B meson potentials in quenched lattice QCD

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National Tsing-Hua University, Oct 11th 2007

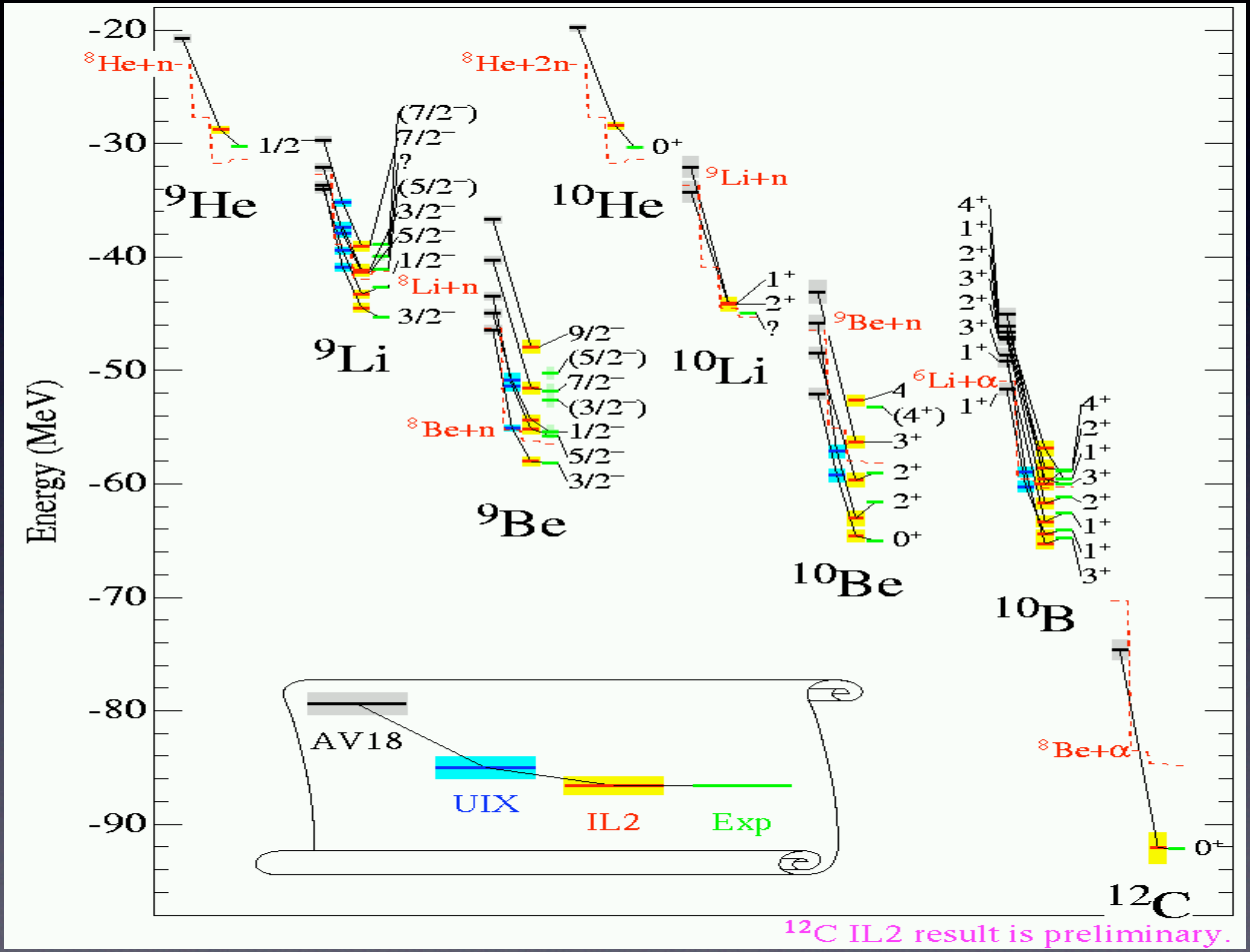
WD, K Orginos and M Savage, *hep-lat/0703009*

- Hadron potentials: NN, YN and BB
- Lattice QCD
- *BB* potentials from lattice QCD
  - Perturbative Coulomb corrections
  - Finite volume effects

Hadron potentials

- Not uniquely defined: r?, unitary equivalence
- Encode of information about scattering amplitude
- Successful phenomenology
  - Fit *NN* potential from scattering data
  - Add *3N*, *4N* forces (tuned to more data)
  - Accurate description of A<10 spectrum

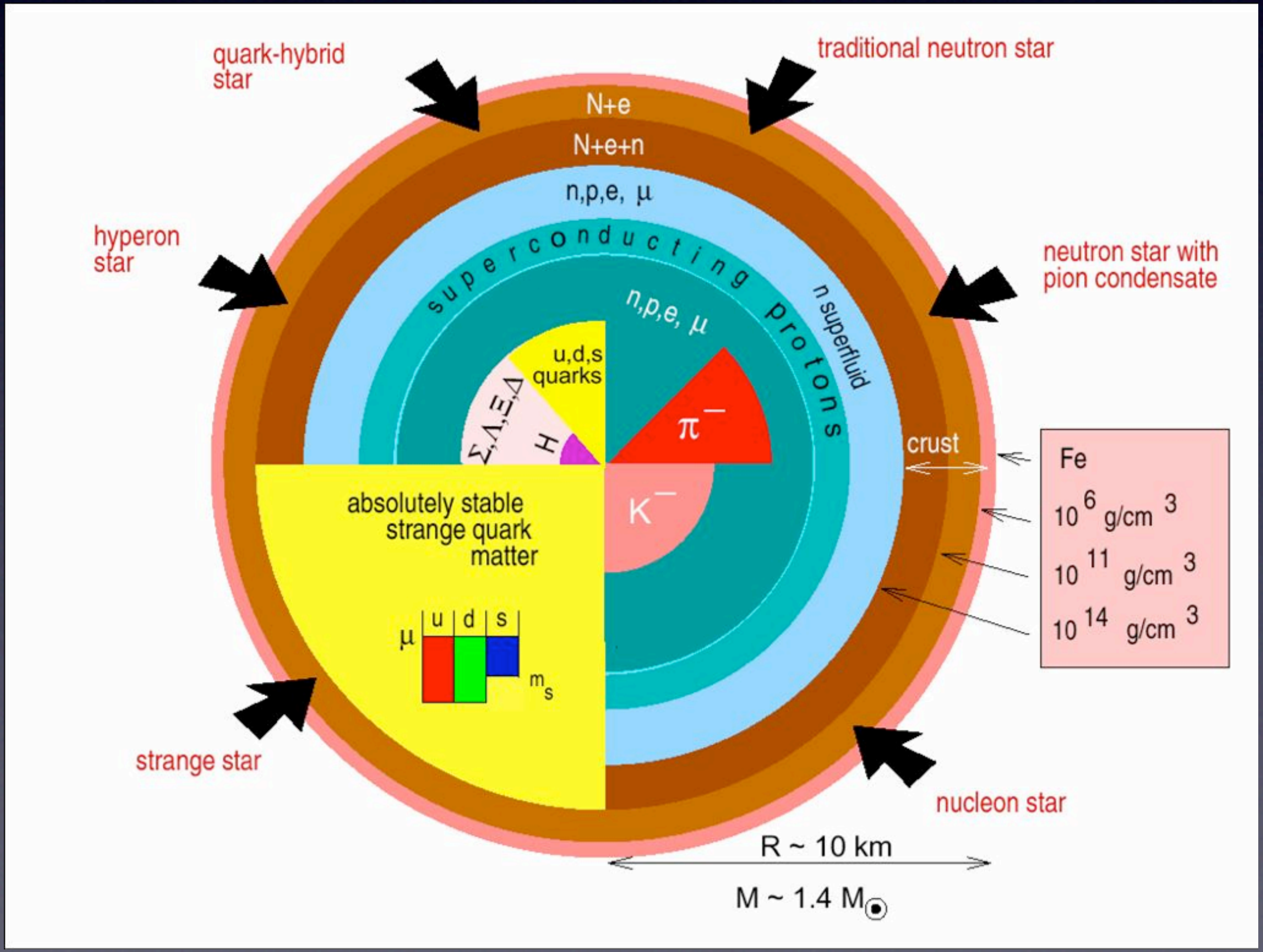
GFMC caclulations

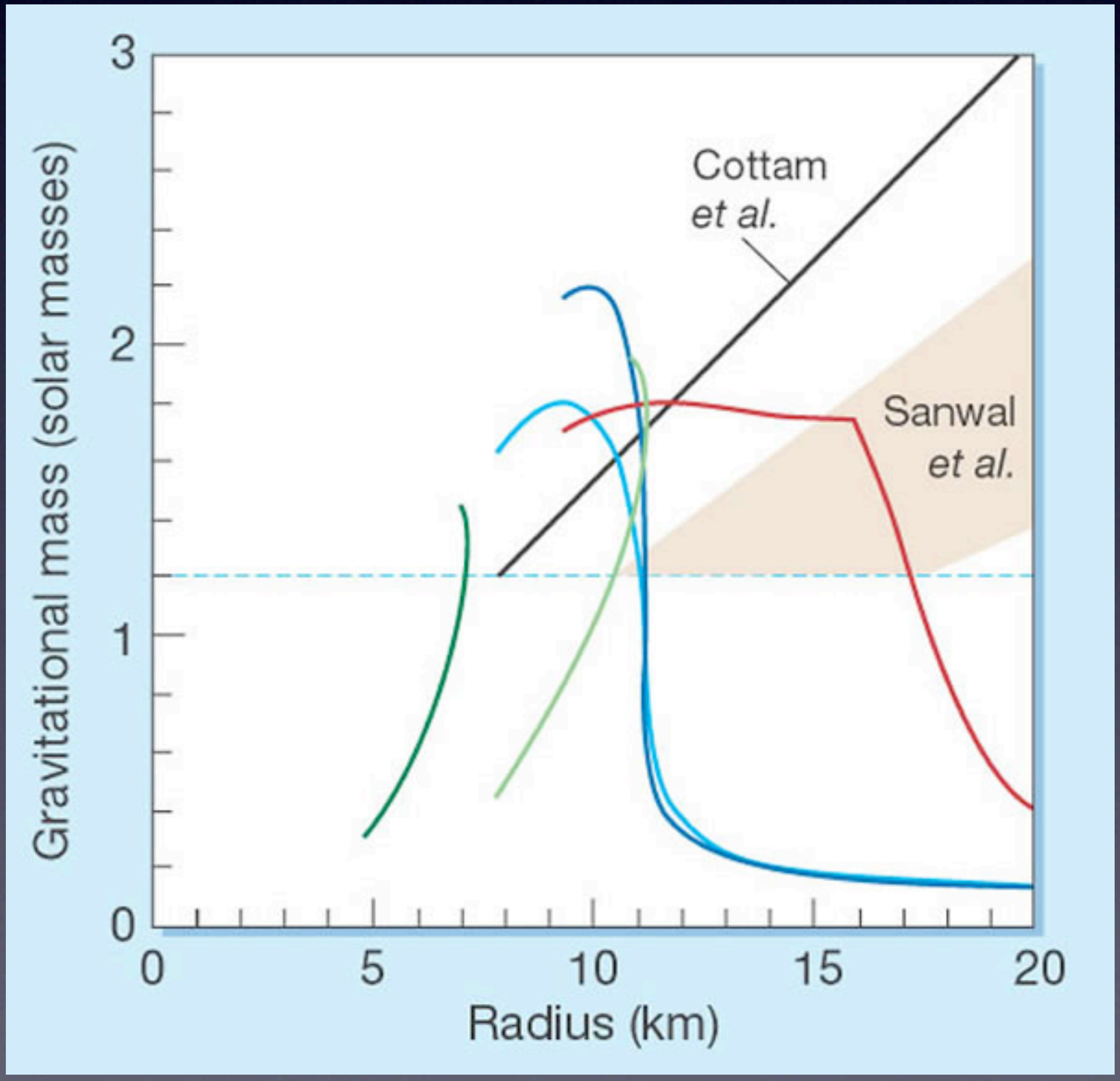


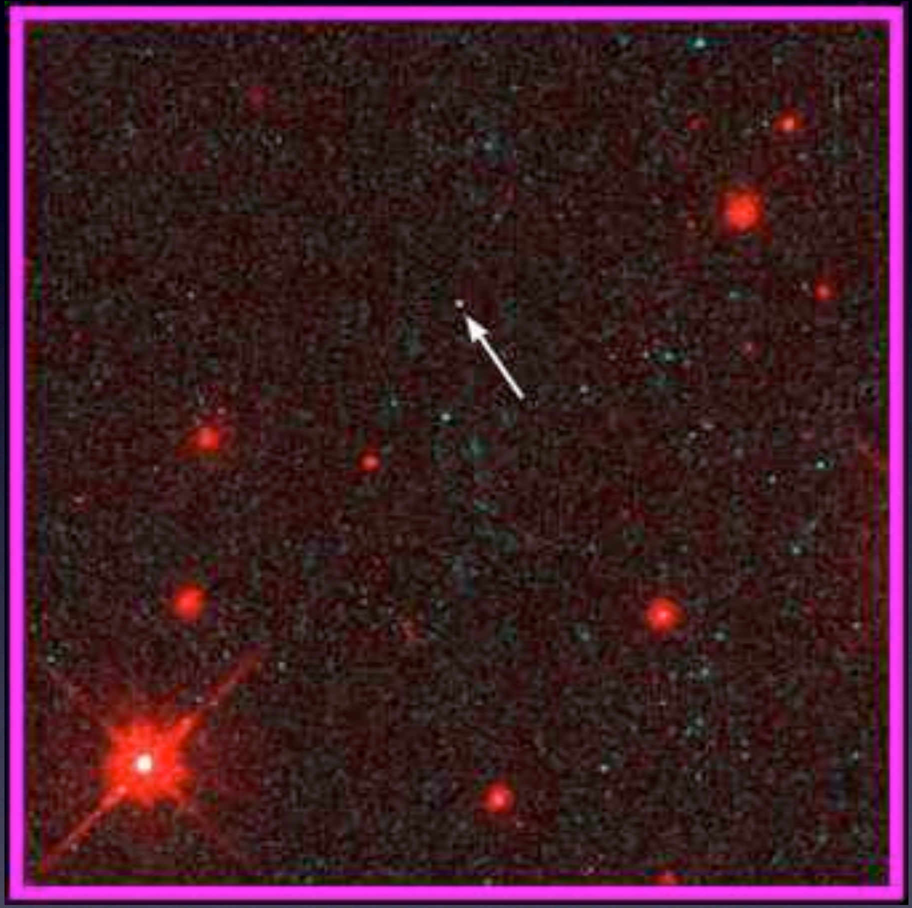
[Carlson, Pieper, Wiringa,...]

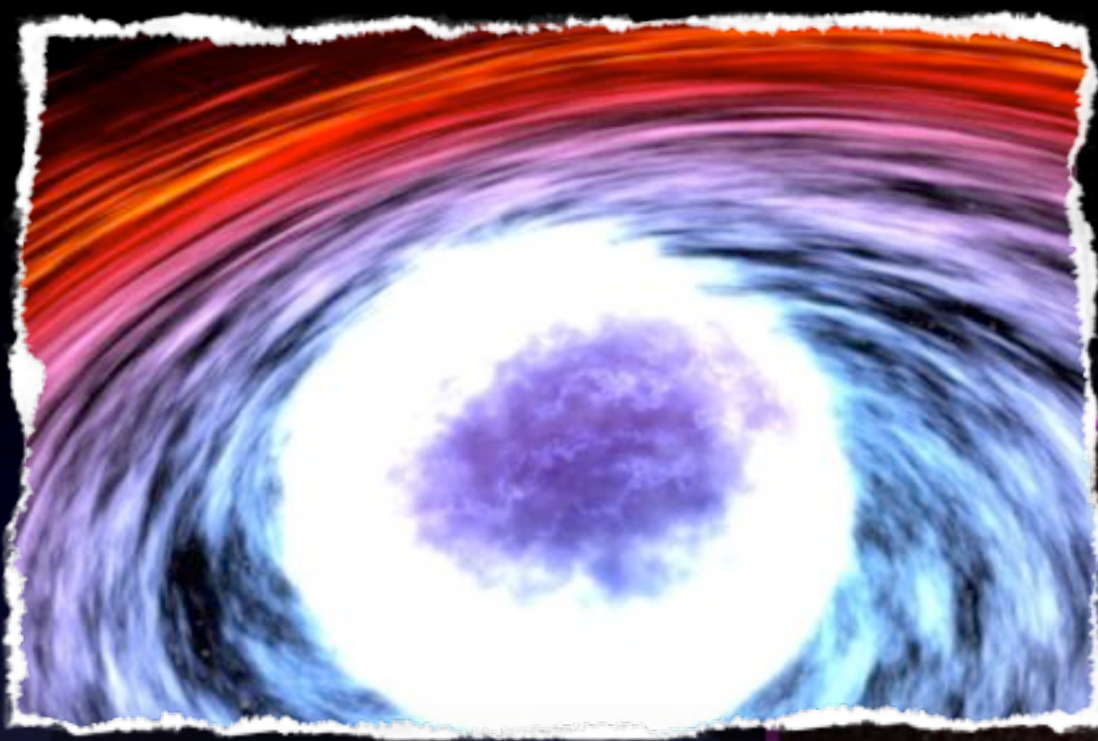
YN scattering

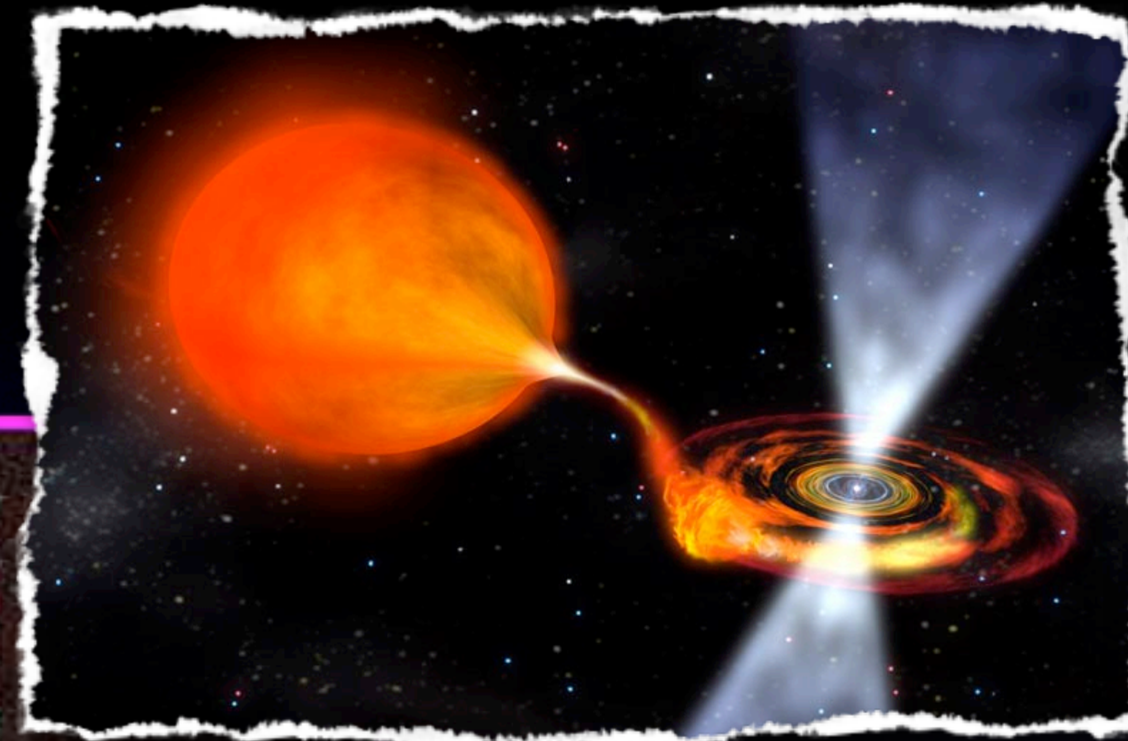
- $\Lambda(\Sigma)$N interactions important in EOS in NS
- Poorly known experimentally

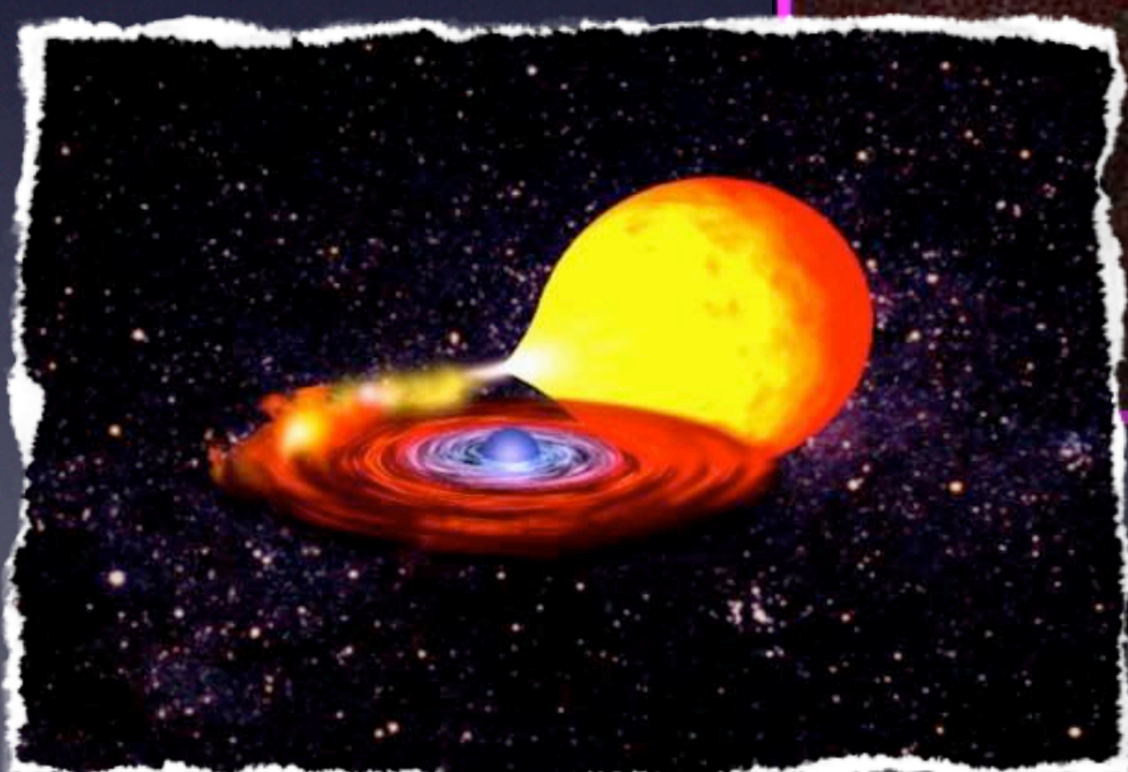


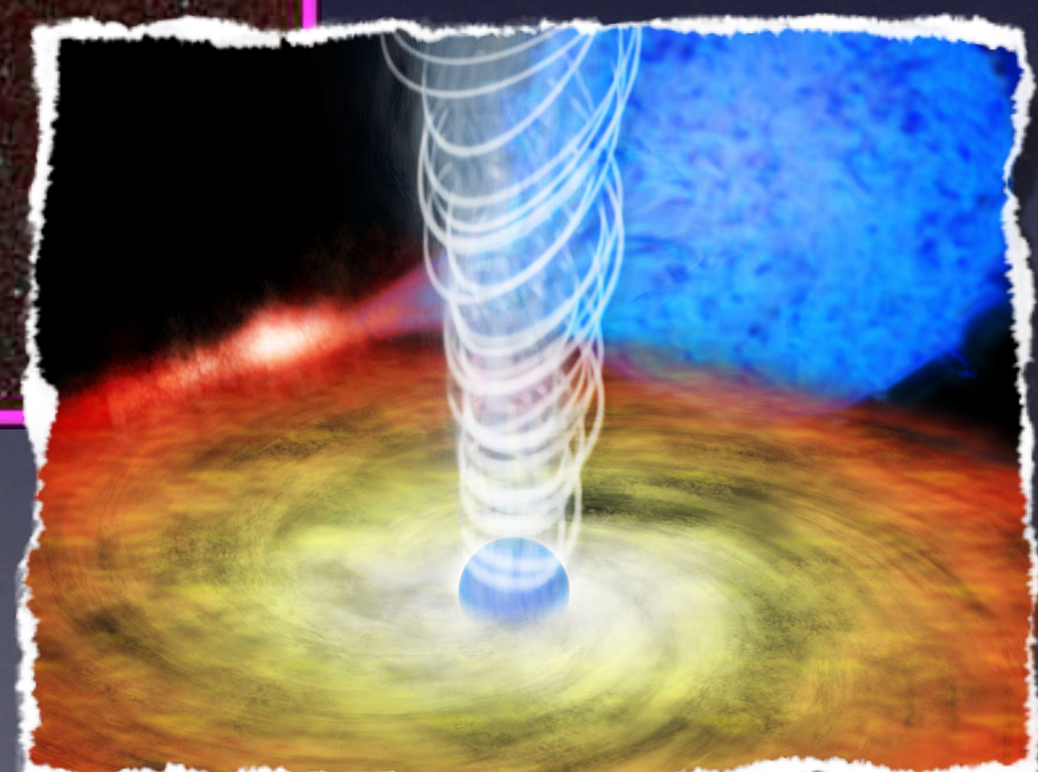












Static heavy quarks

- Static limit for heavy quark: $m_b \to \infty$
- *B* meson ($b\bar{d}$) mass is infinite

$$M_b = m_b + \overline{\Lambda} + \mathcal{O}(1/m_b)$$

- *B* and *B** degenerate
- Heavy quark spin decouples

Static *BB* potentials?

- Static hadrons: defined, observable potential
- LDOF quantum numbers $\supset$ *NN*
- EFT: potential has same form for $|\vec{r}| > \Lambda_\chi^{-1}$

$$V_{NN}(|\vec{r}| > \Lambda_\chi^{-1}) \longrightarrow \# \frac{g_{\pi NN}^2}{f^2} \frac{e^{-m_\pi |\vec{r}|}}{|\vec{r}|}$$

- Short range very different: $1/|\vec{r}|$ Coulomb
- Shallow bound states, molecular states, ...

Static *BB* potentials?

- Static hadrons: defined, observable potential
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$$V_{BB}(|\vec{r}| > \Lambda_\chi^{-1}) \longrightarrow \# \frac{g_{BB^*\pi}^2}{f^2} \frac{e^{-m_\pi |\vec{r}|}}{|\vec{r}|}$$

- Short range very different: $1/|\vec{r}|$ Coulomb
- Shallow bound states, molecular states, ...

Lattice QCD

- Numerical solution of QCD field equations
- QCD partition function

$$\mathcal{Z} \sim \int \mathcal{D}A_\mu \mathcal{D}\psi\, \mathcal{D}\overline{\psi}\, e^{-S_{QCD}[A,\psi,\overline{\psi}]}$$

  - Quark functional integral done exactly

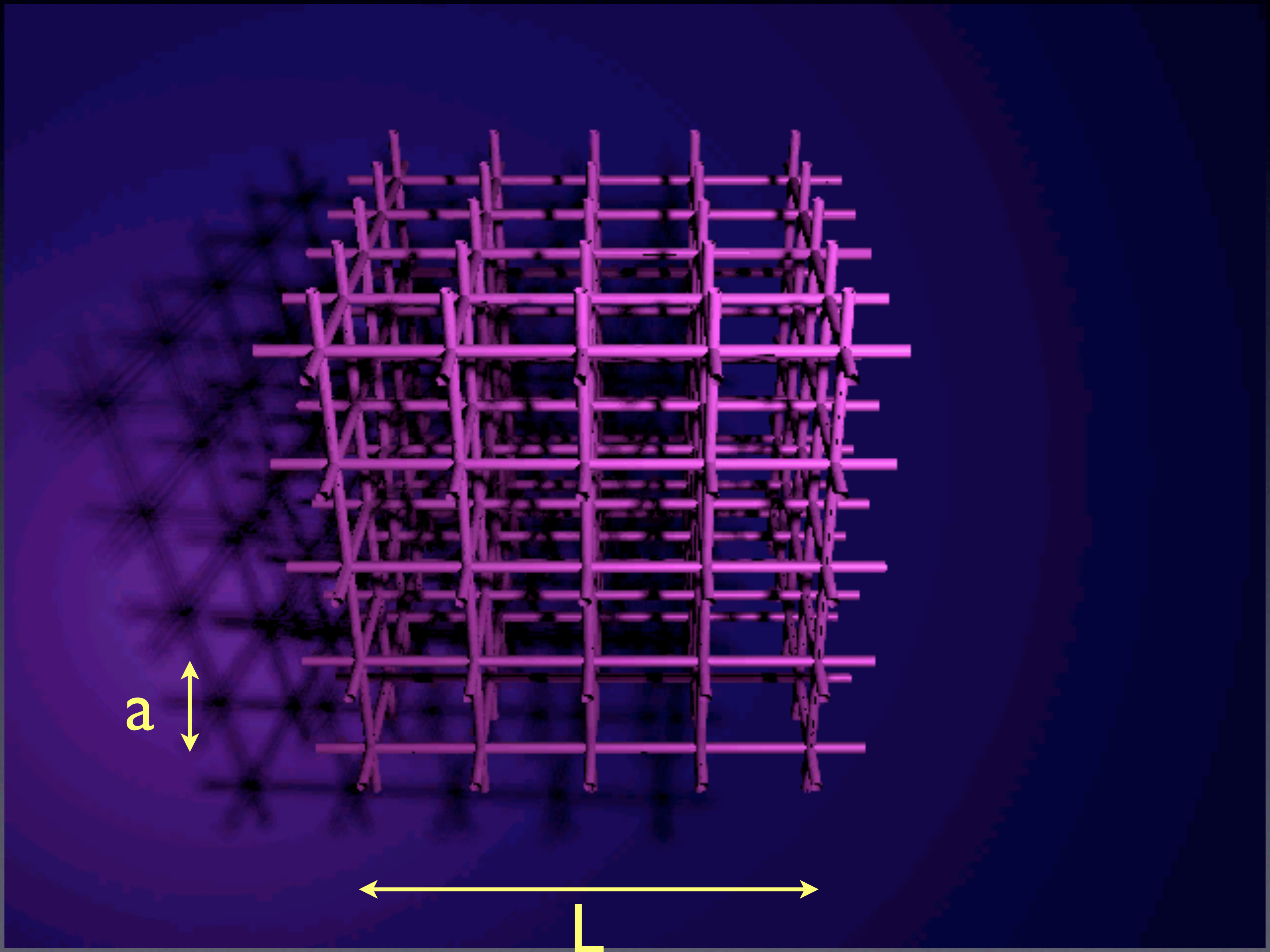
- Observable

$$\langle \mathcal{O} \rangle \sim \frac{1}{\mathcal{Z}} \int \mathcal{D}A_\mu\ \det[\mathcal{M}[A]]\mathcal{O}[A,\mathcal{M}]e^{-S_g[A]}$$

Lattice QCD

- Numerical method to solve QCD non-perturbatively [K Wilson 1974]
- Computers are finite but space-time is infinite
- To make progress
  - Discretise space-time: a
  - Compactify space-time: L
  - Euclidean space

Space-time



Lattice QCD

- Quarks live on lattice sites, gluons on the links between them
- Functional integral is finite dimensional but still too many integrals ($>10^7$!) to do exactly
- Estimate using importance sampling (Monte Carlo)
  - Configurations $\{\phi_i\}$ generated with Boltzmann weight $\det[\mathcal{M}]\exp(-S_{\mathrm{QCD}})$
  - Observable: $\langle\mathcal{O}\rangle \to \frac{1}{N}\sum_{i=1}^{N}\mathcal{O}[\phi_i]$ errors $\sim 1/\sqrt{N}$

Huge computers

- Calculations use worlds largest computers
- Measured in millions of CPU hours
- Specifically designed processors for LQCD

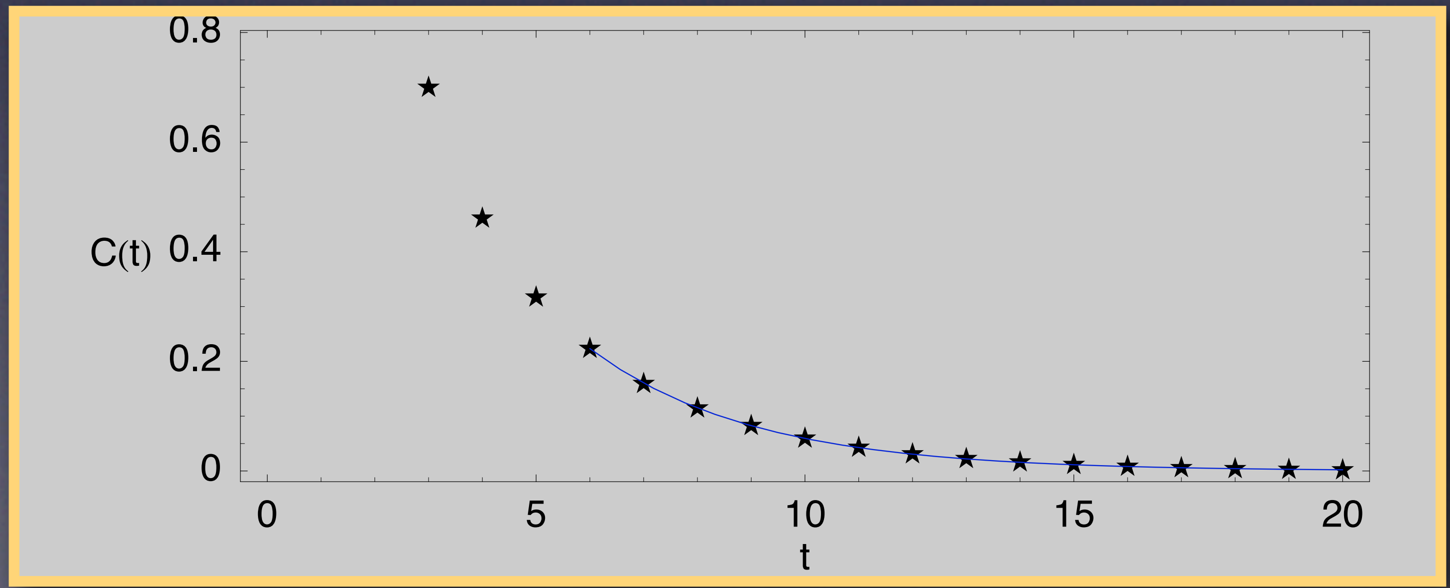
Ex: energy spectrum

- Measure correlator (χ = source with q# of hadron)

$$G_2(\mathbf{p},t) = \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}} \langle 0|\chi(\mathbf{x},t)\overline{\chi}(0,0)|0\rangle$$

- Long times: only ground state survives

$$\xrightarrow{t\to\infty} e^{-E_0(\mathbf{p})t} \langle 0|\chi(0,0)|E_0,\mathbf{p}\rangle\langle E_0,\mathbf{p}|\overline{\chi}(0,0)|0\rangle$$



Extrapolations

- To get real world physics from the lattice calculations we need to take:
  - Lattice spacing to zero
  - Lattice volume to infinity
  - Quark masses to their physical values
    - *Physical masses too demanding*
- *Mostly ignore today...*

Heavy-light systems

Lattice parameters

- $\mathcal{O}(300)$ *quenched* DBW2 lattices: 16^3x32, a=0.1 fm
- Wilson light quark propagators at a single mass
- Light hadron spectrum [MeV]:

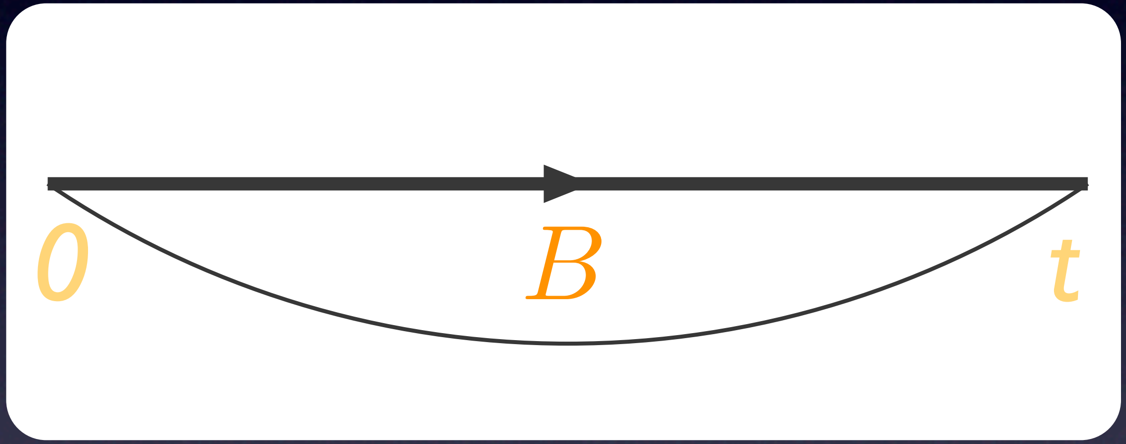
$$m_\pi = 403 \quad m_\rho = 743 \quad m_N = 1140$$

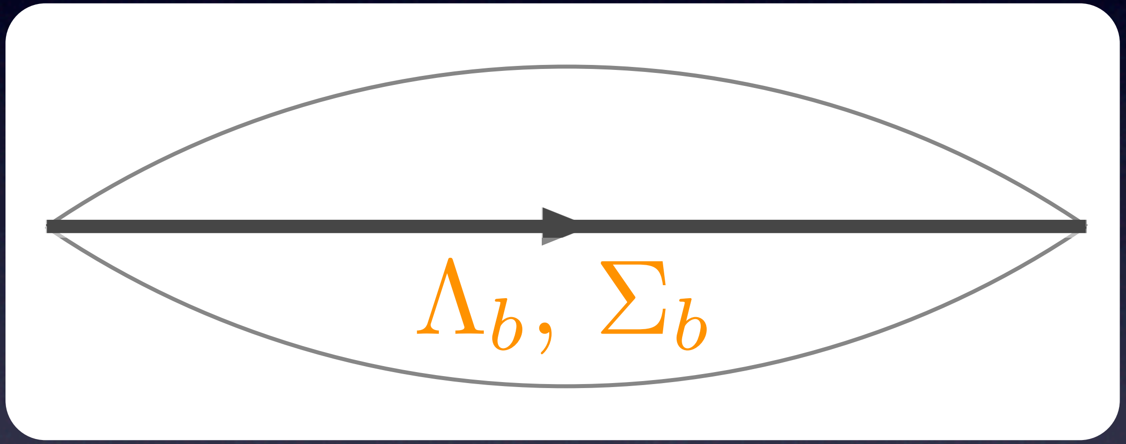
- Static heavy quarks:

$$S_Q(\mathbf{x}, t; t_0) = \left(\frac{1+\gamma_4}{2}\right) \prod_{t'=t_0}^{t} U_4(\mathbf{x}, t')$$

Lattice correlators

- Single particle correlation functions: *C(t)*

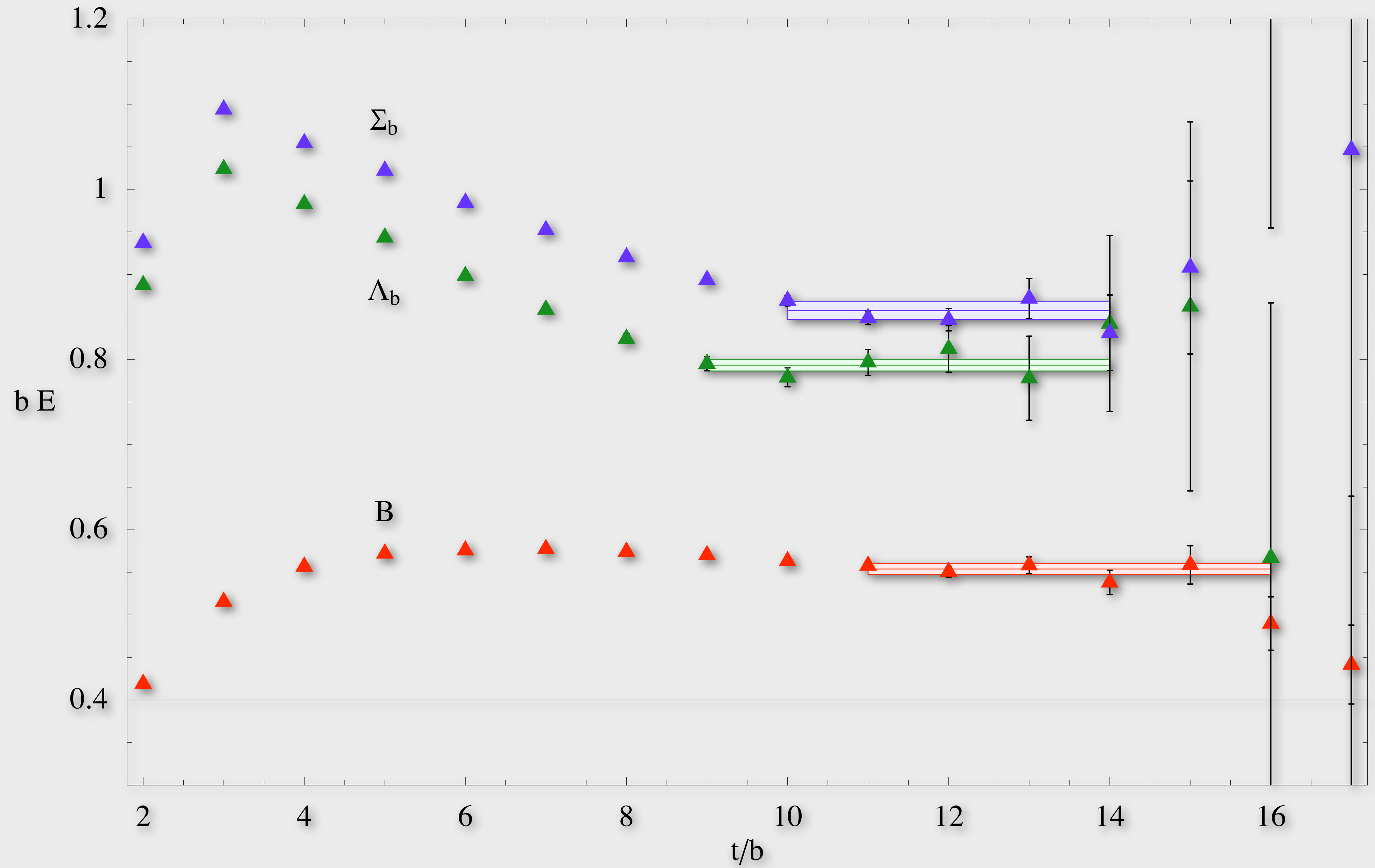




- Energy: plateau in effective mass ratio

$$aE(t) = \log\left[\frac{C(t-1)}{C(t)}\right]$$

Single particle energies



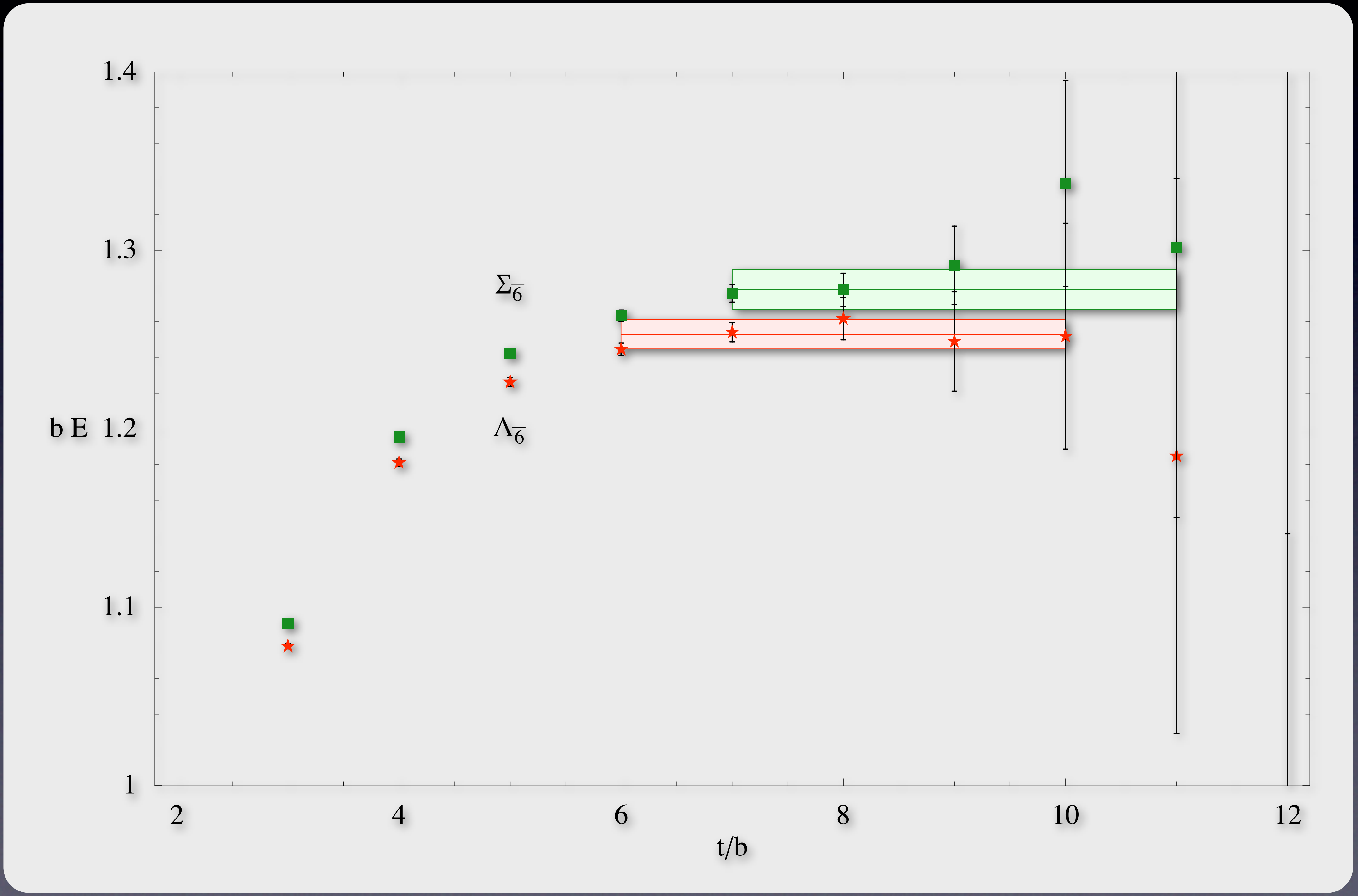
Exotic hadrons

- Byproduct of r=(0,0,0) potential
- Binding energy of exotic hadron states:

$$(\mathbf{3} \otimes \mathbf{3})_Q \otimes (\overline{\mathbf{3}} \otimes \overline{\mathbf{3}})_q \to \overline{\mathbf{3}}_Q \otimes \mathbf{3}_q + \mathbf{6}_Q \otimes \overline{\mathbf{6}}_q$$

- Two exotic states: $\Sigma_{\overline{\mathbf{6}}}$, $\Lambda_{\overline{\mathbf{6}}}$
- Considered long ago: viable BSM
  [Karl, Wilczek&Zee, Marciano,... 70s]

Exotic particle energies

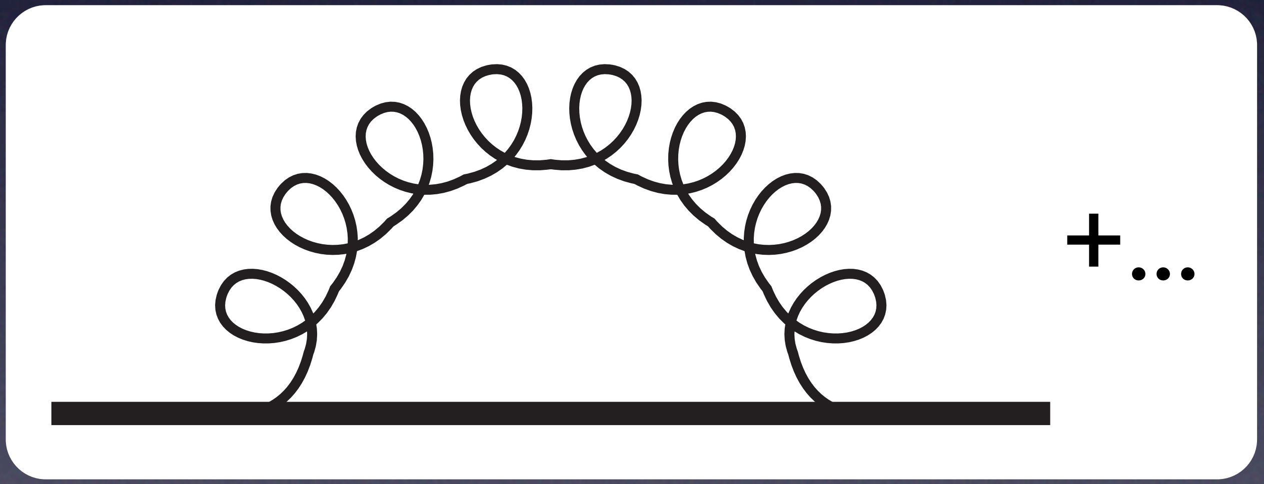


LDOF energies: $\overline{\Lambda}$

- Lattice energies are unphysical ~ a^{-1}

$$E_{latt} = \delta m + \overline{\Lambda}$$

- δm: interaction of static quark with gluons



- $\overline{\Lambda}$: energy of light degrees of freedom

Lattice PT: improved glue

$$G_{\mu\nu}(k) = \frac{1}{(\hat{k}^2)^2}\left(\alpha \hat{k}_\mu \hat{k}_\nu + \sum_\sigma (\hat{k}_\sigma \delta_{\mu\nu} - \hat{k}_\nu \delta_{\mu\sigma})\hat{k}_\sigma A_{\sigma\nu}(k)\right)$$

$$\hat{q}_i = 2\sin\frac{q_i}{2}$$

$$A_{\mu\nu}(k) = A_{\nu\mu}(k) = (1-\delta_{\mu\nu})\Delta(k)^{-1}\left[(\hat{k}^2)^2 - c_1\hat{k}^2\left(2\sum_\rho \hat{k}_\rho^4 + \hat{k}^2 \sum_{\rho\neq\mu,\nu}\hat{k}_\rho^2\right)\right.$$
$$\left. + c_1^2\left(\left(\sum_\rho \hat{k}_\rho^4\right)^2 + \hat{k}^2\sum_\rho \hat{k}_\rho^4 \sum_{\tau\neq\mu,\nu}\hat{k}_\tau^2 + (\hat{k}^2)^2 \prod_{\rho\neq\mu,\nu}\hat{k}_\rho^2\right)\right]$$

$$\Delta(k) = \left(\hat{k}^2 - c_1\sum_\rho \hat{k}_\rho^4\right)\left[\hat{k}^2 - c_1\left((\hat{k}^2)^2 + \sum_\tau \hat{k}_\tau^4\right) + \frac{1}{2}c_1^2\left((\hat{k}^2)^3 + 2\sum_\tau \hat{k}_\tau^6 - \hat{k}^2\sum_\tau \hat{k}_\tau^4\right)\right]$$
$$- 4c_1^3 \sum_\rho \hat{k}_\rho^4 \sum_{\tau\neq\rho}\hat{k}_\tau^2\ .$$

c_1 = Improvement coeff

[Weisz & Wohlert 84]

LDOF energies: $\overline{\Lambda}$

- Lattice energies are unphysical ~ a^{-1}

$$E_{latt} = \delta m + \overline{\Lambda}$$

- δm: interaction of static quark with gluons

$$\delta m_f^{(\alpha)} = \frac{\overline{\alpha}(\mu)}{3\pi^2 a} \int_{BZ} d^3q \, G_{00}^{(f)}(\hat{q}_x, \hat{q}_y, \hat{q}_z, 0)$$

- Evaluate numerically

$$\hat{q}_i = 2\sin\frac{q_i}{2}$$

- $\overline{\Lambda}$: energy of light degrees of freedom

Scale setting

- Scale of one loop contribution not well-defined
- Brodsky-Lepage-Mackenzie procedure
    - Sum vacuum polarisation effects
- Subtlety: depends on choice of action

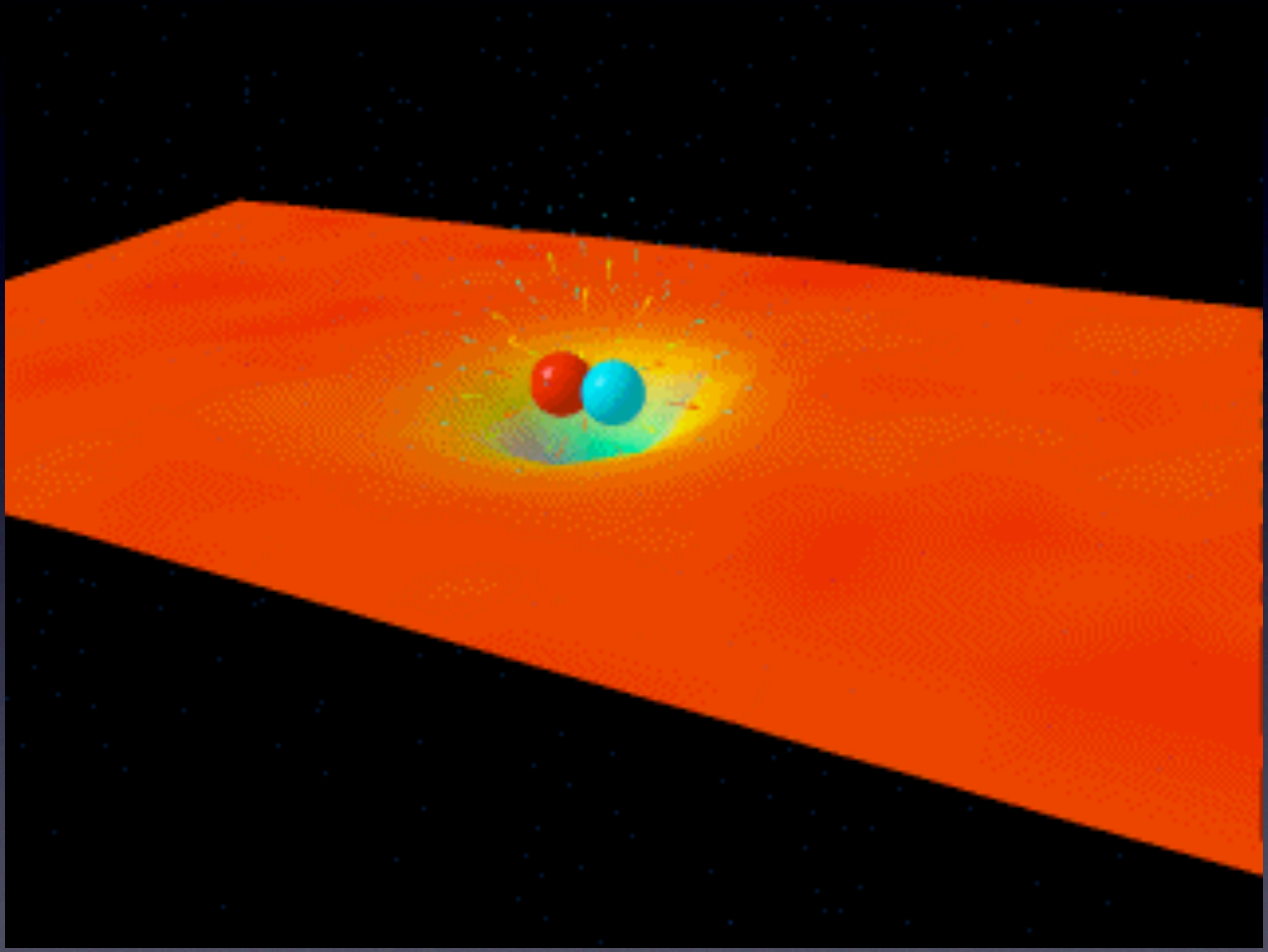
[BLM ‘83/LM ‘93]

Binding energies

- Extracted physical binding energies [MeV]

B	649(31)(10)
Λ_b	1123(36)(04)
Σ_b	1250(38)(15)
$\Lambda_{\bar{6}}$	1364(64)(04)
$\Sigma_{\bar{6}}$	1413(65)(10)

B meson potentials



[cf $b\bar{b}$ potential: Derek Leinweber]

BB potentials

- Lattice energies for BB(I,s_l):

$$V^{latt}_{I,s_l}(\mathbf{r},L) = E^{latt}_{I,s_l}(\mathbf{r},L) - 2E^{latt}_{B}(L)$$

- Perturbative QCD contribution

$$E^{latt}_{I,s_l}(\mathbf{r},L) = \overline{\Lambda}_{I,s_l}(\mathbf{r},L) + 2\delta m - \delta V_{\mathbf{R}}(\mathbf{r},L)$$

- "Continuum" potential

$$V_{I,s_l}(\mathbf{r},L) = \overline{\Lambda}_{I,s_l}(\mathbf{r},L) - 2\overline{\Lambda} = V^{latt}_{I,s_l}(\mathbf{r},L) + \delta V_{\mathbf{R}}(\mathbf{r},L)$$

- Infinite volume continuum potential

Central & tensor potentials

- Separate central and tensor S=1 potentials

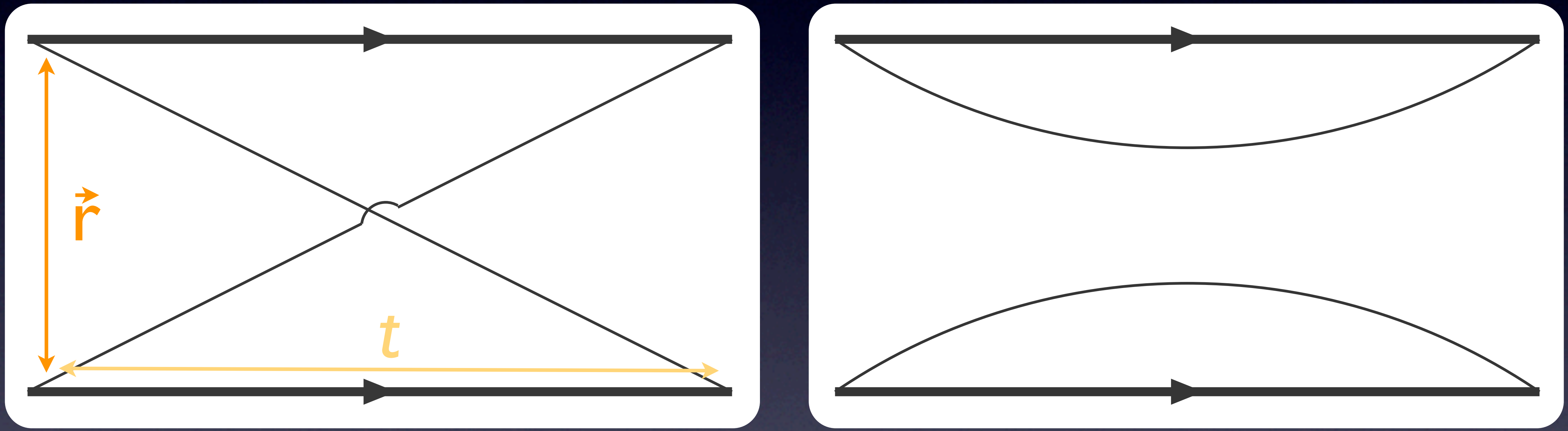
$$V^{(S=1)}(\vec{r}) = V_C(|\vec{r}|) \ + \ \hat{S}_{12}\, V_T(|\vec{r}|)$$

$$\hat{S}_{12} = \tfrac{3}{2}\left(\hat{S}_+(\hat{r}_x - i\hat{r}_y) + \hat{S}_-(\hat{r}_x + i\hat{r}_y) + 2\hat{S}_z\hat{r}_z\right)^2 \ - \ 2\hat{S}^2$$

- Tensor potentials: $|V_T| < 40\,\mathrm{MeV}\ \forall\,\vec{r}$
  - Expect nonzero for NN
  - Treat off-axis points as if central

BB correlators

- Each spin-isospin channel

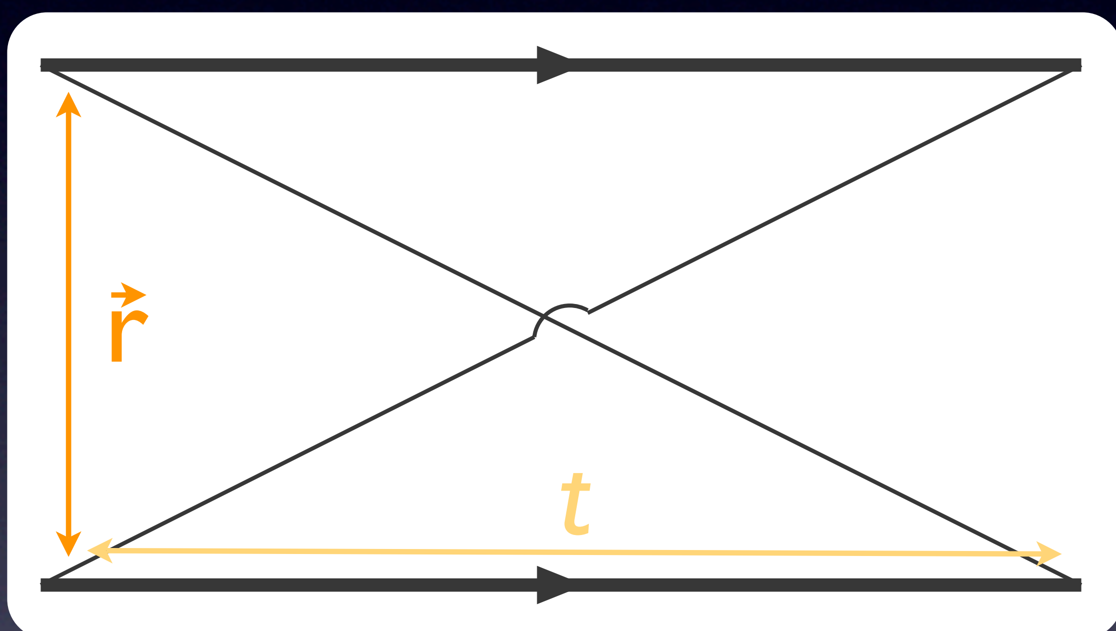


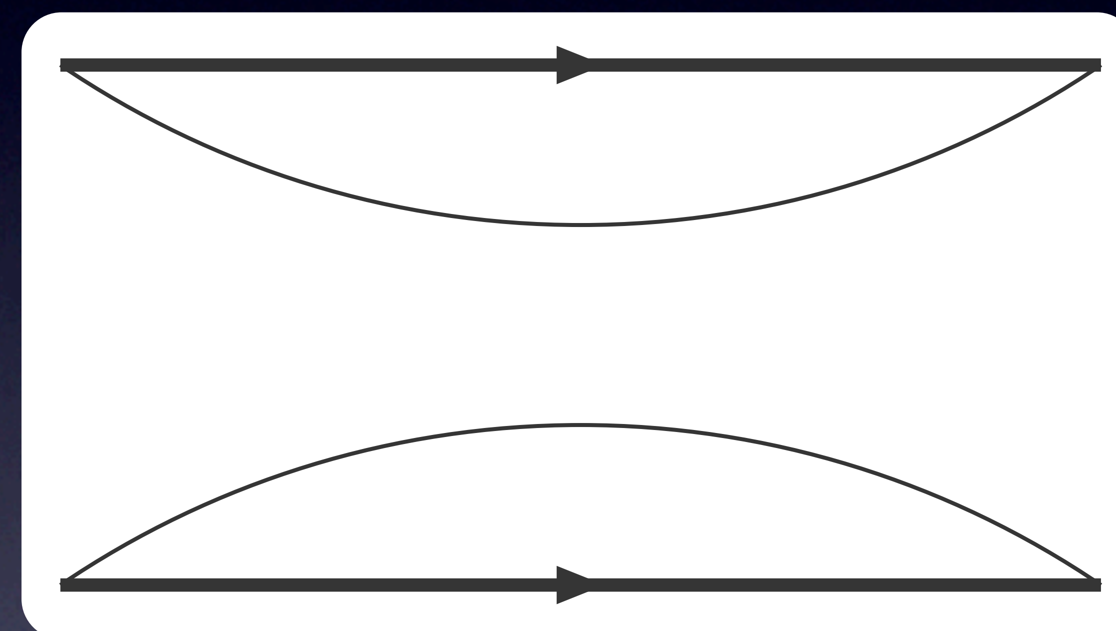
- Calculate for separations

$$\vec{r} = n(1,0,0), n(0,0,1)\ [n = 0,\ldots 8]$$
$$\vec{r} = (1,0,1),\ (2,0,1)$$

BB correlators

- Each spin-isospin channel





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$$\vec{r} = n(1,0,0), n(0,0,1)\, [n = 0, \ldots 8]$$
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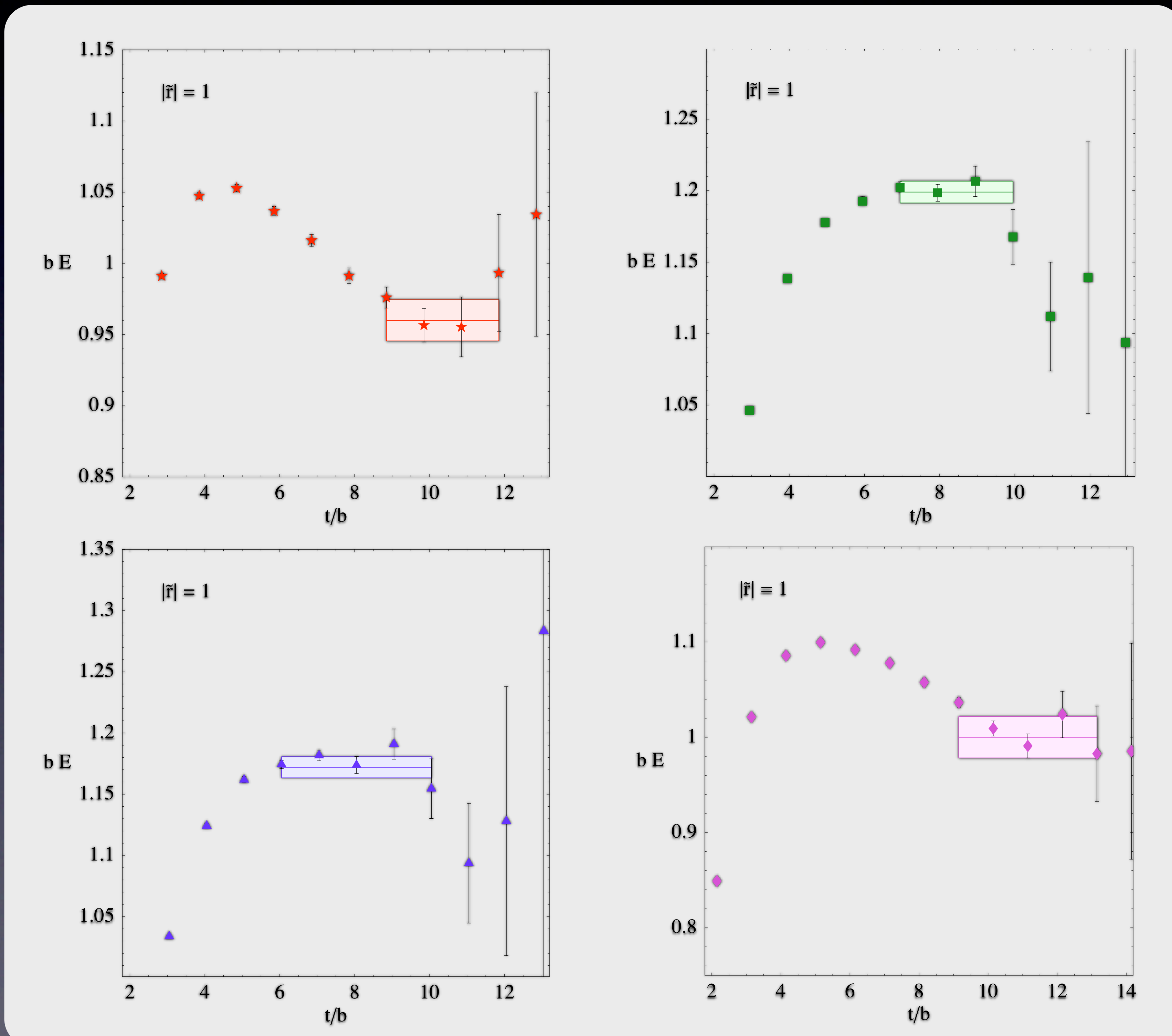
512 propagators/lattice!

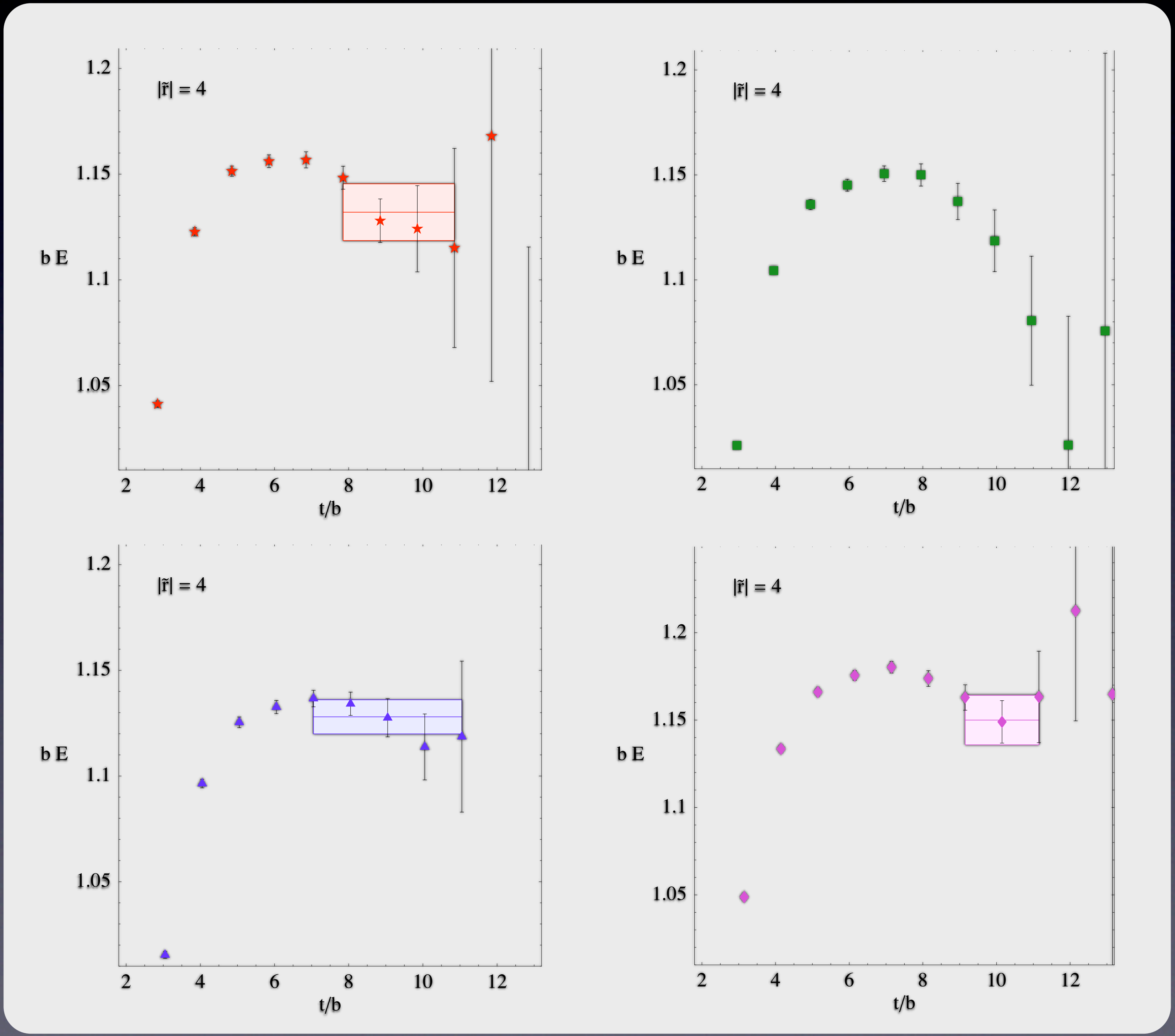
Effective mass plots



$$|\vec{r}| = 1$$

I, s_l	
0,0	1,0
0,1	1,1

Effective mass plots

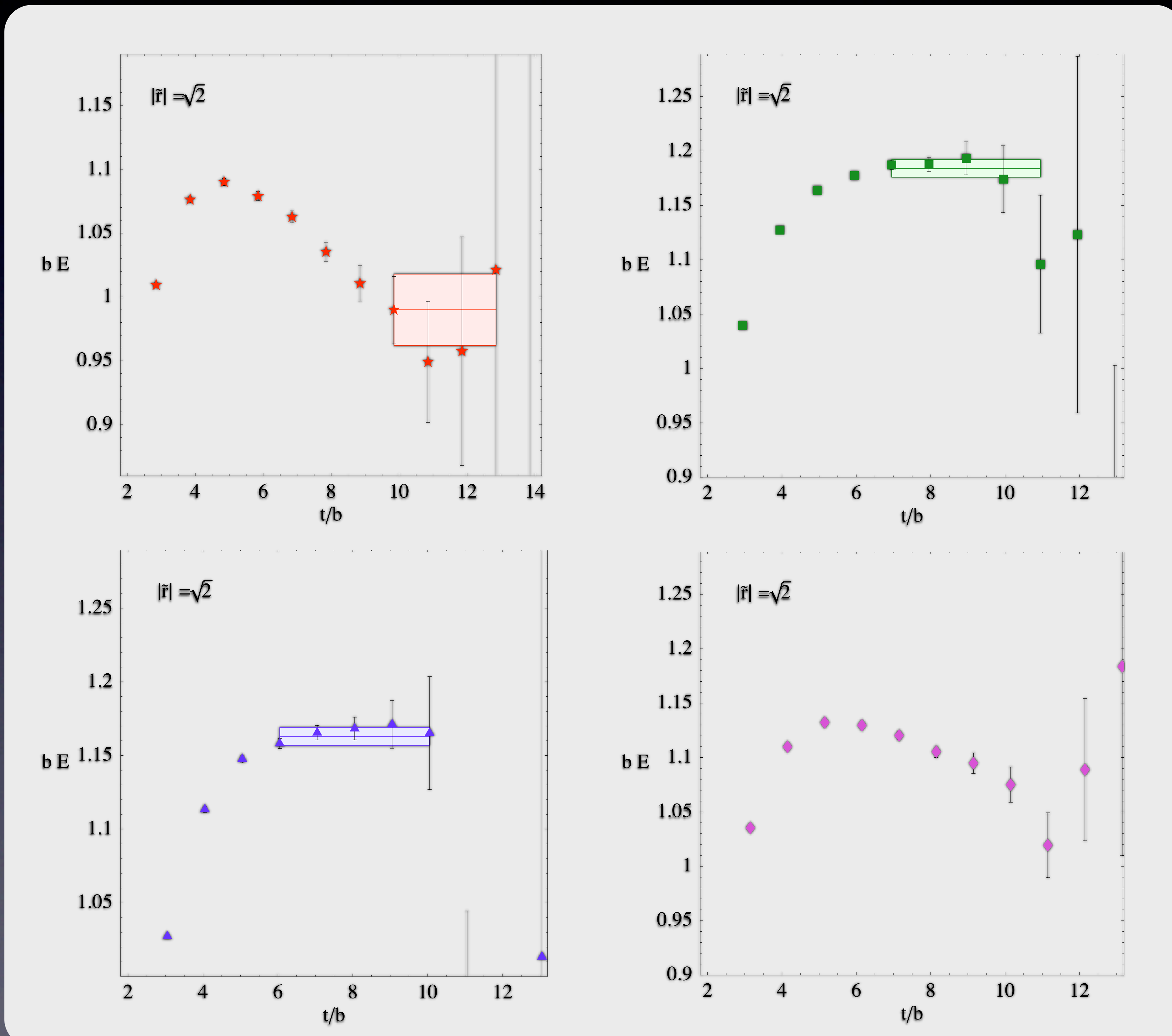


$$|\vec{r}| = 4$$

$$I, s_l$$

0,0	1,0
0,1	1,1

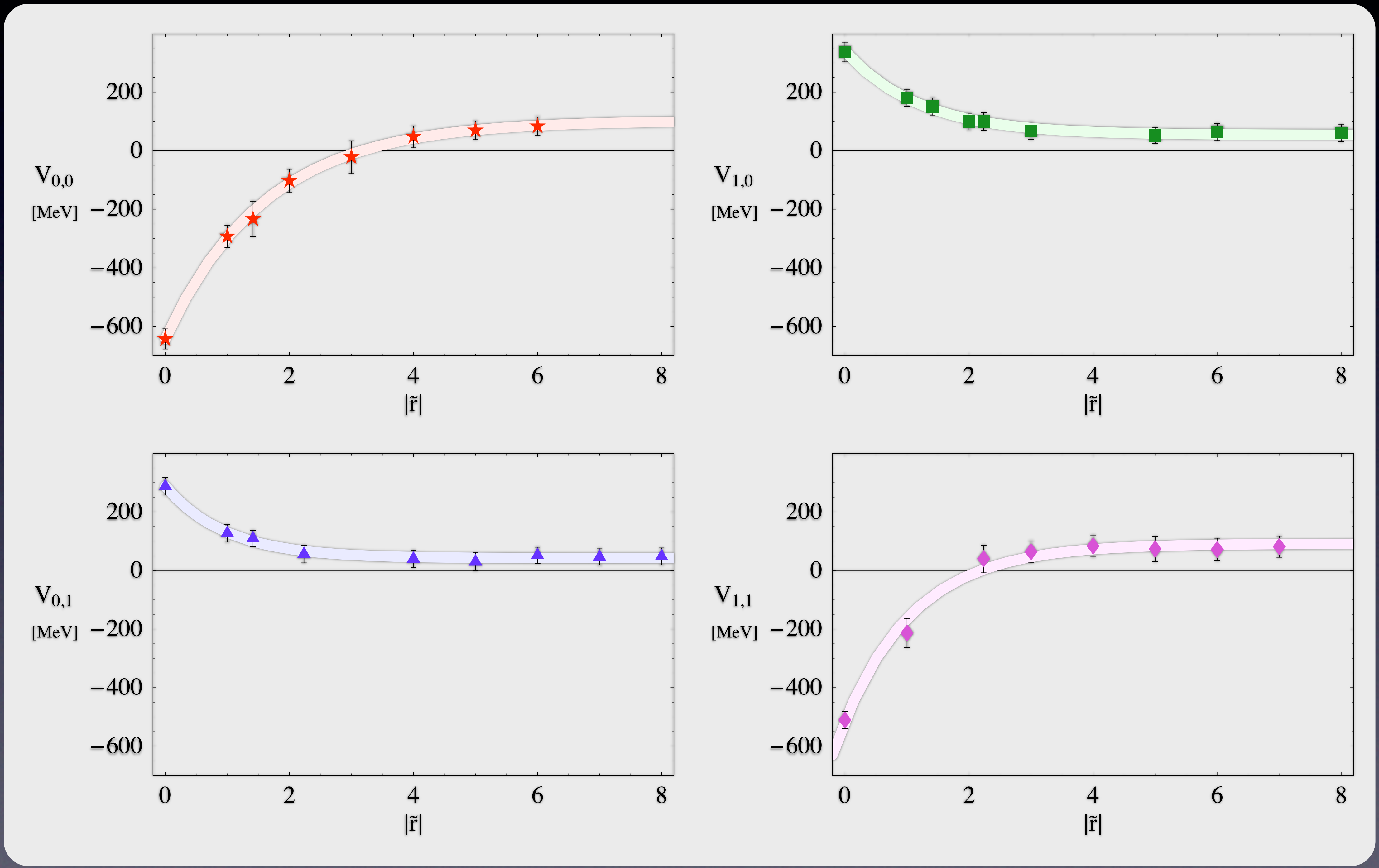
Effective mass plots



$$|\vec{r}| = \sqrt{2}$$

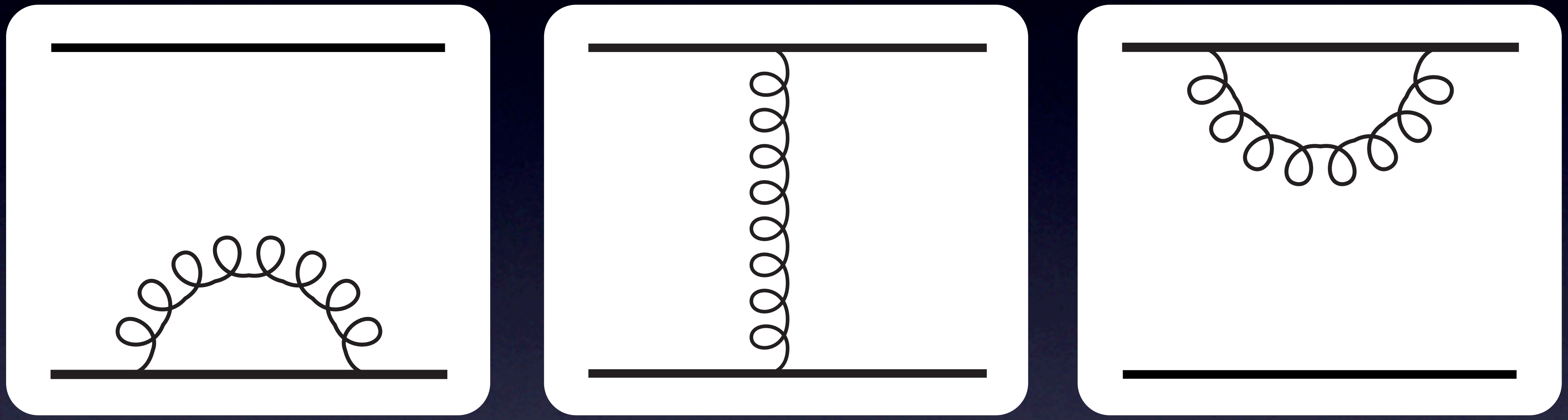
I, s_l	
0,0	1,0
0,1	1,1

Lattice $V = E_{BB} - 2E_B$



Coulomb potential

- OGE Coulomb potential: modified by *a*



- Subtract lattice, add continuum

$$\delta V^{(\alpha)}_{f;\overline{\mathbf{3}}} = \frac{\overline{\alpha}(\mu)}{3\pi^2 a}\left[\int e^{i\mathbf{q}\cdot\mathbf{r}}\frac{d^3q}{|\mathbf{q}|^2} - \int_{BZ} e^{i\mathbf{q}\cdot\mathbf{r}} G^{(f)}_{00}(\hat{q}_x,\hat{q}_y,\hat{q}_z,0)d^3q\right]$$

- BLM improve, FV effects
- Shift by 150 MeV at |r|=1 (∞ at r=0)

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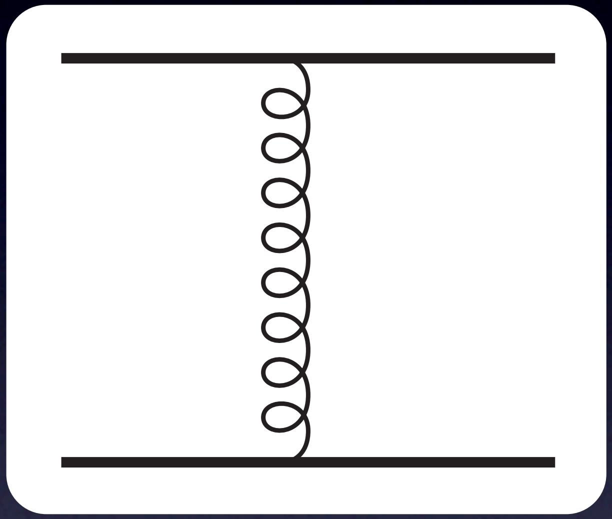

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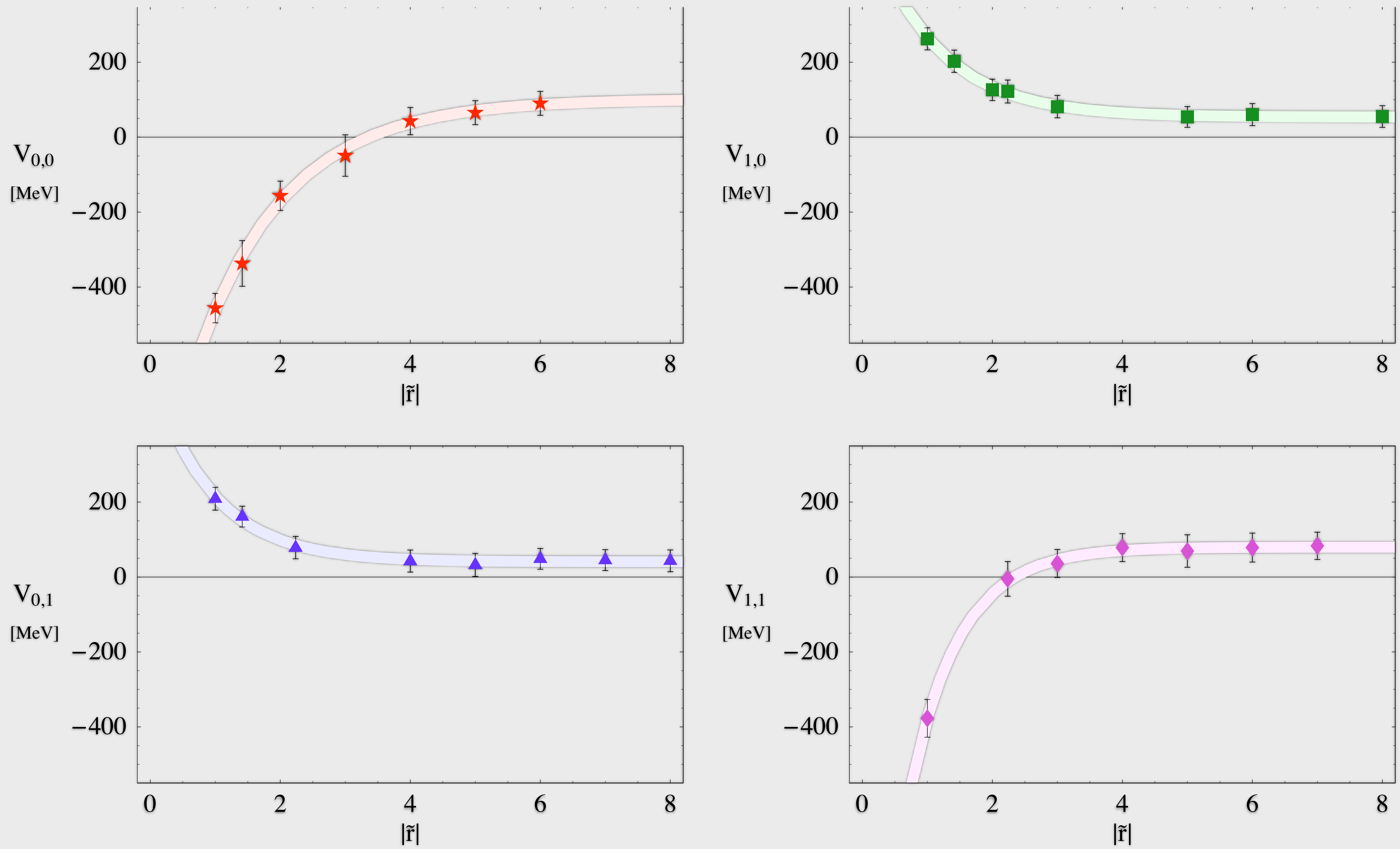
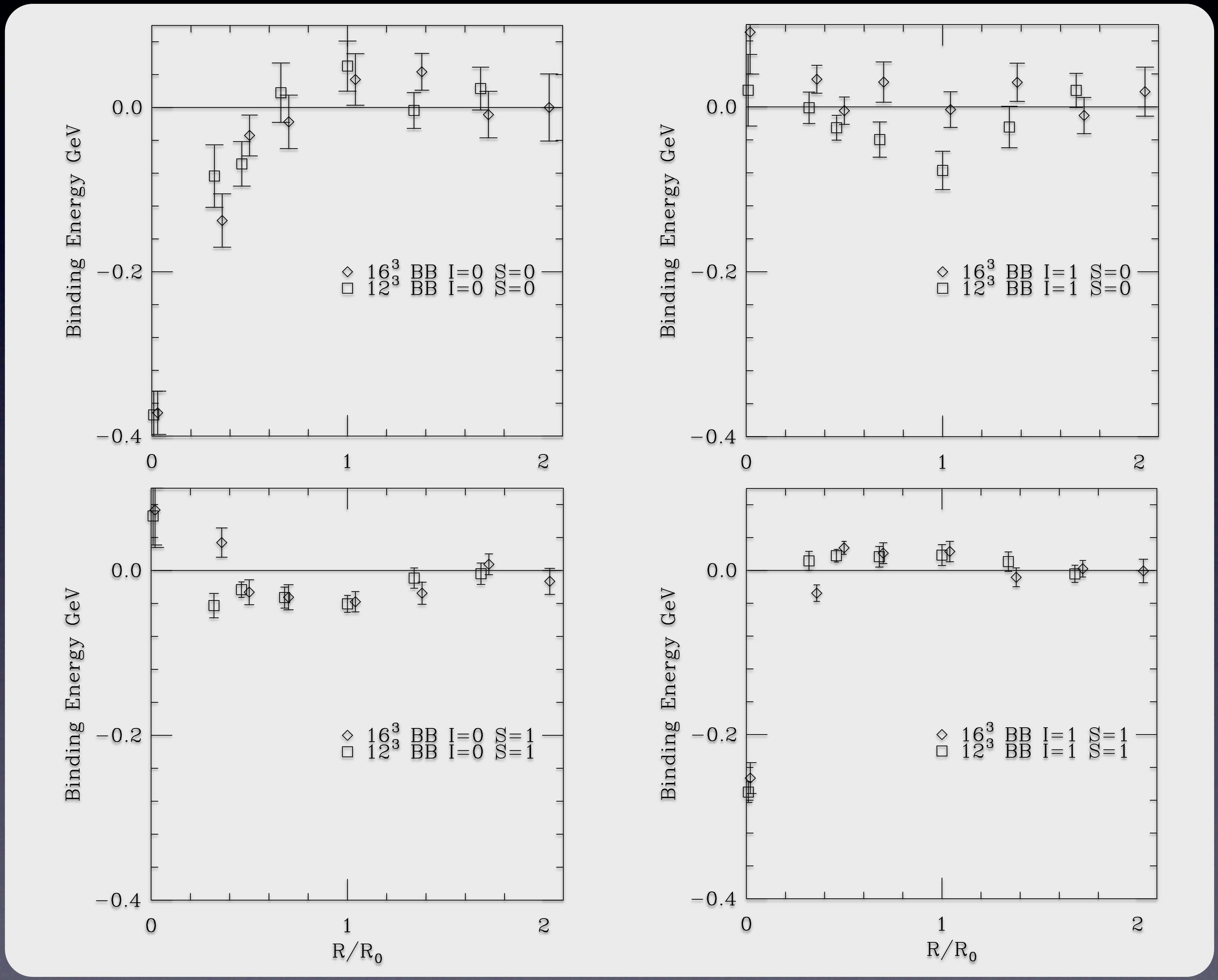
“Continuum” potentials



NB: residual $\mathcal{O}(a, \alpha^2(a)/a)$ effects

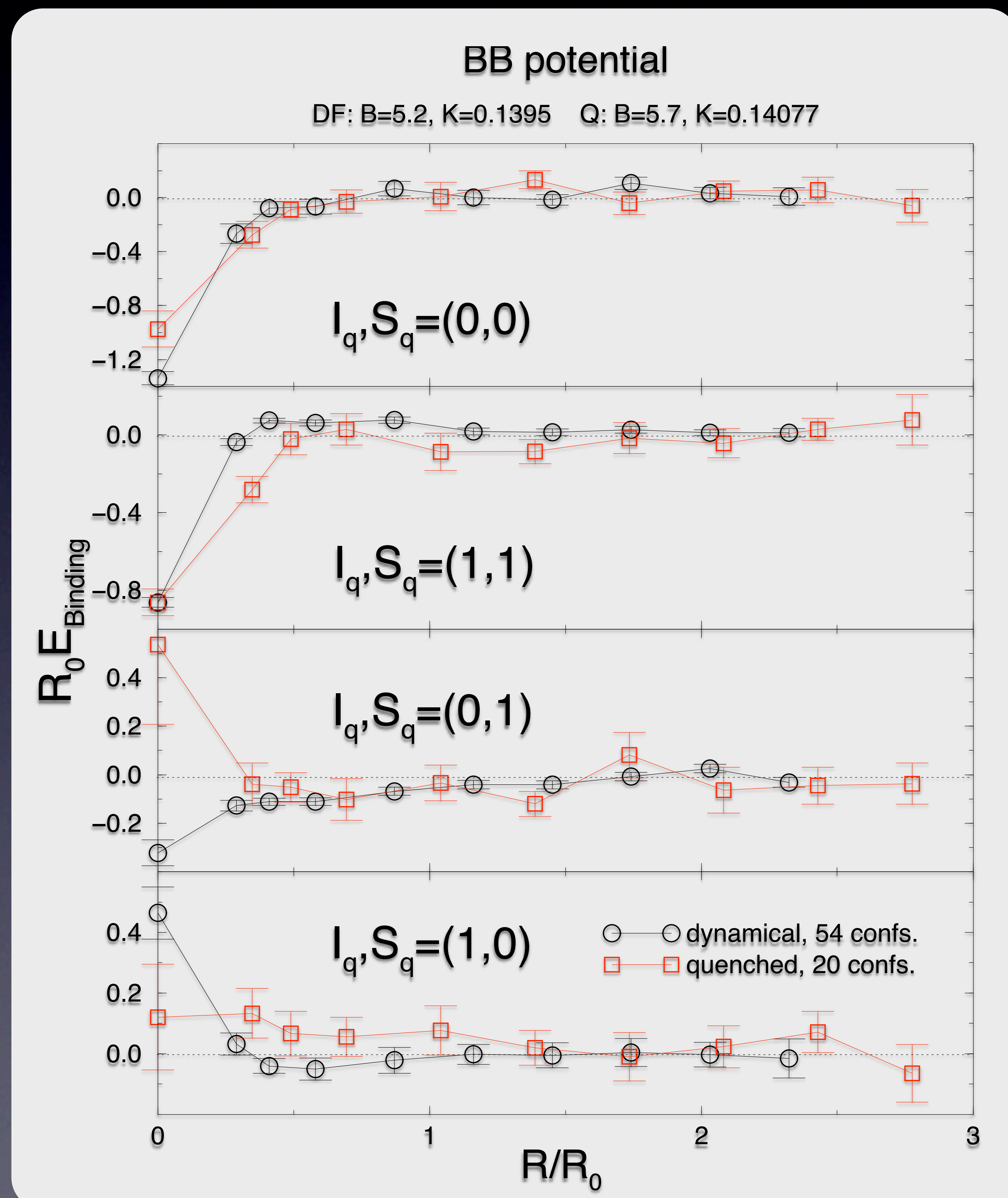
Michael & Pennanen '99



a=0.18 fm
16^3x24
20 cfgs

[Richards… '90, Mihály… '97, Stewart… '98, Fiebig… '02, Takahashi/Doi… '06]

Michael & Pennanen '99



Unquenched
a=0.14 fm
16^3x24
54 cfgs

t-channel potentials

- Potentials with t-channel quantum numbers

$$V_{I,s_l}(|\mathbf{r}|) = V_1 + \ \sigma_1 \cdot \sigma_2 \ V_\sigma + \sigma_1 \cdot \sigma_2 \, \tau_1 \cdot \tau_2 \, V_{\sigma\tau} + \tau_1 \cdot \tau_2 \ V_\tau$$

$$V_{\sigma\tau} = \frac{1}{16}\left(V_{0,0} - V_{0,1} - V_{1,0} + V_{1,1}\ \right)$$

$$V_\tau = \frac{1}{16}\left(-V_{0,0} - 3V_{0,1} + V_{1,0} + 3V_{1,1}\right)$$

$$V_1 = \frac{1}{16}\left(V_{0,0} + 3V_{0,1} + 3V_{1,0} + 9V_{1,1}\right)$$

$$V_\sigma = \frac{1}{16}\left(-V_{0,0} + V_{0,1} - 3V_{1,0} + 3V_{1,1}\right)$$

t-channel potentials

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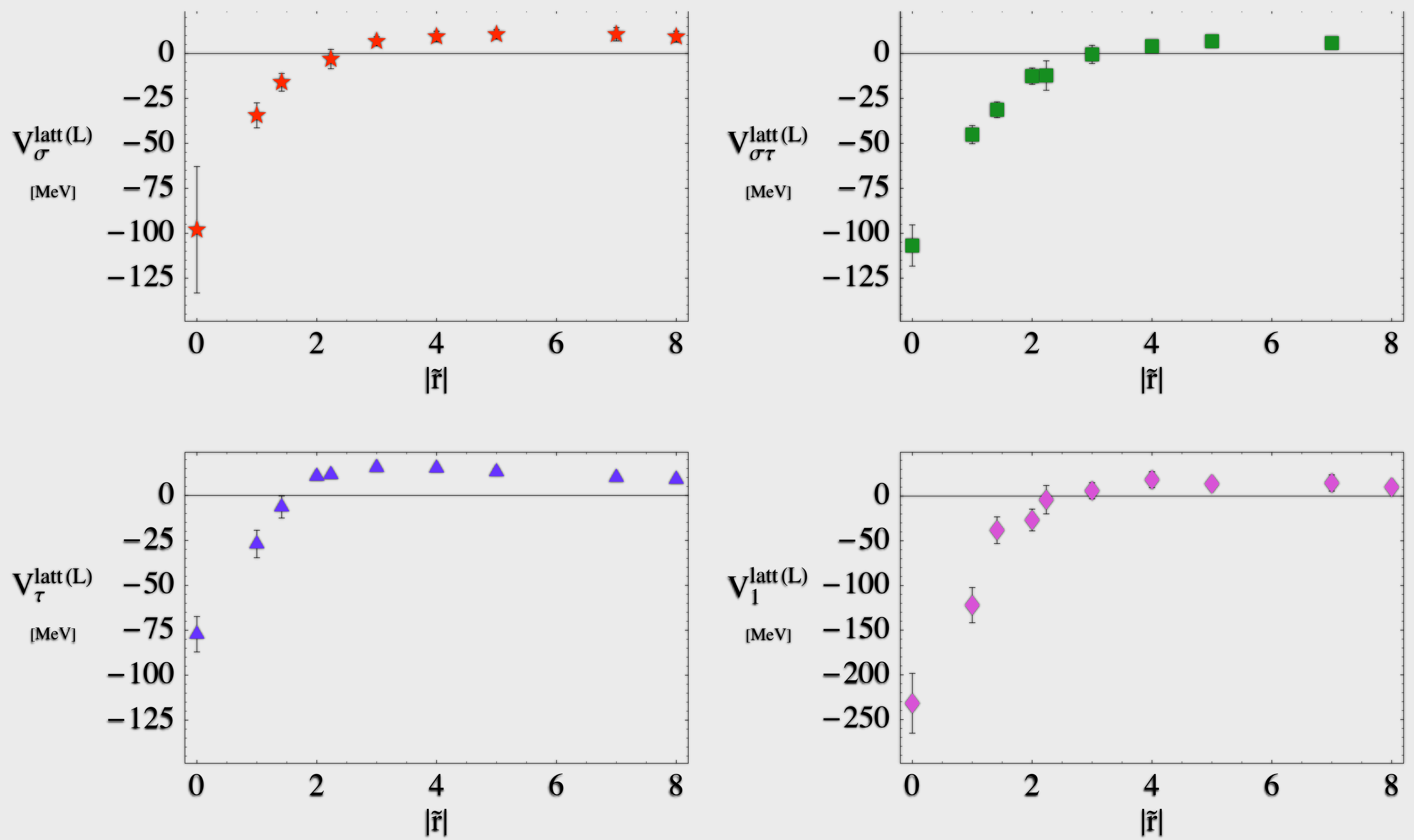
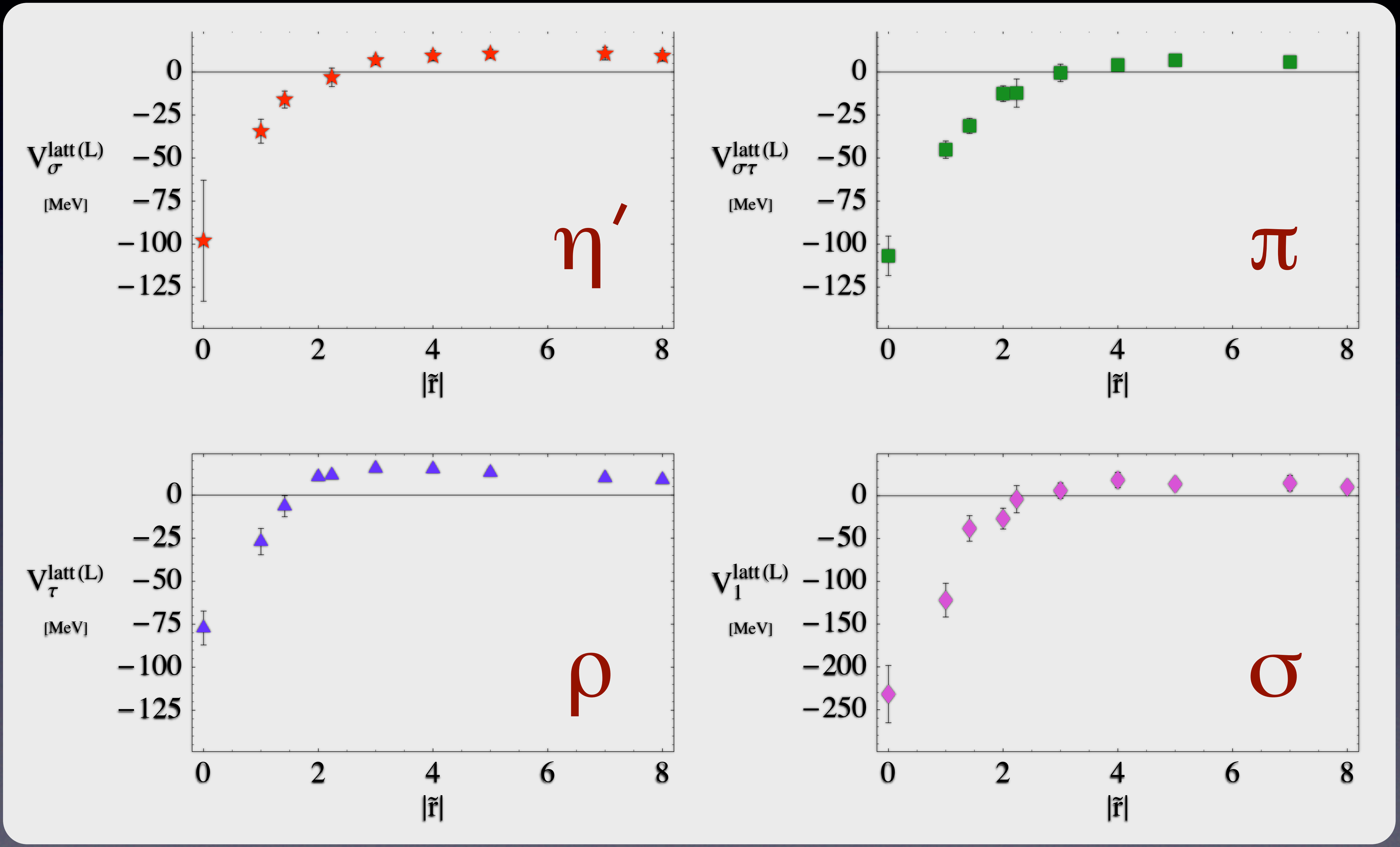
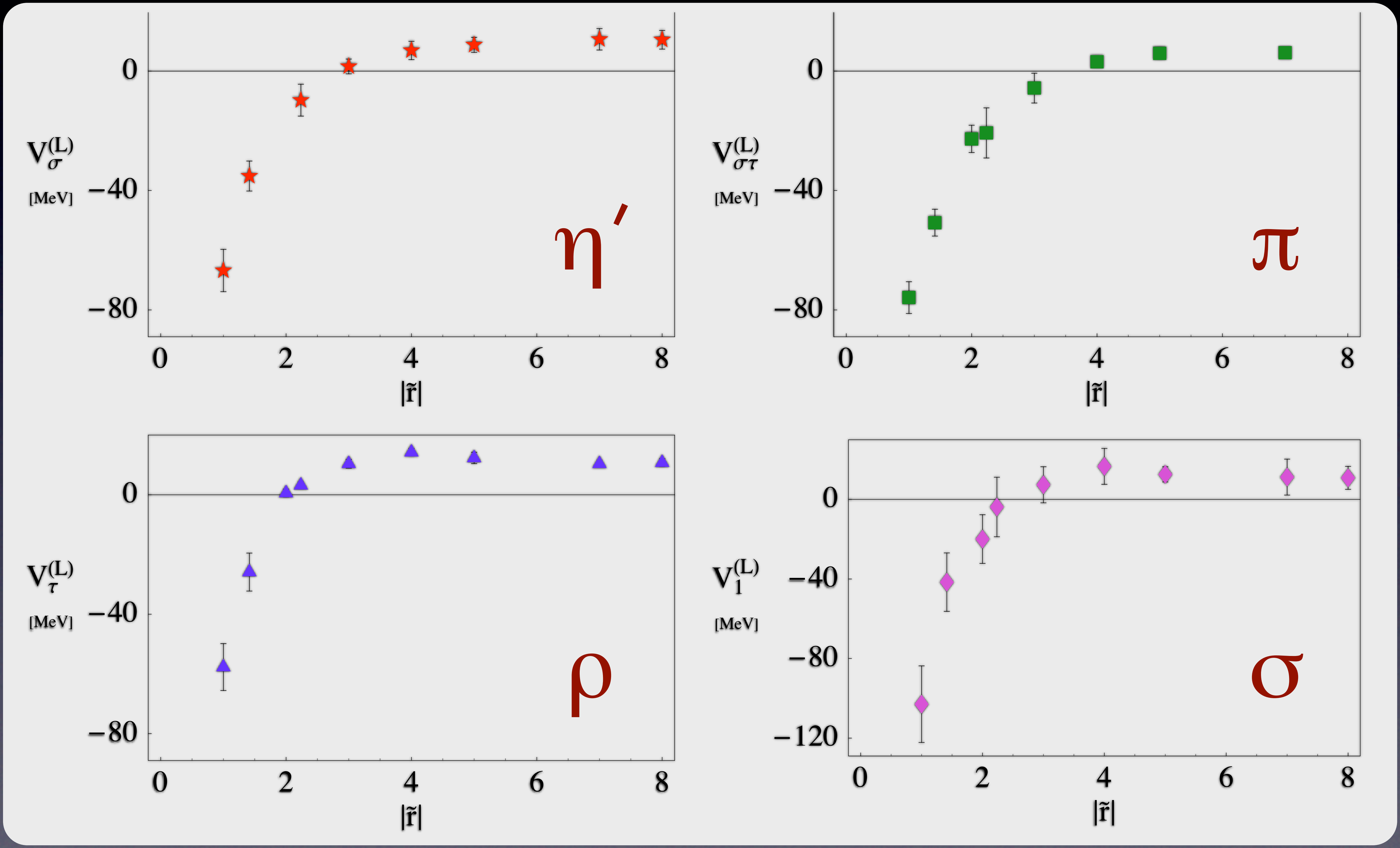
t-channel potentials



t-channel potentials

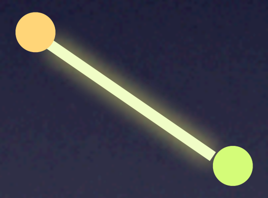


t-channel potentials



Coulomb corrected

Periodic lattice



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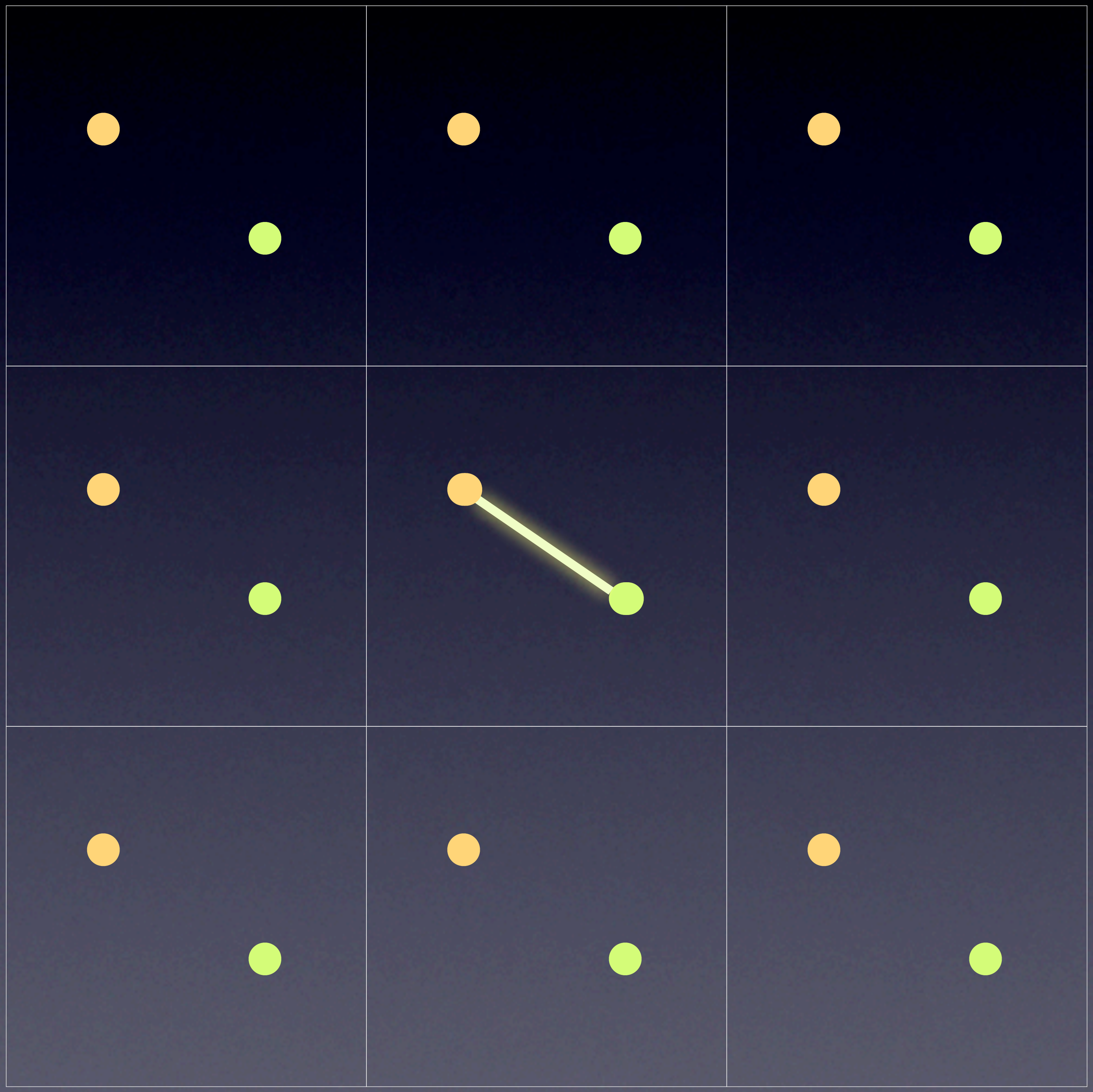

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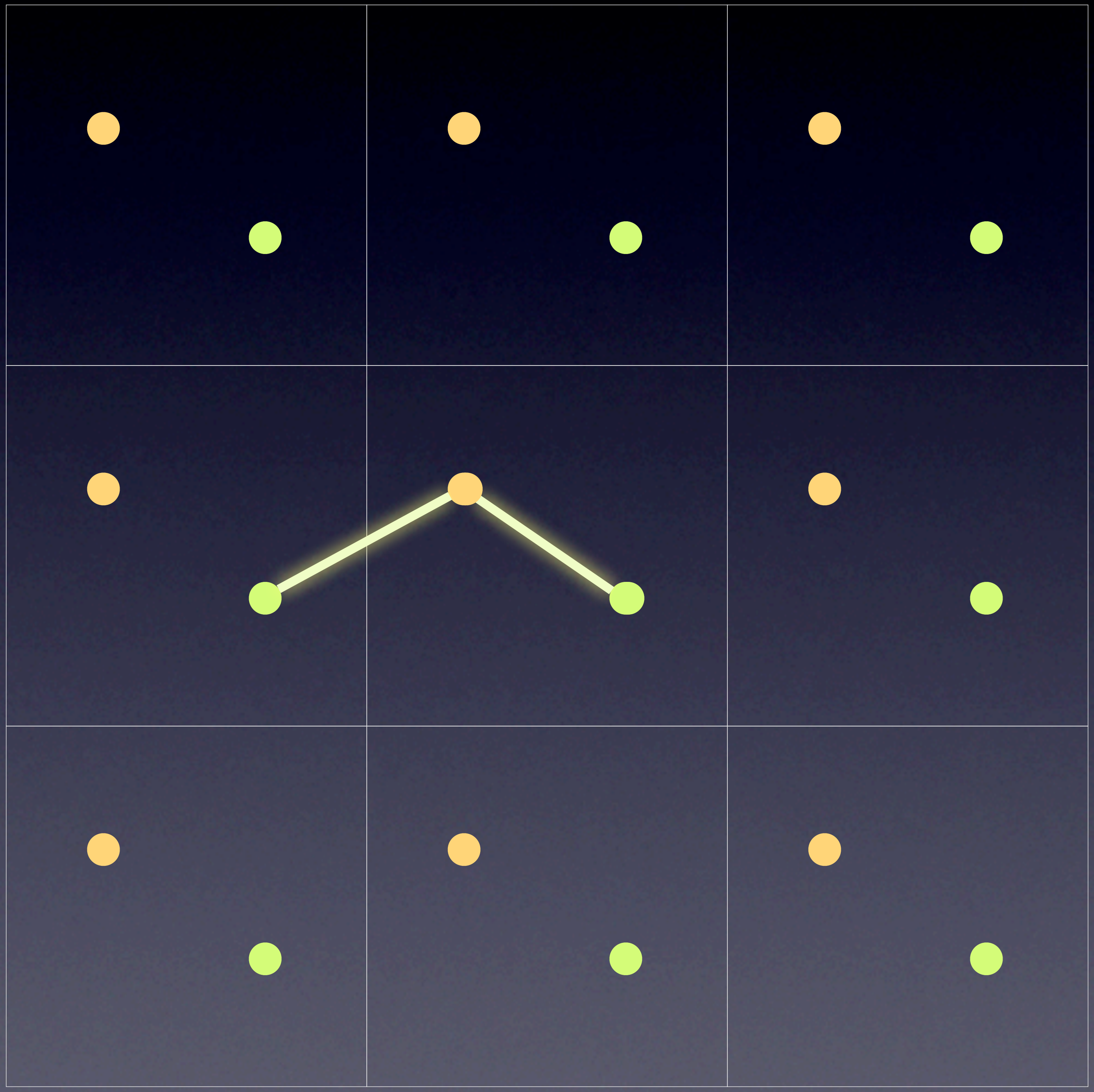
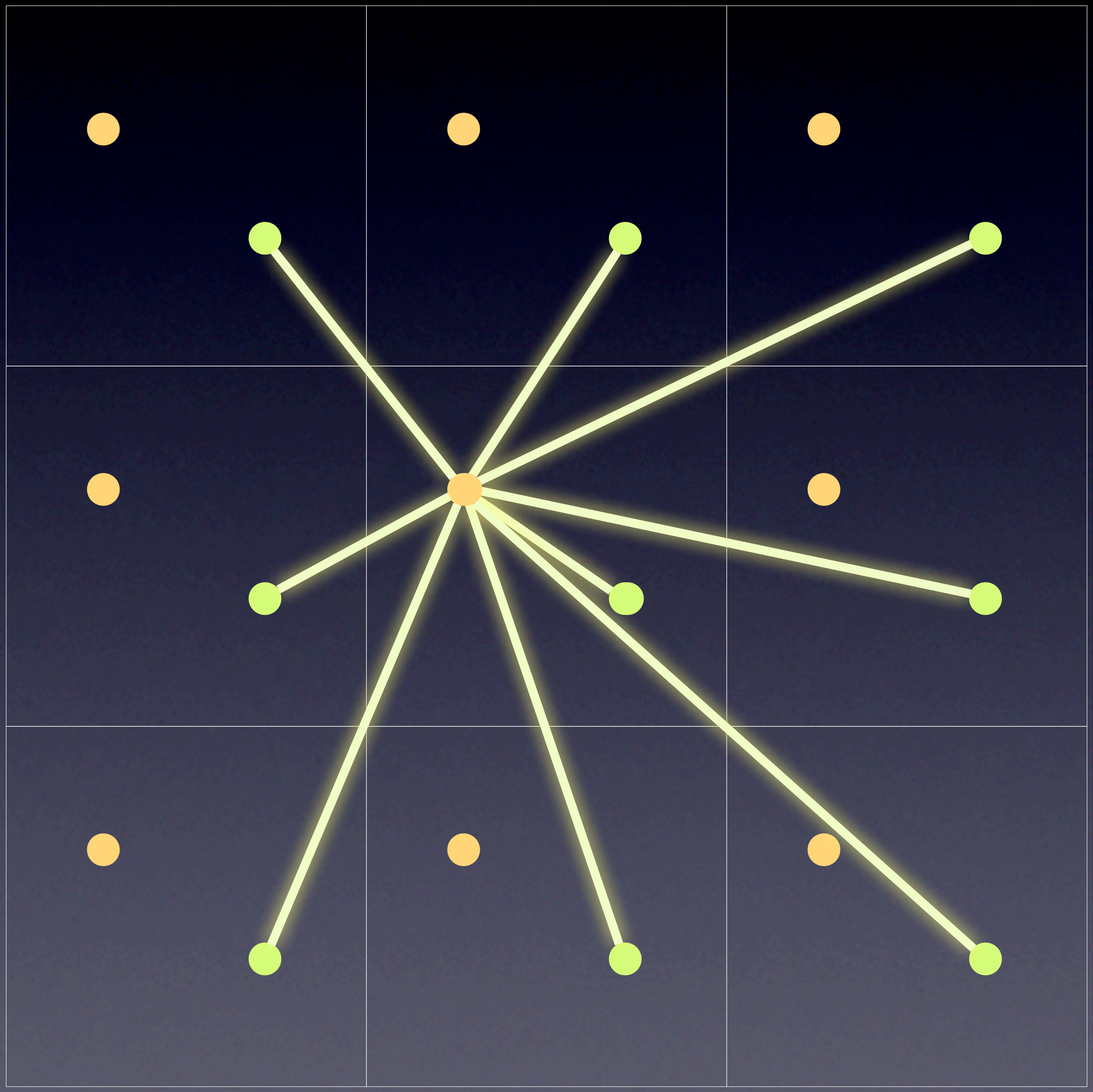


Periodic lattice



Finite volume effects

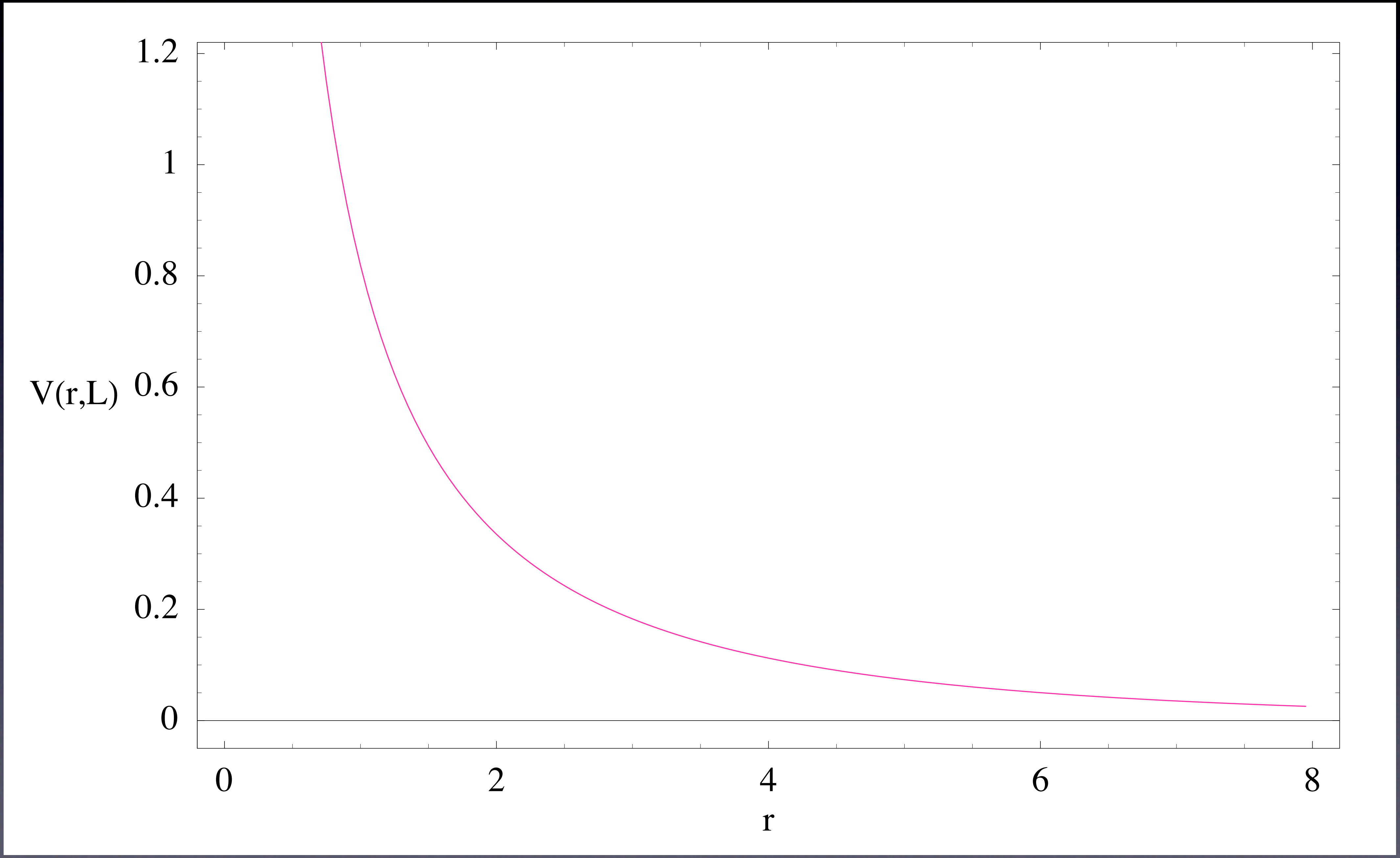
- Periodicity of lattice modifies potentials:

$$V^{(L)}(\mathbf{r}) = V(\mathbf{r}) \; + \sum_{\mathbf{n}\neq\mathbf{0}} V(\mathbf{r}+\mathbf{n}L)$$

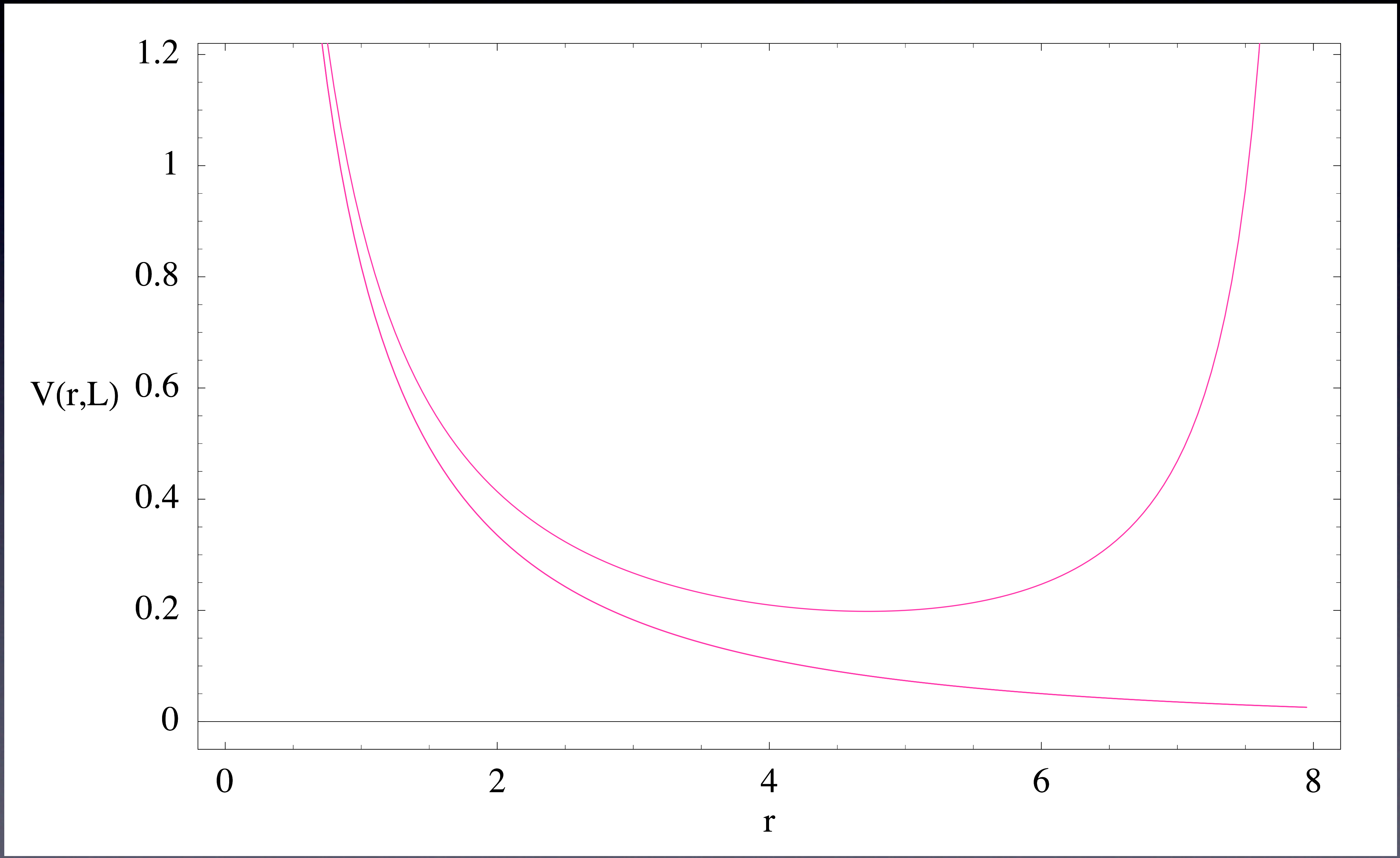
  assuming single particle exchange

- Strictly: *V* from $V^{(L)}$ impossible
  - Long range potential from EFT
  - Short range: FV effects smallest

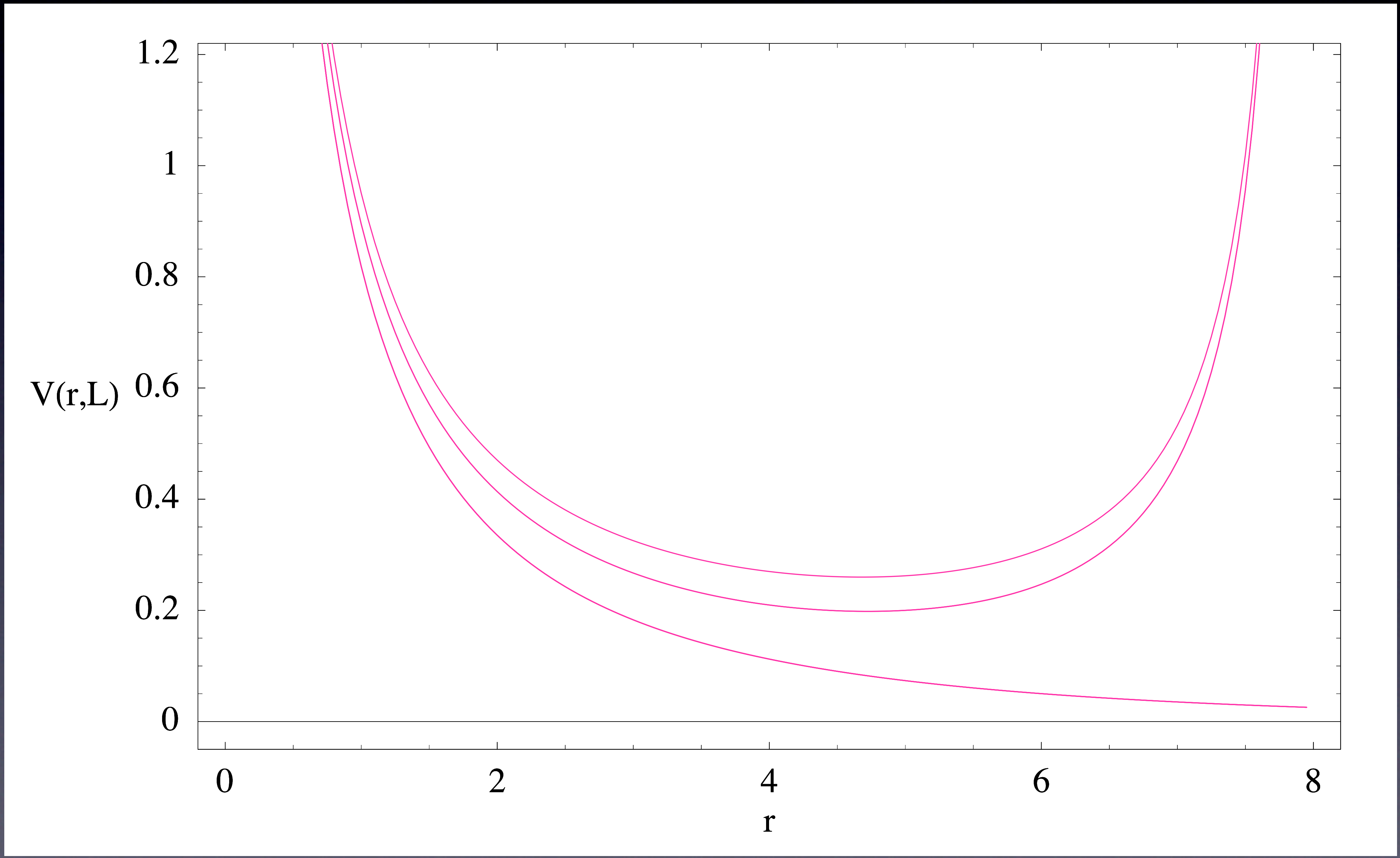
Periodic potential



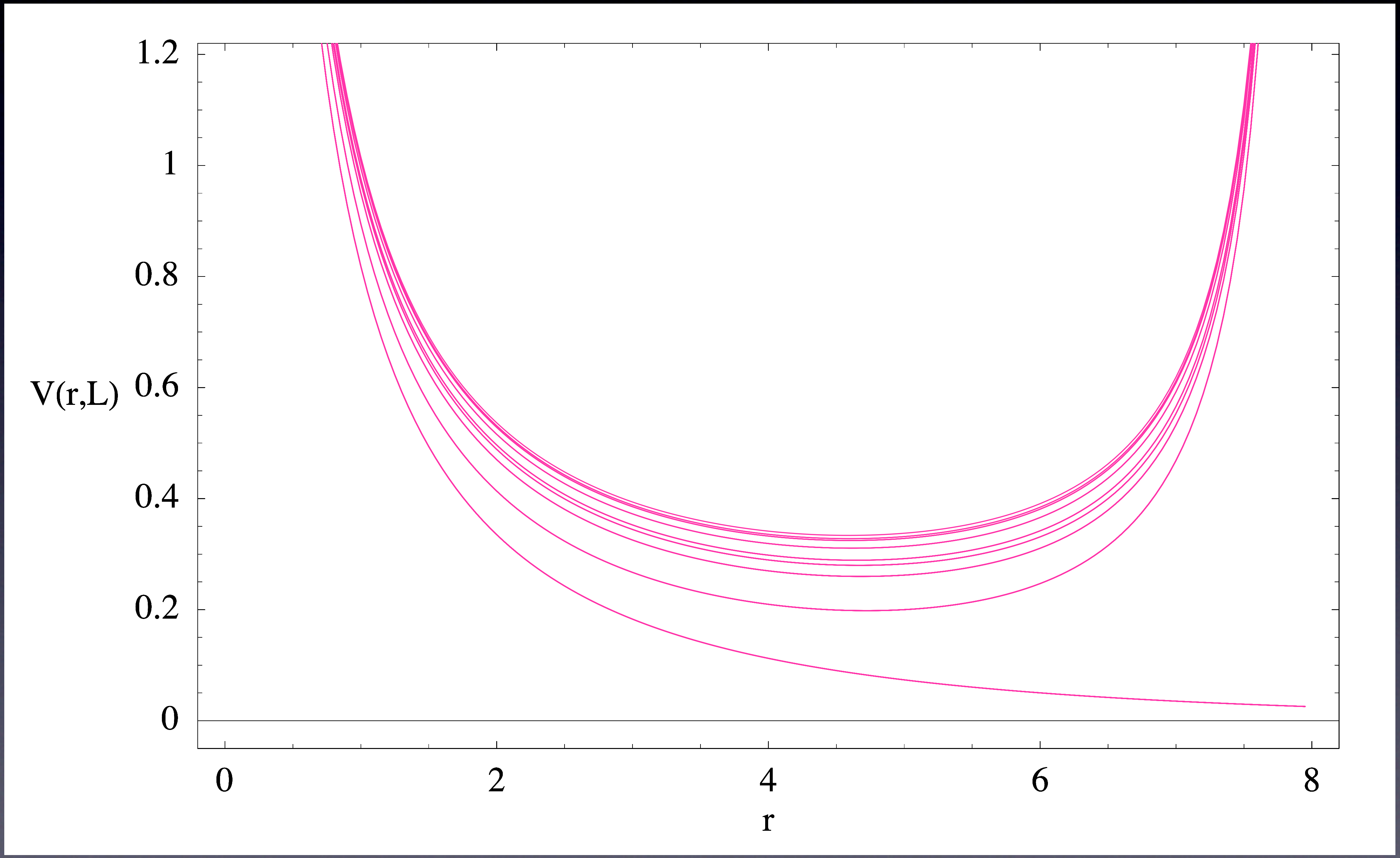
Periodic potential



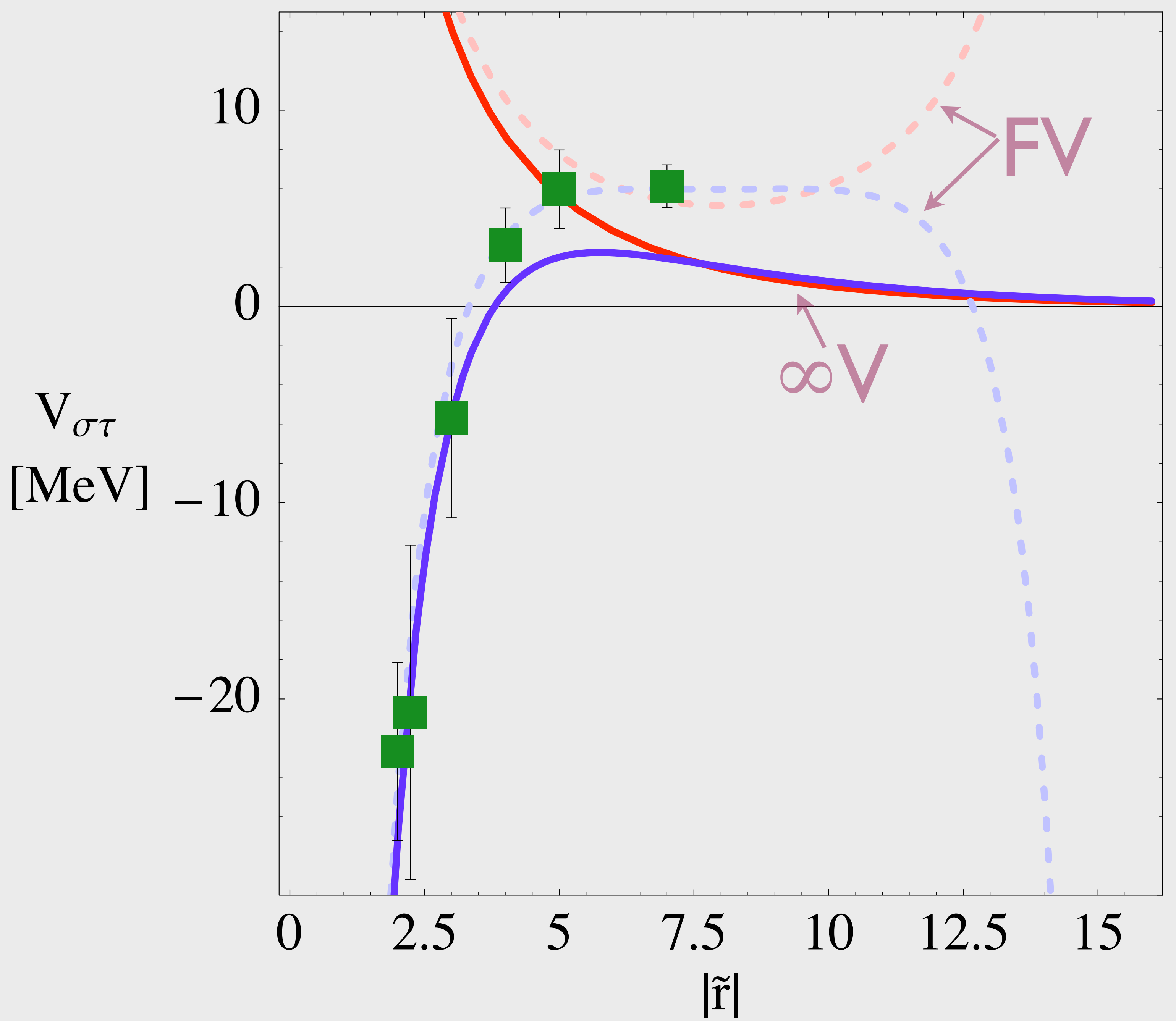
Periodic potential



Periodic potential



Infinite volume: $V_{\sigma\tau}$



Fit 1

$|r| \geq 5$

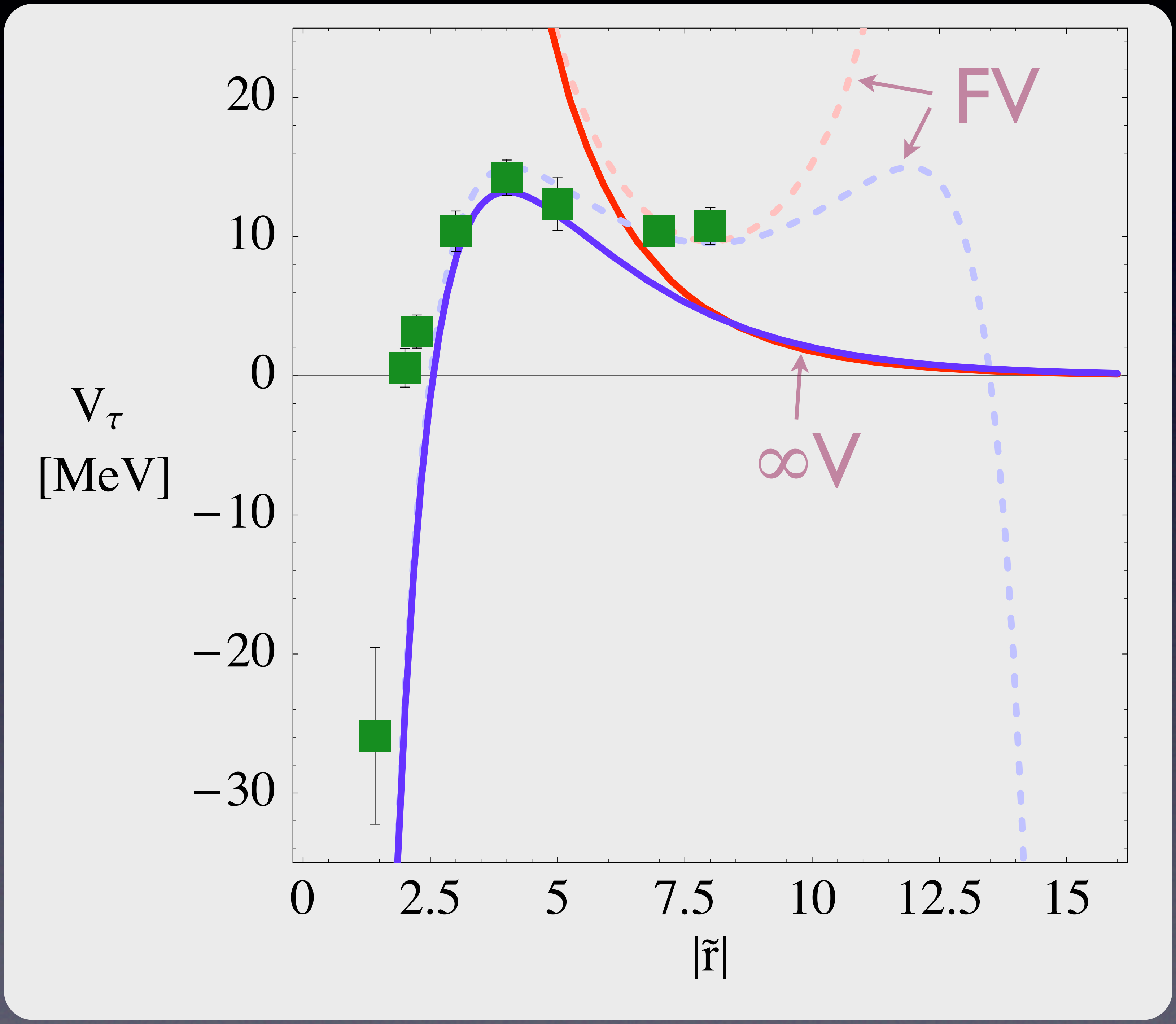
$$\mathcal{FV}\left[A\,\frac{e^{-m_\pi |r|}}{|r|}\right]$$

Fit 2

$|r| \geq 2$

$$+\mathcal{FV}\left[B\,\frac{e^{-\Lambda_\chi |r|}}{|r|}\right]$$

Infinite volume: V_τ



Fit 1

|r|>5

$$\mathcal{FV}\left[\hat{A}\,\frac{e^{-m_\rho|r|}}{|r|}\right]$$

Fit 2

|r|>3

$$+\mathcal{FV}\left[\hat{B}\,\frac{e^{-\Lambda_\chi|r|}}{|r|}\right]$$

Infinite volume

- BBπ/BBρ couplings extracted

$$g_{BB^*\pi} = 0.63 \pm 0.05 \pm 0.06 \qquad g_\rho = 2.6 \pm 0.1 \pm 0.4$$

- $g_{BB^*\pi}$ consistent with direct calculations

$$0.42(4)(8),\ 0.69(18),\ 0.48(3)(11),\ 0.517(16)$$

['98, '02, '03, '06]

- Other channels problematic
  - Quenching effects
  - Noise: vacuum quantum numbers

Summary

- *BB* potentials from lattice QCD
  - *t*-channel potentials measured cleanly
  - Leading lattice spacing artefacts removed
  - Infinite volume extraction attempted
- Future improvements
  - *Unquenched*
  - Multiple volumes/lattice spacings/m_q
  - Heavy baryon potentials