

# Determining the WIMP Mass from Direct Dark Matter Detection Data

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based on [arXiv:0707.0488](https://arxiv.org/abs/0707.0488) [astro-ph]

Reconstructing the velocity distribution function of WIMPs

Deriving  $f_1(v)$  from the scattering spectrum

Reconstructing  $f_1(v)$  from experimental data

Determining the WIMP mass

Summary

## Deriving $f_1(v)$ from the scattering spectrum

- Differential rate for elastic WIMP-nucleus scattering

$$\frac{dR}{dQ} = \mathcal{A} F^2(Q) \int_{v_{\min}}^{\infty} \left[ \frac{f_1(v)}{v} \right] dv$$

Here

$$v_{\min} = \alpha \sqrt{Q}$$

is the minimal incoming velocity of incident WIMPs that can deposit the **recoil energy**  $Q$  in the detector.

$$\mathcal{A} \equiv \frac{\rho_0 \sigma_0}{2m_\chi m_{\text{r}}^2} \quad \alpha \equiv \sqrt{\frac{m_{\text{N}}}{2m_{\text{r}}^2}} \quad m_{\text{r}} = \frac{m_\chi m_{\text{N}}}{m_\chi + m_{\text{N}}}$$

$\rho_0$ : WIMP density near the Earth

$\sigma_0$ : total cross section ignoring the form factor suppression

$F(Q)$ : elastic nuclear form factor

## Deriving $f_1(v)$ from the scattering spectrum

- Normalized one-dimensional velocity distribution function

$$f_1(v) = \mathcal{N} \left\{ -2Q \cdot \frac{d}{dQ} \left[ \frac{1}{F^2(Q)} \left( \frac{dR}{dQ} \right) \right] \right\}_{Q=v^2/\alpha^2}$$

$$\mathcal{N} = \frac{2}{\alpha} \left\{ \int_0^\infty \frac{1}{\sqrt{Q}} \left[ \frac{1}{F^2(Q)} \left( \frac{dR}{dQ} \right) \right] dQ \right\}^{-1}$$

- Moments of the velocity distribution function

$$\langle v^n \rangle = \mathcal{N}(Q_{\text{thre}}) \left( \frac{\alpha^{n+1}}{2} \right) \left[ \frac{2 Q_{\text{thre}}^{(n+1)/2}}{F^2(Q_{\text{thre}})} \left( \frac{dR}{dQ} \right)_{Q=Q_{\text{thre}}} + (n+1) I_n(Q_{\text{thre}}) \right]$$

$$\mathcal{N}(Q_{\text{thre}}) = \frac{2}{\alpha} \left[ \frac{2 Q_{\text{thre}}^{1/2}}{F^2(Q_{\text{thre}})} \left( \frac{dR}{dQ} \right)_{Q=Q_{\text{thre}}} + I_0(Q_{\text{thre}}) \right]^{-1}$$

$$I_n(Q_{\text{thre}}) = \int_{Q_{\text{thre}}}^\infty Q^{(n-1)/2} \left[ \frac{1}{F^2(Q)} \left( \frac{dR}{dQ} \right) \right] dQ$$

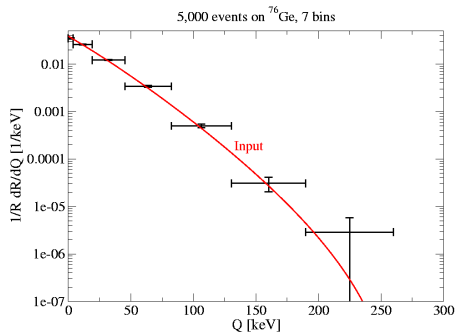
[M. Drees and C. L. Shan, JCAP 0706, 011]

## Reconstructing $f_1(v)$ from experimental data

- Experimental data

$$Q_n - \frac{b_n}{2} \leq Q_{n,i} \leq Q_n + \frac{b_n}{2} \quad i = 1, 2, \dots, N_n, n = 1, 2, \dots, B$$

- Theoretically predicted scattering spectrum



## Reconstructing $f_1(v)$ from experimental data

- Ansatz: in the  $n$ th  $Q$ -bin

$$\left(\frac{dR}{dQ}\right)_n \equiv \left(\frac{dR}{dQ}\right)_{Q \simeq Q_n} = \tilde{r}_n e^{k_n(Q-Q_n)} \equiv r_n e^{k_n(Q-Q_{s,n})}$$

$$\tilde{r}_n \equiv \left(\frac{dR}{dQ}\right)_{Q=Q_n} \qquad r_n \equiv \frac{N_n}{b_n}$$

- Recoil spectrum at  $Q = Q_n$

$$\tilde{r}_n = \frac{N_n}{b_n} \left( \frac{\kappa_n}{\sinh \kappa_n} \right) \qquad \kappa_n \equiv \left( \frac{b_n}{2} \right) k_n$$

- Logarithmic slope and shifted point in the  $n$ th  $Q$ -bin

$$\bar{Q}_n - Q_n = \frac{b_n}{2} \left( \coth \kappa_n - \frac{1}{\kappa_n} \right) \qquad \bar{Q}_n = \frac{1}{N_n} \sum_{i=1}^{N_n} Q_{n,i}$$

$$Q_{s,n} = Q_n + \frac{1}{k_n} \ln \left( \frac{\sinh \kappa_n}{\kappa_n} \right)$$

## Reconstructing $f_1(v)$ from experimental data

- Reconstructing the one-dimensional velocity distribution

$$f_{1,r}(v_{s,\mu}) = \mathcal{N} \left[ \frac{2Q_{s,\mu} r_\mu}{F^2(Q_{s,\mu})} \right] \left[ \frac{d}{dQ} \ln F^2(Q) \Big|_{Q=Q_{s,\mu}} - k_\mu \right]$$

$$v_{s,\mu} = \alpha \sqrt{Q_{s,\mu}}$$

$$\mathcal{N} = \frac{2}{\alpha} \left[ \sum_a \frac{1}{\sqrt{Q_a} F^2(Q_a)} \right]^{-1}$$

- Determining the moments of the velocity distribution

$$\langle v^n \rangle = \alpha^n \left[ \frac{2Q_{\text{thre}}^{1/2} r_{\text{thre}}}{F^2(Q_{\text{thre}})} + I_0 \right]^{-1} \left[ \frac{2Q_{\text{thre}}^{(n+1)/2} r_{\text{thre}}}{F^2(Q_{\text{thre}})} + (n+1)I_n \right]$$

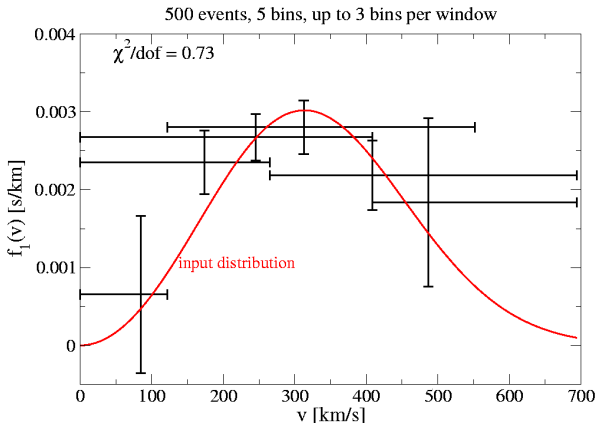
$$I_n = \sum_a \frac{Q_a^{(n-1)/2}}{F^2(Q_a)}$$

$$r_{\text{thre}} = \left( \frac{dR}{dQ} \right)_{Q=Q_{\text{thre}}}$$

[M. Drees and C. L. Shan, JCAP 0706, 011]

## Reconstructing $f_1(v)$ from experimental data

- Reconstruction with simulated experimental data  
 (500 events, 5 bins, up to 3 bins per window)

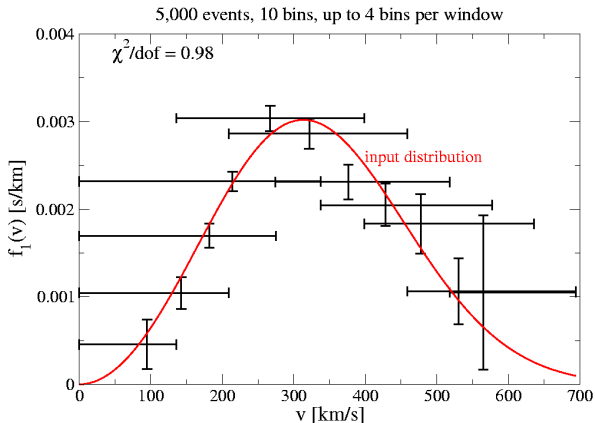


[M. Drees and C. L. Shan, JCAP 0706, 011]



## Reconstructing $f_1(v)$ from experimental data

- Reconstruction with simulated experimental data  
 (5,000 events, 10 bins, up to 4 bins per window)



[M. Drees and C. L. Shan, JCAP 0706, 011]

## Determining the WIMP mass

- Using two different target nuclei

$$\langle v^n \rangle = \alpha_X^n \left[ \frac{(n+1)I_{n,X}}{I_{0,X}} \right] = \alpha_Y^n \left[ \frac{(n+1)I_{n,Y}}{I_{0,Y}} \right]$$

- WIMP mass

$$m_\chi = \frac{\sqrt{m_X m_Y} - m_X \mathcal{R}_n}{\mathcal{R}_n - \sqrt{m_X/m_Y}} \quad \mathcal{R}_n \equiv \frac{\alpha_Y}{\alpha_X} = \left( \frac{I_{n,X}}{I_{0,X}} \cdot \frac{I_{0,Y}}{I_{n,Y}} \right)^{1/n} \quad (n \neq 0, -1)$$

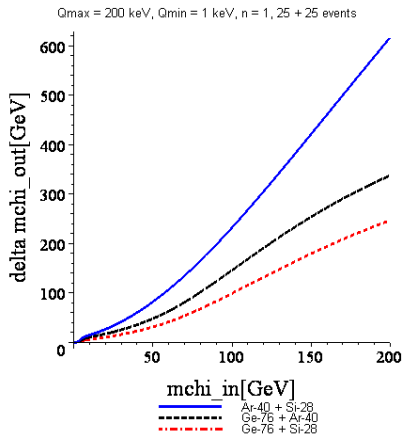
- 1- $\sigma$  statistical error

$$\sigma(m_\chi) = \frac{\mathcal{R}_n \sqrt{m_X/m_Y} |m_X - m_Y|}{\left( \mathcal{R}_n - \sqrt{m_X/m_Y} \right)^2} \times \frac{1}{|n|} \left[ \frac{\sigma^2(I_{n,X})}{I_{n,X}^2} + \frac{\sigma^2(I_{0,X})}{I_{0,X}^2} - \frac{2\text{cov}(I_{0,X}, I_{n,X})}{I_{0,X} I_{n,X}} + (X \rightarrow Y) \right]^{1/2}$$

[C. L. Shan, arXiv:0707.0488]

## Determining the WIMP mass

- $1-\sigma$  statistical error for different combinations  
(1 – 200 keV,  $n = 1, 25 + 25$  events)

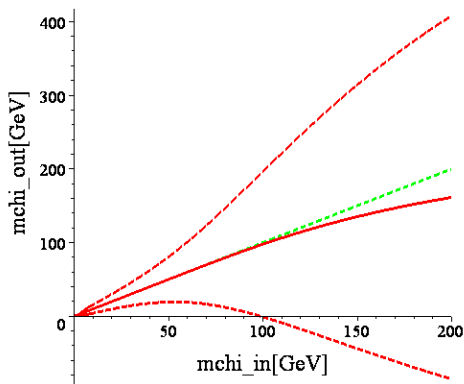


[C. L. Shan, arXiv:0707.0488]

## Determining the WIMP mass

- Reproduced WIMP mass  
(1 – 200 keV,  $n = 1$ ,  $^{76}\text{Ge} + ^{28}\text{Si}$ , 25 + 25 events)

$Q_{\text{max}} = 200 \text{ keV}$ ,  $Q_{\text{min}} = 1 \text{ keV}$ ,  $n = 1$ , 25 + 25 events, Ge-76 + Si-28

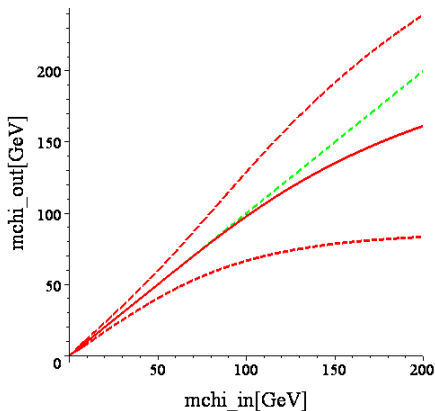


[C. L. Shan, arXiv:0707.0488]

## Determining the WIMP mass

- Reproduced WIMP mass  
(1 – 200 keV,  $n = 1$ ,  $^{76}\text{Ge} + ^{28}\text{Si}$ , 250 + 250 events)

$Q_{\text{max}} = 200$  keV,  $Q_{\text{min}} = 1$  keV,  $n = 1$ , 250 + 250 events, Ge-76 + Si-28



[C. L. Shan, arXiv:0707.0488]

## Determining the WIMP mass

- With  $Q_{\text{thre}} > 0$

$$\mathcal{R}_n(Q_{\text{thre}}) = \left[ \frac{2Q_{\text{thre},X}^{(n+1)/2} r_{\text{thre},X} + (n+1)I_{n,X} F_X^2(Q_{\text{thre},X})}{2Q_{\text{thre},X}^{1/2} r_{\text{thre},X} + I_{0,X} F_X^2(Q_{\text{thre},X})} \right]^{1/n} (X \longrightarrow Y)^{-1}$$

- Choosing  $n = -1$

$$\mathcal{R}_{-1}(Q_{\text{thre}}) = \frac{r_{\text{thre},Y}}{r_{\text{thre},X}} \left[ \frac{2Q_{\text{thre},X}^{1/2} r_{\text{thre},X} + I_{0,X} F_X^2(Q_{\text{thre},X})}{2Q_{\text{thre},Y}^{1/2} r_{\text{thre},Y} + I_{0,Y} F_Y^2(Q_{\text{thre},Y})} \right]$$

$$\begin{aligned} & \sigma(\mathcal{R}_{-1}) \\ &= \mathcal{R}_{-1} \left\{ \left[ \frac{I_{0,X} F_X^2(Q_{\text{thre},X})}{2Q_{\text{thre},X}^{1/2} r_{\text{thre},X} + I_{0,X} F_X^2(Q_{\text{thre},X})} \right]^2 \right. \\ & \quad \left. \times \left[ \frac{\sigma^2(r_{\text{thre},X})}{r_{\text{thre},X}^2} + \frac{\sigma^2(I_{0,X})}{I_{0,X}^2} - \frac{2\text{cov}(r_{\text{thre},X}, I_{0,X})}{r_{\text{thre},X} I_{0,X}} \right] + (X \longrightarrow Y) \right\}^{1/2} \end{aligned}$$

[C. L. Shan, arXiv:0707.0488]

## Summary

- By using experimental data **with different detector materials** we can determine the WIMP mass.
- The **larger the mass difference** between two target nuclei, the **smaller the statistical error** will be.
- Our method is **model-independent** and needs **only measured recoil energies**.
- With **200 keV** maximal measuring energy and **25 events from each experiment**, we can already extract meaningful information about the WIMP mass.

## Summary

A championship for finding new particle(s)  
between direct Dark Matter detection and  
collider experiments has been started.

Thank you very much for your attention.

[<http://www.th.physik.uni-bonn.de/th/People/cshan/>]