_1 \left[(\Phi^\dagger\Phi)_{\mathbf{1}}\right]^2 + \lambda_{1p} (\Phi^\dagger\Phi)_{\mathbf{1}'} (\Phi^\dagger\Phi)_{\mathbf{1}''} \\
& + \lambda_{1ss} \Big((\Phi^\dagger\Phi)_{\mathbf{3}_s} (\Phi^\dagger\Phi)_{\mathbf{3}_s}\Big)_{\mathbf{1}} + \lambda_{1aa} \Big((\Phi^\dagger\Phi)_{\mathbf{3}_a} (\Phi^\dagger\Phi)_{\mathbf{3}_a}\Big)_{\mathbf{1}} \\
& + i\lambda_{1sa} (\Phi^\dagger\Phi)_{\mathbf{3}_s} (\Phi^\dagger\Phi)_{\mathbf{3}_a},
\end{aligned}$$

$$\begin{aligned}
V_2 = {} & \lambda_{2\delta} \left[\mathrm{Tr}(\delta^\dagger\delta)\right]^2 \\
& + \lambda_{2\Delta} \left[\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{1}}\right]^2 + \lambda_{2\Delta p} \mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{1}'}\, \mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{1}''} \\
& + \lambda_{2\Delta ss} \left(\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_s}\, \mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_s}\right)_{\mathbf{1}} + \lambda_{2\Delta aa} \left(\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_a}\, \mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_a}\right)_{\mathbf{1}} \\
& + i\lambda_{2\Delta sa} \left(\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_s}\, \mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_a}\right)_{\mathbf{1}} \\
& + \lambda_{2\delta\Delta 1}\, \mathrm{Tr}(\delta^\dagger\delta)\, \mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{1}} + \lambda_{2\delta\Delta 2}\, (\delta^*_{\beta\alpha}\delta_{\omega\gamma})\, (\Delta_{\beta\alpha}\Delta^*_{\omega\gamma})_{\mathbf{1}} \\
& + \left\{\lambda'_{2\delta\Delta 3}\, (\delta^*_{\beta\alpha}\delta^*_{\omega\gamma})\, \left[\Delta_{\beta\alpha}\Delta_{\omega\gamma}\right]_{\mathbf{1}} + \mathrm{h.c.}\right\} \\
& + \left\{\lambda'_{2\delta\Delta s}\, \delta^*_{\beta\alpha}\, \left[\Delta_{\beta\alpha}(\Delta^*_{\omega\gamma}\Delta_{\omega\gamma})_{\mathbf{3}_s}\right]_{\mathbf{1}} + \mathrm{h.c.}\right\} \\
& + \left\{\lambda'_{2\delta\Delta a}\, \delta^*_{\beta\alpha}\, \left[\Delta_{\beta\alpha}(\Delta^*_{\omega\gamma}\Delta_{\omega\gamma})_{\mathbf{3}_a}\right]_{\mathbf{1}} + \mathrm{h.c.}\right\}.
\end{aligned}$$

A4HTM potential 3

$$V_{\rm A4HTM} \equiv V_m + V_1 + V_2 + V_3 + V_4 + V_5 + V_\mu,$$

$$\begin{aligned}
V_3 = \frac{1}{2}\lambda_{3\delta}&\left\{\left[\mathrm{Tr}(\delta^\dagger\delta)\right]^2 - \mathrm{Tr}\Big(\left[\delta^\dagger\delta\right]^2\Big)\right\}\\
+\frac{1}{2}\lambda_{3\Delta}&\left\{\left[\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{1}}\right]^2 - \mathrm{Tr}\Big(\left[(\Delta^\dagger\Delta)_{\mathbf{1}}\right]^2\Big)\right\}\\
+\frac{1}{2}\lambda_{3\Delta p}&\left\{\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{1}'}\,\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{1}''} - \mathrm{Tr}\Big((\Delta^\dagger\Delta)_{\mathbf{1}'}\,(\Delta^\dagger\Delta)_{\mathbf{1}''}\Big)\right\}\\
+\frac{1}{2}\lambda_{3\Delta ss}&\left\{\Big(\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_s}\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_s}\Big)_{\mathbf{1}} - \mathrm{Tr}\Big((\Delta^\dagger\Delta)_{\mathbf{3}_s}(\Delta^\dagger\Delta)_{\mathbf{3}_s}\Big)_{\mathbf{1}}\right\}\\
+\frac{1}{2}\lambda_{3\Delta aa}&\left\{\Big(\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_a}\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_a}\Big)_{\mathbf{1}} - \mathrm{Tr}\Big((\Delta^\dagger\Delta)_{\mathbf{3}_a}(\Delta^\dagger\Delta)_{\mathbf{3}_a}\Big)_{\mathbf{1}}\right\}\\
+\frac{1}{2}i\lambda_{3\Delta sa}&\left\{\Big(\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_s}\,\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{3}_a}\Big)_{\mathbf{1}} - \mathrm{Tr}\Big((\Delta^\dagger\Delta)_{\mathbf{3}_s}\,(\Delta^\dagger\Delta)_{\mathbf{3}_a}\Big)_{\mathbf{1}}\right\}\\
+\frac{1}{2}\lambda_{3\delta\Delta 1}&\left\{\mathrm{Tr}(\delta^\dagger\delta)\,\mathrm{Tr}(\Delta^\dagger\Delta)_{\mathbf{1}} - \mathrm{Tr}\Big((\delta^\dagger\delta)\,(\Delta^\dagger\Delta)_{\mathbf{1}}\Big)\right\}\\
+\frac{1}{2}\lambda_{3\delta\Delta 2}&\left\{\delta^*_{\beta\alpha}(\Delta_{\beta\alpha}\Delta^*_{\omega\gamma})_{\mathbf{1}}\delta_{\omega\gamma} - \mathrm{Tr}\Big(\delta^\dagger(\Delta\Delta^\dagger)_{\mathbf{1}}\delta\Big)\right\}\\
+\ldots&
\end{aligned}$$

A4HTM potential 4

$$V_{\rm A4HTM} \equiv V_m + V_1 + V_2 + V_3 + V_4 + V_5 + V_\mu,$$

$$V_3 = \dots$$

$$+ \left\{ \frac{1}{2}\lambda'_{3\delta\Delta 3}\left((\delta^*_{\beta\alpha}\delta^*_{\omega\gamma}) \left[\Delta_{\beta\alpha}\Delta_{\omega\gamma}\right]_{\mathbf{1}} - \delta^*_{\beta\alpha}\delta^*_{\omega\gamma}\left[\Delta_{\beta\gamma}\Delta_{\omega\alpha}\right]_{\mathbf{1}}\right) + \text{h.c.}\right\}$$

$$+ \left\{ \frac{1}{2}\lambda'_{3\delta\Delta s}\left(\delta^*_{\beta\alpha} \left[\Delta_{\beta\alpha}(\Delta^*_{\omega\gamma}\Delta_{\omega\gamma})_{\mathbf{3}_s}\right]_{\mathbf{1}} - \delta^*_{\beta\alpha}\left[\Delta_{\beta\gamma}(\Delta^*_{\omega\gamma}\Delta_{\omega\alpha})_{\mathbf{3}_s}\right]_{\mathbf{1}}\right) + \text{h.c.}\right\}$$

$$+ \left\{ \frac{1}{2}\lambda'_{3\delta\Delta a}\delta^*_{\beta\alpha} \left[\Delta_{\beta\alpha}(\Delta^*_{\omega\gamma}\Delta_{\omega\gamma})_{\mathbf{3}_a}\right]_{\mathbf{1}} + \text{h.c.}\right\},$$

$$V_\mu = \frac{1}{\sqrt{2}}\mu_\delta \left[\Phi_\alpha\Phi_\beta\right]_{\mathbf{1}}(i\sigma^2\delta^\dagger)_{\alpha\beta} + \frac{1}{\sqrt{2}}\mu_\Delta\left((\Phi_\alpha\Phi_\beta)_{\mathbf{3}_s}(i\sigma^2\Delta^\dagger)_{\alpha\beta}\right)_{\mathbf{1}} + \text{h.c.}$$

- **Soft A4 breaking terms**

$$\tilde{V}_\mu = \frac{1}{\sqrt{2}}\mu_\delta\left[\Phi_\alpha\Phi_\beta\right]_{\mathbf{1}}(i\sigma^2\delta^\dagger)_{\alpha\beta} + \frac{1}{\sqrt{2}}\mu_{\Delta_x}(2\Phi_{y\alpha}\Phi_{z\beta})(i\sigma^2\Delta^\dagger_x)_{\alpha\beta} + \text{h.c.}$$

Quark mixing

- Unitary transf.

$$\begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} \equiv U_L^\dagger \begin{pmatrix} \Psi_{xL} \\ \Psi_{yL} \\ \Psi_{zL} \end{pmatrix} \qquad \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \psi_{1R}^- \\ \psi_{2R}^- \\ \psi_{3R}^- \end{pmatrix}$$

$$L_\ell \equiv \begin{pmatrix} \nu_{\ell L} \\ \ell_L \end{pmatrix}$$

$$U_L \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Z3 sym. in quark and charged lepton mixing

- **Quark CKM mixing**

$$U_{\rm CKM} = (U_L^d)^\dagger U_L^u = U_L^\dagger U_L = I$$

Unit matrix at LO.
(or Quarks can be coupled with other $\Phi[1]$, next slide)

Possible quark sector

- **Effectively Type-X 2HDM**

Quarks and leptons couple to other Higgs doublet

$$\Phi_1 = \frac{1}{\sqrt{3}}(\Phi_x + \Phi_y + \Phi_z):$$ for leptons

$\Phi_2:$ for quarks

	$\psi_{iR}^{\frac{2}{3}}$	$\psi_{iR}^{-\frac{1}{3}}$	$\Psi_{iQ} = \begin{pmatrix} \psi_{iL}^{\frac{2}{3}} \\ \psi_{iL}^{-\frac{1}{3}} \end{pmatrix}$	Φ_2
A_4	$\underline{\mathbf{1}}$	$\underline{\mathbf{1}}$	$\underline{\mathbf{1}}$	$\underline{\mathbf{1}}$
$SU(2)_L$	singlet	singlet	doublet	doublet
$U(1)_Y$	$4/3$	$-2/3$	$1/3$	1

Neutrino mixing

- Tri-Bi-Maximal mixing: good agreement with experiments.

$$U_{\rm MNS} = U_{\rm TB} \equiv \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sim \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \times \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Z3 symmetry
in charged lepton sector

$$\langle \phi_x^0 \rangle = \langle \phi_y^0 \rangle = \langle \phi_z^0 \rangle = v/\sqrt{6}$$

Z2 symmetry
in neutrino sector

Triplet Yukawa interaction for neutrino masses

$$\left(\overline{(\Psi_{xL})^c},\ \overline{(\Psi_{yL})^c},\ \overline{(\Psi_{zL})^c} \right) \begin{pmatrix} h_\delta i\sigma^2\delta & h_\Delta i\sigma^2\Delta_z & h_\Delta i\sigma^2\Delta_y \\ h_\Delta i\sigma^2\Delta_z & h_\delta i\sigma^2\delta & h_\Delta i\sigma^2\Delta_x \\ h_\Delta i\sigma^2\Delta_y & h_\Delta i\sigma^2\Delta_x & h_\delta i\sigma^2\delta \end{pmatrix} \begin{pmatrix} \Psi_{xL} \\ \Psi_{yL} \\ \Psi_{zL} \end{pmatrix} + \text{h.c.}$$

- 2-3 maximal mixing is preferred in this basis

$$\langle \delta^0 \rangle = \frac{v_\delta}{\sqrt{2}}, \quad \langle \Delta_x^0 \rangle = \frac{v_\Delta}{\sqrt{2}}, \quad \langle \Delta_y^0 \rangle = \langle \Delta_z^0 \rangle = 0$$

$$\frac{1}{\sqrt{2}} \left(\overline{(\psi_{xL}^0)^c},\ \overline{(\psi_{yL}^0)^c},\ \overline{(\psi_{zL}^0)^c} \right) \begin{pmatrix} h_\delta v_\delta & 0 & 0 \\ 0 & h_\delta v_\delta & h_\Delta v_\Delta \\ 0 & h_\Delta v_\Delta & h_\delta v_\delta \end{pmatrix} \begin{pmatrix} \psi_{xL}^0 \\ \psi_{yL}^0 \\ \psi_{zL}^0 \end{pmatrix} + \cdots + \text{h.c.}$$

$$M_\nu = \sqrt{2}\, U_L^T \begin{pmatrix} h_\delta v_\delta & 0 & 0 \\ 0 & h_\delta v_\delta & h_\Delta v_\Delta \\ 0 & h_\Delta v_\Delta & h_\delta v_\delta \end{pmatrix} U_L$$

Neutrino mixing

- Tri-Bi-Maximal mixing: good agreement with experiments.

$$U_{\rm MNS} = U_{\rm TB} \equiv \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sim \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \times \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Z3 symmetry
in charged lepton sector

$$\langle \phi_x^0 \rangle = \langle \phi_y^0 \rangle = \langle \phi_z^0 \rangle = v/\sqrt{6}$$

Z2 symmetry
in neutrino sector

$$\langle \Delta_x^0 \rangle = \frac{v_\Delta}{\sqrt{2}}, \langle \Delta_y^0 \rangle = \langle \Delta_z^0 \rangle = 0$$

Neutrino masses and mixings under A4

- **Diagonalize:** $\mathrm{diag}(m_1 e^{i\alpha_{12}},\ m_2,\ m_3 e^{i\alpha_{32}}) = U_{\mathrm{MNS}}^T M_\nu U_{\mathrm{MNS}}$

$$m_1 e^{i\alpha_{12}} = h_\delta v_\delta + h_\Delta v_\Delta$$

$$m_2 = h_\delta v_\delta \in \mathbf{R}$$

$$m_3 e^{i\alpha_{32}} = -h_\delta v_\delta + h_\Delta v_\Delta$$

$$U_{\mathrm{MNS}} = U_{\mathrm{TB}} \equiv \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

- **A4 sym. + vev alignment → TriBiMaximal mixing**

Good agreement with experiments.

- Note: TB-mixing can be obtained in model without δ[1], but it is required to solve mass degeneracy of m1 & m3.

$$M_\nu = \sqrt{2}\, U_L^T \begin{pmatrix} h_\delta v_\delta & 0 & 0 \\ 0 & h_\delta v_\delta & h_\Delta v_\Delta \\ 0 & h_\Delta v_\Delta & h_\delta v_\delta \end{pmatrix} U_L$$

Approximate symmetry in the broken phase

- Doublet vev is symmetric under T

$$\begin{pmatrix} v/\sqrt{6} \\ v/\sqrt{6} \\ v/\sqrt{6} \end{pmatrix} \xrightarrow{T} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} v/\sqrt{6} \\ v/\sqrt{6} \\ v/\sqrt{6} \end{pmatrix} = \begin{pmatrix} v/\sqrt{6} \\ v/\sqrt{6} \\ v/\sqrt{6} \end{pmatrix}$$

A4 → Z3 sym

- Tiny Triplet vev is symmetric under S

$$\begin{pmatrix} v_\Delta/\sqrt{2} \\ 0 \\ 0 \end{pmatrix} \xrightarrow{S} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} v_\Delta/\sqrt{2} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} v_\Delta/\sqrt{2} \\ 0 \\ 0 \end{pmatrix}$$

A4 → Z2 sym

- **EW precision obs. ρ:** $v \sim 246\text{GeV}, \sqrt{v_\delta^2 + v_\Delta^2} \lesssim 1\text{GeV} \ll v$

→ **approx. Z3 symmetry**
(slightly broken by triplet vev)

All the particle in A4HTM can be classified by approx. Z3 charge !!

Z3 classification

- **Singlets: by default**

$$\underline{\mathbf{1}} : T\,\underline{\mathbf{1}} = \underline{\mathbf{1}}$$
$$\underline{\mathbf{1}}' : T\,\underline{\mathbf{1}}' = \omega\underline{\mathbf{1}}'$$
$$\underline{\mathbf{1}}'' : T\,\underline{\mathbf{1}}'' = \omega^2\underline{\mathbf{1}}''$$

- **Triplets**

$$\underline{\mathbf{3}} : \begin{pmatrix} a_\xi \\ a_\eta \\ a_\zeta \end{pmatrix} \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$$

$$\sqrt{3}a_\xi = a_x + a_y + a_z \xrightarrow{T} a_y + a_z + a_x = \sqrt{3}a_\xi$$
$$\sqrt{3}a_\eta = a_x + \omega^2 a_y + \omega a_z \xrightarrow{T} a_y + \omega^2 a_z + \omega a_x = \omega\sqrt{3}a_\eta$$
$$\sqrt{3}a_\zeta = a_x + \omega a_y + \omega^2 a_z \xrightarrow{T} a_y + \omega a_z + \omega^2 a_x = \omega^2\sqrt{3}a_\zeta$$

	$\underline{\mathbf{1}}$, a_ξ	$\underline{\mathbf{1}}'$, a_η	$\underline{\mathbf{1}}''$, a_ζ
Z_3–charge	1	ω	ω^2

Z3 charges for leptons

	ψ^-_{1R}	ψ^-_{2R}	ψ^-_{3R}	$\Psi_{AL} = \begin{pmatrix} \psi^0_{AL} \\ \psi^-_{AL} \end{pmatrix}$
A_4	$\underline{\mathbf{1}}$	$\underline{\mathbf{1}'}$	$\underline{\mathbf{1}''}$	$\underline{\mathbf{3}}$
$SU(2)_L$	singlet	singlet	singlet	doublet
$U(1)_Y$	-2	-2	-2	-1

Z3 A4

$$\begin{matrix} \mathbf{1} \\ \omega \\ \omega^2 \end{matrix} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \psi^-_{1R} \\ \psi^-_{2R} \\ \psi^-_{3R} \end{pmatrix} \begin{matrix} \underline{\mathbf{1}} \\ \underline{\mathbf{1}'} \\ \underline{\mathbf{1}''} \end{matrix}$$

Z3 A4

$$\begin{matrix} \mathbf{1} \\ \omega \\ \omega^2 \end{matrix} \begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} \equiv U_L^\dagger \begin{pmatrix} \Psi_{xL} \\ \Psi_{yL} \\ \Psi_{zL} \end{pmatrix} \underline{\mathbf{3}}$$

	$L_e,\ e_R$	$L_\mu,\ \mu_R$	$L_\tau,\ \tau_R$
Z_3–charge	1	ω	ω^2

Doubly charged Higgs bosons

- Mass eigenstates can be determined approximately by neglecting tiny effects from triplet vev

$$\begin{pmatrix} H_1^{++} \\ H_2^{++} \\ H_3^{++} \\ H_4^{++} \end{pmatrix} \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{\pm\pm} & s_{\pm\pm} \\ 0 & 0 & -s_{\pm\pm} & c_{\pm\pm} \end{pmatrix} \begin{pmatrix} 1 & \omega & \omega^2 & 0 \\ 1 & \omega^2 & \omega & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & \sqrt{3}\, e^{-i\alpha_{\pm\pm}} \end{pmatrix} \begin{pmatrix} \Delta_x^{++} \\ \Delta_y^{++} \\ \Delta_z^{++} \\ \delta^{++} \end{pmatrix}$$

	$H_3^{++},\ H_4^{++}$	H_2^{++}	H_1^{++}
Z_3–charge	1	ω	ω^2

Doubly charged Higgs Yukawa interaction

$$(h_{i\pm\pm})_{\ell\ell'}\,\overline{(\ell_L)^c}\,\ell'_L\,H_i^{++} + \text{h.c.}$$

1 ω ω^2 ω^2 ω^2

$$\frac{2}{\sqrt{3}} h_\Delta \left\{ -\overline{(e_L)^c}\,\mu_L + \overline{(\tau_L)^c}\,\tau_L \right\} H_1^{++}$$

$$h_{1\pm\pm} = \frac{1}{\sqrt{3}} h_\Delta \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Zeros are consequence of Z3 sym.

$$h_{2\pm\pm} = \frac{1}{\sqrt{3}} h_\Delta \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

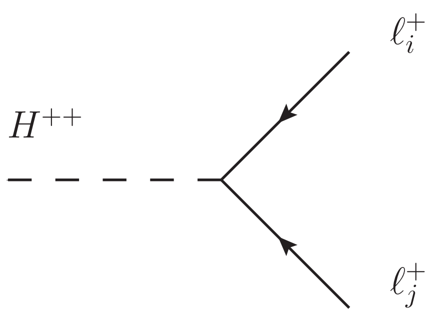
$$h_{3\pm\pm} = \frac{1}{\sqrt{3}} h_\Delta c_{\pm\pm} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + h_\delta s_{\pm\pm} e^{i\alpha_{\pm\pm}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$h_{4\pm\pm} = -\frac{1}{\sqrt{3}} h_\Delta s_{\pm\pm} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + h_\delta c_{\pm\pm} e^{i\alpha_{\pm\pm}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

BRs of doubly charged Higgs bosons

- **A4HTM predicts unique Ratios of BRs.**

	$\mathcal{B}(H^{--} \to \ell\ell')$ $ee(11) : \mu\mu(\omega\omega) : \tau\tau(\omega^2\omega^2) : e\mu(1\omega) : e\tau(\omega^2) : \mu\tau(\omega\omega^2)$
$H_1^{\pm\pm}(\omega)$	$0 : 0 : 2/3 : 1/3 : 0 : 0$
$H_2^{\pm\pm}(\omega^2)$	$0 : 2/3 : 0 : 0 : 1/3 : 0$
$H_3^{\pm\pm}(1)$	$R_3^{\pm\pm}/(1 + R_3^{\pm\pm}) : 0 : 0 : 0 : 0 : 1/(1 + R_3^{\pm\pm})$
$H_4^{\pm\pm}(1)$	$R_4^{\pm\pm}/(1 + R_4^{\pm\pm}) : 0 : 0 : 0 : 0 : 1/(1 + R_4^{\pm\pm})$

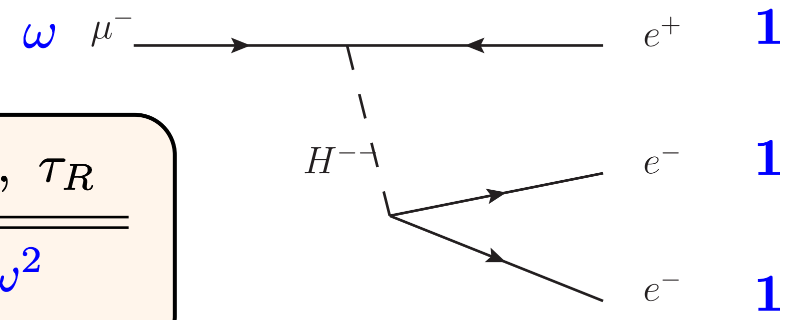


$$R_3^{\pm\pm} \equiv \frac{|2h_\Delta c_{\pm\pm} + \sqrt{3}\, h_\delta s_{\pm\pm} e^{i\alpha_{\pm\pm}}|^2}{2|h_\Delta c_{\pm\pm} - \sqrt{3}\, h_\delta s_{\pm\pm} e^{i\alpha_{\pm\pm}}|^2}$$

$$R_4^{\pm\pm} \equiv \frac{|2h_\Delta s_{\pm\pm} - \sqrt{3}\, h_\delta c_{\pm\pm} e^{i\alpha_{\pm\pm}}|^2}{2|h_\Delta s_{\pm\pm} + \sqrt{3}\, h_\delta c_{\pm\pm} e^{i\alpha_{\pm\pm}}|^2}$$

- **A4HTM (approx. Z3 sym.) forbids specific LFV modes**

In particular, $\mu^- \not\to e^- e^- e^+$



	$L_e,\ e_R$	$L_\mu,\ \mu_R$	$L_\tau,\ \tau_R$
Z_3–charge	1	ω	ω^2

Same for

$$\tau^- \not\to e^+ e^- e^-,\ \mu^+ \mu^- \mu^-,\ e^+ e^- \mu^-,\ \mu^+ \mu^- e^-$$

ω^2 1 1 1 ω^2 ω ω 1 1 ω ω^2 ω 1

- **A4HTM predicts specific LFV tau decays**

$$\tau^- \to e^+ \mu^- \mu^-,\ \mu^+ e^- e^-$$

ω^2 1 ω ω ω^2 1 1

(Triplet-like) Singly charged Higgs Yukawa interaction

$$\sqrt{2}(h_{i\pm})_{\ell\ell'}\,\overline{(\nu_L)^c}\,\ell'_L\,H_i^+ + \text{h.c.}$$

$$h_{1\pm} = h_{1\pm\pm} = \frac{1}{\sqrt{3}} h_\Delta \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$h_{2\pm} = h_{2\pm\pm} = \frac{1}{\sqrt{3}} h_\Delta \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$h_{3\pm} = \frac{1}{\sqrt{3}} h_\Delta \underline{c_\pm} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + h_\delta \underline{s_\pm e^{i\alpha_\pm}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

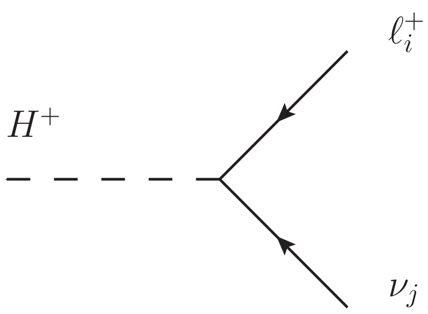
$$h_{4\pm} = -\frac{1}{\sqrt{3}} h_\Delta \underline{s_\pm} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + h_\delta \underline{c_\pm e^{i\alpha_\pm}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

Decays of singly charged Higgs bosons

- **Ratios of BRs**

$$\mathcal{B}(H^- \to \ell\nu) \equiv \sum_i \mathcal{B}(H^- \to \ell\nu_i)$$

	$\mathcal{B}(H^- \to \ell\nu)$ $e\nu : \mu\nu : \tau\nu$
$H_1^\pm$	$1/6 : 1/6 : 2/3$
$H_2^\pm$	$1/6 : 1/6 : 2/3$
$H_3^\pm$	$R_3^\pm/(1+R_3^\pm) : \frac{1}{2}/(1+R_3^\pm) : \frac{1}{2}/(1+R_3^\pm)$
$H_4^\pm$	$R_4^\pm/(1+R_4^\pm) : \frac{1}{2}/(1+R_4^\pm) : \frac{1}{2}/(1+R_4^\pm)$



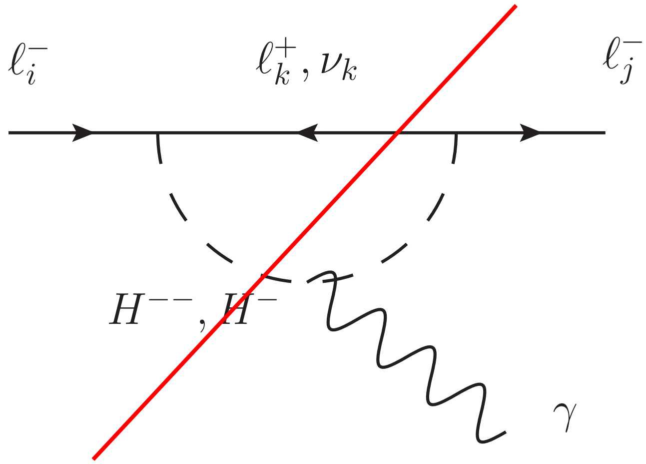
$$R_3^\pm \equiv \frac{|2h_\Delta c_\pm + \sqrt{3}\, h_\delta s_\pm e^{i\alpha_\pm}|^2}{2|h_\Delta c_\pm - \sqrt{3}\, h_\delta s_\pm e^{i\alpha_\pm}|^2}$$

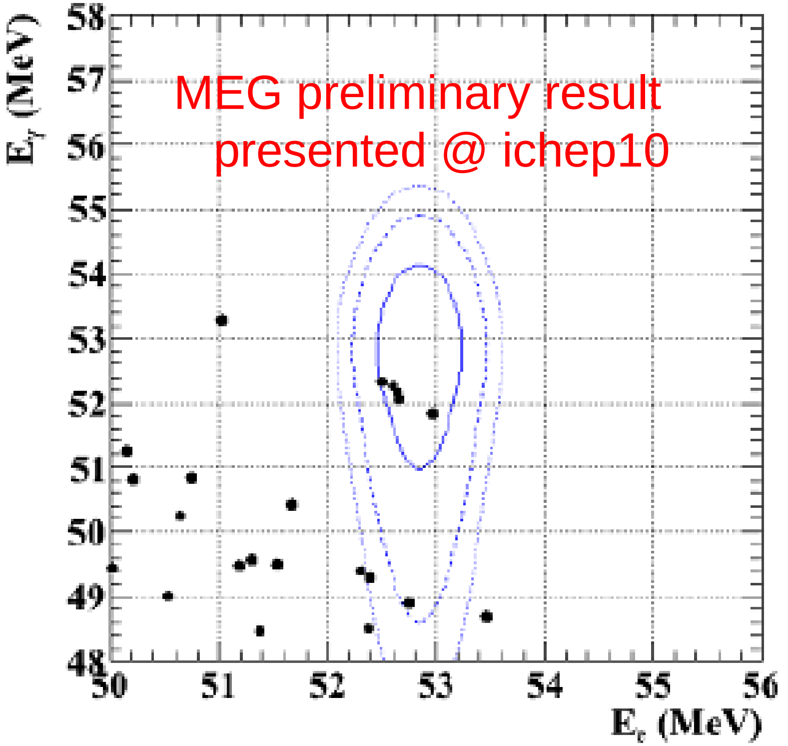
$$R_4^{\pm\pm} \equiv \frac{|2h_\Delta s_\pm - \sqrt{3}\, h_\delta c_\pm e^{i\alpha_\pm}|^2}{2|h_\Delta s_\pm + \sqrt{3}\, h_\delta c_\pm e^{i\alpha_\pm}|^2}$$

Lepton flavor violation $\ell_i^- \not\to \ell_j^- \gamma$

▫ Natural suppression

	$L_e,\ e_R$	$L_\mu,\ \mu_R$	$L_\tau,\ \tau_R$
Z_3–charge	1	ω	ω^2



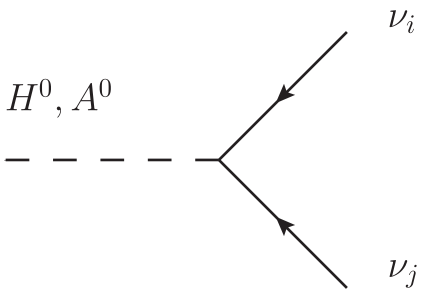


Discovery of μ → e γ excludes A4HTM

It can be excluded by coming MEG data

Neutral Higgs phenomenology

- **Only couple to neutrinos**
  - H0, A0 phenomenology may be pooooooooor



Summary

- **HTM provides new source for neutrino mass.**
- **A4 sym. can give large neutrino mixing and small quark mixing even in HTM.**
- **Remaining Z3 sym. plays an important role in A4HTM.**
  - Unique predictions of triplet Higgs decays
  - Natural suppression of muon LFV

Thank you very much for your attention.

Back up

Lepton universality

- **Charged Higgs contributes to leptonic decay**

$$2\sqrt{2}(G_W + G_{\mu e\ell\ell'})(\bar{\nu}_\ell \gamma_\mu P_L \mu)(\bar{e}\gamma^\mu P_L \nu_{\ell'})$$

$$G_{\mu e\ell\ell'} = \sum_i \frac{(h_{i\pm})_{\ell'\mu}(h^*_{i\pm})_{\ell e}}{2\sqrt{2} m^2_{H_i^+}}$$

Non-standard neutrino interactions

- **Charged Higgs contributes to NSI**

$$2\sqrt{2}G_F\epsilon^{fX}_{\ell\ell'}(\bar{f}\gamma_\mu P_X f)(\bar{\nu}_\ell\gamma^\mu P_L\nu_{\ell'})$$

$$\epsilon^{eL}_{\ell\ell} = \sum_i \frac{(h_{i\pm})_{\ell'\mu}(h^*_{i\pm})_{\ell e}}{2\sqrt{2}G_F m^2_{H_i^+}}$$

Doublet like charged Higgs bosons

- Doublet-triplet mixing is suppressed by vev ratio

$$\begin{pmatrix} H_{1D}^+ \\ H_{2D}^+ \\ H_{NG}^+ \end{pmatrix} \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \phi_x^+ \\ \phi_y^+ \\ \phi_z^+ \end{pmatrix}$$