B Meson Physics on the Lattice

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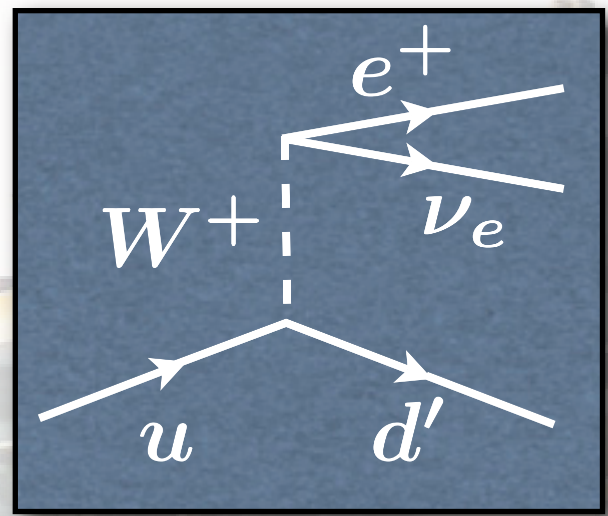
Quark flavor mixing



- CKM mechanism in the Standard Model
- Weak interactions change quark flavors

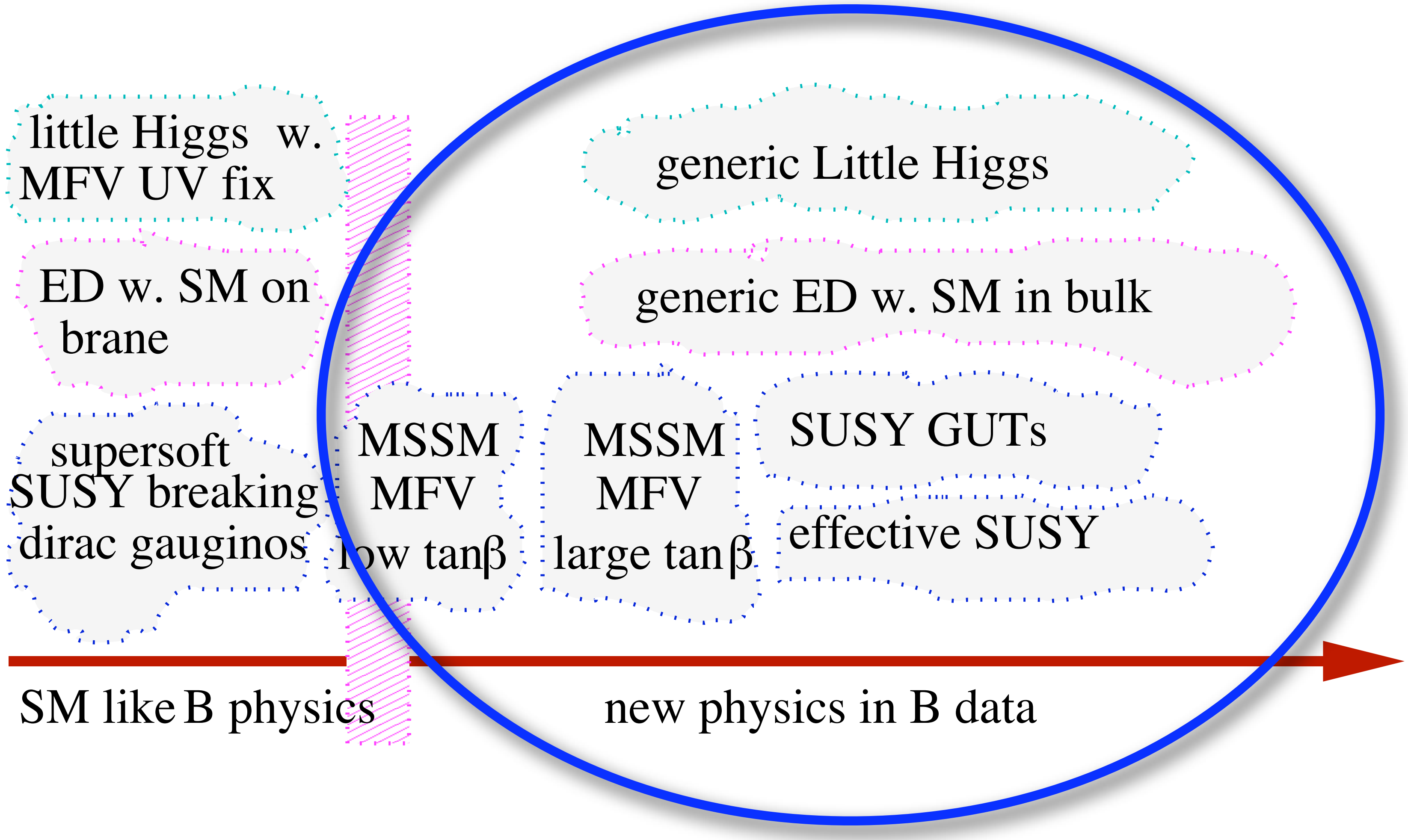
$$\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix} \begin{pmatrix} t \\ b' \end{pmatrix}$$

- Flavor mixing

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- V is the CKM matrix. Unitarity implies 4 undetermined parameters
- For other reasons (Higgs fine-tuning, gauge coupling-unification) we expect new physics at the TeV scale
- Success of CKM model tightly constrains new models of EWSB

New physics models & quark flavor



Discovery potential of flavor physics experiments **or**
Nondiscovery rules out/tightly constrains these models

Figure from G. Hiller, hep-ph/0207121

Wherefore LQCD for Flavor Physics?

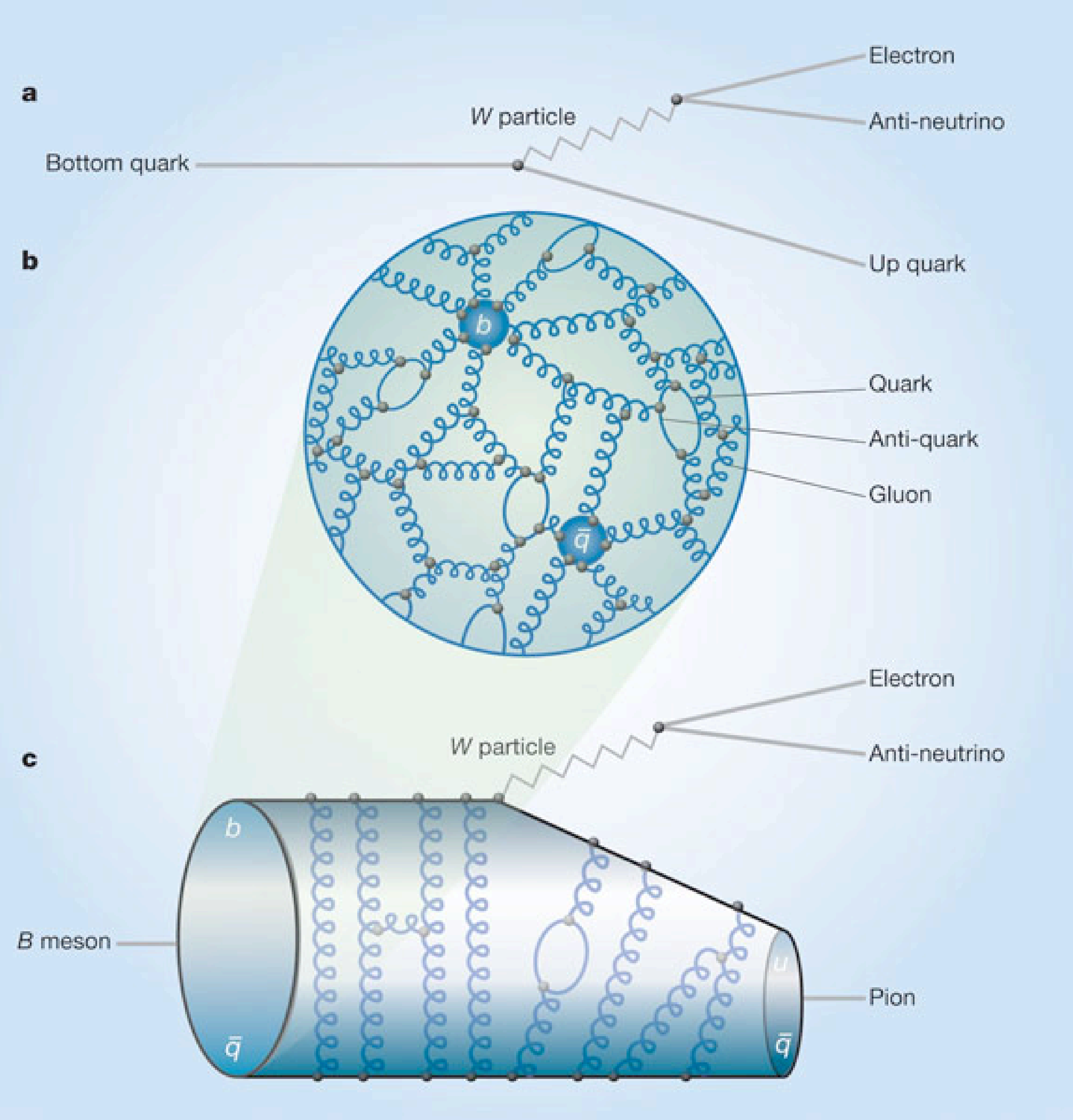


Illustration from I. Shipsey, Nature 427, 591 (2004)

a. Want to study weak decay of b to u

b. **Confinement:** Nature shows us the B *meson*, not just the b *quark*

c. Expt. sees $B^- \longrightarrow \pi^0 e^- \overline{\nu}_e$ LQCD brings us from meson level to quark level

Wolfenstein Parameterization

Expansion based on empirical observation

$$|V_{us}| = 0.22 \ll 1$$

$$|V_{cb}| \approx |V_{us}|^2$$

$$|V_{ub}| \ll |V_{cb}|$$

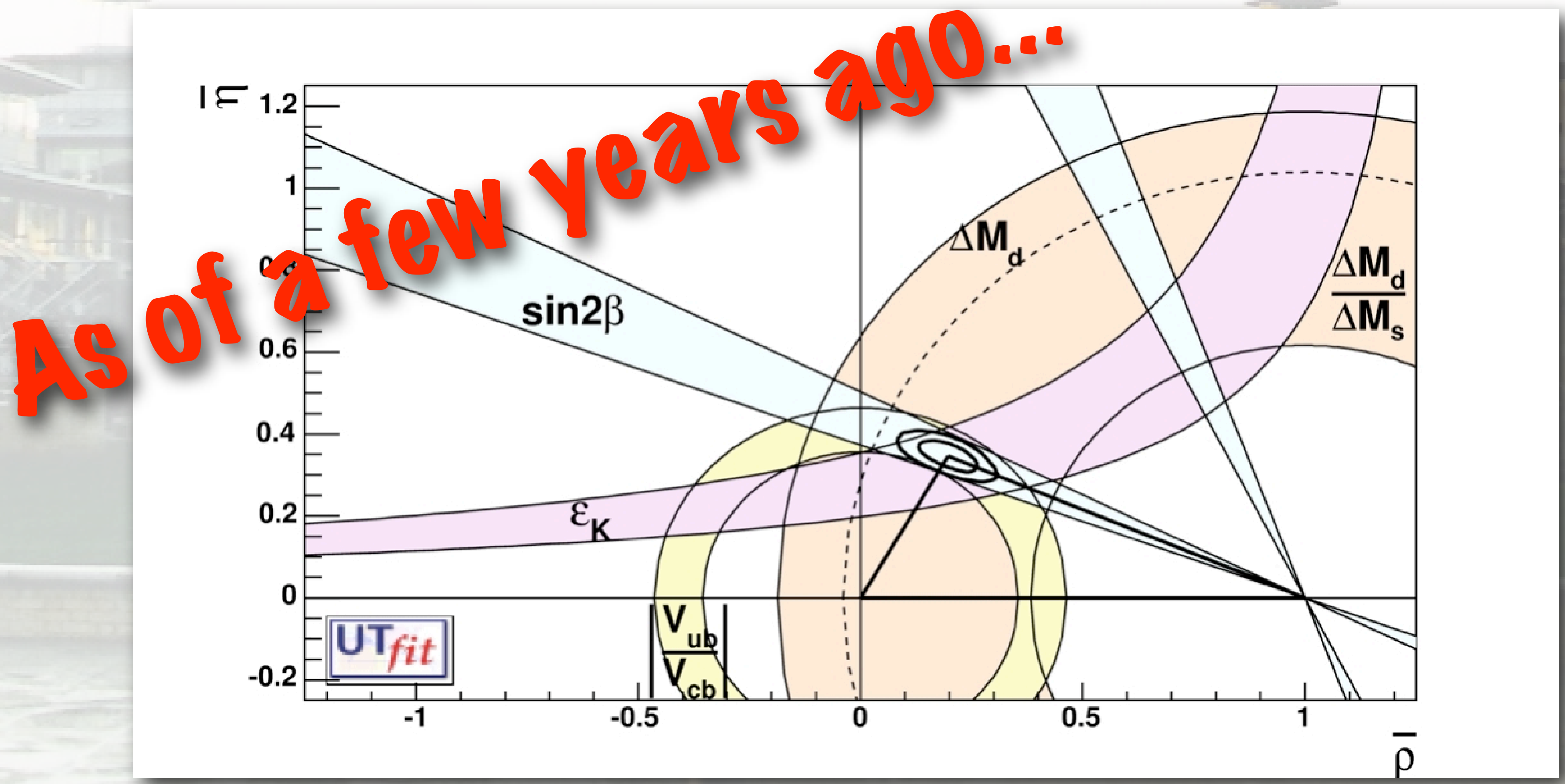
$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

In practice, go to next order

$$\bar{\rho} = \rho\left(1 - \frac{\lambda^2}{2}\right) \qquad \bar{\eta} = \eta\left(1 - \frac{\lambda^2}{2}\right)$$

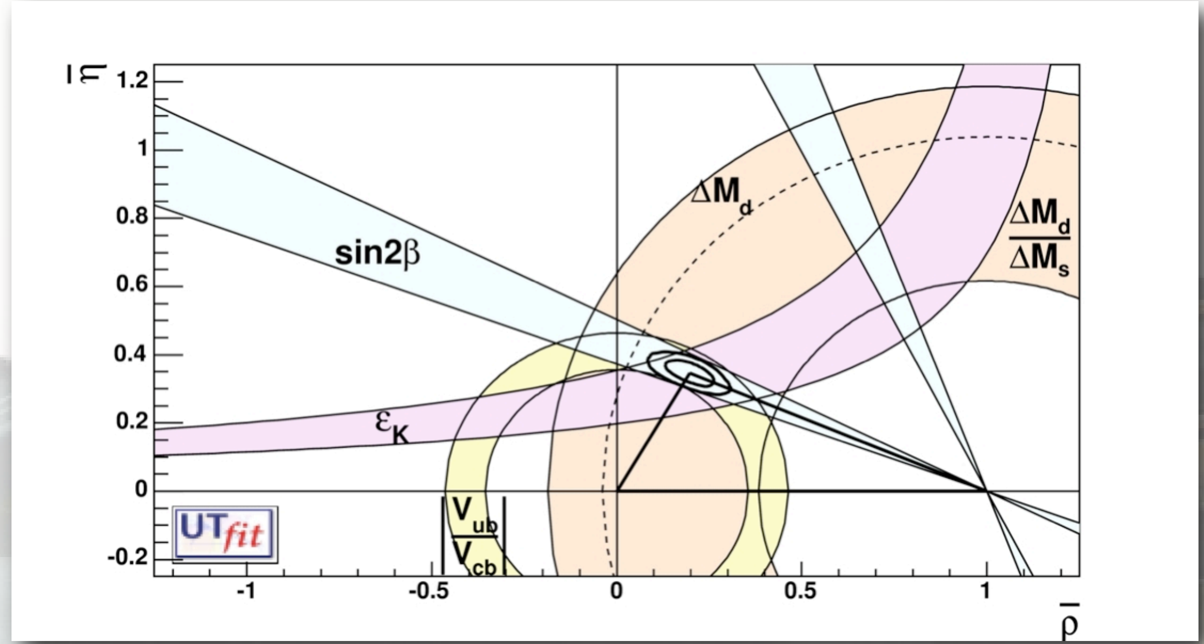
Wolfenstein Parameterization

$\lambda = 0.2205 \pm 0.0018 (0.8\%)$ $\quad$ $A = 0.824 \pm 0.075 (9\%)$



$\bar{\rho} = 0.196 \pm 0.045 (23\%)$ $\quad$ $\bar{\eta} = 0.347 \pm 0.025 (7\%)$

Experimental Constraints

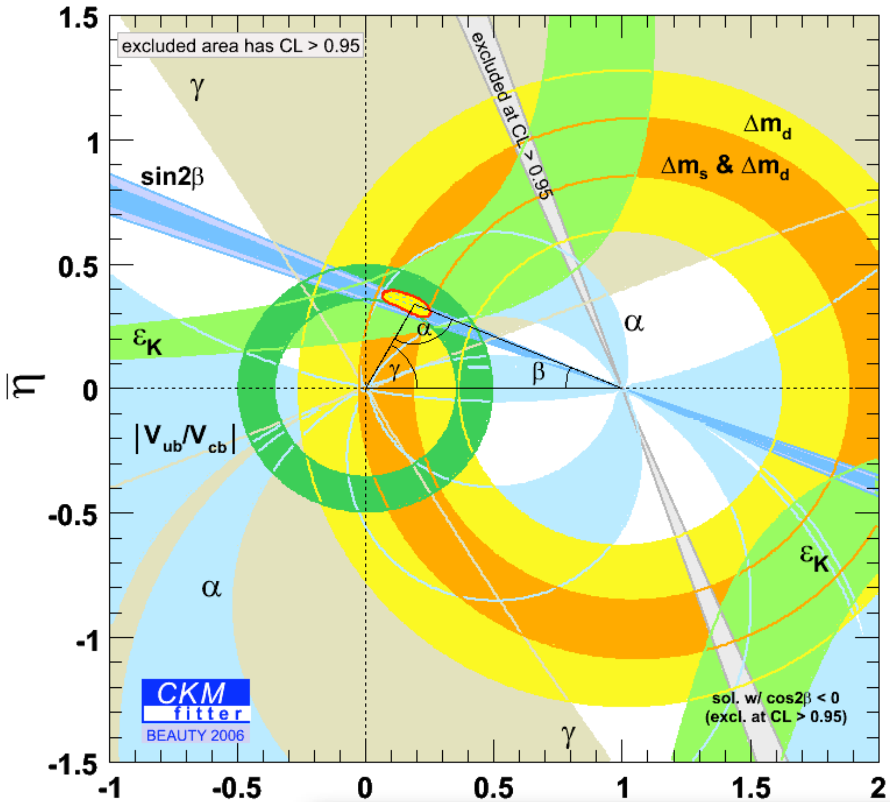


- $\sin 2\beta$ from $B \to (J/\psi)K$
- ε_K from $K^0 \leftrightarrow \overline{K^0}$
- ΔM from $B^0 \leftrightarrow \overline{B^0}$
- V_{ub} and V_{cb} from exclusive semileptonic decays $B \to \pi\ell\nu$, $B \to D\ell\nu$

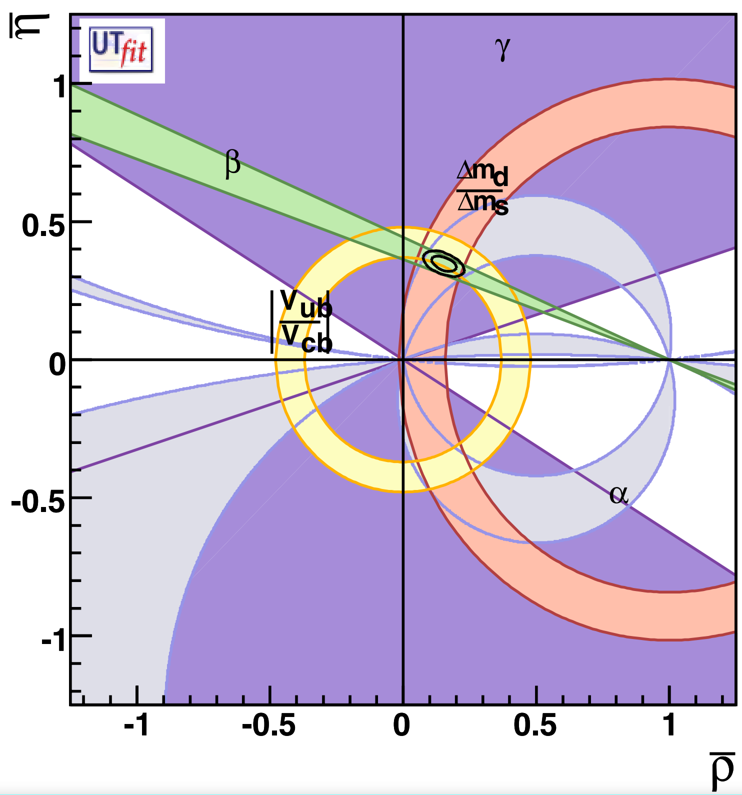
} need LQCD input

Persistent Standard Model

CKMfitter Group (J. Charles et al.)
http://www.ckmfitter.in2p3.fr
version of Oct 2006



UTfit collaboration
PRL **97**, 151803 (2006)



A couple discrepencies at 3σ: Hints or fluctuations?

New flavor physics

$$A(\text{Decay}) = \sum_i B_i \eta^i_{\rm QCD} V^i_{\rm CKM}[F^i_{\rm SM} + F^i_{\rm New}] + \sum_k B^{\rm New}_k [\eta^k_{\rm QCD}]^{\rm New} V^k_{\rm New}[G^k_{\rm New}]$$

- A. Buras' master formula
- Various classifications (5) of new physics models
- MFV: only new Inami-Lin--like functions $F_{\rm New}$
- Complementary for direct searches for new physics
- Requires high precision experiment and theory
- ***B factors are hadronic matrix elements, need LQCD***

Big picture

- We expect physics beyond the Standard Model, e.g. to explain the hierarchy problem, unify forces, give a good dark matter candidate.
    - Directly create new particles at the LHC: measure masses
- New particles couple to Standard Model particles
    - Discern new coupling constants from precise flavor experiments (BaBar, Belle, Tevatron, LHC, ...)
    - Most new models have new sources of flavor changing interactions, even having a ***flavor problem***
- ***Flavor physics is LHC-era physics***
- Lattice QCD connects meson measurements to quark couplings through systematically improvable first principles calculations

Outline

- ☑ Importance of flavor physics and role of lattice QCD
- Our strategy for achieving accurate results ***now***
- Recent results
    - $B \to \pi\, l\, \nu$
    - Neutral B mixing
- New effort: radiative & semileptonic penguin decays ($b \to s$)

Lattice QCD in a nutshell

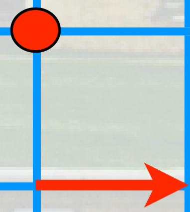
- QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} - \sum_q \overline{\psi}_q \left[\gamma^\mu (\partial_\mu - i g A^a_\mu t^a) + m_q \right] \psi_q$$

$$= \mathcal{L}_g - \overline{\psi} Q \psi$$

- Break-up spacetime into a grid
- Maintains gauge invariance
- Breaking of rotational/translational Lorentz invariance at short distances is controllable and removable

Quarks on sites



Glue on links

Lattice QCD in a nutshell

QFT : Euclidean space path integral

$$\langle J(z')J(z)\rangle = \frac{1}{Z}\int [d\psi][d\bar{\psi}][dU]\, J(z')J(z)\, e^{-S_E}$$

SFT : Sum over all microstates

$$\langle J(z')J(z)\rangle = \frac{1}{Z}\,\mathrm{Tr}\left[J(z')J(z)\, e^{-\beta H}\right]$$

Use same numerical methods!

Monte Carlo Simulation : Find and use field "configurations" which dominate the integral/sum

Lattice QCD in a nutshell

Gluonic expectation values

$$\langle\Theta\rangle = \frac{1}{Z}\int [d\psi][d\bar\psi][dU]\,\Theta[U]\,e^{-S_g[U]-\bar\psi Q[U]\psi}$$

$$= \frac{1}{Z}\int [dU]\,\Theta[U]\,\det Q[U]\,e^{-S_g[U]}$$

Probability weight

Fermionic expectation values

$$\langle\bar\psi\Gamma\psi\rangle = \int [dU]\,\frac{\delta}{\delta\bar\zeta}\Gamma\frac{\delta}{\delta\zeta}\,e^{-\bar\zeta Q^{-1}[U]\zeta}\,\det Q[U]e^{-S_g[U]}\Big|_{\zeta,\bar\zeta\to 0}$$

Determinant in probability weight difficult

1) Requires nonlocal updating; 2) Matrix

Quenched approximation

Set $\det Q = 1$

Partial quenching =

different mass for valence Q^{-1} than for sea $\det Q$

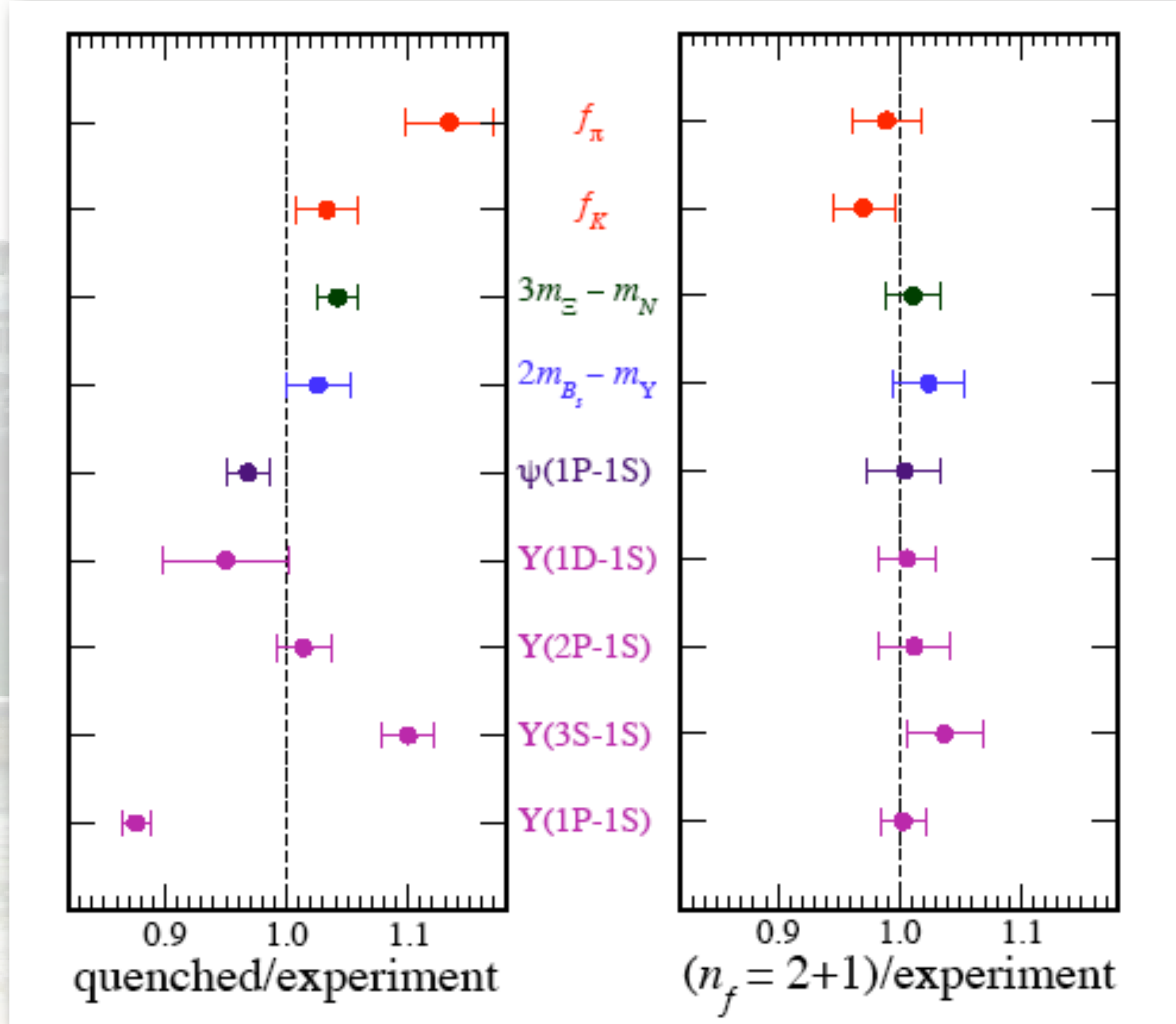
Challenges (*viz* systematic errors)

- Lattice volume must be big enough
- Lattice spacing must be smaller than physically relevant length scales
  - Cost increases quickly as a decreases: $a^{-4} \times a^{-(\sim 2.5)}$
  - Heavy quarks have small Compton wavelengths
- Singular behavior at light quark masses requires extrapolations from feasible masses to physical masses
  - Need mild mass dependence or trustworthy theory (chiral PT)

Summary of our strategy

- The goal: to address all systematic errors simultaneously
- (Improved) staggered fermion formulation in order to be in chiral regime
- Nonrelativistic bottom quark to avoid extrapolations in m_b -- treats heavy quark effects through effective field theory
- Discretization errors treated via Symanzik effective field theory
- Perturbation theory -- automation
- Some critics think they can do better in the future with other methods. GOOD! That is progress
- This approach has been very successful in improving lattice results for phenomenology (compare: quenched, outside chiral regime)

Light quark effects are important

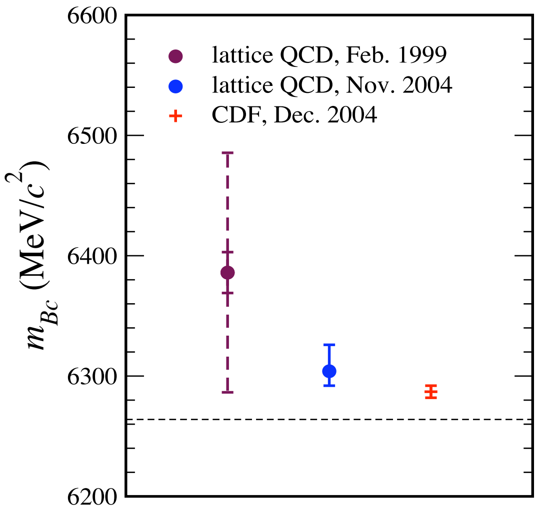


C. Davies, *et al.*, PRL 92 (2004)

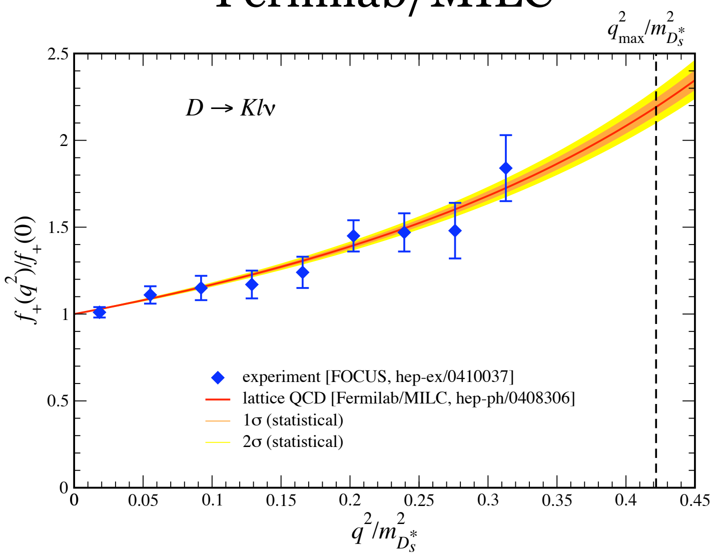
Other checks and predictions

- ☑ B_c meson mass (Fermilab/HPQCD LQCD; CDF expt)
- ☑ D&D_s meson decay constants (Fermilab/MILC LQCD; CLEO-c expt)
- ☑ $D \longrightarrow K\ell\nu$ form factor (Fermilab/MILC LQCD; BES, FOCUS expt's)
- ☑ QCD coupling and quark masses (HPQCD)

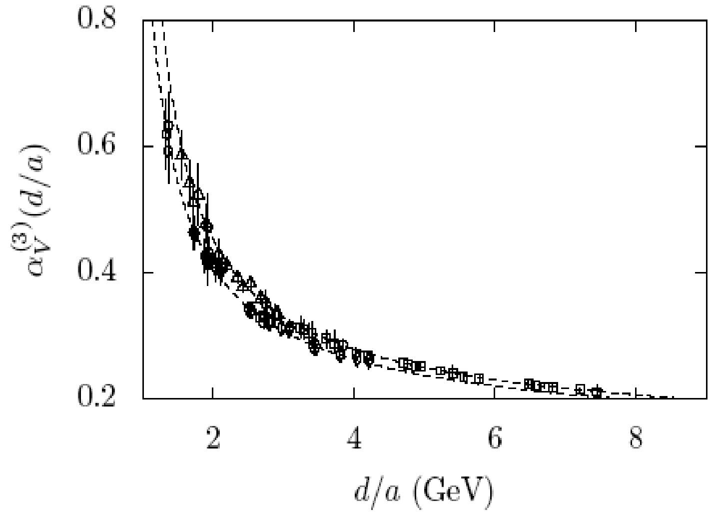
Fermilab/HPQCD



Fermilab/MILC



HPQCD



Logic of 4th root hypothesis

- Hypothesize that 4th root procedure is QCD in continuum limit
  - A testable hypothesis
  - Comparable to hypotheses of quark mass extrapolation from outside the chiral regime
- Empirical tests
  - So far so good -- obviously better than quenched LQCD
- Skeptics welcome
  - Also invited to look hard at non-lattice CKM uncertainties
  - Progress in understanding 4th root (Shamir, Bernard, Golterman)
- All approaches should be pushed hard

Lattice NRQCD for heavy quark

$$S_0 = \int d^4x \, \Psi^\dagger \left(iD_t + \frac{|\vec{D}|^2}{2m_Q} \right) \Psi$$

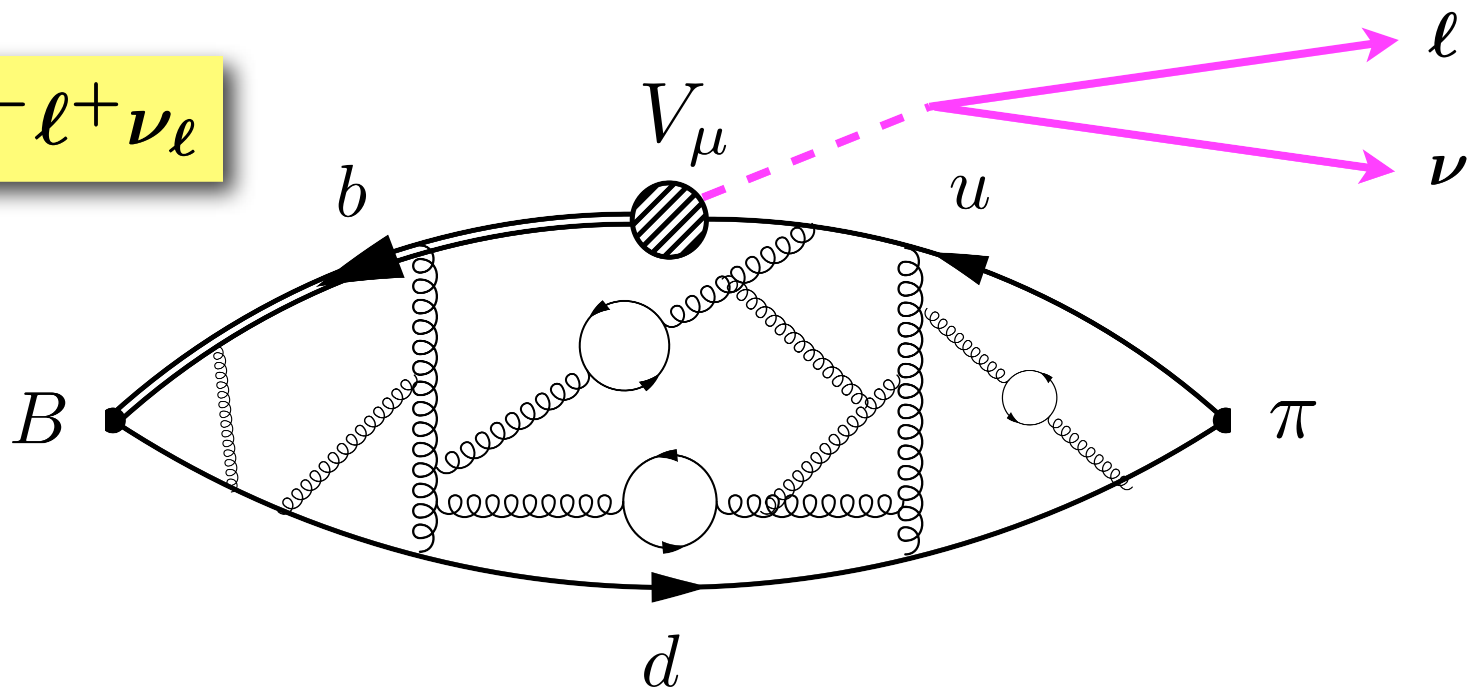
- Foldy-Wouthuysen-Tani (FWT) transformation
- Take lattice action as given: can analyze just like continuum HQET
- Requires $am_Q > 1$, satisfied for b quark on present and near future unquenched lattices
- The following phrase is often uttered: "The continuum limit cannot be taken."
- In theory, there are no lattice artifacts on the renormalized trajectory
- In practice, discretization errors are short distance effects, systematically removed using Symanzik's EFT
- Improvement & matching rely on perturbation theory
- Nonperturbative methods preferable in principle, *when practical & precise*

Overview of simulation parameters

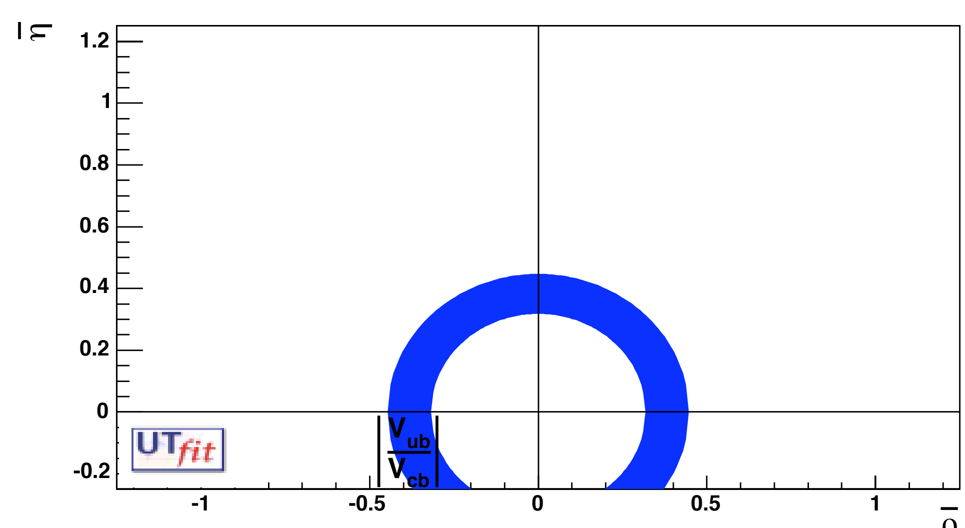
- MILC collaboration's 2+1 flavor configurations (AsqTad staggered)
- "coarse" $a = 0.13\ \mathrm{fm}$ and "fine" $a = 0.09\ \mathrm{fm}$
- Spatial volume $(2.5\ \mathrm{fm})^3$
- Lightest up/down mass $m_s/8$
- We compute at both unquenched and partially quenched masses
- NRQCD action for bottom, correct through $O(\Lambda^2_{\mathrm{QCD}}/m^2_Q)$

Semileptonic B to pi decay

$B^0 \to \pi^- \ell^+ \nu_\ell$



Combine with
$B^0 \to D^- \ell^+ \nu_\ell$



work done with

E. (Gulez) Dalgic (Simon Fraser)
A. Gray (EPCC)
J. Shigemitsu (Ohio State)
C. T. H. Davies (Glasgow)
G. P. Lepage (Cornell)

(PART OF THE HPQCD COLLABORATION)

Semileptonic Decays

$$B^0 \to \pi^- \ell^+ \nu_\ell$$

$$\frac{1}{|V_{ub}|^2} \frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3} |\vec{p'}|^3 |f_+(q^2)|^2$$

$$\langle \pi(p') | V^\mu | B(p) \rangle = f_+(q^2)(p^\mu + p'^\mu) + f_-(q^2)(p^\mu - p'^\mu)$$

Currents in EFT

Temporal components

$$J_0^{(0)}(x) = \bar{q}(x)\,\Gamma_0\,Q(x),$$

$$J_0^{(1)}(x) = -\frac{1}{2M_0}\bar{q}(x)\,\Gamma_0\,\boldsymbol{\gamma}\cdot\boldsymbol{\nabla}\,Q(x),$$

$$J_0^{(2)}(x) = -\frac{1}{2M_0}\bar{q}(x)\,\boldsymbol{\gamma}\cdot\overleftarrow{\boldsymbol{\nabla}}\,\gamma_0\,\Gamma_0\,Q(x).$$

$$\Gamma_\mu \equiv \begin{cases} \gamma_\mu & \text{for } V_\mu \\ \gamma_\mu\gamma_5 & \text{for } A_\mu \end{cases}$$

Spatial components

$$J_k^{(0)}(x) = \bar{q}(x)\,\Gamma_k\,Q(x),$$

$$J_k^{(1)}(x) = -\frac{1}{2M_0}\bar{q}(x)\,\Gamma_k\,\boldsymbol{\gamma}\cdot\boldsymbol{\nabla}\,Q(x),$$

$$J_k^{(2)}(x) = -\frac{1}{2M_0}\bar{q}(x)\,\boldsymbol{\gamma}\cdot\overleftarrow{\boldsymbol{\nabla}}\,\gamma_0\,\Gamma_k\,Q(x),$$

$$J_k^{(3)}(x) = -\frac{1}{2M_0}\bar{q}(x)\,\nabla_k\,Q(x)$$

$$J_k^{(4)}(x) = \frac{1}{2M_0}\bar{q}(x)\,\overleftarrow{\nabla}_k\,Q(x),$$

Perturbative matching

For example,

$$\langle A_0 \rangle_{\rm QCD} = (1 + \alpha_s \tilde{\rho}_0) \langle J_0^{(0)} \rangle + (1 + \alpha_s \rho_1) \langle J_0^{(1),sub} \rangle + \alpha_s \rho_2 \langle J_0^{(2),sub} \rangle$$

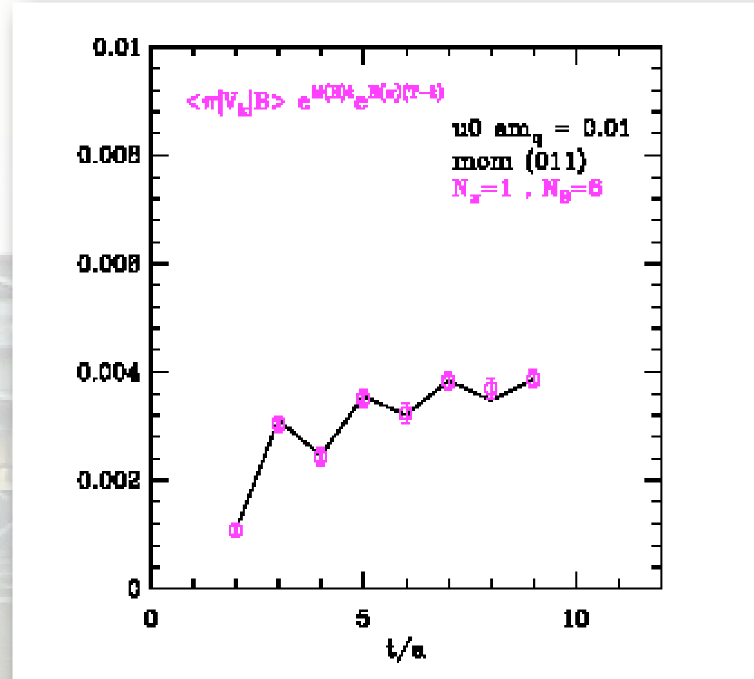
$$J^{(i),sub} = J^{(i)} - \alpha_s \zeta_{10} J^{(0)}$$

Turns out to be leading uncertainty

Perturbative coefficients computed in

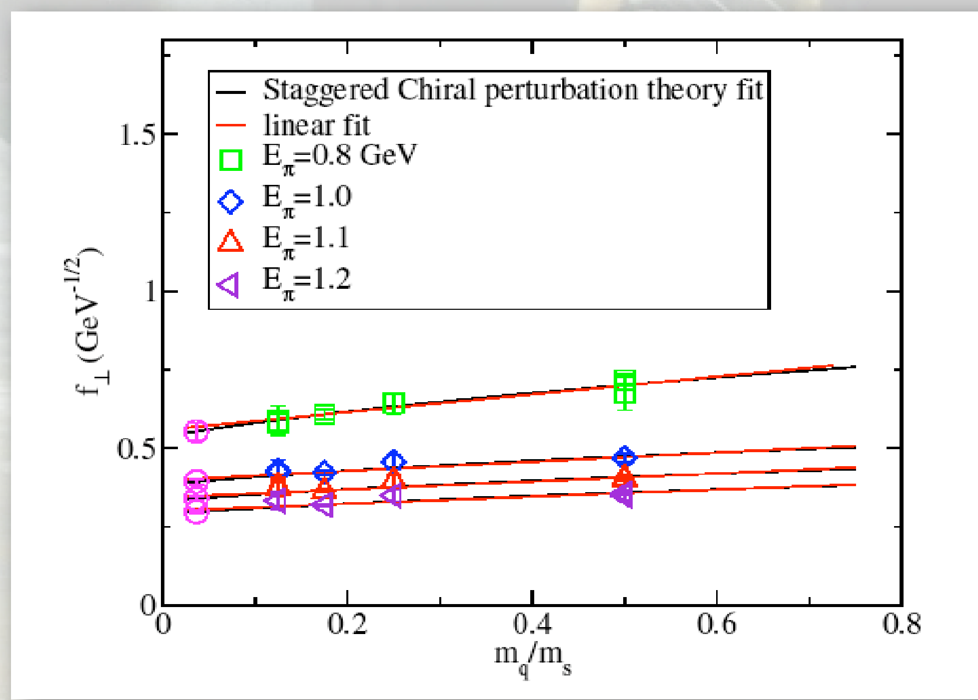
E. Gulez, J. Shigemitsu, M.W., PRD 69, 074501 (2004)

Fits, fits, fits, ...



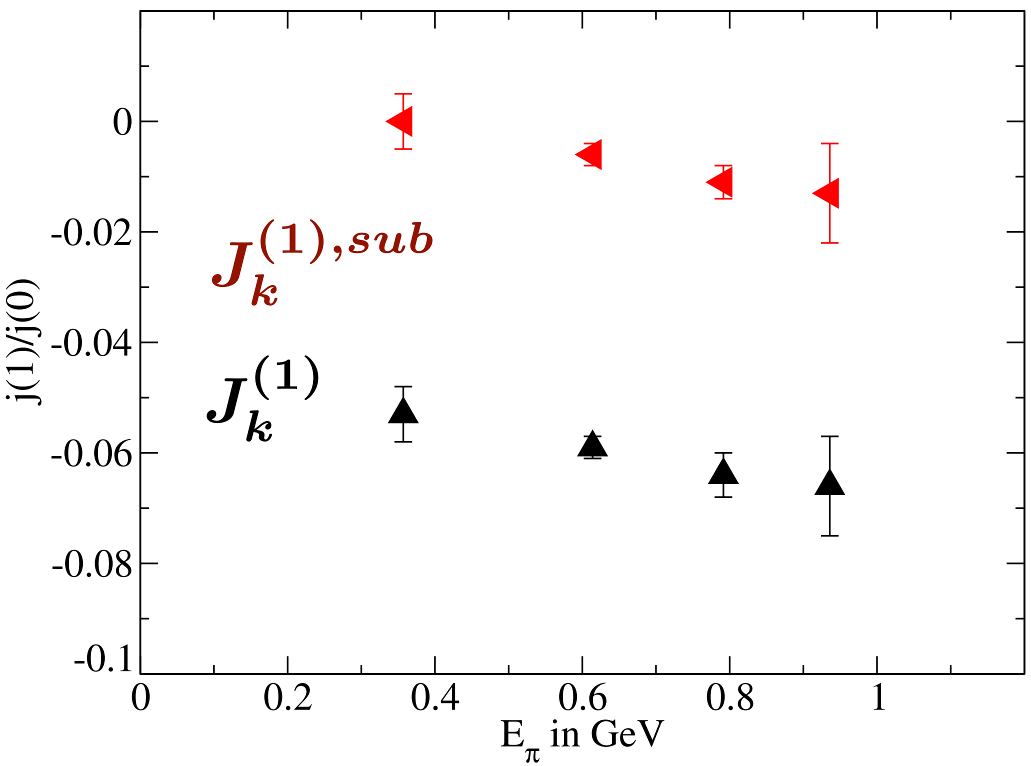
1) Fit 3-point correlators
2) Combine LO and NLO
3) Interpolate to fixed E_π
4) Extrapolate in quark mass

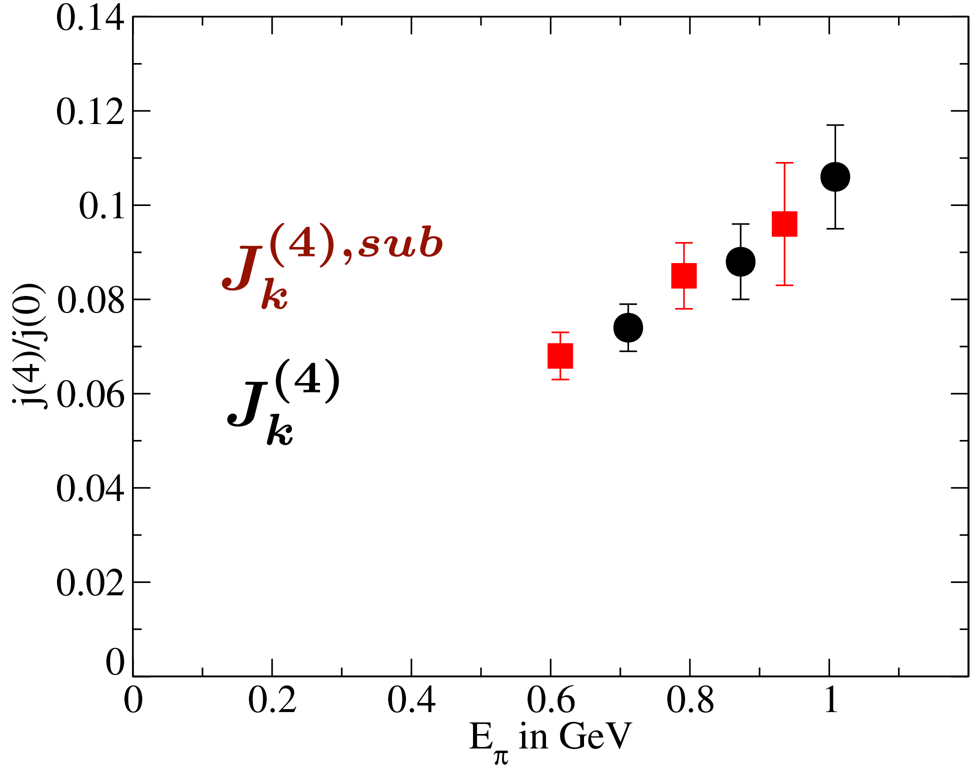
$$
\begin{aligned}
\langle \pi | V^0 | B \rangle &= \sqrt{2 m_B}\, f_{\parallel} \\
\langle \pi | V^k | B \rangle &= \sqrt{2 m_B}\, p_\pi^k\, f_{\perp}
\end{aligned}
$$



$1/m_b$ corrections

aM_0	n	$\tilde{\rho}_k^{(0)}$	$\rho_k^{(1)}$	$\rho_k^{(2)}$	$\rho_k^{(3)}$	$\rho_k^{(4)}$	$\zeta_{10,k}$
4.00	2	0.256	0.484(3)	0.340(6)	0.244(3)	−0.137(3)	0.041
2.80	2	0.270	0.349(3)	0.169(6)	0.218(4)	−0.029(4)	0.055
1.95	2	0.332	0.232(3)	0.121(8)	0.161(4)	0.063(3)	0.073





Form factor shape

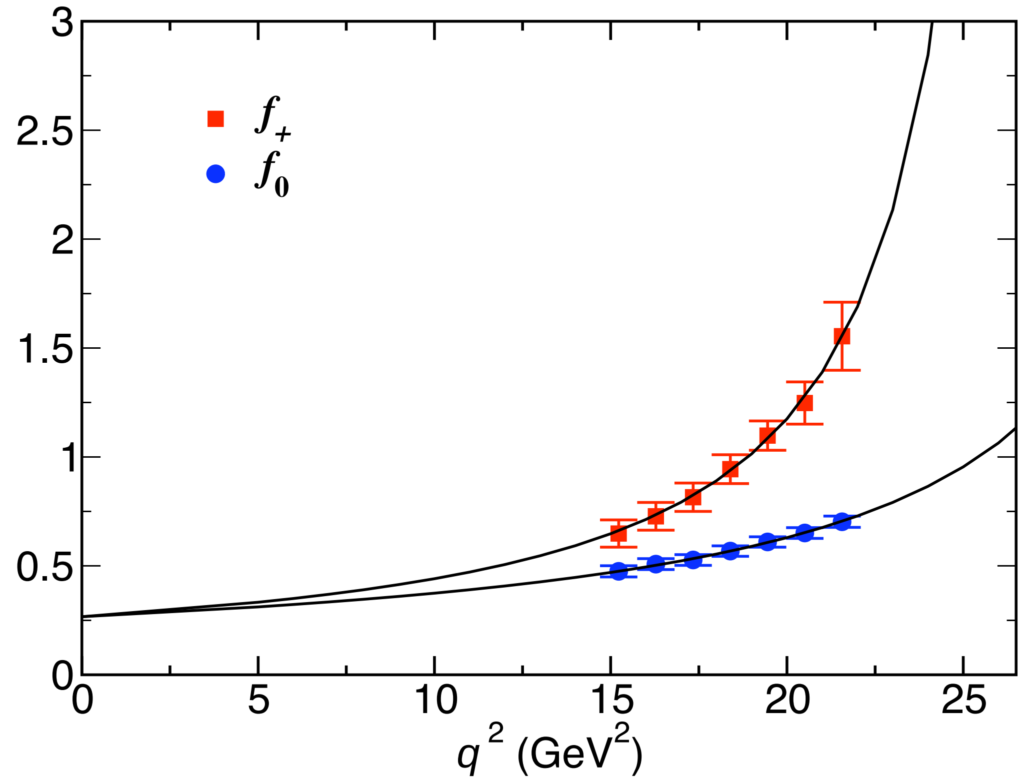
Ball-Zwicky, 4-parameter Becirevic-Kaidalov

$$f_+(q^2) = \frac{f_+(0)}{1-\tilde{q}^2} + \frac{r\tilde{q}^2}{(1-\tilde{q}^2)(1-\alpha\tilde{q}^2)} \qquad \tilde{q}^2 \equiv q^2/m_{B^*}^2$$

$$f_0(q^2) = \frac{f_+(0)}{1-\tilde{q}^2/\beta}$$

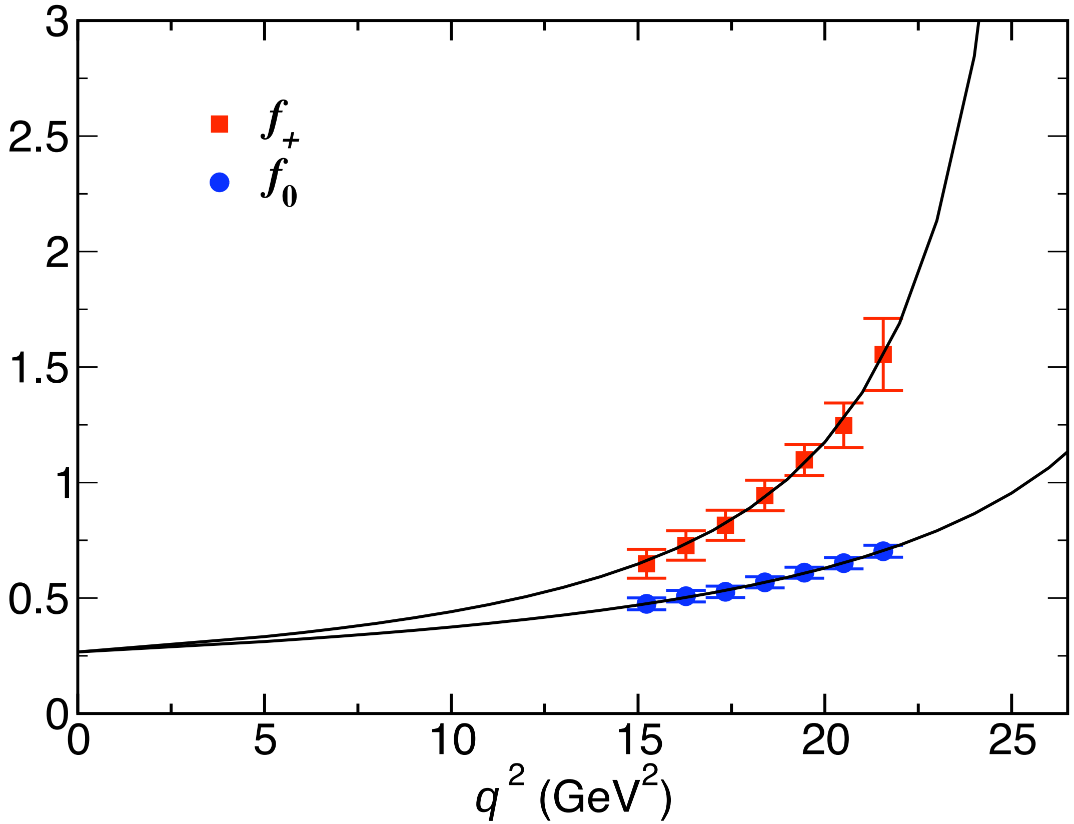
Considering analyticity and unitarity constraints (...) could remove model dependence.

Bourrely et. al; Boyd et al; Fukunaga & Onogi; Lellouch; Boyd & Savage; Arnesen et al; R. Hill; P. Mackenzie; P. Ball; Flynn & Nieves; ...



Experiment + Lattice QCD

$$\frac{1}{|V_{ub}|^2}\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3}|\vec{p'}|^3|f_+(q^2)|^2$$



$|V_{ub}| =$

$$(3.55 \pm 0.25 \pm 0.50) \times 10^{-3}$$

expt. LQCD

using HFAG avg b.f. with $q^2 > 16$ GeV^2
EPS 2005

Inclusive $B \to X_u \ell\nu$

$|V_{ub}| =$

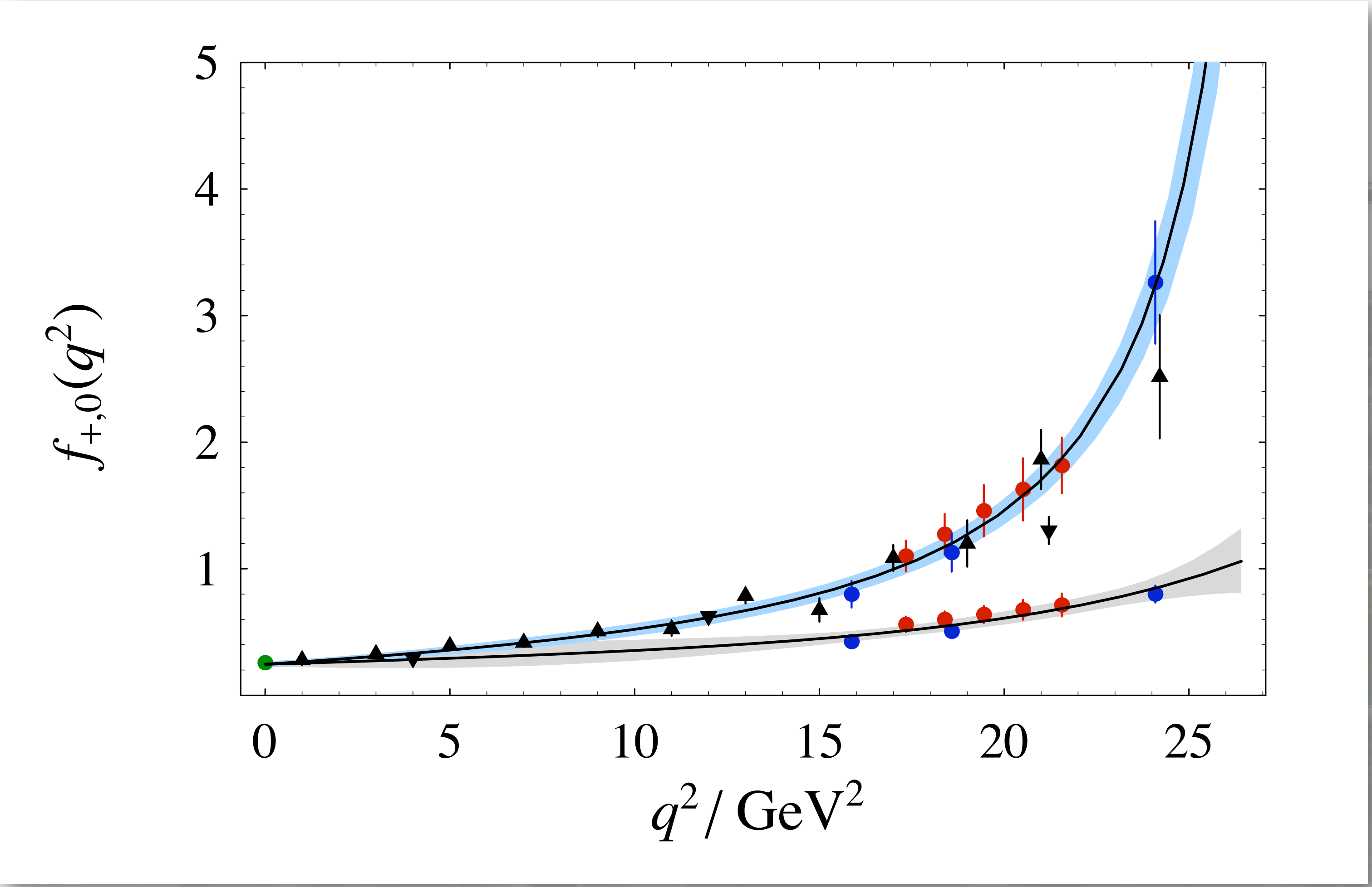
$$(4.46 \pm 0.20 \pm 0.20) \times 10^{-3}$$

expt. th.

HFAG avg (DGE) summer 2006

E. Gulez, *et al.*, PRD 73, 074502 (2006), erratum ibid **75**, 119906 (2007)

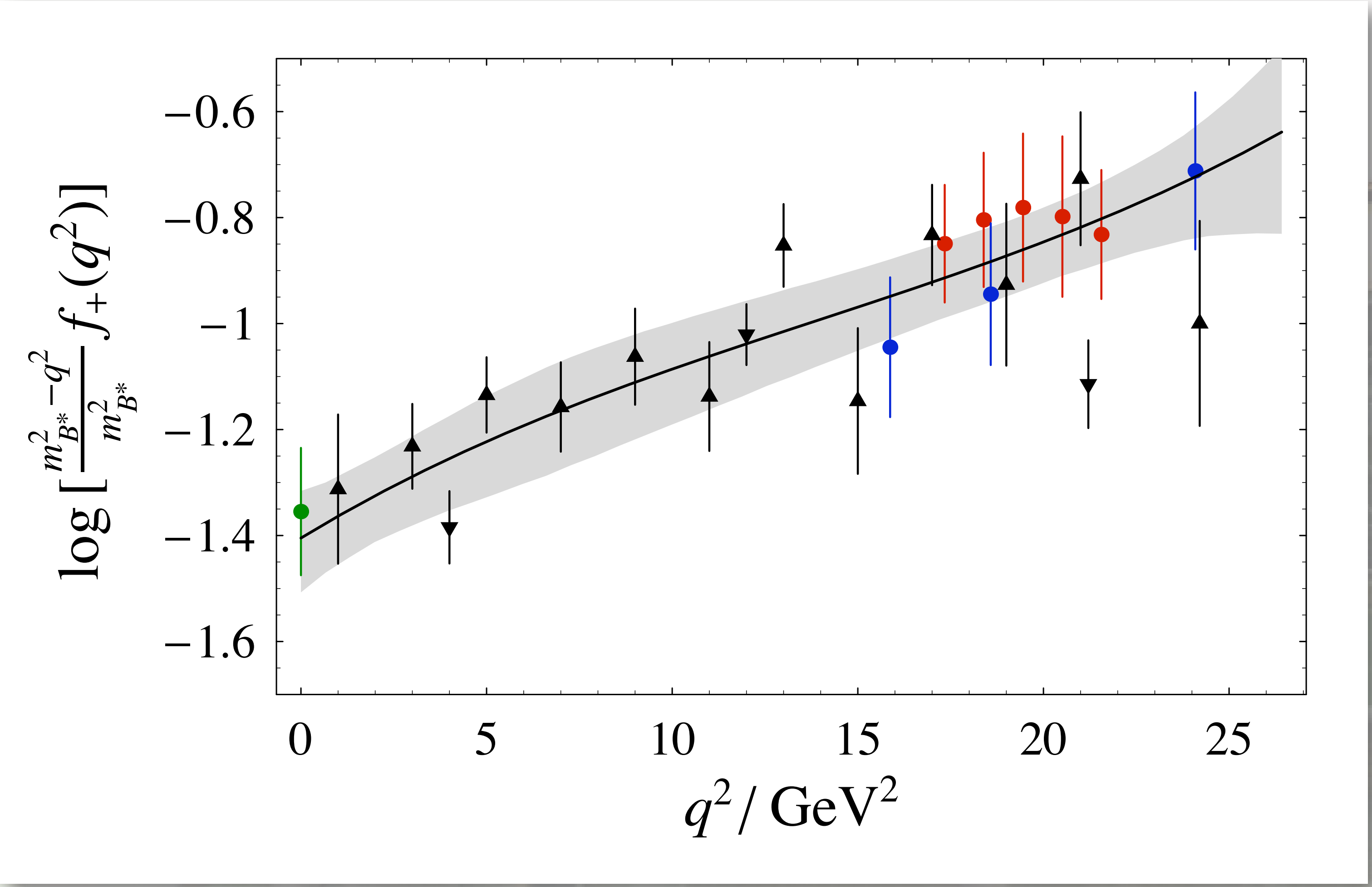
Comparison



Green: LCSR, Red: HPQCD, Blue: FNAL/MILC, Black: experiment*norm,
Curves & bands: fit to Omnès parametrisation (Flynn & Nieves)

Plot from J. M. Flynn and J. Nieves, arXiv:0705.3553

Comparison



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Error budgets

Estimate of percentage errors in $f_+(q^2)$ for $q^2 > 16$ G

source of error	size of error
statistics + chiral extrapolations	10
two-loop matching	9
discretization	3
relativistic	1
Total	14

From E. Gulez, et al., Phys. Rev. D **73**, 074502 (2006), erratum ibid **75**, 119906 (2

HPQCD

Improvement seen with use of random wall sources: K. Wong *et al*, Lattice 2007

Table 1: Systematic errors for CKM matrix elements from the semileptonic decay are obtained from the integration with $q^2_{\text{min}} = 16\ \text{GeV}^2$.

decay	$D \to \pi(K)l\nu$	$B \to$
CKM matrix element	$\lvert V_{cd(s)}\rvert$	$\lvert$
discretization effect	9%	9%
fitting 3- and 2-point functions	3%	3%
chiral extrapolation	3%(2%)	4%
q^2 dependence (BK parameterization)	2%	4%
current renormalization	0%	1%
a uncertainty	1%	1%
total systematic	10%	11%

FNAL/MILC

Improved action: M. Oktay and A. S. Kronfeld, 2008

From P. Mackenzie, Lattice 2005

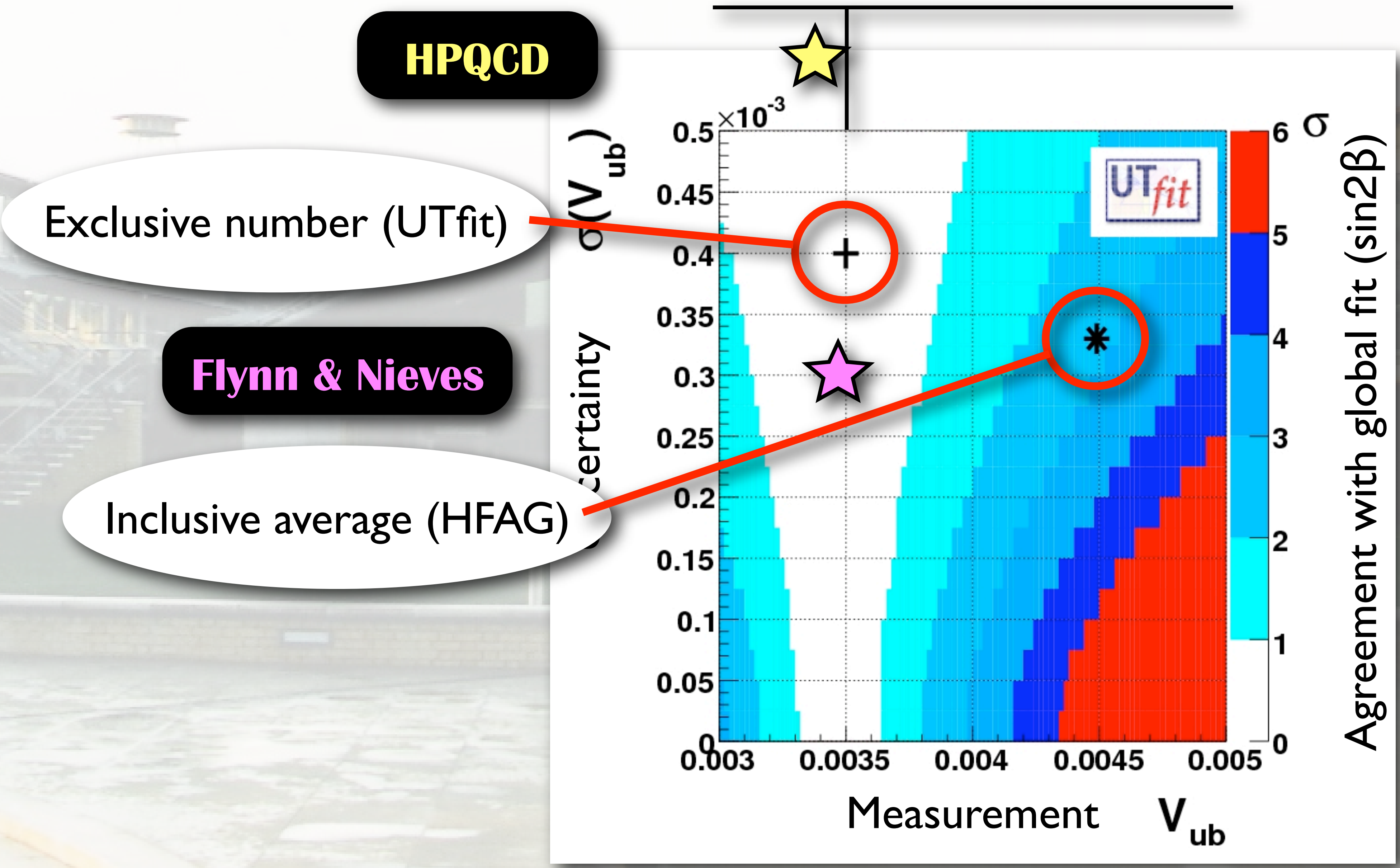
Updating the experimental br. frac.

HFAG update	B.F. ($q^2 > 16$ GeV^2) $* 10^4$	$V_{ub} * 10^3$ (HPQCD)
EPS 2005	0.40(4)(4)	3.55(25)(50)
LP 2007	0.35(3)(3)	3.33(21)($^{+58}_{-38}$)

V_{ub}($*10^3$): FNAL 3.6(2)($^{+6}_{-4}$), BZ 3.4(1)($^{+6}_{-4}$) vs. Inclusive 4.5(2)(2)

Tension between inclusive and exclusive determinations is a continuing story, demonstrating the challenges of precision physics.

Semileptonic decay vs. sin2β



From UTfit website, version of 28 September 2006

plus my decorations

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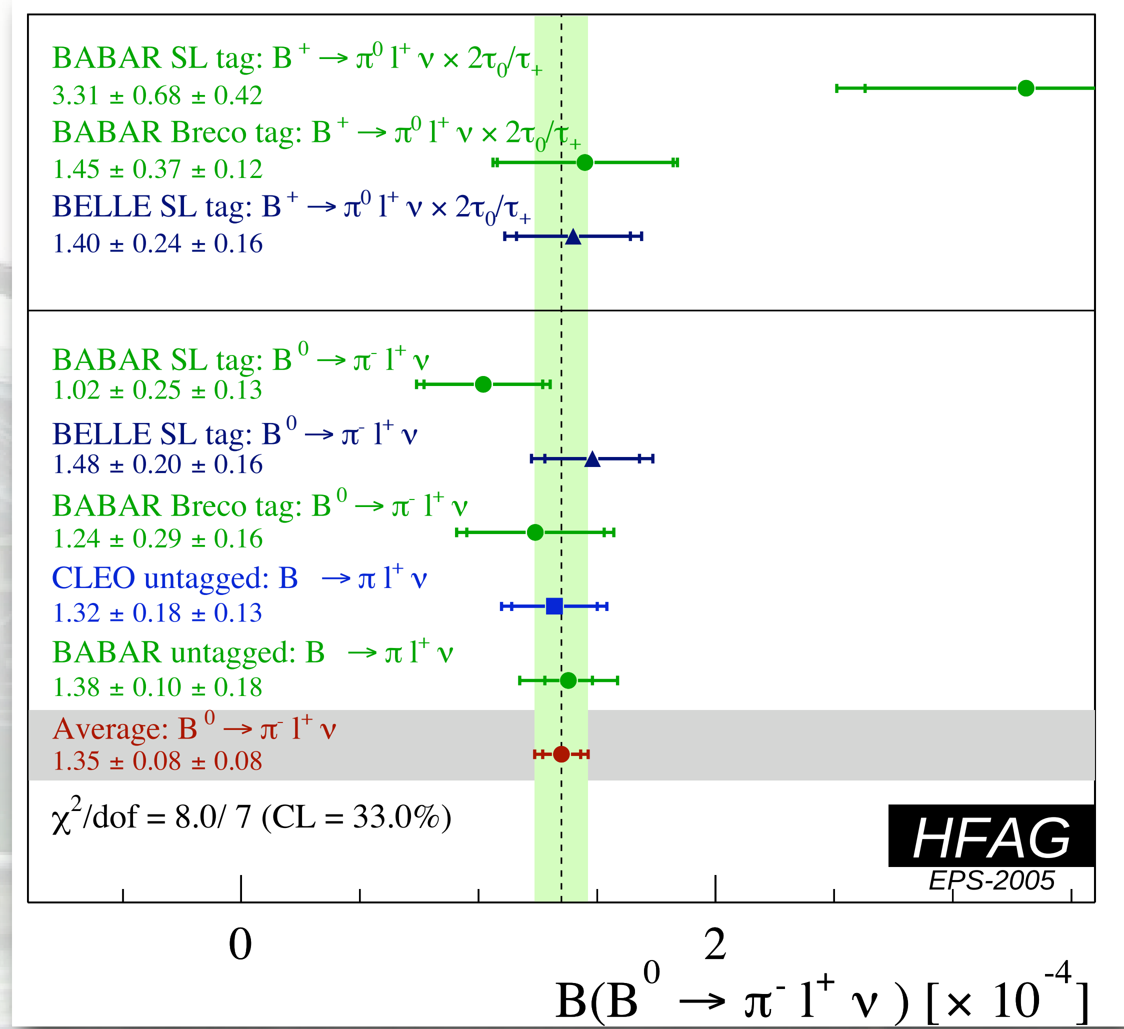
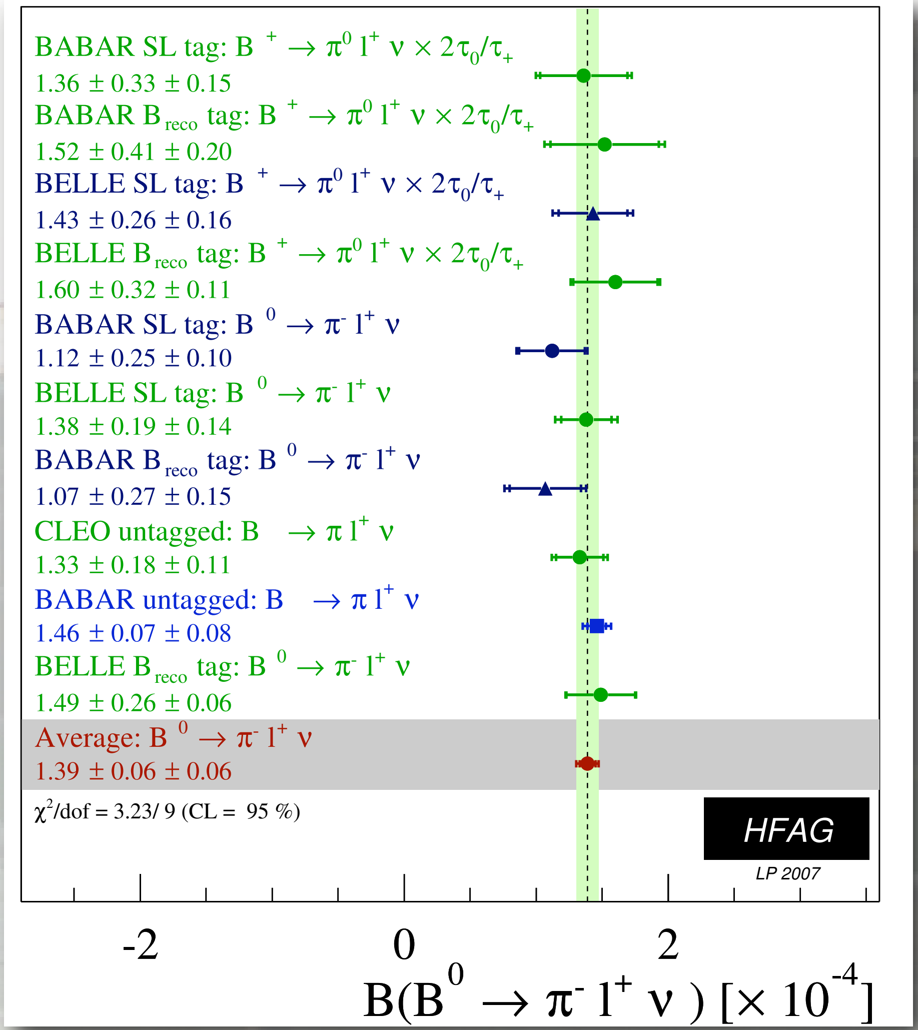
Total branching fraction





The total branching fraction has not changed much, while the b.f. with $q^2 > 16\ \mathrm{GeV}^2$ has moved ~1 sigma. This highlights the need for LQCD to extend its kinematic reach.

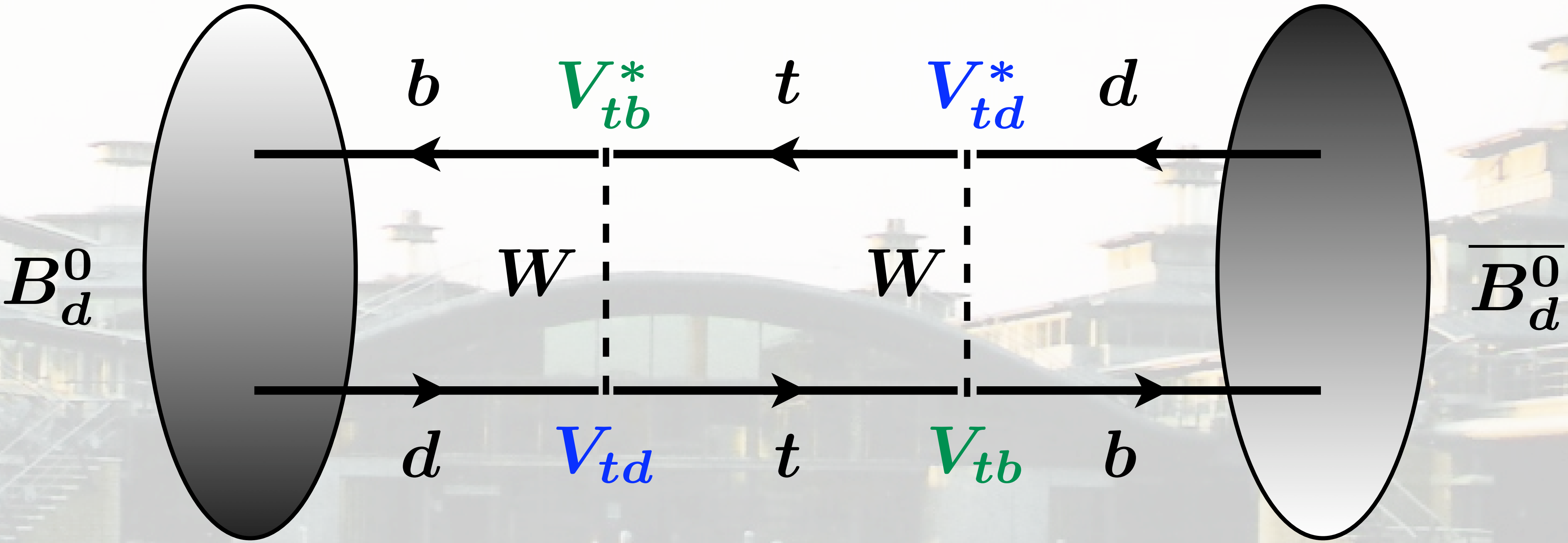
Lower q^2 on the lattice

- Low q^2 implies large pion recoil
- But pion momentum must be small compared to inverse lattice spacing in lattice rest frame
- So far lattice and B frames roughly coincide in all calculations
- Progress can be made by discretizing in a frame which is boosted relative to the B

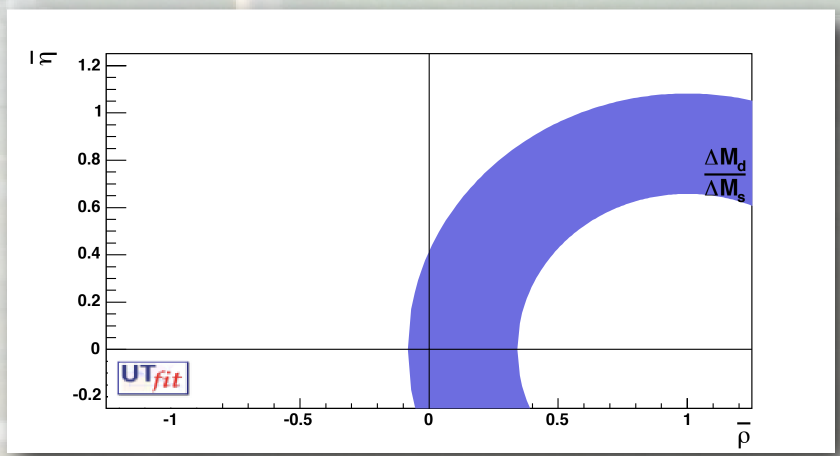
$$\boldsymbol{p} = m_b \boldsymbol{u} + \boldsymbol{k}$$

- Extending the range of q^2 will remove model dependence of shape and reduce statistical uncertainties due to better overlap with experimental signal
- Preliminary tests: S. Meinel, *et al.*, PoS(Lattice 2007)377, arXiv:0710.3101

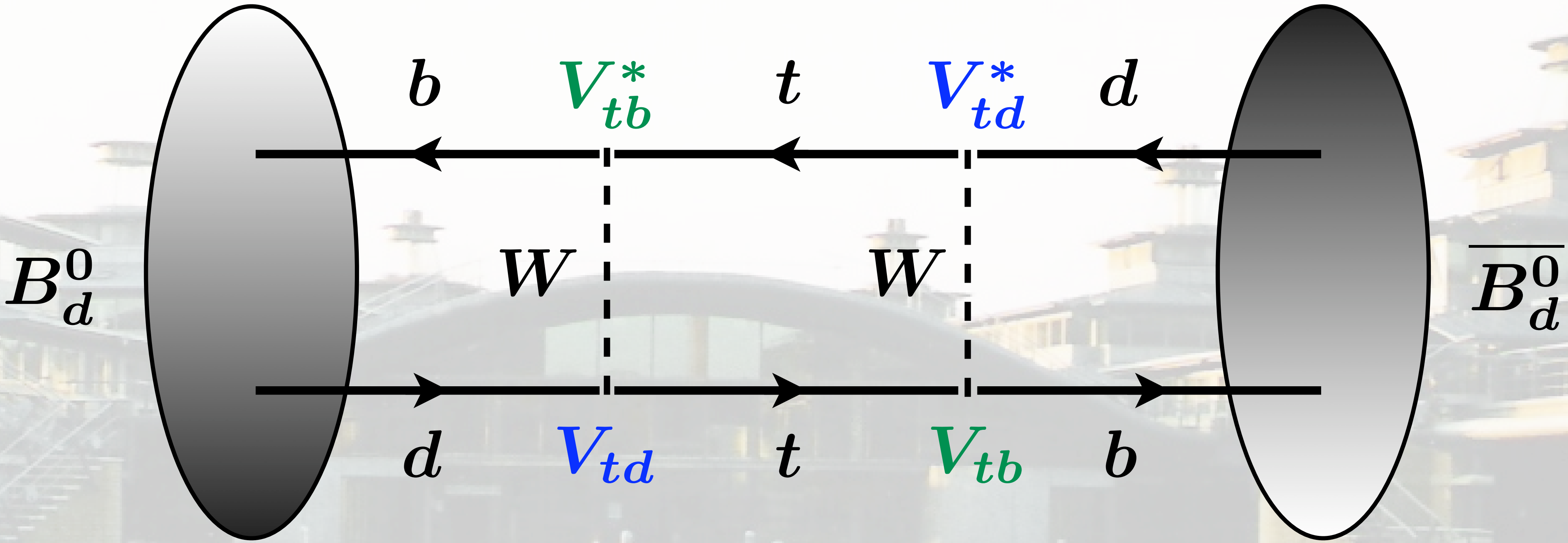
$B^0 - \overline{B^0}$ Mixing

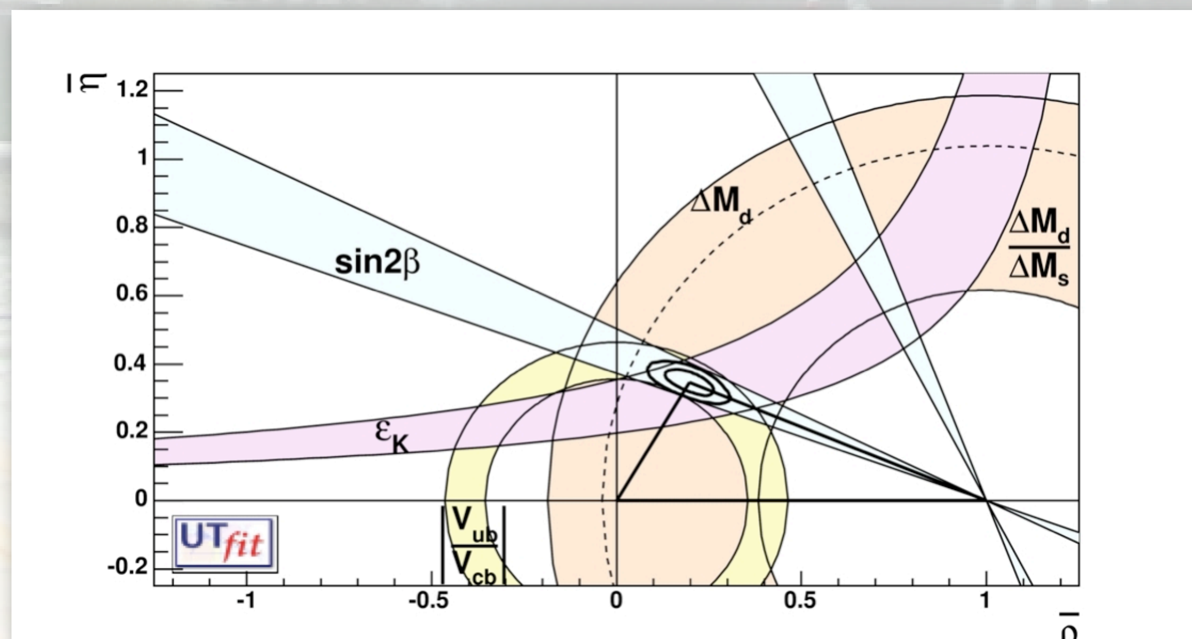


$$|V_{td}|^2 \propto [(1-\bar{\rho})^2 + \bar{\eta}^2]$$



$B^0 - \overline{B^0}$ Mixing





- Constraint complements $\sin 2\beta$ & $|V_{ub}|/|V_{cb}|$
- More likely ***New Physics*** contributions

work done with

E. Dalgic (Simon Fraser)
E. Gámiz (Illinois)
A. Gray (Ohio State, Edinburgh)
J. Shigemitsu (Ohio State)
C. T. H. Davies (Glasgow)
G. P. Lepage (Cornell)

(part of the HPQCD Collaboration)

$B^0 - \overline{B^0}$ Mixing

Only $B_d^0 - \overline{B_d^0}$

$$\Delta m_d = \frac{G_F^2}{6\pi^2} m_W^2 \eta_B S(x_t) m_{B_d} f_{B_d}^2 B_{B_d} |V_{td} V_{tb}^*|^2$$

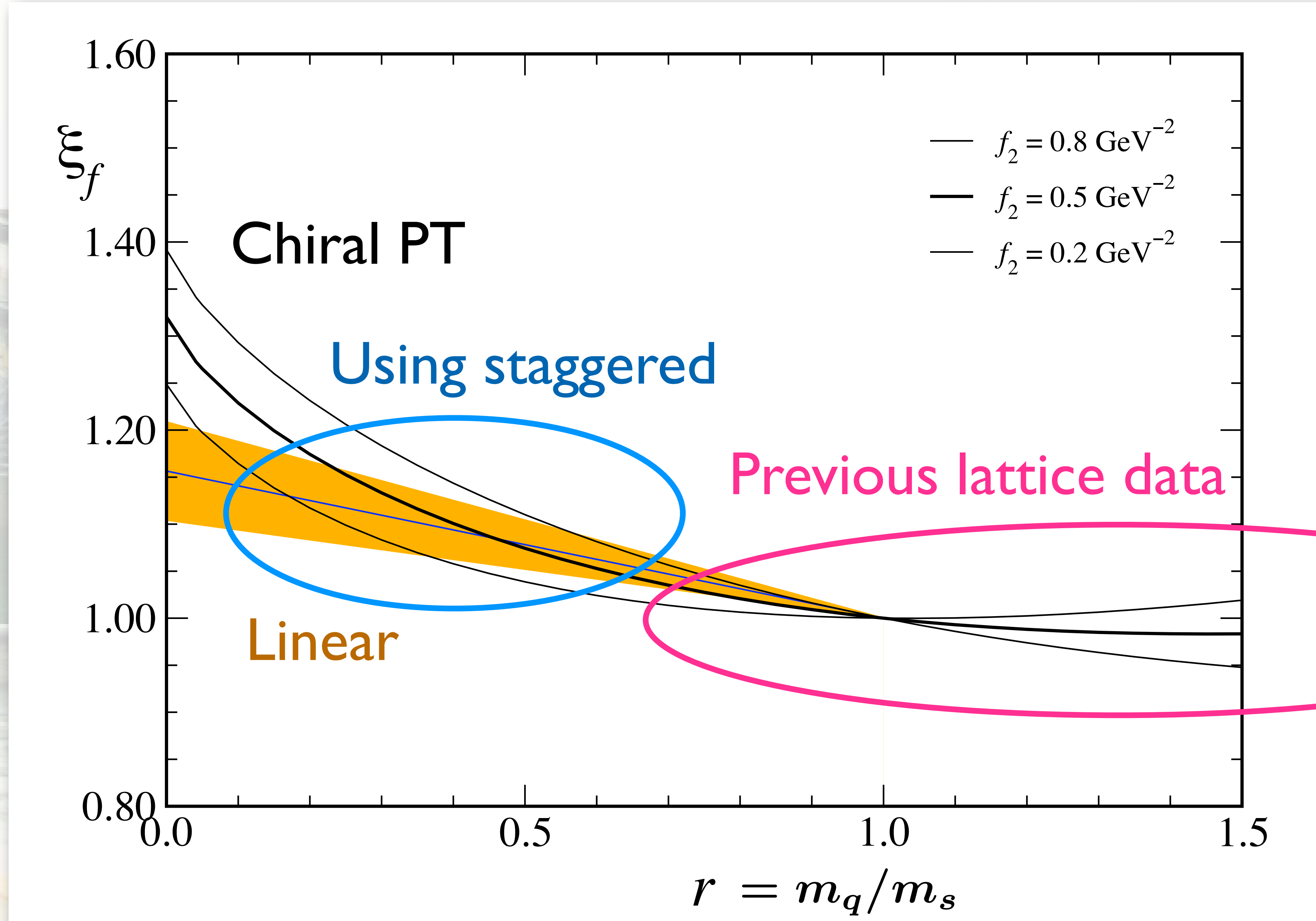
Including $B_s^0 - \overline{B_s^0}$

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \xi^2 \left| \frac{V_{ts}}{V_{td}} \right|^2$$

$$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$

Most of the mass dependence, & simplest to compute on lattice

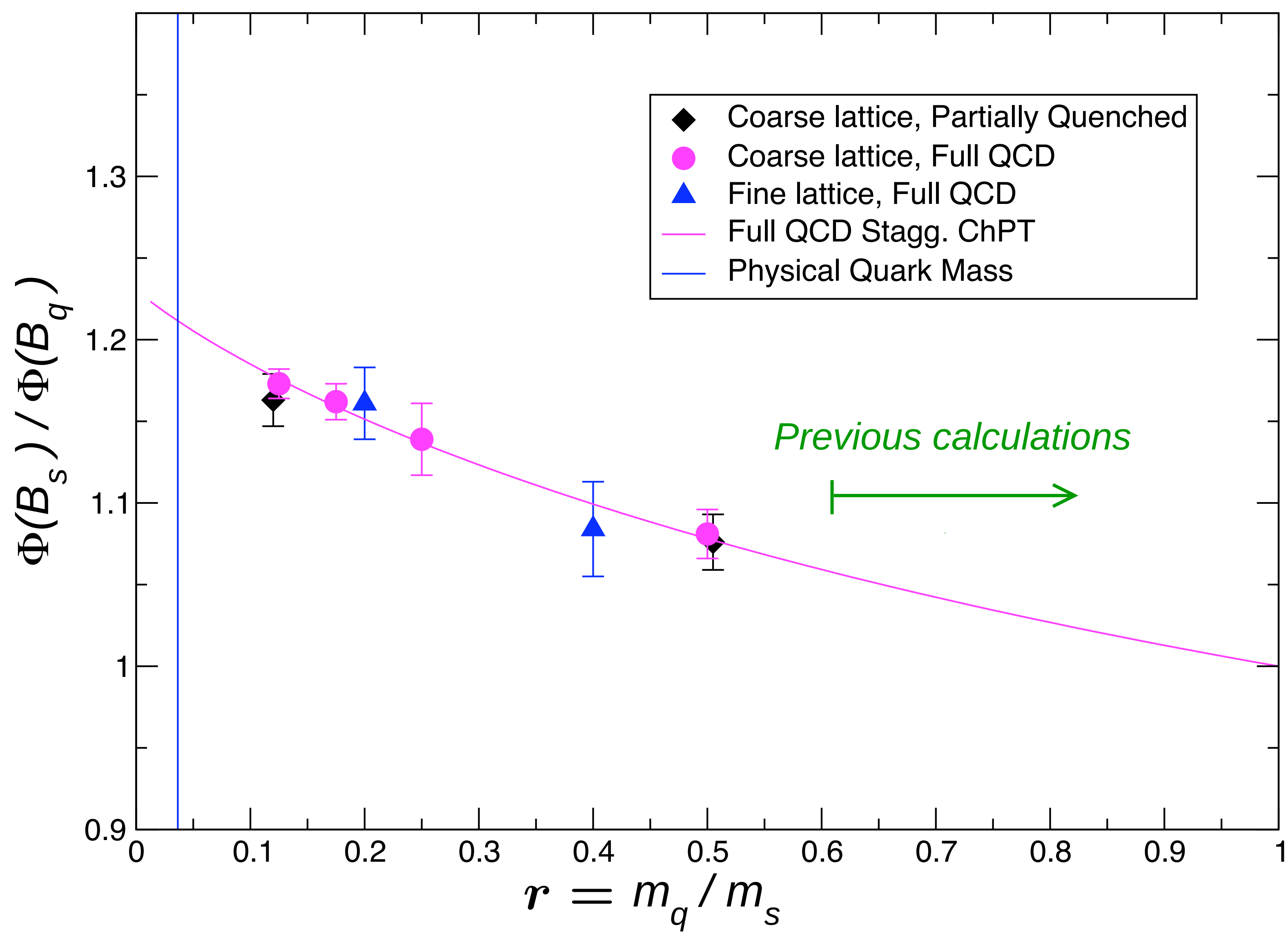
$B^0 - \overline{B^0}$ Mixing 2002



Kronfeld & Ryan, (also JLQCD)

$B^0 - \overline{B^0}$ Mixing 2005

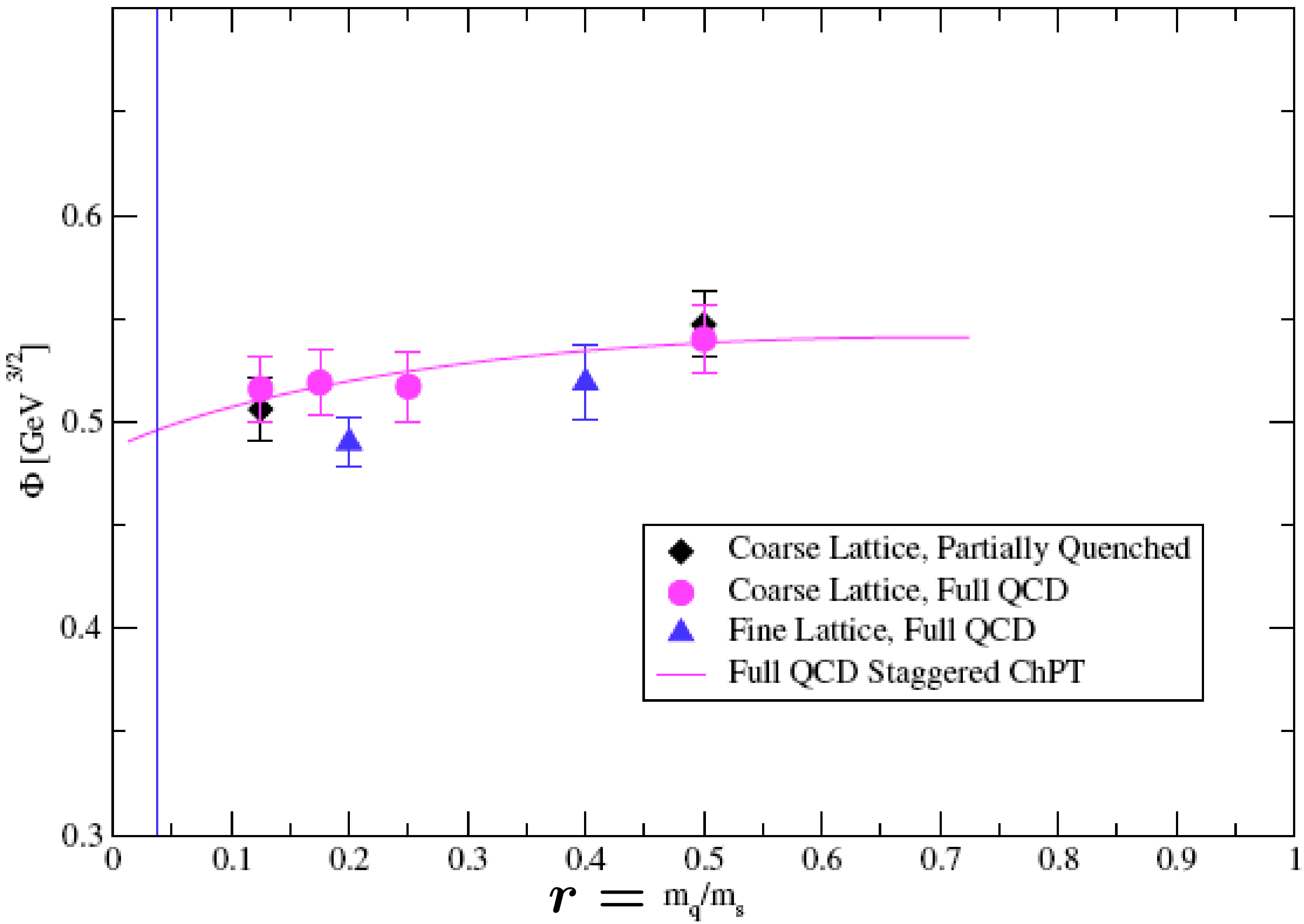
A. Gray, *et al* (HPQCD) PRL 95, 212001 (2005)



$\Phi(B) \equiv f_B\sqrt{m_B}$

$B^0 - \overline{B^0}$ Mixing 2005

A. Gray, *et al* (HPQCD) PRL 95, 212001 (2005)



$\Phi(B) \equiv f_B\sqrt{m_B}$

Decay constant results

$$f_{B_s} = 260 \pm 7|_{\text{stat}} \pm 26_{\text{match}} \pm 8_{\text{hq}} \pm 5_{\text{disc}} \text{ MeV}$$

$$f_B = 216 \pm 9|_{\text{stat}} \pm 19_{\text{match}} \pm 6_{\text{hq}} \pm 4_{\text{disc}} \text{ MeV}$$

Most errors cancel in the ratio

$$\frac{f_{B_s}}{f_B} = 1.20 \pm 0.03 \pm 0.01$$

M. W., *et al* (HPQCD) PRL 92 (2004); A. Gray, *et al* (HPQCD) PRL 95 (2005)

$B \to \tau\nu$

Belle, hep-ex/0604018

$$f_B|V_{ub}| = 0.77 \begin{pmatrix} +12 \\ -10 \end{pmatrix}_{\text{stat}} \begin{pmatrix} +7 \\ -6 \end{pmatrix}_{\text{sys}} \text{ MeV}$$

$$\frac{f_B|V_{ub}|}{|V_{ub}|_{\text{DGS}}} = 175 \pm 37 \text{ MeV} \qquad \frac{f_B|V_{ub}|}{|V_{ub}|_{\text{HPQCD}}} = 193 \pm 46 \text{ MeV}$$

BaBar, hep-ex/0611019

$$f_B|V_{ub}| = 0.70 \begin{pmatrix} +23 \\ -36 \end{pmatrix}_{\text{stat}} \begin{pmatrix} +4 \\ -5 \end{pmatrix}_{\text{sys}} \text{ MeV}$$

Full 4-quark matrix elements

3 LO operators

$$OL \equiv [\overline{b^i}\, s^i]_{V-A}[\overline{b^j}\, s^j]_{V-A},$$
$$OS \equiv [\overline{b^i}\, s^i]_{S-P}[\overline{b^j}\, s^j]_{S-P},$$
$$O3 \equiv [\overline{b^i}\, s^j]_{S-P}[\overline{b^j}\, s^i]_{S-P}.$$

$$\langle OL\rangle^{\overline{MS}}_{(\mu)} \equiv \langle \overline{B}_s|OL|B_s\rangle^{\overline{MS}}_{(\mu)} \equiv \frac{8}{3} f^2_{B_s}\, B_{B_s}(\mu)\, M^2_{B_s}.$$
$$\langle OS\rangle^{\overline{MS}}_{(\mu)} \equiv -\frac{5}{3} f^2_{B_s}\, \frac{B_S(\mu)}{R^2}\, M^2_{B_s},$$
$$\langle O3\rangle^{\overline{MS}}_{(\mu)} \equiv \frac{1}{3} f^2_{B_s}\, \frac{\tilde{B}_S(\mu)}{R^2}\, M^2_{B_s},$$

$$\frac{1}{R^2} \equiv \frac{M^2_{B_s}}{(\overline{m}_b + \overline{m}_s)^2}.$$

NLO operators

$$OLj1 \equiv \frac{1}{2M}\left\{[\vec{\nabla}\overline{b^i}\cdot\vec{\gamma}\, s^i]_{V-A}[\overline{b^j}\, s^j]_{V-A} + [\overline{b^i}\, s^i]_{V-A}[\vec{\nabla}\overline{b^j}\cdot\vec{\gamma}\, s^j]_{V-A}\right\}$$

Together

$$\frac{a^3}{2M_{B_s}}\langle OX\rangle^{\overline{\mathrm{MS}}} = [1+\alpha_s\cdot\rho_{XX}]\langle OX\rangle + \alpha_s\cdot\rho_{XY}\langle OY\rangle + \left[\langle OXj1\rangle - \alpha_s(\zeta^{XX}_{10}\langle OX\rangle + \zeta^{XY}_{10}\langle OY\rangle)\right]$$

Results

E. Dalgic, *et al*, Phys. Rev. D **76**, 011501, hep-lat/0610104

u/d sea quark mass

	$m_f/m_s = 0.25$	$m_f/m_s = 0.50$
$f_{B_s}\sqrt{\hat{B}_{B_s}}$ [GeV]	0.281(21)	0.289(22)
$f_{B_s}\sqrt{B_{B_s}(m_b)}$ [GeV]	0.227(17)	0.233(17)
$f_{B_s}\frac{\sqrt{B_S(m_b)}}{R}$ [GeV]	0.295(22)	0.301(23)
$f_{B_s}\frac{\sqrt{\tilde{B}_S(m_b)}}{R}$ [GeV]	0.305(23)	0.310(23)

Theory $\quad \Delta m_s = 20.3 \pm 3.0 \pm 0.8 \text{ ps}^{-1}$ $\quad$ (LQCD)(V_{ts})

Experiment $\quad \Delta m_s = 17.77 \pm 0.10 \pm 0.07 \text{ ps}^{-1}$ $\quad$ (stat)(syst)

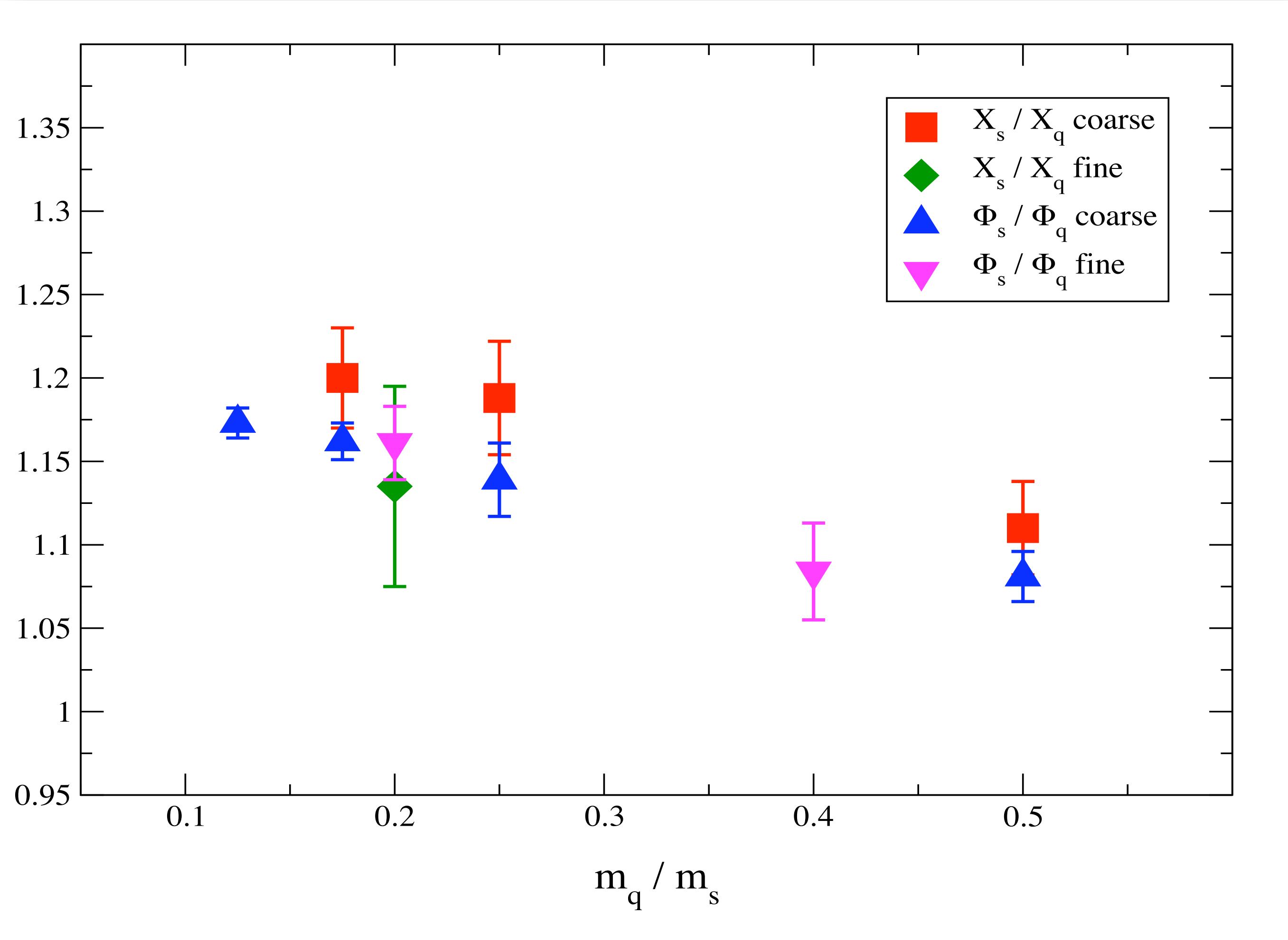
Error budget

$$f_{B_s}^2 \, B_{B_s}, \quad f_{B_s}^2 \, \frac{B_S}{R^2}, \quad f_{B_s}^2 \, \frac{\tilde{B}_S}{R^2}. \tag{15}$$

TABLE II: Error budget for quantities listed in (15).

Statistical + Fitting	9 %
Higher Order Matching	9 %
Discretization	4 %
Relativistic	3 %
Scale (a^{-3})	5 %
Total	15 %

New data for ratios

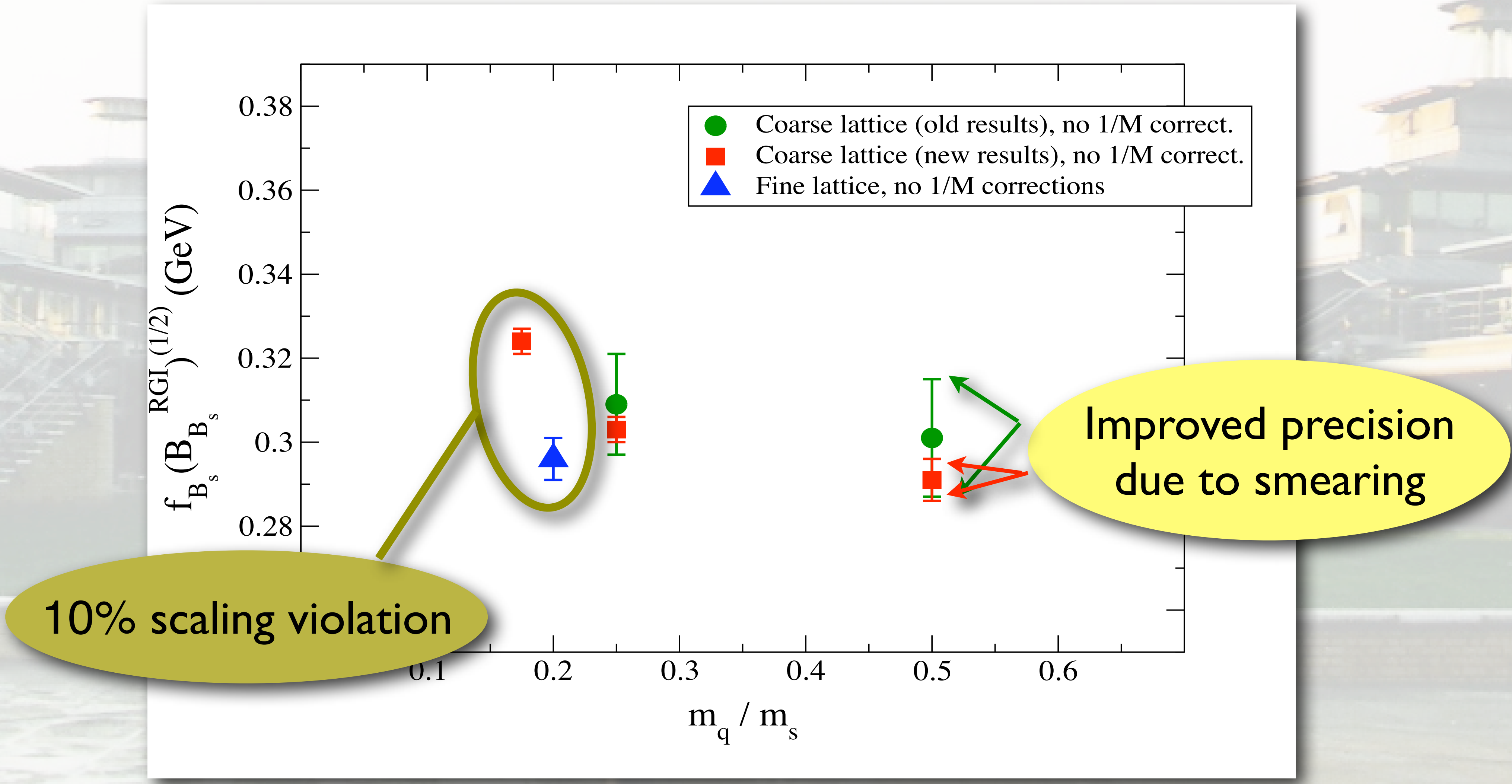


$$X_q \equiv f_{B_q}\sqrt{B_{B_q}M_{B_q}}$$

$$\Phi_q \equiv f_{B_q}\sqrt{M_{B_q}}$$

E. Gámiz, *et al.*, PoS(Lattice 2007)349, arXiv:0710.0646

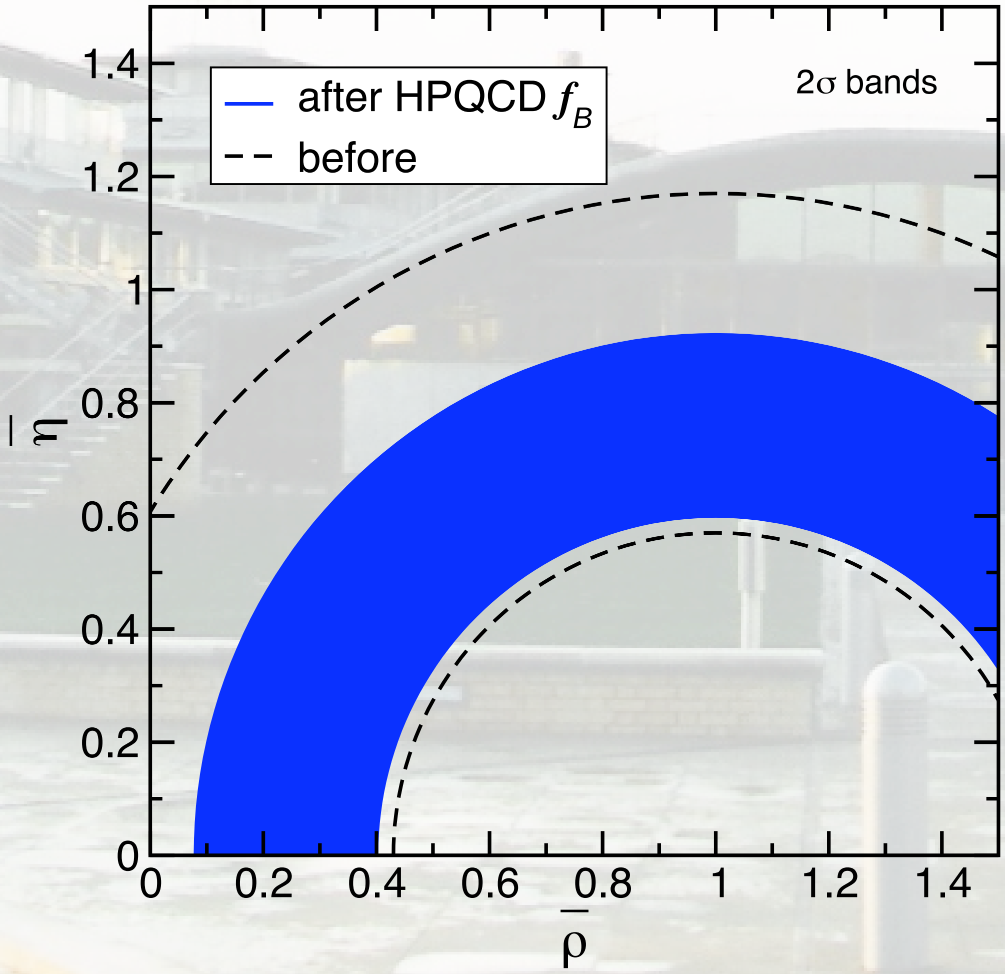
Sea quark/discretization effect for B_s



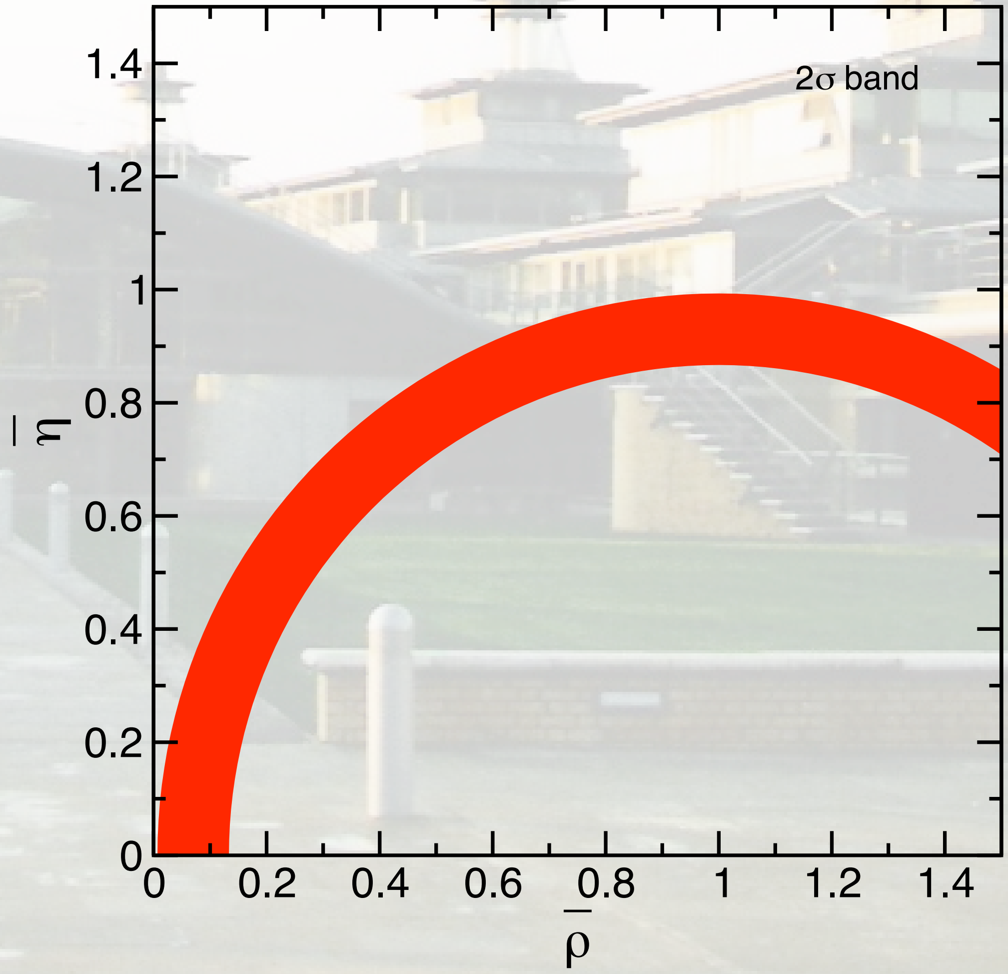
E. Gámiz, *et al.*, PoS(Lattice 2007)349, arXiv:0710.0646

Tightening V_{td} constraint

(a) $(\bar{\rho}, \bar{\eta})$ from Δm_d only



(b) $(\bar{\rho}, \bar{\eta})$ from Δm_d and Δm_s



mNRQCD & Form Factors

L. Khomskii, R. R. Horgan, S. Meinel, L. Storoni, M.W.
(Cambridge)
C. T. H. Davies, *et al* (Glasgow)
A. Hart & E. Müller (Edinburgh)

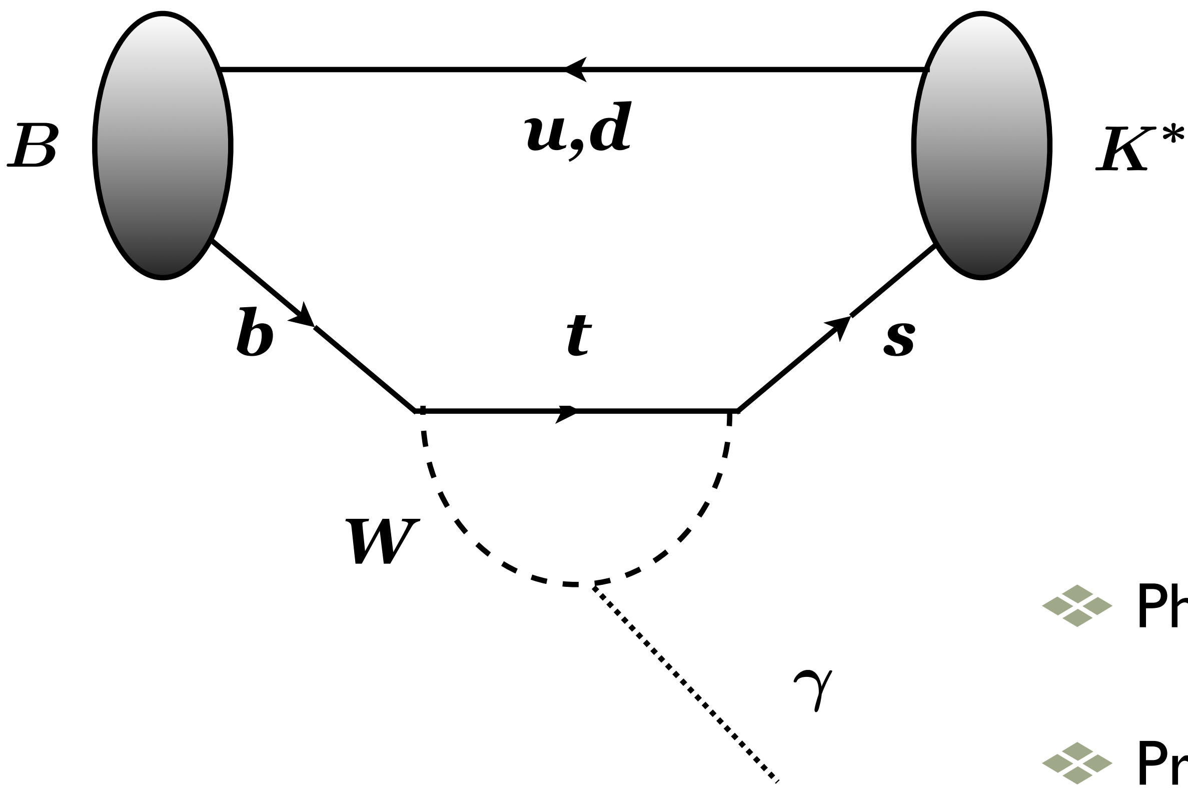
(PART OF THE HPQCD COLLABORATION)

Motivation

- Increase precision in form factors by extending q^2 range
- Increase list of observables (lesson from inclusive/exclusive V_{ub})
- More direct focus on standard vs. nonstandard FCNC
- Complement future progress on LHC measurements of, e.g.

$$B \to K^* \gamma \qquad B \to K^* \mu^+ \mu^-$$

Rare B decays



- Physical point $q^2 = 0$
- Progress w/ mNRQCD?
- Independent way to get V_{ts}
- Also V_{td}, but worry about weak annihilation contrib.

Full set of form factors

Change spectator mass:
$B_s \to \phi$

Matrix element	Form fact	
$\langle P \vert \bar{q}\gamma^{\mu} b \vert B\rangle$	f_+, f_0	$B \to \pi\ell\nu$ $B \to K\ell^+\ell^-$
$\langle P \vert \bar{q}\sigma^{\mu\nu} q_\nu b \vert B\rangle$	f_T	$B \to K\ell^+\ell^-$
$\langle V \vert \bar{q}\gamma^{\mu} b \vert B\rangle$	V	$\left\{ \begin{array}{l} B \to (\rho/\omega)\ell\nu \\ B \to K^*\ell^+\ell^- \end{array} \right.$
$\langle V \vert \bar{q}\gamma^{\mu}\gamma^5 b \vert B\rangle$	A_0, A_1, A_2	
$\langle V \vert \bar{q}\sigma^{\mu\nu} q_\nu b \vert B\rangle$	T_1	$\left\{ \begin{array}{l} B \to K^*\gamma \\ B \to K^*\ell^+\ell^- \end{array} \right.$
$\langle V \vert \bar{q}\sigma^{\mu\nu}\gamma^5 q_\nu b \vert B\rangle$	T_2, T_3	

Summary

- LQCD calculations of B decay/mixing matrix elements ***contribute*** to the study of physics ***beyond*** the Standard Model
- Taken approaches which allow us to ***address all systematic errors simultaneously*** (within 4th root hypothesis)
- List of postdictions and predictions having positive impact in flavor physics community
- V_{ub} from $B\to\pi$ semileptonic decay consistently lower and in better agreement with CKM fits ($\sin 2\beta$) than inclusive B semileptonic decays
- Chiral extrapolation of 4-quark operators underway
- Further improvement of actions, mNRQCD, automated lattice perturbation theory
- Many alternatives which will check and probably do better in the future. Need balance between perfection and ***timeliness***.