Theoretical strategies for constraining dark energy

: challenges and pitfalls

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Outline

- Evidences for the current accelerating expansion
  - Geometrical tests : H(z), SNe, CMB, BAO
  - Dynamical tests : Linear growth factor (EG), Nonlinear growth (SCM), Cluster numbers
- Optimal strategies
  - Parametrizations of ω
  - Pitfalls
- Future work : SZe, WL
- Summary

Make sense ?


Of course these are false colors. The reds are really greens, the greens are reds, the blues are oranges and the oranges are blues.
MAKES SENSE

Geometrical Probes

Geometrical probes : SNe Type Ia

Dataset	Redshift Range	# of SN	Filtered subsets	Released
SNLS1	$0.015 \leq z \leq 1.01$	115	SNLS, LR	2005
Gold06	$0.024 \leq z \leq 1.76$	182	SNLS1, HST, SCP, HZSST	2006
ESSENCE	$0.016 \leq z \leq 1.76$	192	SNLS1, HST, ESSENCE	2007
Union	$0.015 \leq z \leq 1.55$	307	Gold06, ESSENCE	2008
Constitution	$0.015 \leq z \leq 1.55$	397	Union, CfA3	2009
SDSS	$0.022 \leq z \leq 1.55$	288	Nearby, SDSS-II, ESSENCE, SNLS, HST	2009

- luminosity distance :

$d_L(z) = c(1+z)\int_0^z \frac{dz'}{H(z')}$, where

$$H(z) = H_0\left(\Omega_{m0}(1+z)^3 + (1-\Omega_{m0}) \times \exp[3\int_0^z (1+\omega(x))\, d\ln(1+x)]\right)^{\frac{1}{2}}$$

- distance modulus :

$$\mu_{th} = m_{th} - M$$
$$= 5\log_{10}[d_L(z)] + 42.38$$

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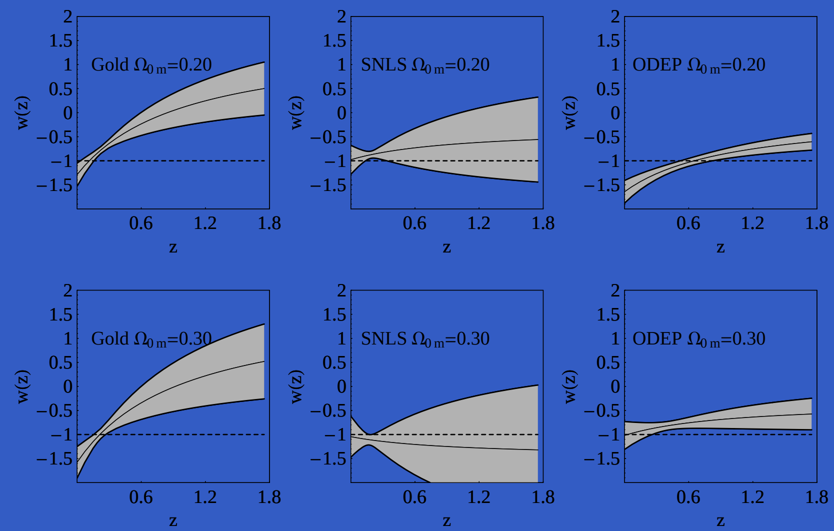

- Consistency with standard rulers : SNLS1 > SDSS > Constitution > Union > ESSENCE > Golden06

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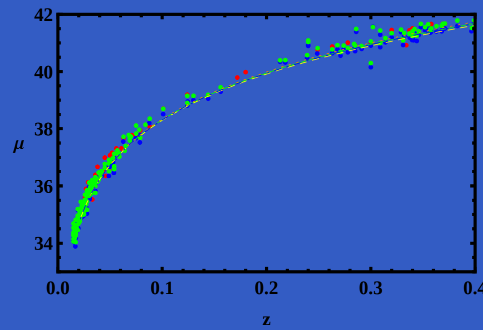
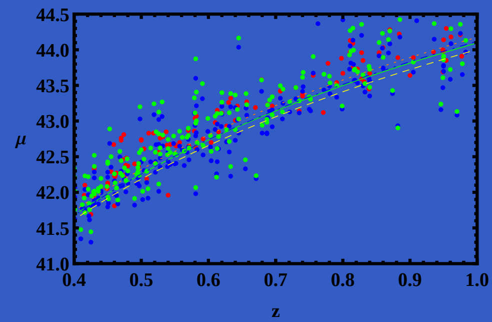




- Models : $\omega = -1.2$(orange), -1(green), -0.8(yellow), and $(\omega_0, \omega_a) = (-0.897, -0.885)$ for CPL (blue)

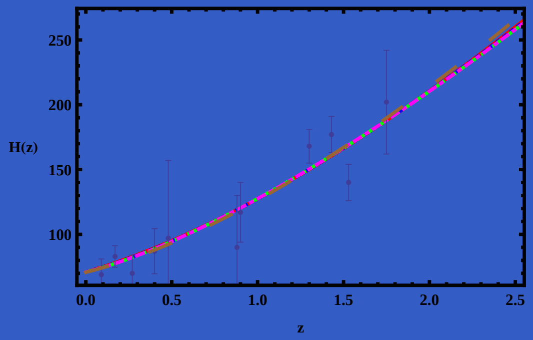
Geometrical probes : H(z)

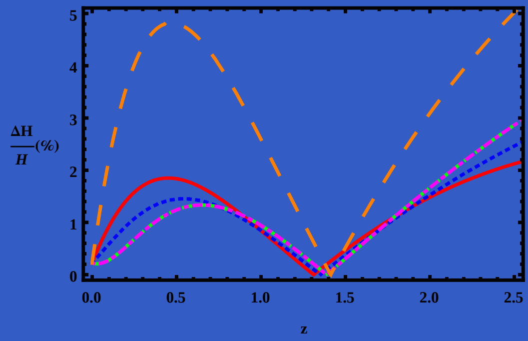
- H(z) from passively evolving galaxies data :

  D.Stern *et.al.* [2010]

z	0.09	0.17	0.27	0.4	0.48
H(z)	69 ± 12	83 ± 8	77 ± 14	95 ± 17	97 ± 62
0.88	0.9	1.3	1.43	1.53	1.75
90 ± 40	117 ± 23	168 ± 17	177 ± 18	140 ± 14	202 ± 40

Geometrical probes : H(z)





- H(z) from different models. Orange(PCA), Red(linear), Blue(logarithmic), Magenta(CPL), Skyblue($\tanh$)
- Relative errors of different model w.r.t ΛCDM : SL[2011]

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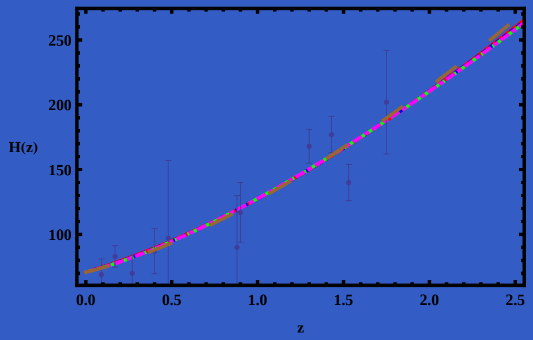

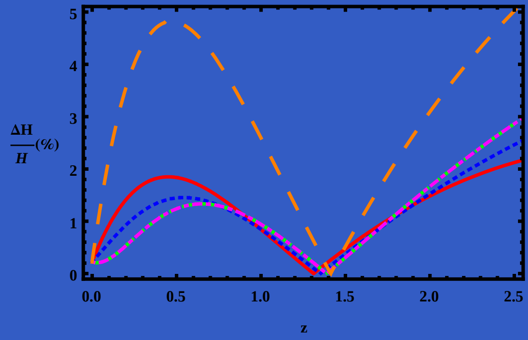

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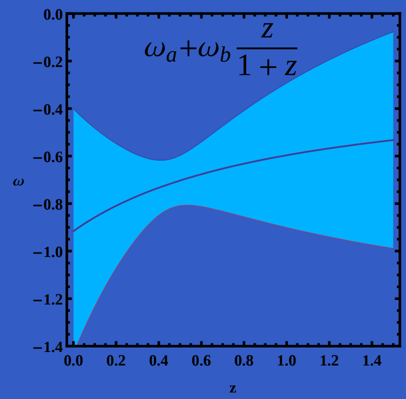

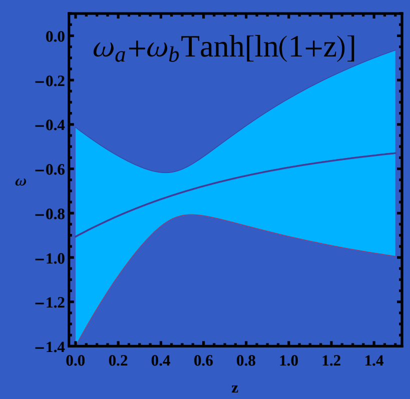

- Models : $\omega = \omega_a + \omega_b \frac{z}{1+z}$ and $\omega_a + \omega_b \tanh[\ln(1+z)]$
- 1-σ error

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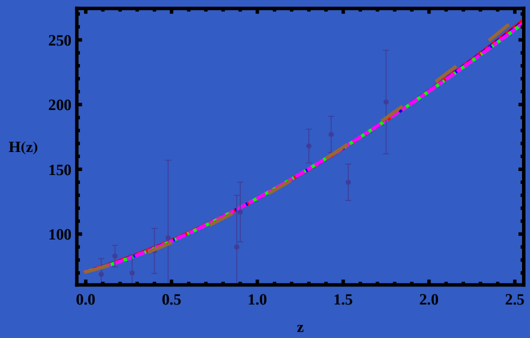

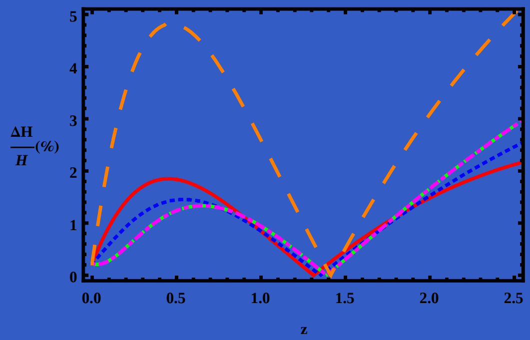

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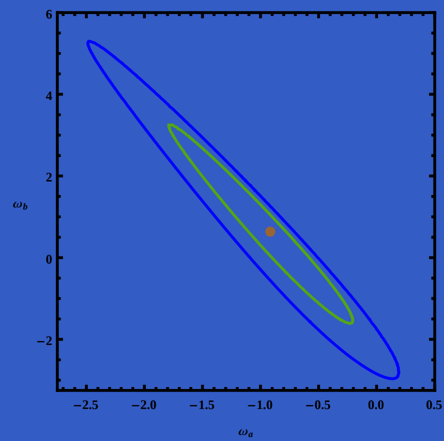

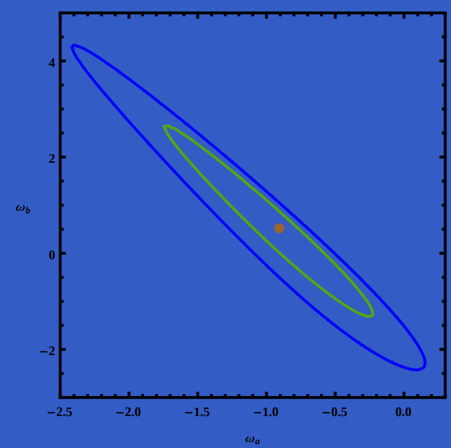

- Contour plots of ω_a and ω_b for the corresponding models 1 and 2-σ

Geometrical probes : CMB & BAO

- CMB :
  - Shift parameter R : the ratio of the location of the first acoustic peak of a reference flat SCDM model to one of a fiducial model : J.R.Bond *et.al.* [1997] , $R_{\mathrm{WMAP}} = \frac{l_1'}{l_1} = 1.123 \pm 0.03$
  - $R = \frac{2}{\sqrt{\Omega_{\mathrm{m}0}}} \frac{q(\Omega_r', a_{rec})}{H_0 r(z)}$, where $q \equiv \left(\sqrt{\Omega_r' + 1} - \sqrt{a_{rec}' + \Omega_r'}\right)$
  - Both CMB and BAO also provide the dynamical probes : ISW effect SL [MPLA,2008] $\Theta_l(k, \eta_0) = (2l+1) \int_{\eta_{rec}}^{\eta_0} d\eta e^{-\tau} 2\dot{\Phi} j_l[k(\eta_0 - \eta)]$ and changing amplitude of BAO SL *et.al.* [PRDR,2010]

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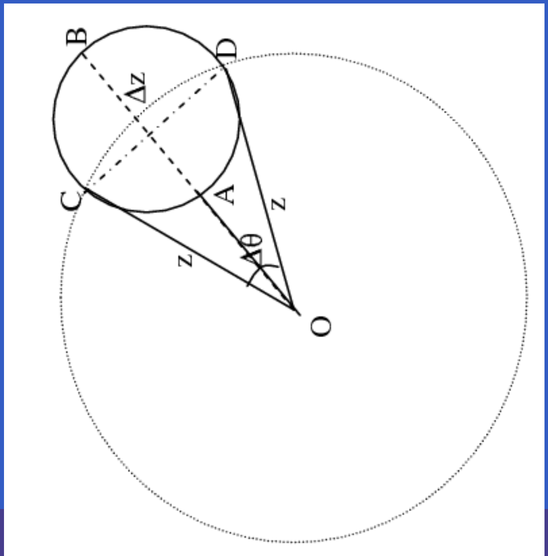

- BAO :
  - Radial size : $AB = \Delta r = \frac{\Delta t}{a} = \frac{\Delta z}{H(z)}$
  - Transverse size : $CD = r\Delta\theta = \Delta\theta \int_0^z \frac{dz}{H(z)}$
  - Dilation scale :
    $D_V(z) = \left[\left(\int_0^{z_{\rm BAO}} \frac{dz}{H(z)}\right)^2 \frac{z_{\rm BAO}}{H(z_{\rm BAO})}\right]^{\frac{1}{3}}$
  - SDSS : $z_{\rm BAO} \simeq 0.35$: D.J. Einstein *et.al.* [2005]
    $D_V(z_{\rm BAO}) = 1370 \pm 64$ Mpc
  - Similar to Alcock-Pazcynski (AP) test $\frac{\Delta z}{\Delta \theta} = H(z) r(z)$

Dynamical Probes

Linear growth factor

- at sub-horizon scale, matter density perturbation $\delta_m(\vec{k},a) = \delta_0(k)D_g(a)$ grows uniformly as long as DE does not cluster : $\frac{d^2D}{da^2} + \left(\frac{d\ln H}{da} + \frac{3}{a}\right)\frac{dD}{da} - \frac{3}{2}\frac{\Omega_{\rm m0}}{a^5}f(k,a)D = 0$
- $D(a) = c_1\left(\frac{\Omega_{\rm m0}}{\Omega^0_{\rm de}}\right)^{\frac{3\omega-1}{6\omega}} a^{\frac{3\omega-1}{2}} F\left[\frac{1}{2} - \frac{1}{2\omega}, \frac{1}{2} + \frac{1}{3\omega}, \frac{3}{2} - \frac{1}{6\omega}, -\frac{\Omega_{\rm m0}}{\Omega^0_{\rm de}}a^{3\omega}\right] +$ $c_2 F\left[-\frac{1}{3\omega}, \frac{1}{2\omega}, \frac{1}{2} + \frac{1}{6\omega}, -\frac{\Omega_{\rm m0}}{\Omega^0_{\rm de}}a^{3\omega}\right]$: SL *et.al.* [PRD2010, PLB2010]

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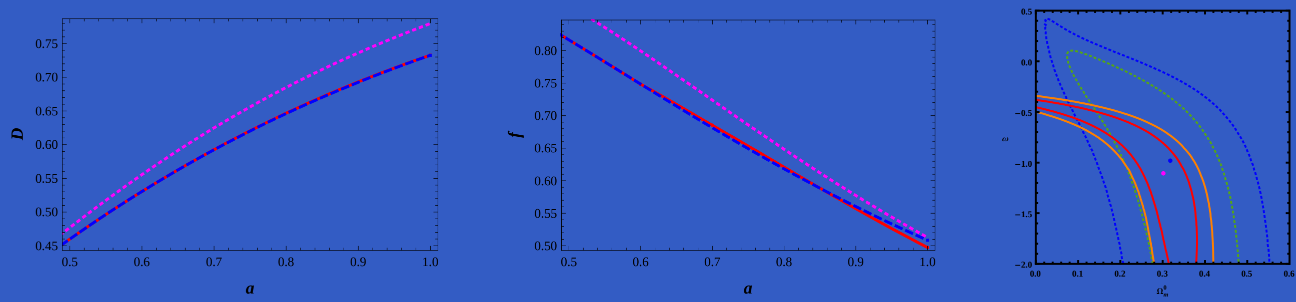

- $f(a) = \frac{d \ln D}{d \ln a} \equiv \Omega_m(a)^{\gamma}$ SL *et.al.* [2009]
- $\gamma_L^0 = \frac{\ln\left[-\frac{3}{2}\Omega_m^0 + (\Omega_m^0)^{\frac{3}{2}} \frac{\Gamma[\frac{11}{6}]/\Gamma[\frac{5}{6}]}{F[\frac{3}{2},\frac{5}{6},\frac{11}{6},-\Omega_{de}^0/\Omega_m^0]} \right]}{\ln \Omega_m^0}$
- $E_G = \frac{\Omega_{\rm m0}}{f(a)}$ SL *et.al.* [in preparation], SL [JCAP 2011]

Spherical collapse model

- spherical collapse model $\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\left[\rho_{\rm cluster} + (1+3\omega)\rho_{\rm dec}\right]$, $\dot{\rho}_{\rm cluster} + 3\left(\frac{\dot{R}}{R}\right)\rho_{\rm cluster} = 0$
$\dot{\rho}_{\rm dec} + 3(1+\omega)\left(\frac{\dot{R}}{R}\right)\rho_{\rm dec} = \alpha\Gamma$ where $\Gamma = 3(1+\omega)\left(\frac{\dot{R}}{R} - \frac{\dot{a}}{a}\right)\rho_{\rm dec}$ with $0 \leq \alpha \leq 1$ $\zeta \equiv \frac{\rho_{\rm cluster}}{\rho_m}\Big|_{ta}$
- $D(a) = c_1\left(\frac{\Omega_{\rm m0}}{\Omega^0_{\rm de}}\right)^{\frac{3\omega-1}{6\omega}} a^{\frac{3\omega-1}{2}} F\left[\frac{1}{2} - \frac{1}{2\omega}, \frac{1}{2} + \frac{1}{3\omega}, \frac{3}{2} - \frac{1}{6\omega}, -\frac{\Omega_{\rm m0}}{\Omega^0_{\rm de}} a^{3\omega}\right] +$
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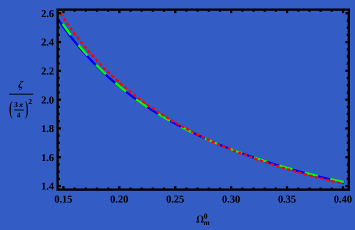

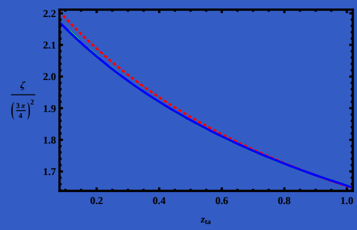

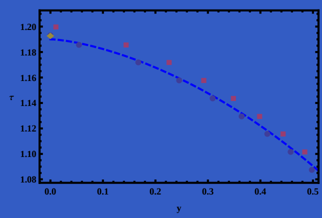

- $\zeta_{\rm sk} = \left(\frac{3\pi}{4}\right)^2 \Omega_{\rm mta}^{-0.724+0.157\Omega_{\rm mta}+\alpha(1+\omega_{\rm de})(1+3\omega_{\rm de})(0.064-0.368\Omega_{\rm mta})}$ SL *et.al.* [JCAP 2010]
- $\delta_{\rm lin}(z_{\rm vir}) = \frac{3}{5}(\sqrt{\zeta})^{\frac{2}{3}}\left|\left(\frac{3}{4} + \frac{9\pi}{8}\right)\frac{1}{\sqrt{\zeta}}\right|^{\frac{2}{3}} = \frac{3}{20}(6+9\pi)^{\frac{2}{3}} \simeq 1.58$
- $\Delta(z_{\rm vir}) = \frac{\rho_{\rm cluster}}{\rho_{\rm m}}\Big|_{z_{\rm vir}} = \zeta\left(\frac{x_{\rm vir}}{y_{\rm vir}}\right)^3 = 18\pi^2 \frac{x^3_{\rm vir}}{4(F\left[\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, -\frac{x_{\rm vir}}{Q_{\rm ta}}\right])^2} \rightarrow 147$ instead of 178

Cluster number

- linear perturbation of DE : $\delta\ddot{Q} + 3H\delta\dot{Q} + (k^2 + V_{,QQ})\delta Q = -\frac{1}{2}\dot{h}\dot{Q}_0$
- linear power spectrum of δ_m : $P(k,a) = A_Q k^{n_s} T_Q^2(k)\left(\frac{D(a)}{D(a_0)}\right)^2$, where $A_Q = 2\pi^2\delta_H^2(c/H_0)^{n_s+3}$,

  $\delta_H = 2.05 \times 10^{-5}\alpha_0^{-1}(\Omega_m)^{c_1+c_2\ln\Omega_m}\exp[c_3(n_s-1)+c_4(n_s-1)^2]$ with
  $c_1 = -0.789|\omega|^{0.0754-0.211\ln|\omega|}$, $c_2 = -0.118 - 0.0727\omega$, $c_3 = -1.037$, $c_4 = -0.138$,
  $\alpha = (-\omega)^s$,
  $s = (0.012 - 0.036\omega - 0.017\omega^{-1})\Big(1 - \Omega_m(a)\Big) + (0.098 + 0.029\omega - 0.085\omega^{-1})\ln\Omega_m(a)$: Ma *et.al.* [1999], SL *et.al.* [2010]

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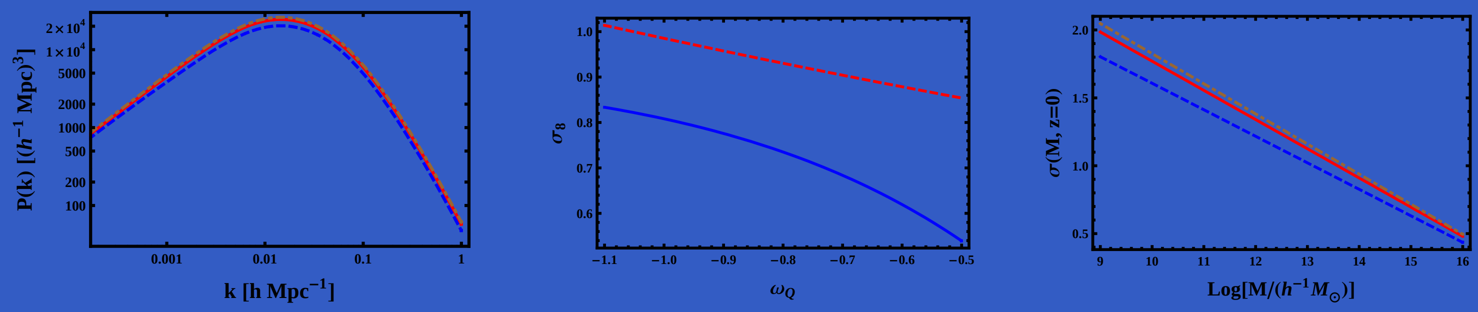

- rms linear mass fluctuation : $\sigma_R^2(a) \equiv \left\langle \left|\frac{\delta M}{M(R,a)}\right|^2 \right\rangle = \frac{1}{2\pi^2}\int_0^\infty k^2 P(k,a)\left|W(kR)\right|^2 dk$
- $\omega = -1.1$ (brown), -1.0 (red), -0.8 (blue)
- $\sigma(M,z) \simeq (-\omega)^{0.72+0.36\omega}\left(3.90 - 0.215\log\left[\frac{M}{h^{-1}M_\odot}\right]\right)\left(\frac{D_g(z)}{D_g(z_0)}\right)$

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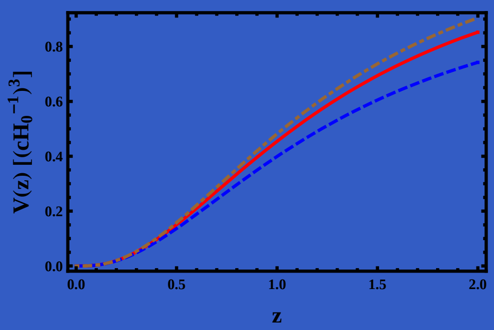
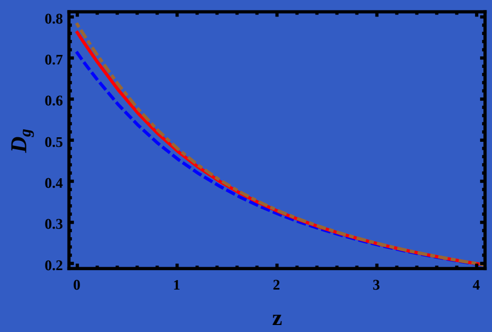




- $V(z) = \int_0^z 4\pi d_A^2(z')\left|\frac{cdt}{dz}\right|(z')dz'$
- linear growth factor : D_g

Cluster number

- linear perturbation of DE : $\delta\ddot{Q} + 3H\delta\dot{Q} + (k^2 + V_{,QQ})\delta Q = -\frac{1}{2}\dot{h}\dot{Q}_0$

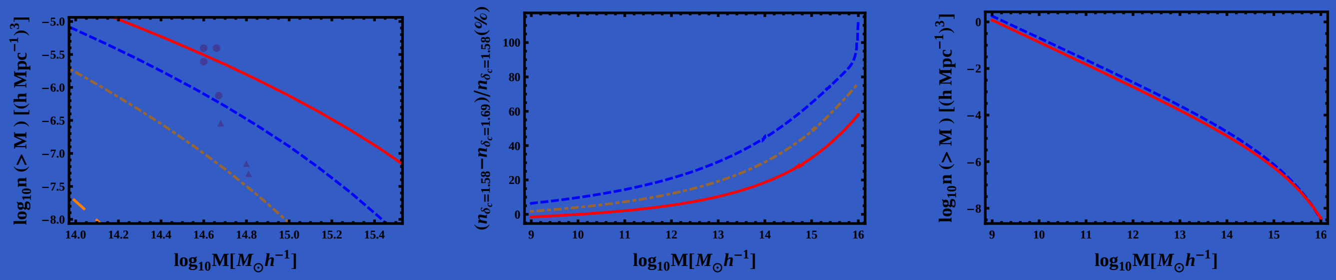
- linear power spectrum of δ_m : $P(k,a) = A_Q k^{n_s} T_Q^2(k)\left(\frac{D(a)}{D(a_0)}\right)^2$, where $A_Q = 2\pi^2\delta_H^2(c/H_0)^{n_s+3}$,

  $\delta_H = 2.05 \times 10^{-5}\alpha_0^{-1}(\Omega_m)^{c_1+c_2\ln\Omega_m}\exp[c_3(n_s-1)+c_4(n_s-1)^2]$ with

  $c_1 = -0.789|\omega|^{0.0754-0.211\ln|\omega|}$, $c_2 = -0.118 - 0.0727\omega$, $c_3 = -1.037$, $c_4 = -0.138$,

  $\alpha = (-\omega)^s$,

  $s = (0.012 - 0.036\omega - 0.017\omega^{-1})\left(1-\Omega_m(a)\right) + (0.098 + 0.029\omega - 0.085\omega^{-1})\ln\Omega_m(a)$: Ma *et.al.* [1999], SL *et.al.* [2010]



- comoving number density for different $z = 0$, 0.5, 1.0, 2.0 (from top to bottom) when $\omega = -1$ and $\delta_c = 1.58$ Data from R.G. Carlberg *et.al.* [1997] errors of n when $\delta_c = 1.69$ PS : $dn(M,z) = \sqrt{\frac{2}{\pi}}\frac{\rho_m^0}{M^2}\left|\frac{d\ln\sigma}{d\ln M}\right|\frac{\delta_c}{\sigma}\exp\left[-\frac{\delta_c^2}{2\sigma^2}\right]dM$

- $n(>M)$ from PS with $\delta_c = 1.58$ (blue) and ST with 1.69 (red) $f_{\rm ST}(\sigma) = A\sqrt{\frac{2b}{\pi}}\exp\left[-\frac{b\delta_c^2}{2\sigma^2}\right]\left[1+\left(\frac{\sigma^2}{b\delta_c^2}\right)^p\right]\frac{\delta_c}{\sigma}$

Cluster number

- linear perturbation of DE : $\delta\ddot{Q} + 3H\delta\dot{Q} + (k^2 + V_{,QQ})\delta Q = -\frac{1}{2}\dot{h}\dot{Q}_0$
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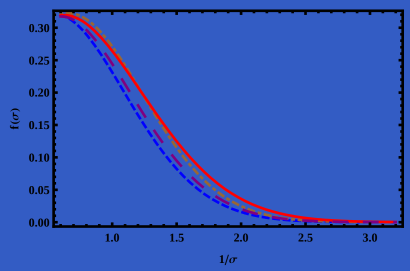
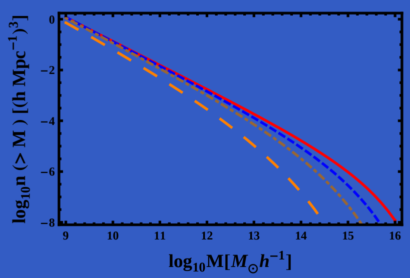
  $\delta_H = 2.05 \times 10^{-5}\alpha_0^{-1}(\Omega_m)^{c_1+c_2\ln\Omega_m}\exp[c_3(n_s-1)+c_4(n_s-1)^2]$ with

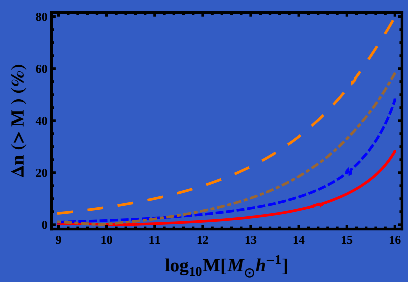
  $c_1 = -0.789|\omega|^{0.0754-0.211\ln|\omega|}$, $c_2 = -0.118 - 0.0727\omega$, $c_3 = -1.037$, $c_4 = -0.138$,

  $\alpha = (-\omega)^s$,

  $s = (0.012 - 0.036\omega - 0.017\omega^{-1})\left(1-\Omega_m(a)\right) + (0.098 + 0.029\omega - 0.085\omega^{-1})\ln\Omega_m(a)$: Ma *et.al.* [1999], SL *et.al.* [2010]







- different mass functions : $f_{\rm ST}$, f_{Manera}, f_{LN}, f_{mod} (from top to bottom) , Using original :

  $$f_{\rm LN}(\sigma,z) = 0.32\sqrt{\frac{2(0.67)}{\pi}}\exp\left[-\frac{0.67\delta_c^2}{2\sigma^2}\right]\left[1+\left(\frac{\sigma^2}{0.67\delta_c^2}\right)^{0.32}\right]\frac{\delta_c}{\sigma}$$

- $n(>M)$ with $f_{\rm LN}$ for $\omega = -1$ at $z = 0$, 0.5, 1, 2 (from top to bottom) relative errors of $n(>M)$ between $\omega = -1.1$ and -1.0 at $z = 0$(solid) and 1(dashed) and between $\omega = -0.8$ and -1.0 at $z = 0$(dot-dashed) and 1(long dashed)

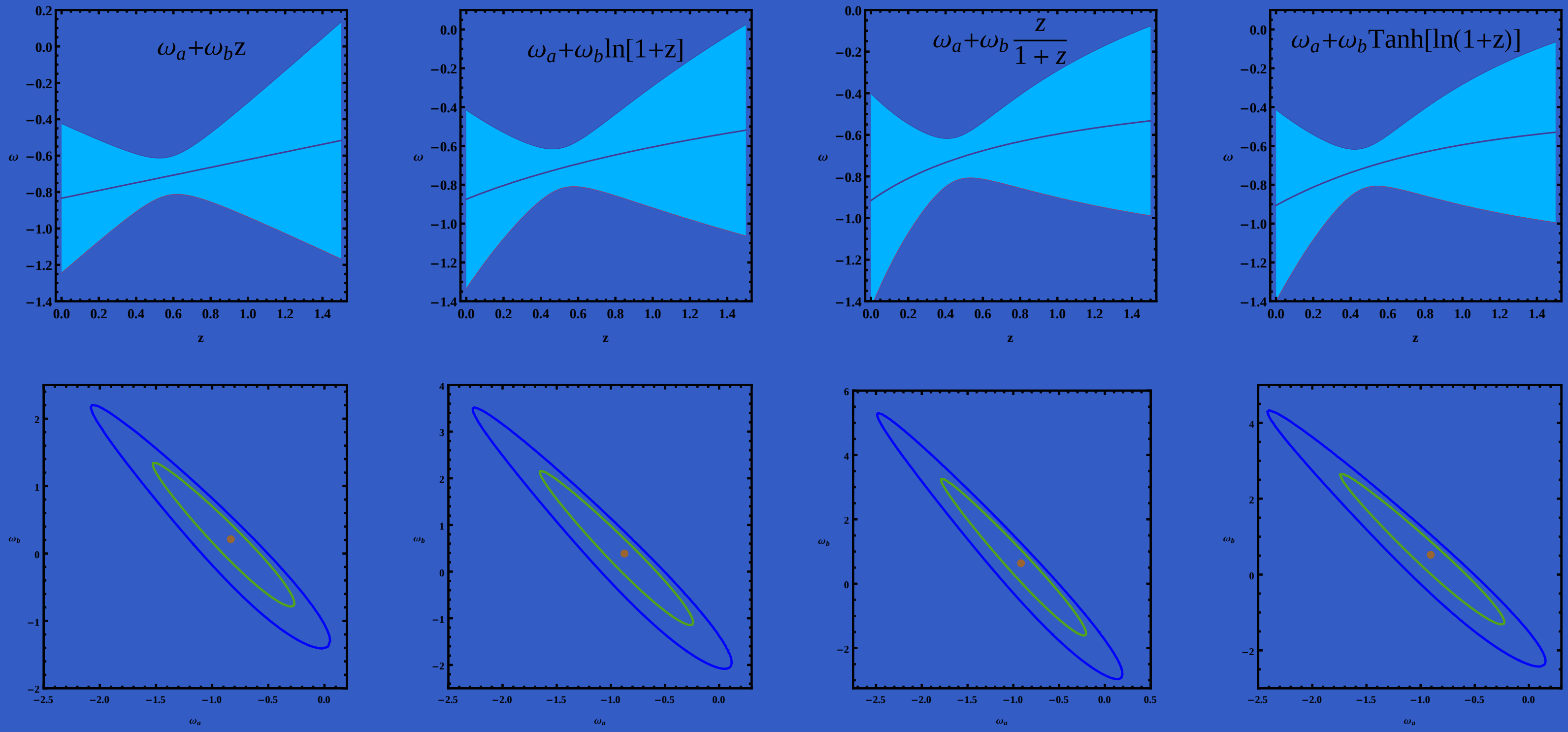
Optimal Strategies

Parametrization of ω

- Absence of compelling model requires the parametrization of ω :
  - Principal component analysis (PCA) : $\omega = \sum_i \omega_i \Theta(z - z_i)$ Tegmark *et.al.* [ApJ 1997]
  - so-called CPL parametrization : $\omega = \omega_a + \omega_b \frac{z}{1+z}$
  - $\omega = \omega_a + \omega_b \tanh[\ln[1+z]]$ SL [2011]
- Fisher matrix : $F_{lm} = -\sum \frac{\mathcal{O}_i - \mathcal{O}(z_i,\vec{p})}{\sigma_i^2} \frac{\partial^2 \mathcal{O}(z_i,\vec{p})}{\partial p_l \partial p_m} + \sum \frac{1}{\sigma_i^2} \frac{\partial \mathcal{O}(z_i,\vec{p})}{\partial p_l} \frac{\partial \mathcal{O}(z_i,\vec{p})}{\partial p_m} \simeq$
  $\sum \frac{1}{\sigma_i^2} \frac{\partial \mathcal{O}(z_i,\vec{p})}{\partial p_l} \frac{\partial \mathcal{O}(z_i,\vec{p})}{\partial p_m} \equiv \widetilde{F}_{lm}$
  - when $\rho_{\rm de}$ is a direct variable of $\mathcal{O}$:
    $\widetilde{F}_{lm} = \sum \frac{1}{\sigma_i^2} \left(3 \frac{\partial \mathcal{O}(z_i,\vec{p})}{\partial \ln[\rho_{\rm de}(z_i,\vec{p})]}\right)^2 \left(\int_0^{z_i} \frac{\partial \omega}{\partial p_l} d\ln(1+x)\right)\left(\int_0^{z_i} \frac{\partial \omega}{\partial p_m} d\ln(1+x)\right) \equiv$
    $\sum G_i^2 \left(\int_0^{z_i} \frac{\partial \omega}{\partial p_l} d\ln(1+x)\right)\left(\int_0^{z_i} \frac{\partial \omega}{\partial p_m} d\ln(1+x)\right)$
- $\det(\widetilde{F})_H = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} G_i^2 G_j^2 \left(\ln[1+z_i] \int_0^{z_j} f(x) d\ln[1+x] - \ln[1+z_j] \int_0^{z_i} f(x) d\ln[1+x]\right)^2 =$

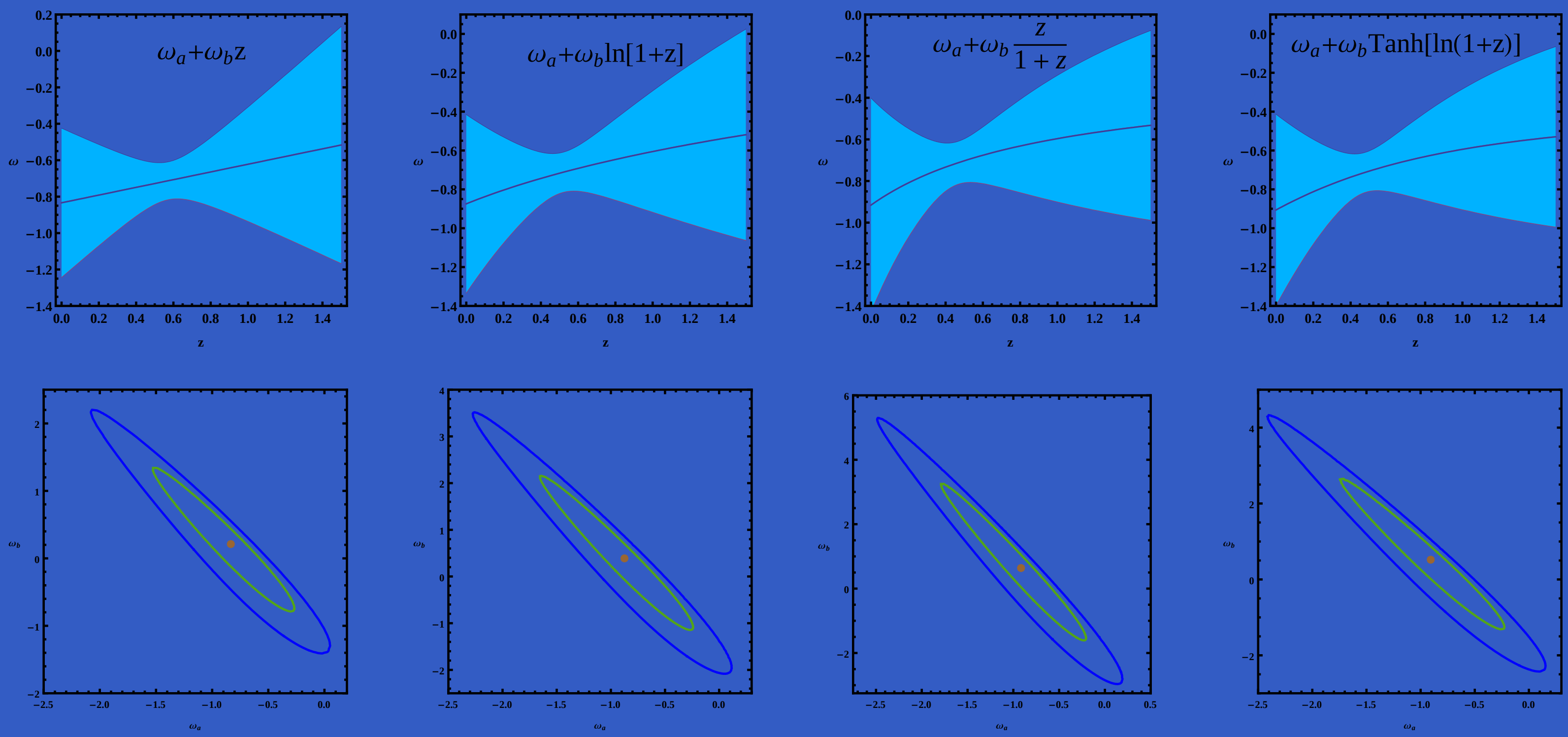
$$\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} G_i^2 G_j^2 \begin{cases} \left(\ln[1+z_i] z_j - z_i \ln[1+z_j]\right)^2 & \text{if } f(z) = z \\ \left(\frac{1}{2} \ln[1+z_i] \ln[1+z_j] \ln\left[\frac{1+z_j}{1+z_i}\right]\right)^2 & \text{if } f(z) = \ln[1+z] \\ \left(\frac{z_i}{1+z_i} \ln[1+z_j] - \frac{z_j}{1+z_j} \ln[1+z_i]\right)^2 & \text{if } f(z) = \frac{z}{1+z} \end{cases}$$

Parametrization of ω



- $\sigma_\omega = \sum_{l=1}^{n}\left(\frac{\partial\omega}{\partial\omega_l}\right)^2 C_{ll} + 2\sum_{l=1}^{n}\sum_{m=l+1}^{n}\left(\frac{\partial\omega}{\partial\omega_l}\right)\left(\frac{\partial\omega}{\partial\omega_m}\right)C_{lm} =$
  $C_{aa} + C_{bb} f(z)^2 + 2C_{ab} f(z)$

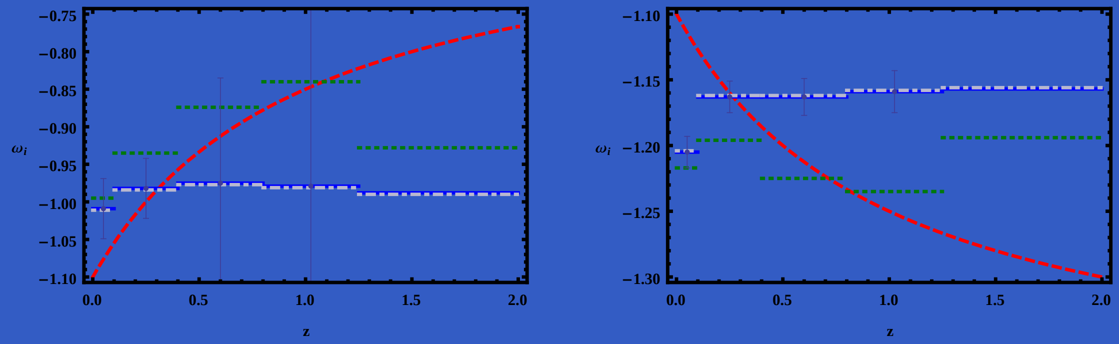
Parametrization of ω



$f(z)$	$\sum \frac{\mathcal{O}_i-\mathcal{O}(z_i,\vec{p})}{\sigma_i^2}\frac{\partial^2\mathcal{O}(z_i,\vec{p})}{\partial\omega_a\partial\omega_b}$			$\sum \frac{1}{\sigma_i^2}\frac{\partial\mathcal{O}(z_i,\vec{p})}{\partial\omega_a}\frac{\partial\mathcal{O}(z_i,\vec{p})}{\partial\omega_b}$			ω_a^*	ω_b^*	$\chi^2_{\min}$	$\det(F)$	Ref
	(a,a)	(a,b)	(b,b)	(a,a)	(a,b)	(b,b)					
z	0.19	-0.66	-1.04	92.51	52.19	31.29	-0.834	0.211	9.486	149.3	[?]
$\ln[1+z]$	-0.12	-0.46	-0.45	93.14	38.52	16.72	-0.875	0.388	9.446	65.07	[?]
$\frac{z}{1+z}$	-0.34	-0.33	-0.21	93.66	29.65	9.75	-0.916	0.638	9.412	31.19	[?, ?]
$\tanh[\ln[1+z]]$	-0.34	-0.41	-0.31	93.63	34.66	13.38	-0.906	0.520	9.413	46.35	

Pitfalls

- determinant of $\widetilde{F}$:
  - $\det(\widetilde{F}) = \sum_{i=1}^{N-n+1} \sum_{j=i+1}^{N-n+2} \cdots \sum_{l=n}^{N} G_i^2 G_j^2 \cdots G_l^2 \left(\varepsilon_{ij\ldots l} W_a(z_i) W_b(z_j) \cdots W_n(z_l)\right)^2$
  - $\binom{N}{n}$ number of components : $\binom{N}{n} = N(N-1)(N-2)\cdots(N-n+1)/n!$
  - PCA number of components decreases



- a) The fiducial model is $\omega = -1.1 + 0.5\frac{z}{1+z}$ (dashed) and the obtained values of ω_is from PCA with H data (dotted), D_L data (dot-dashed), and $H + D_L$ data (solid). Error bars are obtained from the analysis of $H + D_L$ data.
- b) The fiducial model is $\omega = -1.1 - 0.3\frac{z}{1+z}$ (dashed) and the obtained values of ω_is from PCA with H data (dotted), D_L data (dot-dashed), and $H + D_L$ data (solid)

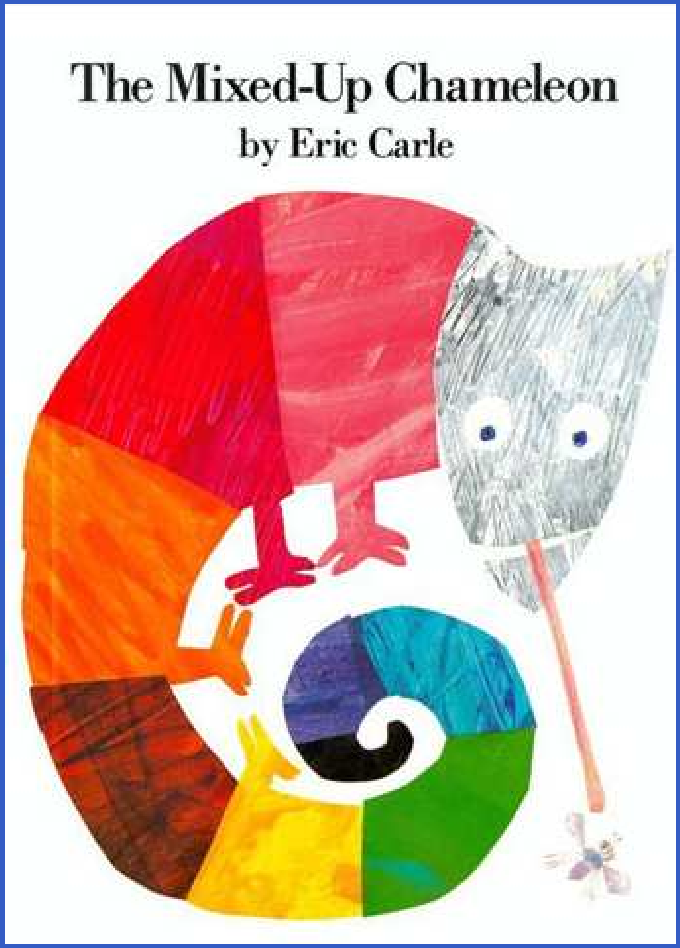
Future works

Sunyaev-Zel$'$dovich effect

- angular power spectrum of the SZe using halo formalism:
  $$C_l = g_\nu^2 \int_0^{z\max} dz \frac{dV}{dz} \int_{M_{min}}^{M_{max}} dM \frac{dn(M,z)}{dM} |\tilde{y}_l(M,z)|^2$$
  - g_ν : spectral function of SZe (-2 in Rayleigh-Jeans limi)
  - $dn(M,z)/dM$: comoving DM halo mass function
  - $\tilde{y}_l(M,z)$: 2D Fourier transform of the projected Compton y-parameter
- redshift distribution of C_l for a given l : $\frac{d\ln C_l}{d\ln z} = \frac{z\frac{dV}{dz}\int dM \frac{dn}{dM}|\tilde{y}_l|^2}{\int dz \frac{dV}{dz}\int dM \frac{dn}{dM}|\tilde{y}_l|^2}$
  - haloes at $z \sim 1$ determined C_l at $l \sim 3000$ E. Komatsu *et.al.* [2002]
  - haloes at $z \sim 2$ determined C_l at $l \sim 10000$
- mass distribution of C_l for a given l : $\frac{d\ln C_l}{d\ln M} = \frac{M\int dz \frac{dV}{dz}\frac{dn}{dM}|\tilde{y}_l|^2}{\int dz \frac{dV}{dz}\int dM \frac{dn}{dM}|\tilde{y}_l|^2}$
  - haloes $10^{14}h^{-1}M_\odot < M < 10^{14}h^{-1}M_\odot$ dominate C_l at $l \sim 3000$ with peak at $3\times 10^{14}h^{-1}M_\odot$
  - $l = 1000$ peak at $5\times 10^{14}h^{-1}M_\odot$
  - $l = 10000$ peak at $10^{14}h^{-1}M_\odot$

Who am I？

Who am I ?



- After too much adoption from other animals, she even can′t catch a fly.
- How about in Cosmology ? (Growth factor, Nonlinear model, mass function, $\cdots$)
- Need to be consistent