On the Instability of the Lee-Wick Bounce

Taotao Qiu

Chung Yuan Christian University, Taiwan

2011-06-21

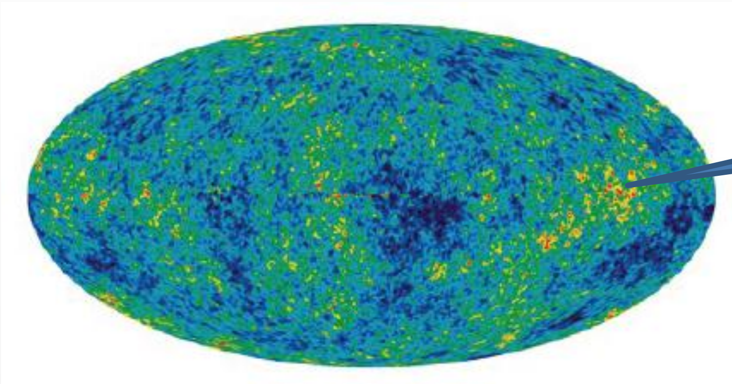
Based on:

Y. Cai, T. Qiu, R. Brandenberger and X. Zhang, Phys. Rev. D 80, 023511 (2009)

J. Karouby, R. Brandenberger, Phys.Rev.D82:063532,2010.

J. Karouby, T. Qiu and R. Brandenberger, e-Print: arXiv:1104.3193 [hep-th]

The Early Universe



Temperature fluctuations

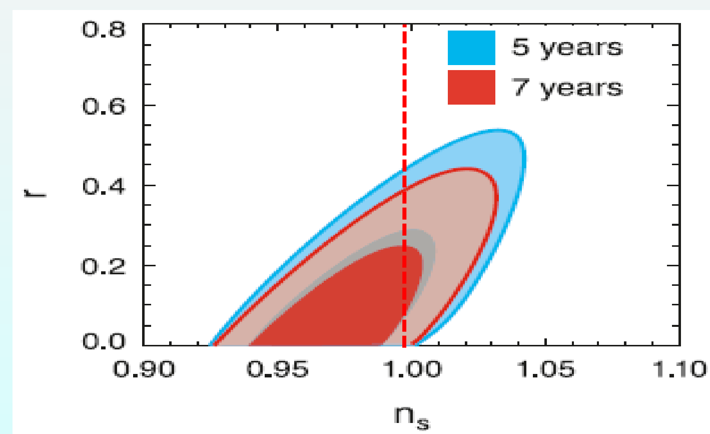
$$\frac{\Delta T}{T} \sim 10^{-5}$$

Variables for testing perturbations: Power spectrum

$$ds^2 = (1 + 2\Phi)dt^2 - a^2(t)(1 - 2\Psi)dx^{i2}$$

Power spectrum: $$P_\Phi \equiv \frac{k^3}{2\pi^2}|\Phi_k^2|$$

Spectral index: $$n_\Phi \equiv \frac{d \ln \mathcal{P}_\Phi}{d \ln k} + 1$$



n_s 0.963 ± 0.014

D. Larson et al. [WMAP collaboration], arXiv:1001.4635 [astro-ph.CO].

Inflation

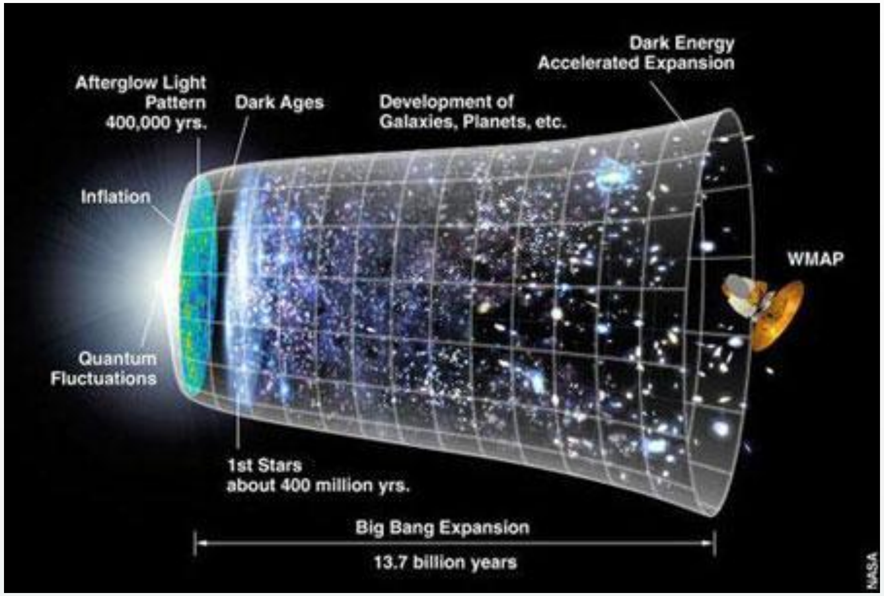
Definition: $\frac{\ddot{a}}{a} > 0$

"Slow roll":

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \ll 1$$

$$|\delta| \equiv |\frac{\ddot{\phi}}{H\dot{\phi}}| \ll 1$$

$$|\xi| \equiv |\epsilon - \dot{\delta}| \ll 1$$



✓the Horizon problem
✓the Flatness problem
✓the unwanted relics problem
✓Give rise to the right amount of perturbations
✓The singularity problem

The Alternatives of inflation

- Pre-big bang Scenario
- Ekpyrotic Scenario
- String gas/Hagedorn Scenario
- Non-local SFT Scenario
- Bouncing Scenario

The basic idea of bounce

contraction → IR size with Low energy scale → expansion

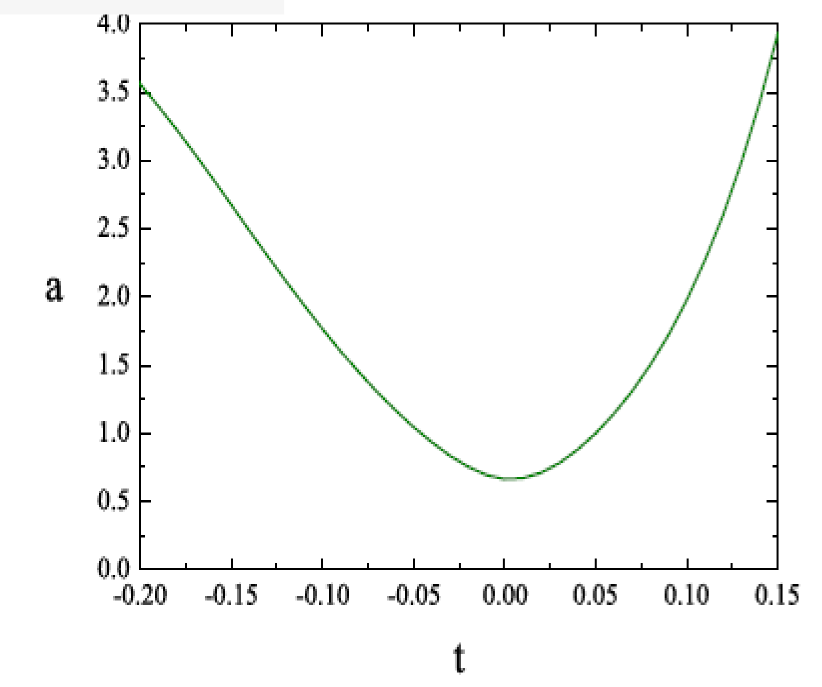
Singularity problem avoided!

Formalism:

Contraction: $H < 0$ Expansion: $H > 0$

Bouncing Point: $H = 0$ ($\rho = 0$) At the Neiborhood: $\dot{H} > 0$

$$\dot{H} = -4\pi G(\rho + p) \Rightarrow w < -1$$



In order to connect this process to the observable universe (radiation dominant, matter dominant, etc), w should come back to above -1 again!

Y. Cai, T. Qiu, Y. Piao, M. Li and X. Zhang, JHEP 0710:071, 2007

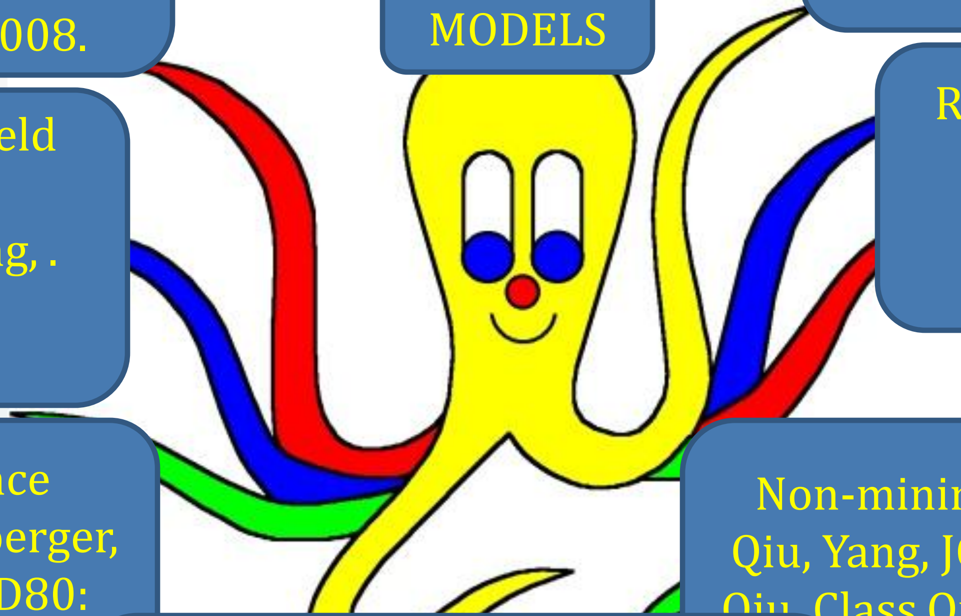
The Zoo of Bounce models

Bounce + Large field inflation
Cai, Qiu, Brandenberger, Piao, Zhang,
JCAP 0803: 013,2008.

Bounce + Small field inflation
Cai, Qiu, Xia, Zhang, . Phys.Rev.D79: 021303,2009.

Lee-Wick Bounce
Cai, Qiu, Brandenberger, Zhang, Phys.Rev.D80: 023511,2009.

BOUNCE MODELS



Holographic Bounce
Cai, Xue, Brandenberger, Zhang, JCAP 0906: 037,2009.

Radiation Bounce
Brandenberger, Karouby, Qiu, in preparation.

Non-minimal coupling Bounce
Qiu, Yang, JCAP 1011: 012, 2010;
Qiu, Class.Quant.Grav.27: 215013, 2010.

Others:
Bouncing in Modified Gravity
New Ekpyrotic model
K-Bounce
....

The Lee-Wick Bounce Model

The Lagrangian:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\hat{\phi}\partial^\mu\hat{\phi} - \frac{1}{2M^2}(\partial^2\hat{\phi})^2 - \frac{1}{2}m^2\hat{\phi}^2 - V(\hat{\phi})$$

Or equivalently:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} + \frac{1}{2}M^2\tilde{\phi}^2 - \frac{1}{2}m^2(\phi - \tilde{\phi})^2 - V(\phi - \tilde{\phi})$$

Regular Higgs $\phi \equiv \hat{\phi} + \tilde{\phi}$ LW partner $\tilde{\phi} \equiv \frac{\partial L}{\partial \Box \hat{\phi}}$

Y. Cai, T. Qiu, R. Brandenberger and X. Zhang, Phys. Rev. D 80, 023511 (2009)

Equations of Motion

- Metric of space-time:

$$ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2)$$

- Friedmann Equation:

$$H^2 = \frac{8\pi G}{3}[\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\dot{\tilde{\phi}}^2 + \frac{1}{2}m^2\phi^2 - \frac{1}{2}M^2\tilde{\phi}^2]$$

- In addition, there are the coupled Klein-Gordon equations for ϕ and $\tilde{\phi}$.

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

$$\ddot{\tilde{\phi}} + 3H\dot{\tilde{\phi}} + M^2\tilde{\phi} = 0$$

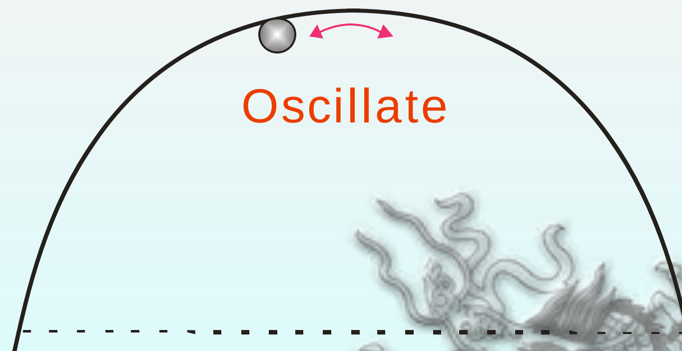
The Field Evolution

Contracting: ϕ dominates

Near the bounce:

$\phi_f \simeq \frac{M_{pl}}{\sqrt{12\pi}}$ ϕ freezes $\tilde{\phi}$ still oscillates

A bounce happens:

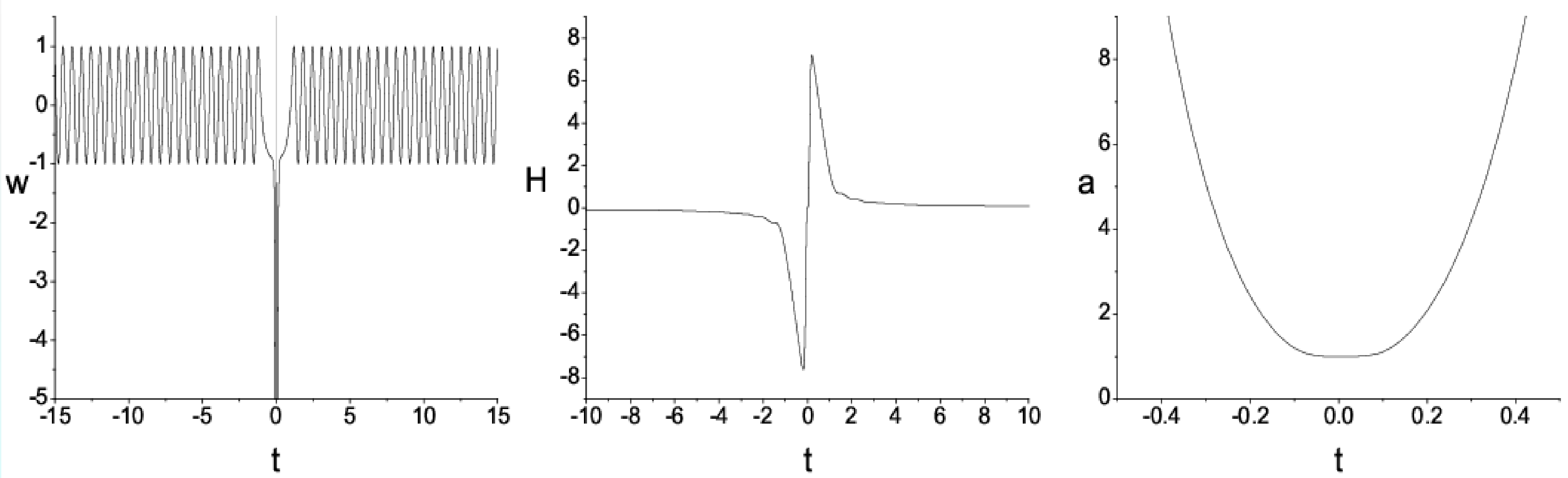


when the contribution of LW scalar to the energy density catches up to that of the normal scalar

Expanding: ϕ dominates again

Background Description of LW Matter Bounce

Background parameters:



Perturbations in LW Matter Bounce

1. Analytical Analysis:

Perturbed Einstein Equations:

$$\Phi'' - c_s^2 \nabla^2 \Phi + \frac{2\mathcal{H}^3 - 4\mathcal{H}\mathcal{H}' + \mathcal{H}''}{\mathcal{H}^2 - \mathcal{H}'} \Phi' + \frac{\mathcal{H}\mathcal{H}'' - 2\mathcal{H}'^2}{\mathcal{H}^2 - \mathcal{H}'} \Phi = 0$$

In the matter-dominant era: $w = 0$

$\Phi_k = D + S(\pm\eta)^{2\nu}$ with $\nu = -\frac{5}{2}$

Near the bounce point: $\mathcal{H} \simeq \alpha(\eta - \eta_B)$

$\Phi_k^b = e^{-y(\eta-\eta_B)^2}\left\{E_k H_l[\sqrt{y}(\eta - \eta_B)] + F_k \, {}_1F_1[-\frac{l}{2}, \frac{1}{2}, y(\eta - \eta_B)^2]\right\}$ where $y = \frac{12}{\pi}\alpha a_B^2$, $l \equiv -\frac{2}{3} + \frac{c_s^2 k^2}{2y}$

Initial condition: Bunch-Davies vacuum $\Phi_i \sim \frac{1}{\sqrt{2k^3}} e^{-ik\eta}$

Perturbations in LW Matter Bounce

1. Analytical Analysis:

Before bounce:

$$\Phi_k^c = \bar{D}_- + \frac{\bar{S}_-}{(\eta - \tilde{\eta}_{B-})^{2\nu_c}}$$

After bounce:

$$\Phi_k^e = \bar{D}_+ + \frac{\bar{S}_+}{(\eta - \tilde{\eta}_{B+})^{2\nu_e}}$$

Matching condition:

$$\hat{E}_k\sqrt{y}(\eta_{B-} - \eta_B) = -\left(\frac{1}{3} + 2l\right)\Phi_k^c - \hat{\zeta}_k^c|_{B-},$$
$$\hat{F}_k = \left(\frac{4}{3} + 2l\right)\Phi_k^c + \hat{\zeta}_k^c|_{B-}.$$

$$\hat{E}_k\sqrt{y}(\eta_{B+} - \eta_B) = -\left(\frac{1}{3} + 2l\right)\Phi_k^e - \hat{\zeta}_k^e|_{B+},$$
$$\hat{F}_k = \left(\frac{4}{3} + 2l\right)\Phi_k^e + \hat{\zeta}_k^e|_{B+}.$$

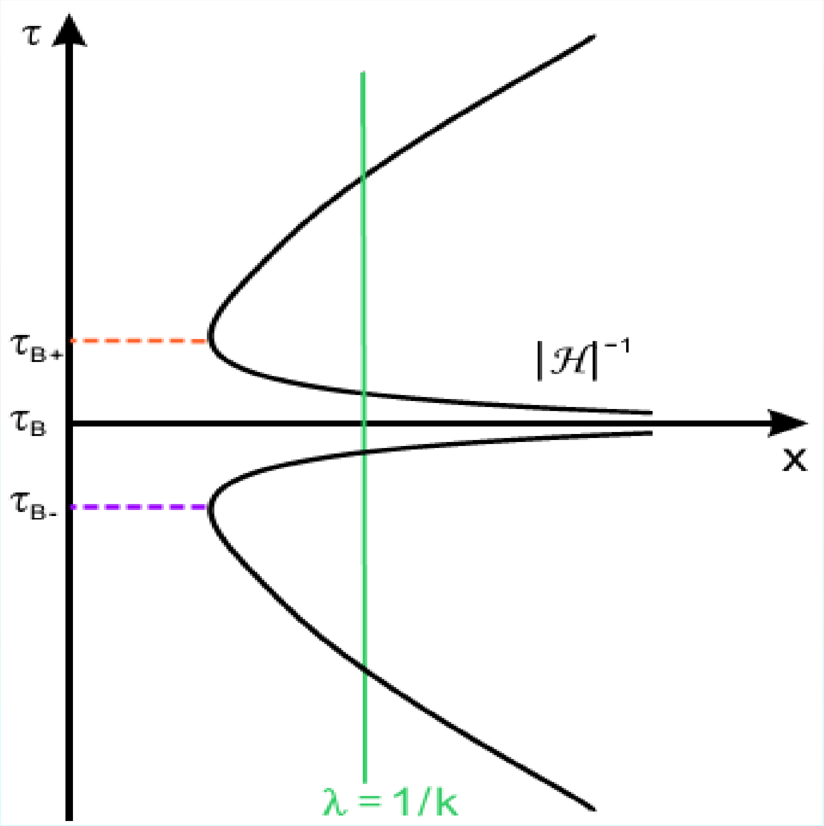
Power spectrum:

$$\mathcal{P}_\Phi = \frac{k^3}{2\pi^2}|D_+|^2 = \frac{\rho_{B-}}{(20\pi)^2 M_p^4}$$

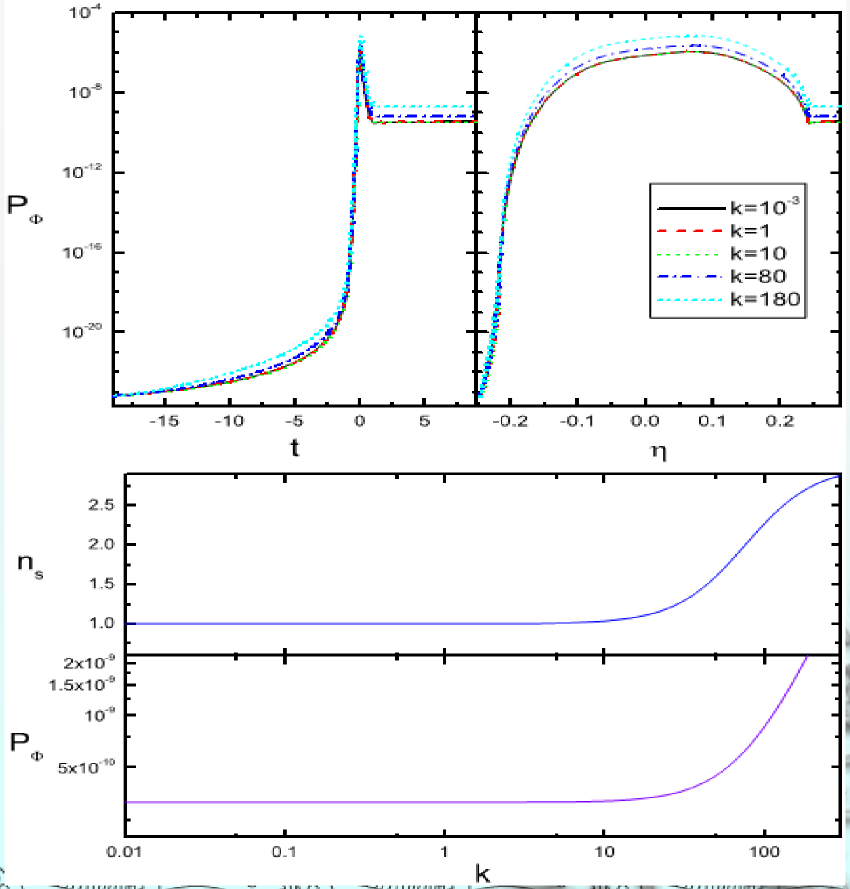
Perturbations in LW Matter Bounce

2. Numerical Calculation:

Sketch plot of perturbation:

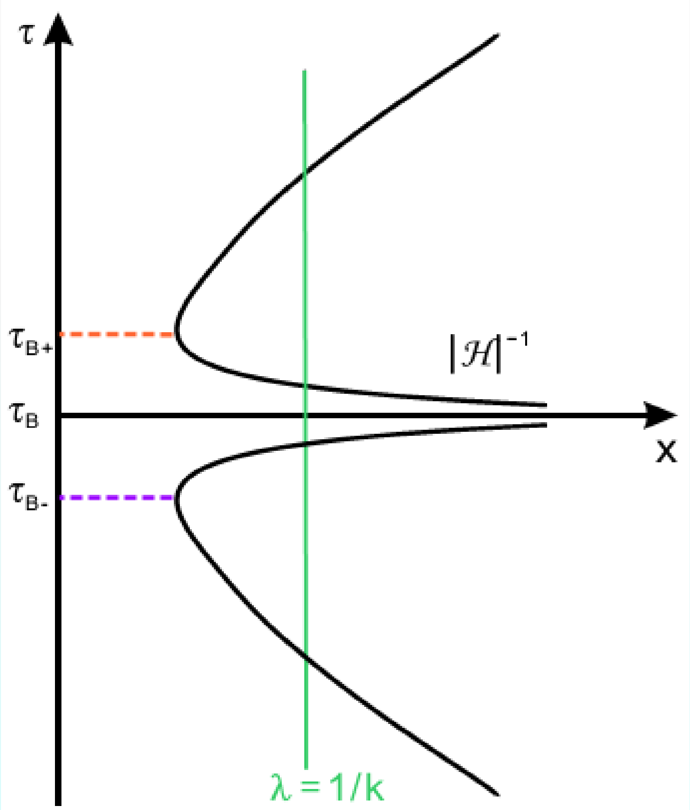


Power spectrum and index:



How does Bounce solve other cosmological problems?

Horizon problem: the horizon in the far past in contracting phase is very large;



Flatness problem:

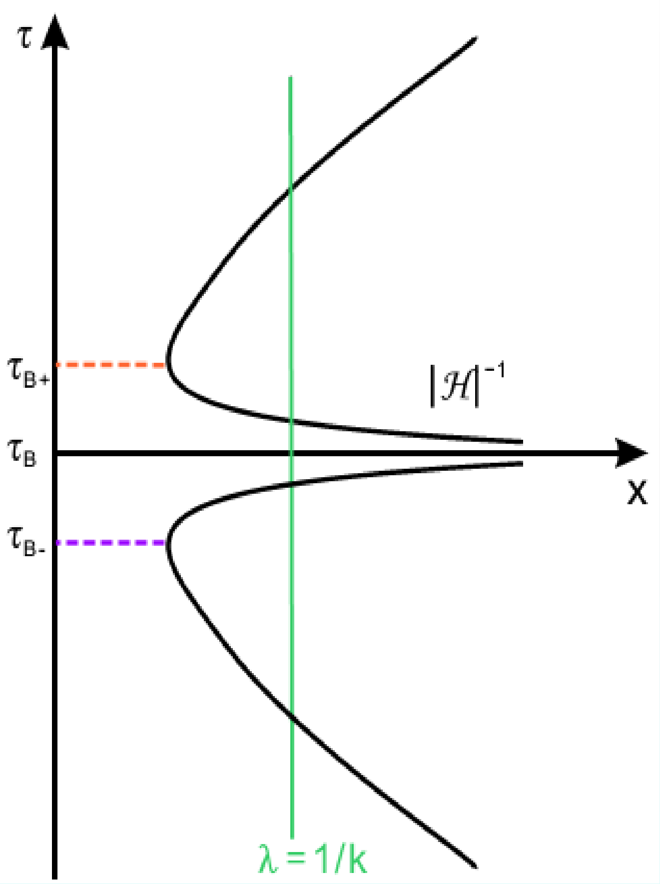
$$\Omega_{tot}(t) - 1 = \frac{k}{a^2 H^2}$$

$$\frac{|\Omega_{tot}(t) - 1|_i}{|\Omega_{tot}(t) - 1|_0} \sim \left(\frac{a_i^2}{a_0^2}\right) \sim \left(\frac{T_0^2}{T_i^2}\right)$$

avoided if the spatial curvature at temperatures in the contracting phase comparable to the current temperature is not larger than the current spatial curvature;

How does Bounce solve other cosmological problems?

Trans-Planckian and Unwanted relics problem:



✓If the energy density at the bounce point is given by the Grand Unification scale ($\sim 10^{16} GeV$), then

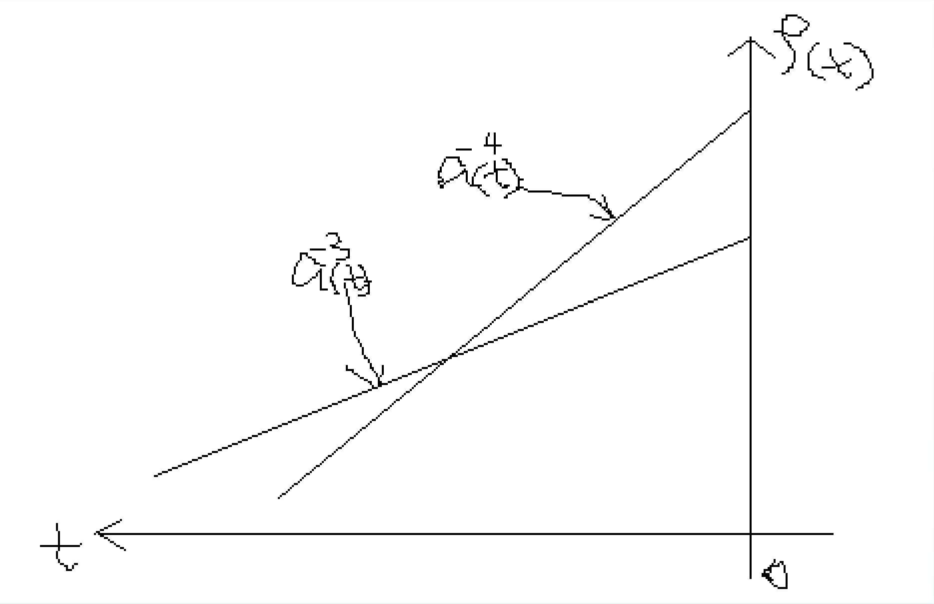
$$\rho \sim 10^{64} GeV^4 \qquad H \sim 10^{13} GeV^{-1}$$

and the wavelength of a perturbation mode is about

$$H^{-1} \sim 1mm.$$

✓Unwanted relics can also be avoided because of the low energy scale

The Problem of LW matter bounce



Radiation:

$$\rho_r \sim a^{-4}(t)$$

Matter:

$$\rho_m \sim a^{-3}(t)$$

Instability in presence of radiation in contracting phase!

Adding the LW gauge partner

The Lagrangian:

$$L_{hd} = -\frac{1}{4}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} + \frac{1}{2M_{\tilde{A}}^2}\mathcal{D}^{\mu}\hat{F}_{\mu\nu}\mathcal{D}^{\lambda}\hat{F}^{\nu}_{\lambda}$$

Or equivalently:

$$L = -\frac{1}{4}(F_{\mu\nu}F^{\mu\nu} - \tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}) + cF_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{M_{\tilde{A}}^2}{2}\tilde{A}_a\tilde{A}^a$$

U(1) symmetry breaking!

Expectation: have $\rho_{\tilde{A}}$ (which is negative) cancel ρ_A, in order to protect the bounce.

But can it succeed?

J. Karouby, R. Brandenberger, Phys.Rev.D82:063532,2010.

The Dynamics

The energy density:

$$\rho_A = \frac{1}{4}(F^2 + F_0{}^\lambda F_{0\lambda})$$

$$\rho_{\tilde{A}} = -\frac{1}{4}(\tilde{F}^2 + \tilde{F}_0{}^\lambda \tilde{F}_{0\lambda}) - M_{\tilde{A}}^2\left(\frac{\tilde{A}^2}{2} + \tilde{A}_0{}^2\right)$$

$$\rho_{A-\tilde{A}} = -c(F_{\lambda\sigma}\tilde{F}^{\lambda\sigma} + 4F_{0\lambda}\tilde{F}_0^\lambda)$$

Equations of motion:

$$\partial_\mu F^{\mu\nu} + 3HF^{0\nu} = \frac{2cM_{\tilde{A}}^2}{1+4c^2}\tilde{A}^\nu$$ sourced by $\tilde{A}^\nu$

$$\partial_\mu \tilde{F}^{\mu\nu} + 3H\tilde{F}^{0\nu} - \frac{M_{\tilde{A}}^2}{1+4c^2}\tilde{A}^\nu = 0.$$ unsourced

The reason of the difference is that we required mass of LW gauge field, while normal gauge field is massless!

The solutions

The energy density: $A_1(k,\eta) = u(\eta)\cos(kz)\;, \tilde{A}_1(k,\eta) = v(\eta)\cos(kz)\;.$

$$\rho_A(\eta,k) = \frac{1}{4a(\eta)^4}[u'(\eta)^2 + k^2u(\eta)^2] \qquad \rho_{\tilde{A}}(\eta,k) = \frac{-1}{4a(\eta)^4}\left[v'(\eta)^2 + \left[k^2 + \frac{M_{\tilde{A}}^2}{2}a(\eta)^2\right]v(\eta)^2\right]$$

$$\rho_{A-\tilde{A}}(\eta,k) = \frac{-c}{a(\eta)^4}[u'(\eta)v'(\eta) + k^2u(\eta)v(\eta)]$$

$c = 0:$

$$\begin{aligned} u''(\eta) + k^2u(\eta) &= 0\,, \\ v''(\eta) + [k^2 + a(t)^2M_{\tilde{A}}^2]v(\eta) &= 0\,. \end{aligned}$$

$u(\eta)$ and $v(\eta)$ are undamping oscillating functions and hence $\rho_A \sim \rho_{\tilde{A}} \sim a^{-4}(t)$

$c \neq 0:$

$$u''(\eta) + k^2u(\eta) = -a(t)^2\frac{2c}{1+4c^2}M_{\tilde{A}}^2v(\eta)$$

$$v''(\eta) + \left[k^2 + a(t)^2\frac{M_{\tilde{A}}^2}{1+4c^2}\right]v(\eta) = 0$$

$u(\eta)$ depends on the unsourced solution $u_0(\eta)$ and $v(\eta)$, hence depend on the initial phase.

The solutions

For example:

If $u_0(\eta) = \mathcal{A}\cos(k\eta)$ and $v(\eta) = v_0\sin(k\eta)$

Then
$$u(\eta) \simeq \left(\mathcal{A} - \frac{cv_0}{1+4c^2}\frac{M_A^2}{4k}(\eta - \eta_I)\right)\cos(k\eta)$$

and $u(\eta)$ will lose energy due to $v(\eta)$ for positive c.

If $u_0(\eta) = \mathcal{A}\sin(k\eta)$ and $v(\eta) = v_0\cos(k\eta)$

Then
$$u(\eta) \simeq \left(\mathcal{A} + \frac{cv_0}{1+4c^2}\frac{M_A^2}{4k}(\eta - \eta_I)\right)\sin(k\eta)$$

and $u(\eta)$ will get energy due to $v(\eta)$ for positive c.

Even with only one Fourier mode excited, the average over all phases (angles) will have the total effect of energy transfer vanish!

So the conclusion is that it is impossible to transfer energy from the gauge field to its LW partner, and bounce would not happen, unless severely fine-tuned!

Our model

Introducing interactions between scalar and gauge fields:

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 + \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 - \frac{1}{2}m^2\phi_1^2 + \frac{1}{2}M^2\phi_2^2$$
$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}(c\phi_1^2 + d\phi_2^2)F_{\mu\nu}F^{\mu\nu} ,$$

Equations of motion:

$$\Box\phi_1 - (m^2 - \frac{c}{2}F^2)\phi_1 = 0$$

$$\Box\phi_2 - (M^2 + \frac{d}{2}F^2)\phi_2 = 0$$

$$(1 - c\phi_1^2 - d\phi_2^2)(\partial_\nu F^{\mu\nu} + 3HF^{\mu 0}) - 2(c\phi_1\partial_\nu\phi_1 + d\phi_2\partial_\nu\phi_2)F^{\mu\nu} = 0$$

J. Karouby, T. Qiu and R. Brandenberger, e-Print: arXiv:1104.3193 [hep-th]

Born Approximation

Ref: http://en.wikipedia.org/wiki/Born_approximation

For scalar fields:

$$\phi_1(t,z) \; = \; \phi_1^{(0)}(t) + \epsilon\phi_1^{(1)}(t,z) + \epsilon^2\phi_1^{(2)}(t)$$

$$\phi_2(t,z) \; = \; \phi_2^{(0)}(t) + \epsilon\phi_2^{(1)}(t,z) + \epsilon^2\phi_2^{(2)}(t)$$

For vector field:

$$A_1(k,t) \; = \; f(t)cos(kz) \equiv \gamma(k,t) \qquad A^1(k,t) \; = \; a(t)^{-2}\gamma(k,t)$$

with other components of A_μ vanishing.

The Energy Momentum Tensor

The energy density:

$$\begin{aligned}
\rho^{(0)} &= \frac{1}{2}(\dot{\phi}_1^{(0)^2} + m^2\phi_1^{(0)^2}) - \frac{1}{2}(\dot{\phi}_2^{(0)^2} + M^2\phi_2^{(0)^2})\ , \\
\rho^{(1)} &= (\dot{\phi}_1^{(0)}\dot{\phi}_1^{(1)} + m^2\phi_1^{(0)}\phi_1^{(1)}) - (\dot{\phi}_2^{(0)}\dot{\phi}_2^{(1)} + M^2\phi_2^{(0)}\phi_2^{(1)})\ , \\
\rho^{(2)} &= \frac{1}{2}(\dot{\phi}_1^{(1)^2} + \dot{\phi}_1^{(0)}\dot{\phi}_1^{(2)} + \frac{k^2}{a^2}\phi_1^{(1)^2} + m^2\phi_1^{(1)^2} + m^2\phi_1^{(0)}\phi_1^{(2)}) - \frac{1}{2}(\dot{\phi}_2^{(1)^2} + \dot{\phi}_2^{(0)}\dot{\phi}_2^{(2)} \\
&\quad + \frac{k^2}{a^2}\phi_2^{(1)^2} + M^2\phi_2^{(1)^2} + M^2\phi_2^{(0)}\phi_2^{(2)}) + (1 - c\phi_1^{(0)^2} - d\phi_2^{(0)^2})(\frac{k^2}{2a^4}\gamma^2 + \frac{\dot{\gamma}^2}{2a^2})
\end{aligned}$$

The pressure:

Direction-dependent!

$$\begin{aligned}
p_i^{(0)} &= \frac{1}{2}(\dot{\phi}_1^{(0)^2} - m^2\phi_1^{(0)^2}) - \frac{1}{2}(\dot{\phi}_2^{(0)^2} - M^2\phi_2^{(0)^2})\ , \\
p_i^{(1)} &= (\dot{\phi}_1^{(0)}\dot{\phi}_1^{(1)} - m^2\phi_1^{(0)}\phi_1^{(1)}) - (\dot{\phi}_2^{(0)}\dot{\phi}_2^{(1)} - M^2\phi_2^{(0)}\phi_2^{(1)})\ , \\
p_i^{(2)} &= \frac{1}{2}(\dot{\phi}_1^{(1)^2} + \dot{\phi}_1^{(0)}\dot{\phi}_1^{(2)} - \frac{k^2}{a^2}\phi_1^{(1)^2} + m^2\phi_1^{(1)^2} + m^2\phi_1^{(0)}\phi_1^{(2)}) - \frac{1}{2}(\dot{\phi}_2^{(1)^2} + \dot{\phi}_2^{(0)}\dot{\phi}_2^{(2)} - \frac{k^2}{a^2}\phi_2^{(1)^2} \\
&\quad + M^2\phi_2^{(1)^2} + M^2\phi_2^{(0)}\phi_2^{(2)}) - (1 - c\phi_1^{(0)^2} - d\phi_2^{(0)^2})(\frac{F^2}{4} - \frac{F_{i\lambda}F_i^{\lambda}}{a^2}) + \boxed{\frac{\partial_i\phi_1\partial_i\phi_1}{a^2} - \frac{\partial_i\phi_2\partial_i\phi_2}{a^2}},
\end{aligned}$$

with $$p_{eff}^{(2)} = \frac{p_1^{(2)} + p_2^{(2)} + p_3^{(2)}}{3}$$

Dynamics

For ^ (0) components:

Equations:
$$\begin{cases} \ddot{\phi}_1^{(0)} + 3H\dot{\phi}_1^{(0)} + m^2\phi_1^{(0)} = 0 \,, \\ \ddot{\phi}_2^{(0)} + 3H\dot{\phi}_2^{(0)} + M^2\phi_2^{(0)} = 0 \,. \end{cases}$$

Note: the vector field (radiation) is of first order!

Solutions:

For $a|\eta| \gg m^{-1}, M^{-1}$

$$\phi_1^{(0)} \sim |\eta|^{\frac{3p}{2(p-1)}} \sqrt{\frac{2}{(1-p)\pi a_0^{\frac{3}{2}} m}} \cos((1-p)a_0 m|\eta|^{\frac{1}{1-p}} + \theta_1^{(0)}) \,,$$
$$\phi_2^{(0)} \sim |\eta|^{\frac{3p}{2(p-1)}} \sqrt{\frac{2}{(1-p)\pi a_0^{\frac{3}{2}} M}} \cos((1-p)a_0 M|\eta|^{\frac{1}{1-p}} + \theta_2^{(0)}) \,.$$

For $a|\eta| \ll m^{-1}, M^{-1}$

$$\phi_1^{(0)} \sim \frac{((1-p)a_0 m)^{\frac{1-3p}{2}} |\eta|^{\frac{1-3p}{1-p}}}{a_0\Gamma(\frac{3(1-p)}{2})} + \frac{((1-p)a_0 m)^{\frac{3p-1}{2}}}{a_0\Gamma(\frac{1+3p}{2})} \,,$$
$$\phi_2^{(0)} \sim \frac{((1-p)a_0 M)^{\frac{1-3p}{2}} |\eta|^{\frac{1-3p}{1-p}}}{a_0\Gamma(\frac{3(1-p)}{2})} + \frac{((1-p)a_0 M)^{\frac{3p-1}{2}}}{a_0\Gamma(\frac{1+3p}{2})} \,,$$

Dynamics

For ^ (1) components:

Scalar fields:

Equations:

$$\begin{cases} \ddot{\phi}_1^{(1)} + 3H\dot{\phi}_1^{(1)} + (\frac{k^2}{a^2} + m^2)\phi_1^{(1)} = 0 \,, \\ \ddot{\phi}_2^{(1)} + 3H\dot{\phi}_2^{(1)} + (\frac{k^2}{a^2} + M^2)\phi_2^{(1)} = 0 \,, \end{cases}$$

Solutions:

For $k|\eta| > 1$

$$\phi_1^{(1)} \sim |\eta|^{\frac{3p}{2(p-1)}} \sqrt{\frac{2}{(1-p)\pi m a_0^3}} \cos((1-p)a_0 m|\eta| + \theta_1^{(1)}) \,,$$

$$\phi_2^{(1)} \sim |\eta|^{\frac{3p}{2(p-1)}} \sqrt{\frac{2}{(1-p)\pi M a_0^3}} \cos((1-p)a_0 M|\eta| + \theta_2^{(1)}) \,.$$

For $k|\eta| \ll 1$

$$\phi_1^{(1)} \sim \frac{((1-p)a_0 m)^{\frac{1-3p}{2}} |\eta|^{\frac{1-3p}{1-p}}}{a_0 \Gamma(\frac{3(1-p)}{2})} + \frac{((1-p)a_0 m)^{\frac{3p-1}{2}}}{a_0 \Gamma(\frac{1+3p}{2})} \,,$$

$$\phi_2^{(1)} \sim \frac{((1-p)a_0 M)^{\frac{1-3p}{2}} |\eta|^{\frac{1-3p}{1-p}}}{a_0 \Gamma(\frac{3(1-p)}{2})} + \frac{((1-p)a_0 M)^{\frac{3p-1}{2}}}{a_0 \Gamma(\frac{1+3p}{2})} \,,$$

Dynamics

For ^ (1) components:

Gauge fields:

Equations: $$\gamma'' + k^2\gamma - \frac{2(c\phi_1^{(0)}\phi_1^{(0)\prime} + d\phi_2^{(0)}\phi_2^{(0)\prime})}{1 - c{\phi_1^{(0)}}^2 - d{\phi_2^{(0)}}^2}\gamma' = 0\ .$$

Let $\gamma \simeq \gamma_0 + \delta\gamma\,,$ while γ_0 is the solution with c=d=0.

Solutions: $\gamma_0 \sim \cos(k|\eta| + \theta_\gamma)\ .$

For $a|\eta| \gg m^{-1}, M^{-1}$

$$\delta\gamma \sim C_1|\eta|^{\frac{1-4p}{1-p}}$$

For $a|\eta| \ll m^{-1}, M^{-1}$

$$\delta\gamma \sim C_2|\eta|^{\frac{3-7p}{1-p}}\ , \quad p > \frac{1}{3}$$

$$\delta\gamma \sim C_3\cos(k|\eta| + \theta_{\delta\gamma})\ . \quad p < \frac{1}{3}$$

$$p = \frac{2}{3(1+w)}$$

Dynamics

For ^(2) components:

Equations:
$$\begin{cases} \ddot{\phi}_1^{(2)} + m^2\phi_1^{(2)} - \frac{c}{2} < F_{\mu\nu}F^{\mu\nu} > \phi_1^{(0)} = 0 \ , \\ \ddot{\phi}_2^{(2)} + M^2\phi_2^{(2)} + \frac{d}{2} < F_{\mu\nu}F^{\mu\nu} > \phi_2^{(0)} = 0 \ . \end{cases}$$

Solutions:
$$\begin{aligned} \phi_1^{(2)} \sim & -a^{-1}(t)\cos(\omega_m\eta)\int_{\eta_I}^{\eta}\frac{d\eta k^2}{a^3(t)m}|\eta|^{\frac{p}{2(p-1)}}\sin(\omega_m\eta)\cos(2k|\eta|+2\theta_\gamma)\cos((1-p)am|\eta|+\theta_1^{(0)}) \\ & +\sin(\omega_m\eta)\int_{\eta_I}^{\eta}\frac{d\eta k^2}{a^3(t)m}|\eta|^{\frac{p}{2(p-1)}}\cos(\omega_m\eta)\cos(2k|\eta|+2\theta_\gamma)\sin((1-p)am|\eta|+\theta_1^{(0)}) \ , \\ \phi_2^{(2)} \sim & -a^{-1}(t)\cos(\omega_M\eta)\int_{\eta_I}^{\eta}\frac{d\eta k^2}{a^3(t)M}|\eta|^{\frac{p}{2(p-1)}}\sin(\omega_M\eta)\cos(2k|\eta|+2\theta_\gamma)\cos((1-p)aM|\eta|+\theta_1^{(0)}) \\ & +\sin(\omega_M\eta)\int_{\eta_I}^{\eta}\frac{d\eta k^2}{a^3(t)M}|\eta|^{\frac{p}{2(p-1)}}\cos(\omega_M\eta)\cos(2k|\eta|+2\theta_\gamma)\sin((1-p)aM|\eta|+\theta_1^{(0)}) \ . \end{aligned}$$

Or if we only care about their scalings:

$$\phi_{1,2}^{(2)} \propto |\eta|^{\frac{11p-2}{2(p-1)}} \propto a^{-\frac{11}{2}+\frac{1}{p}}$$

Scalings of Each Component

For ^ (0) components:

Terms	${\dot{\phi}_1^{(0)}}^2$	$m^2{\phi_1^{(0)}}^2$	$-{\dot{\phi}_2^{(0)}}^2$	$-M^2{\phi_2^{(0)}}^2$
Behavior	$a^{-3-\frac{2}{p}}(a\|\eta\| \gg m^{-1})$	$a^{-3}(a\|\eta\| \gg m^{-1})$	$a^{-3-\frac{2}{p}}(a\|\eta\| \gg m^{-1})$	$a^{-3}(a\|\eta\| \gg M^{-1})$
	$a^{-6}\left(\begin{matrix} a\|\eta\| \ll m^{-1} \\ p > \frac{1}{3}\end{matrix}\right)$	$a^{-6+\frac{2}{p}}\left(\begin{matrix} a\|\eta\| \ll m^{-1} \\ p > \frac{1}{3}\end{matrix}\right)$	$a^{-6}\left(\begin{matrix} a\|\eta\| \ll m^{-1} \\ p > \frac{1}{3}\end{matrix}\right)$	$a^{-6+\frac{2}{p}}\left(\begin{matrix} a\|\eta\| \ll M^{-1} \\ p > \frac{1}{3}\end{matrix}\right)$
	$0\left(\begin{matrix} a\|\eta\| \ll m^{-1} \\ p < \frac{1}{3}\end{matrix}\right)$	$a^{0}\left(\begin{matrix} a\|\eta\| \ll m^{-1} \\ p < \frac{1}{3}\end{matrix}\right)$	$0\left(\begin{matrix} a\|\eta\| \ll m^{-1} \\ p < \frac{1}{3}\end{matrix}\right)$	$a^{0}\left(\begin{matrix} a\|\eta\| \ll M^{-1} \\ p < \frac{1}{3}\end{matrix}\right)$
Sign	Positive Definite	Positive Definite	Negative Definite	Negative Definite

Scalings of Each Component

For ^ (1) components:

Terms	$\dot{\phi}_1^{(0)}\dot{\phi}_1^{(1)}$	$\phi_1^{(0)}\phi_1^{(1)}$	$-\dot{\phi}_2^{(0)}\dot{\phi}_2^{(1)}$
Behavior	$a^{-3-\frac{2}{p}}(\lvert\eta\rvert \gg Max\{k^{-1},(am)^{-1}\})$	$a^{-3}(\lvert\eta\rvert \gg Max\{k^{-1},(am)^{-1}\})$	$a^{-3-\frac{2}{p}}(\lvert\eta\rvert \gg Max\{k^{-1},(aM)^{-1}\})$
	$a^{-\frac{9}{2}-\frac{1}{p}}\left(\begin{matrix}\lvert\eta\rvert \in [k^{-1},(am)^{-1}] \\ p > \frac{1}{3}\end{matrix}\right)$	$a^{-\frac{9}{2}+\frac{1}{p}}\left(\begin{matrix}\lvert\eta\rvert \in [k^{-1},(am)^{-1}] \\ p > \frac{1}{3}\end{matrix}\right)$	$a^{-\frac{9}{2}-\frac{1}{p}}\left(\begin{matrix}\lvert\eta\rvert \in [k^{-1},(aM)^{-1}] \\ p > \frac{1}{3}\end{matrix}\right)$
	$a^{-6}\left(\begin{matrix}\lvert\eta\rvert \ll Min\{k^{-1},(am)^{-1}\} \\ p > \frac{1}{3}\end{matrix}\right)$	$a^{-6+\frac{2}{p}}\left(\begin{matrix}\lvert\eta\rvert \ll Min\{k^{-1},(am)^{-1}\} \\ p > \frac{1}{3}\end{matrix}\right)$	$a^{-6}\left(\begin{matrix}\lvert\eta\rvert \ll Min\{k^{-1},(aM)^{-1}\} \\ p > \frac{1}{3}\end{matrix}\right)$
	$0\left(\begin{matrix}\lvert\eta\rvert \in [k^{-1},(am)^{-1}] \\ p < \frac{1}{3}\end{matrix}\right)$	$a^{-\frac{3}{2}}\left(\begin{matrix}\lvert\eta\rvert \in [k^{-1},(am)^{-1}] \\ p < \frac{1}{3}\end{matrix}\right)$	$0\left(\begin{matrix}\lvert\eta\rvert \in [k^{-1},(aM)^{-1}] \\ p < \frac{1}{3}\end{matrix}\right)$
	$0\left(\begin{matrix}\lvert\eta\rvert \ll Min\{k^{-1},(am)^{-1}\} \\ p < \frac{1}{3}\end{matrix}\right)$	$a^{0}\left(\begin{matrix}\lvert\eta\rvert \ll Min\{k^{-1},(am)^{-1}\} \\ p < \frac{1}{3}\end{matrix}\right)$	$0\left(\begin{matrix}\lvert\eta\rvert \ll Min\{k^{-1},(aM)^{-1}\} \\ p < \frac{1}{3}\end{matrix}\right)$
Sign	Indefinite	Indefinite	Indefinite

Terms	$-\phi_2^{(0)}\phi_2^{(1)}$
Behavior	$a^{-3}(\lvert\eta\rvert \gg Max\{k^{-1},(aM)^{-1}\})$
	$a^{-\frac{9}{2}+\frac{1}{p}}\left(\begin{matrix}\lvert\eta\rvert \in [k^{-1},(aM)^{-1}] \\ p > \frac{1}{3}\end{matrix}\right)$
	$a^{-6+\frac{2}{p}}\left(\begin{matrix}\lvert\eta\rvert \ll Min\{k^{-1},(aM)^{-1}\} \\ p > \frac{1}{3}\end{matrix}\right)$
	$a^{-\frac{3}{2}}\left(\begin{matrix}\lvert\eta\rvert \in [k^{-1},(aM)^{-1}] \\ p < \frac{1}{3}\end{matrix}\right)$
	$a^{0}\left(\begin{matrix}\lvert\eta\rvert \ll Min\{k^{-1},(aM)^{-1}\} \\ p < \frac{1}{3}\end{matrix}\right)$
Sign	Indefinite

Scalings of Each Component

For ^ (2) components:

Terms	$\dot{\phi}_1^{(1)^2}$	$\dot{\phi}_1^{(0)}\dot{\phi}_1^{(2)}$	$\frac{k^2}{a^2}\phi_1^{(1)^2}$	$m^2\phi_1^{(1)^2}$	$m^2\phi_1^{(0)}\phi_1^{(2)}$
Behavior	$a^{-3-\frac{2}{p}}(k\|\eta\| \gg 1)$ $a^{-6}\left(\begin{array}{c} k\|\eta\| \ll 1 \\ p > \frac{1}{3}\end{array}\right)$ $0\ \left(\begin{array}{c} k\|\eta\| \ll 1 \\ p < \frac{1}{3}\end{array}\right)$	$a^{-7-\frac{1}{p}}(a\|\eta\| \gg m^{-1})$ $a^{-\frac{17}{2}}\left(\begin{array}{c} a\|\eta\| \ll m^{-1} \\ p > \frac{1}{3}\end{array}\right)$ $0\ \left(\begin{array}{c} a\|\eta\| \ll m^{-1} \\ p < \frac{1}{3}\end{array}\right)$	$a^{-5}(k\|\eta\| \gg 1)$ $a^{-8+\frac{2}{p}}\left(\begin{array}{c} k\|\eta\| \ll 1 \\ p > \frac{1}{3}\end{array}\right)$ $a^{-2}\ \left(\begin{array}{c} k\|\eta\| \ll 1 \\ p < \frac{1}{3}\end{array}\right)$	$a^{-3}(k\|\eta\| \gg 1)$ $a^{-6+\frac{2}{p}}\left(\begin{array}{c} k\|\eta\| \ll 1 \\ p > \frac{1}{3}\end{array}\right)$ $a^{0}\ \left(\begin{array}{c} k\|\eta\| \ll 1 \\ p < \frac{1}{3}\end{array}\right)$	$a^{-7+\frac{1}{p}}(a\|\eta\| \gg m^{-1})$ $a^{-\frac{17}{2}+\frac{2}{p}}\left(\begin{array}{c} a\|\eta\| \ll m^{-1} \\ p > \frac{1}{3}\end{array}\right)$ $a^{-\frac{11}{2}+\frac{1}{p}}\ \left(\begin{array}{c} a\|\eta\| \ll m^{-1} \\ p < \frac{1}{3}\end{array}\right)$
Sign	Positive Definite	Indefinite	Positive Definite	Positive Definite	Indefinite

Terms	$-\dot{\phi}_2^{(1)^2}$	$-\dot{\phi}_2^{(0)}\dot{\phi}_2^{(2)}$	$\frac{k^2}{a^2}\phi_2^{(1)^2}$	$-M^2\phi_2^{(1)^2}$	$-M^2\phi_2^{(0)}\phi_2^{(2)}$
Behavior	$a^{-3-\frac{2}{p}}(k\|\eta\| \gg 1)$ $a^{-6}\left(\begin{array}{c} k\|\eta\| \ll 1 \\ p > \frac{1}{3}\end{array}\right)$ $0\ \left(\begin{array}{c} k\|\eta\| \ll 1 \\ p < \frac{1}{3}\end{array}\right)$	$a^{-7-\frac{1}{p}}(a\|\eta\| \gg M^{-1})$ $a^{-\frac{17}{2}}\left(\begin{array}{c} a\|\eta\| \ll M^{-1} \\ p > \frac{1}{3}\end{array}\right)$ $0\ \left(\begin{array}{c} a\|\eta\| \ll M^{-1} \\ p < \frac{1}{3}\end{array}\right)$	$a^{-5}(k\|\eta\| \gg 1)$ $a^{-8+\frac{2}{p}}\left(\begin{array}{c} k\|\eta\| \ll 1 \\ p > \frac{1}{3}\end{array}\right)$ $a^{-2}\ \left(\begin{array}{c} k\|\eta\| \ll 1 \\ p < \frac{1}{3}\end{array}\right)$	$a^{-3}(k\|\eta\| \gg 1)$ $a^{-6+\frac{2}{p}}\left(\begin{array}{c} k\|\eta\| \ll 1 \\ p > \frac{1}{3}\end{array}\right)$ $a^{0}\ \left(\begin{array}{c} k\|\eta\| \ll 1 \\ p < \frac{1}{3}\end{array}\right)$	$a^{-7+\frac{1}{p}}(a\|\eta\| \gg M^{-1})$ $a^{-\frac{17}{2}+\frac{2}{p}}\left(\begin{array}{c} a\|\eta\| \ll M^{-1} \\ p > \frac{1}{3}\end{array}\right)$ $a^{-\frac{11}{2}+\frac{1}{p}}\ \left(\begin{array}{c} a\|\eta\| \ll M^{-1} \\ p < \frac{1}{3}\end{array}\right)$
Sign	Negative Definite	Indefinite	Negative Definite	Negative Definite	Indefinite

Terms	$a^{-4}k^2\gamma_0^2 + a^{-2}\dot{\gamma}_0^2$	$(-c\phi_1^{(0)^2} - d\phi_2^{(0)^2})(a^{-4}k^2\gamma_0^2 + a^{-2}\dot{\gamma}_0^2)$	$a^{-4}k^2\gamma_0\delta\gamma$	$a^{-2}\dot{\gamma}\dot{\delta\gamma}$
Behavior	a^{-4}	$a^{-7}(a\|\eta\| \gg m^{-1})$ $a^{-10+\frac{2}{p}}\left(\begin{array}{c} a\|\eta\| \ll m^{-1} \\ p > \frac{1}{3}\end{array}\right)$ $a^{-4}\ \left(\begin{array}{c} a\|\eta\| \ll m^{-1} \\ p < \frac{1}{3}\end{array}\right)$	$a^{-8+\frac{1}{p}}(a\|\eta\| \gg m^{-1})$ $a^{-11+\frac{3}{p}}\left(\begin{array}{c} a\|\eta\| \ll m^{-1} \\ p > \frac{1}{3}\end{array}\right)$ $a^{-4}\ \left(\begin{array}{c} a\|\eta\| \ll m^{-1} \\ p < \frac{1}{3}\end{array}\right)$	$a^{-7}(a\|\eta\| \gg m^{-1})$ $a^{-10+\frac{2}{p}}\left(\begin{array}{c} a\|\eta\| \ll m^{-1} \\ p > \frac{1}{3}\end{array}\right)$ $a^{-4}\ \left(\begin{array}{c} a\|\eta\| \ll m^{-1} \\ p < \frac{1}{3}\end{array}\right)$
Sign	Positive Definite	Indefinite (Depending only on c and d)	Indefinite	Indefinite

Conditions for a bounce to happen

3 types of terms: positive definite, negative definite, indefinite

energy densities of normal scalar and gauge field

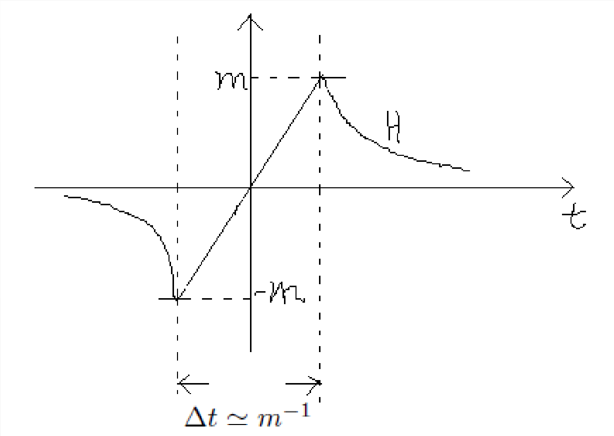
energy densities of the ghost scalar

couplings among scalars and gauge field

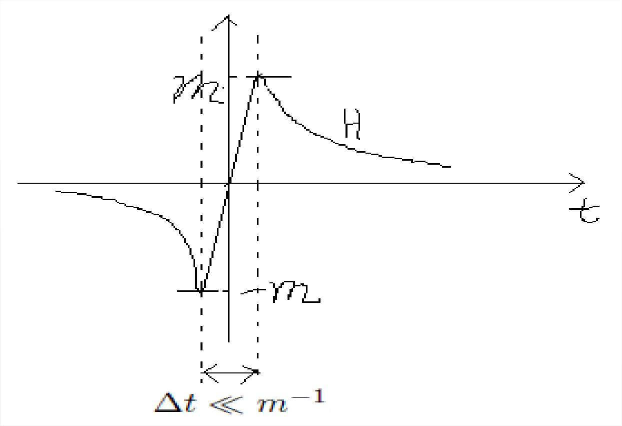
The basic condition for a bounce to happen: the negative definite terms scales more negative w.r.t. a(t), then grows faster!

- In the absence of coupling: impossible! (the radiation part will always grow faster than the scalars)
- In the presence of coupling: can be achieved by setting coupling coefficients c and d.

Conditions for a bounce to happen



Slow bounce: only solutions of $a|\eta| \gg m^{-1}$ before the bounce

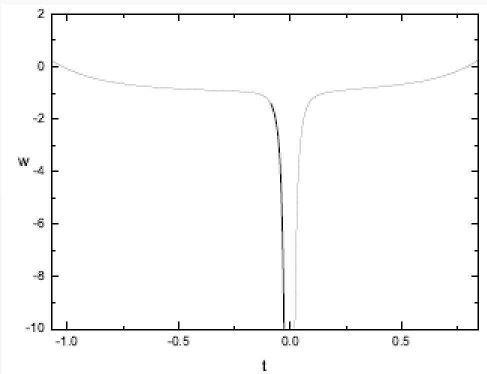


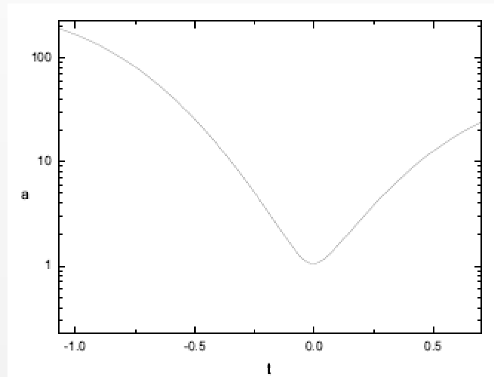
Fast bounce: both solutions of $a|\eta| \gg m^{-1}$ and $a|\eta| \ll m^{-1}$ before the bounce

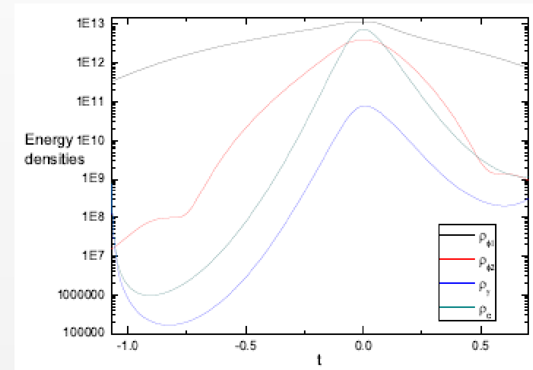
- For slow bounce: $w < 1$
- For fast bounce: $w > -7/6$

Results

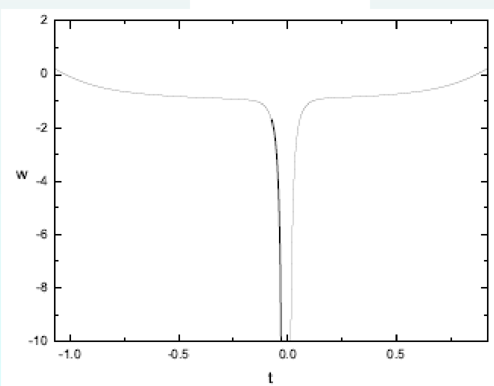
For the configuration which a single Fourier mode is excited: $k \simeq 0.01hMpc^{-1}$

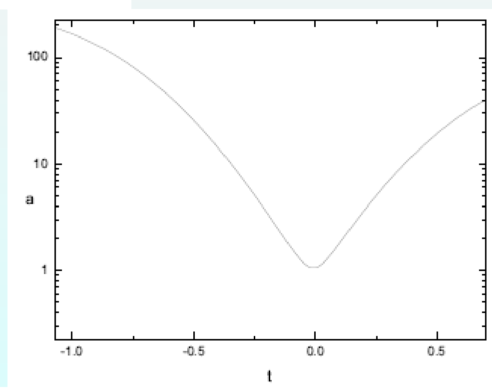


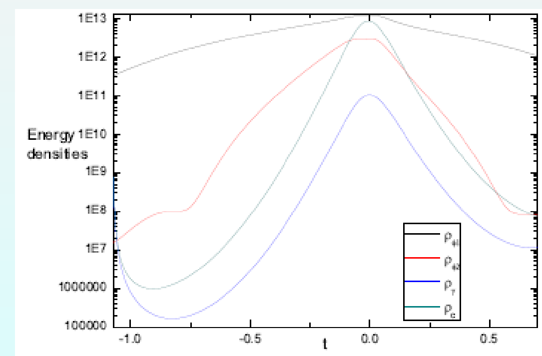




For $c > 0$ and $d > 0$.







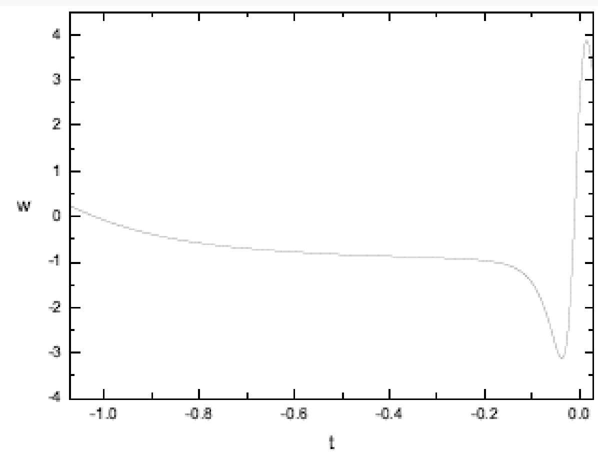
For $c > 0$ and $d < 0$

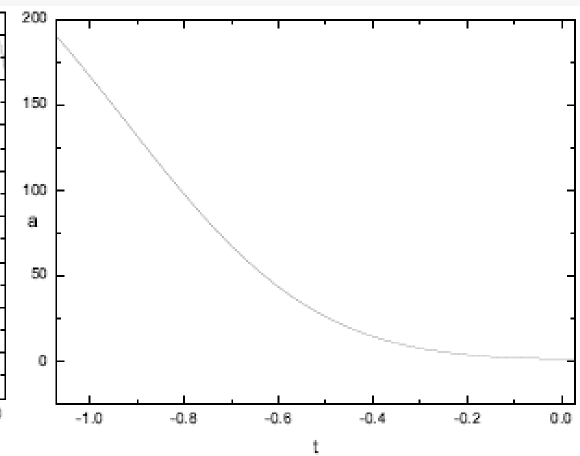
Results

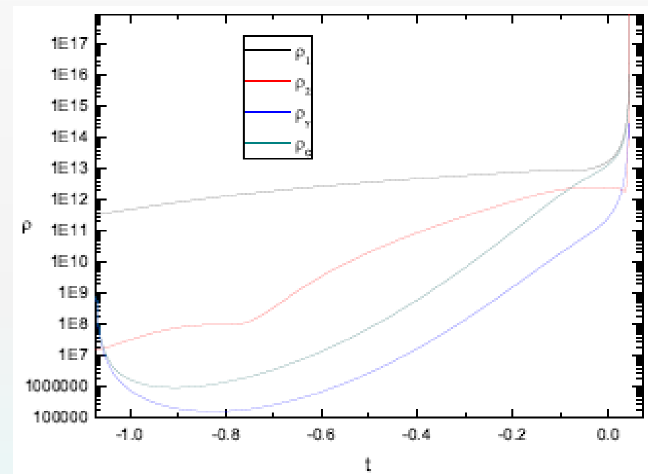
For the configuration which two Fourier modes are excited:

$$k \simeq 0.01 hMpc^{-1}$$

$$k \simeq 0.04 hMpc^{-1}$$







For $c > 0$ and $d > 0$.

So the conclusion is that for more than one Fourier modes excited, it is also difficult to get a bounce, and fine-tuning is also needed!

Conclusions

- The Lee-Wick theory can give rise to a bounce, however naively the presence of radiation will prevent the bounce because of the larger negative scaling w.r.t. $a(t)$.
- With the presence of LW partner of the gauge field, bounce needs severe fine-tuning of initial phase.
- With the coupling between gauge field and scalar, conditions for bounce derived for one Fourier mode, but for infinite modes fine-tuning also needed!

Thank you!!!

Perturbations in inflationary cosmology

Perturbed metric in conformal Newtonian gauge:

$$ds^2 = a^2(\eta)[(1+2\Phi)d\eta^2 - (1-2\Psi)dx^i dx^i]$$

Perturbation Equations for metric:

$$\Phi'' - c_s^2\nabla^2\Phi + \frac{2\mathcal{H}^3 - 4\mathcal{H}\mathcal{H}' + \mathcal{H}''}{\mathcal{H}^2 - \mathcal{H}'}\Phi' + \frac{\mathcal{H}\mathcal{H}'' - 2\mathcal{H}'^2}{\mathcal{H}^2 - \mathcal{H}'}\Phi = 0$$

where $\mathcal{H}$ is the conformal Hubble parameter and prime denotes derivative w.r.t. η

The general solution is:

$$\Phi_k = S\frac{\mathcal{H}}{a^2} + D(\frac{1}{a}\int a dt)'$$

where S and D are constants.

Perturbations in inflationary cosmology

For background with constant equation of state w: $a(t) \sim t^{\frac{2}{3(1+w)}}$

$$\Phi_k = D + S(\pm\eta)^{2\nu} \quad \text{where} \quad \nu \equiv -\frac{5+3w}{2(1+3w)}$$

Defining curvature perturbation: $\zeta_k \equiv \Phi_k + \frac{1}{\epsilon}(\Phi_k + \frac{\Phi_k'}{\mathcal{H}})$

One have: $v_k'' + (k^2 - \frac{z''}{z})v_k = 0$ where $v_k \equiv z\zeta\ , \quad z \equiv a\sqrt{\epsilon}\ .$

The solution for adiabatic perturbation is: $v_k \to \frac{1}{\sqrt{2k}}e^{ik\eta}\ , k \to \infty$

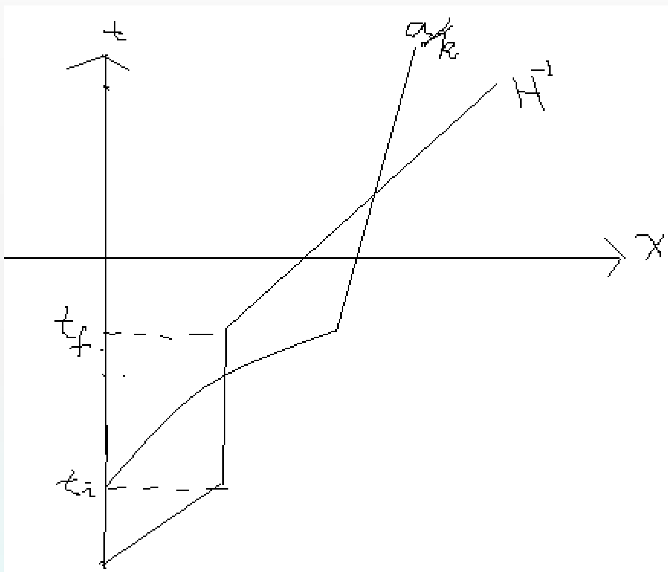
and $v_k \to e^{i(\nu-\frac{1}{2})\frac{\pi}{2}}2^{\nu-\frac{3}{2}}\frac{\Gamma(\nu)}{\Gamma(\frac{3}{2})}\frac{1}{\sqrt{2k}}(-k\eta)^{\frac{1}{2}-\nu}\ , k \to 0$

The spectrum: $\mathcal{P}_\zeta = \frac{H^2}{64\pi^3 M_{pl}^2 \epsilon}$ with index: $n_s = 1 - 6\epsilon + 2\xi$

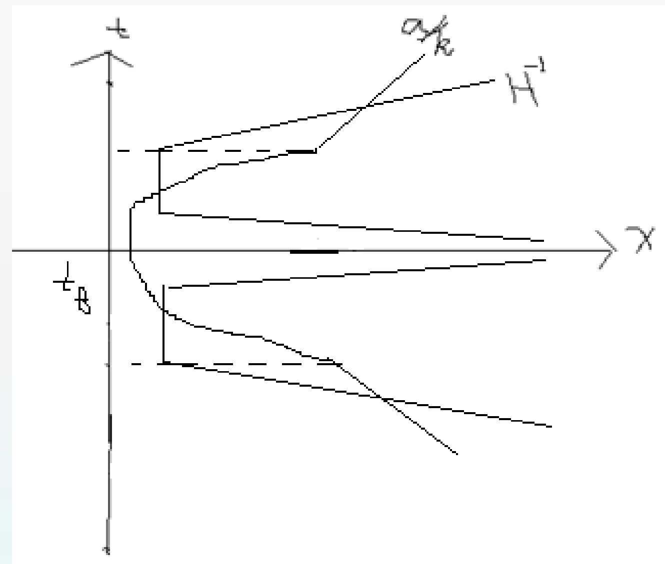
For inflationary cosmology where $\epsilon, \xi \ll 1$ the spectrum is scale invariant.

The differences between perturbations in inflationary and bounce cosmologies

1. There is pre-evolution in contracting time, when horizon was crossed



Inflationary cosmology



bounce cosmology

NOTE: the initial perturbation was still deep in horizon and can also can be seed for structure formation, so the structure formation problem can also be solved!

The differences between perturbations in inflationary and bounce cosmologies

2. Evolutions of different stages are connected via matching conditions

Hwang-Vishiniac (Deruelle-Mukhanov) matching conditions (in Conformal Newtonian Gauge):

$$
\begin{aligned}
[\Phi_k]_\pm &= 0 \\
\left[\zeta_k - \frac{k^2\Phi_k}{3(\mathcal{H}' - \mathcal{H}^2)}\right]_\pm &= 0
\end{aligned}
$$

where $[]_\pm$ denotes variable difference between (+) phase and (-) phase.

According to this, we have: $D_+ = AD_- + Bk^2(S_+ - S_-)$

where + and – stands for expanding and contracting phases, and A and B are constants.

J. C. Hwang and E. T. Vishniac, Astrophys. J. 382, 363 (1991);
N. Deruelle and V. F. Mukhanov, Phys. Rev. D 52, 5549 (1995);
R. Brandenberger and F. Finelli, JHEP 0111, 056 (2001).

The differences between perturbations in inflationary and bounce cosmologies

3. Since in bounce cosmology, there can be no inflationary phase which redshifts all the pre-existing classical matter, thermodynamic generation of the perturbations is also possible!

For example, see

J. Magueijo and L. Pogosian, Phys. Rev. D 67, 043518 (2003) [arXiv:astro-ph/0211337];

J. Magueijo and P. Singh, Phys. Rev. D 76, 023510 (2007) [arXiv:astro-ph/0703566].

Matter Bounce Cosmology: Introduction

- Background
- Vacuum production
- Thermal production

Perturbations in Matter Bounce (II): thermal production

From perturbed Einstein Equation:

$$\nabla^2\Phi - 3\mathcal{H}(\Phi' + \mathcal{H}\Phi) = 4\pi G a^2 \delta\rho$$

We get the amplitude of Newtonian potential:

$$|\Phi(k)|^2 = 16\pi^2 G^2 k^{-4} a^4 < \delta\rho(k)^2 >$$ where $$< \delta\rho(k)^2 > \sim k^{-3} < \delta\rho(x)^2 >$$

And the power spectrum at the Hubble radius crossing:

$$\mathcal{P}_\Phi(k) \sim \frac{a^4}{k^4} < \delta\rho(x)^2 > \sim \frac{< \delta\rho(x)^2 >}{H^4}$$

where $< \delta\rho(x)^2 > = C_V \frac{T^2}{R^6}$ and $C_V \equiv \frac{\partial E}{\partial T}$

Perturbations in Matter Bounce (II): thermal production

Assuming that the fluctuations are generated in a holographic way:

$$< E > = T R^2 M_p^2\,, \quad \text{and} \quad C_V(R) \sim R^2 M_p^2$$

T corresponds to Gibbons-Hawking temperature:

$$T = \frac{1}{R_H} \sim H$$

The spectrum: $P_\Phi(k,t) \sim k^4 \eta_H(k)^{4\nu} k^{\frac{2p}{1-p}+2}$ with $\nu \equiv -\frac{5+3w}{2(1+3w)}$, $p \equiv \frac{2}{3(1+w)}$

where $\eta_H(k) \sim k^{-1}$

For matter bounce (w=0), one have $\nu = -\frac{5}{2}$ and $p = \frac{2}{3}$, and a scale-invariant spectrum could be obtained.

Y. F. Cai, W. Xue, R. Brandenberger, X. m. Zhang, JCAP 0906:037,2009.

Summary on bouncing cosmology

- Can solve the singularity problem as well as other problems that are encountered by Big Bang theory;
- Have different generation (vacuum and thermodynamical) and evolution mechanisms of perturbations from inflationary cosmology;
- Can give rise to scale-invariant power spectrum of primordial perturbations.

What can inflation not do

- *If the universe has a beginning before inflation, the beginning point will be geodesically incomplete where classical General Relativity cannot be implied.*

  S.W. HAWKING, G.F.R. ELLIS, CAMBRIDGE UNIVERSITY PRESS, CAMBRIDGE, 1973.

- *Even there is no beginning and inflation is eternal to the past, there will also be singularities in the far past.*

  BORDE AND VILENKIN, PHYS.REV.LETT.72,3305 (1994).

- *As long as the universe go beyond Planckian scale, the Robustness of high energy effects are inevitable.*

J. MARTIN, AND R.BRANDENBERGER, PHYS.REV.D63:123501 (2001).

Outline

- Preliminary: Bouncing Scenario As an Alternative of Inflation
- Perturbations of Bouncing Cosmology vs. Inflationary Cosmology
- Matter Bounce

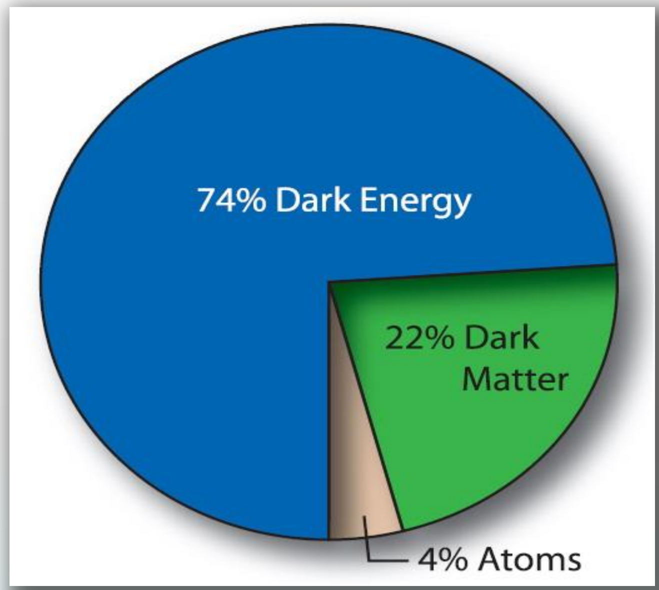
Precision Cosmology

Beginning from the observational data:

BBN,CMB(COBE) → Big Bang

SN → Dark Energy

WMAP, SDSS → LCDM



Concordance model:

BIG BANG + INFLATION + BARYON + DARK MATTER + DARK ENERGY

Others: such as PLANCK etc.