Lectures on exact renormalization group-2

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Progress of Theoretical Physics 109 (2003) 751

Progress of Theoretical Physics 110 (2003) 563

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Collaborated with K.Higashijima and T.Higashi (Osaka U.)

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Plan to talk

First lecture:

Wilsonian renormalization group

Second lecture:

Scalar field theory (sigma model)

Third lecture:

Gauge theory

There are some Wilsonian renormalization group equations.

- Wegner-Houghton equation (sharp cutoff)

K-I. Aoki, H. Terao, K.Higashijima…

local potential, Nambu-Jona-Lasinio, NL σ M

- Polchinski equation (smooth cutoff)

→ Lecture-3

T.Morris, K. Itoh, Y. Igarashi, H. Sonoda, M. Bonini,…

YM theory, QED, SUSY…

- Exact evolution equation (for 1PI effective action)

C. Wetterich, M. Reuter, N. Tetradis, J. Pawlowski,…

quantum gravity, Yang-Mills theory,

higher-dimensional gauge theory…

The WRG equation (Wegner-Houghton equation) describes the variation of effective action when energy scale Λ is changed to $\Lambda(\delta t)=\Lambda \exp[-\delta t]$.

$$\frac{d}{dt}S[\Omega;t] = \frac{1}{2\delta t}\int_{p'} tr \ln\left(\frac{\delta^2 S}{\delta\Omega^i \delta\Omega^j}\right)$$
$$-\frac{1}{2\delta t}\int_{p'}\int_{q'} \frac{\delta S}{\delta\Omega^i(p')}\left(\frac{\delta^2 S}{\delta\Omega^i(p')\delta\Omega^j(q')}\right)^{-1}\frac{\delta S}{\delta\Omega^j(q')}$$
$$+\left[D-\sum_{\Omega_i}\int_p \hat{\Omega}_i(p)\left(d_{\Omega_i}+\gamma_{\Omega_i}+\hat{p}^\mu\frac{\partial}{\partial\hat{p}^\mu}\right)\frac{\delta}{\delta\hat{\Omega}_i(p)}\right]\hat{S}$$

Sigma model approximation

We expand the most generic action as

$$S[\varphi] = \int d^D x V[\varphi] + \frac{1}{2} K[\varphi](\partial_\mu \varphi)^2 + H_1[\varphi](\partial_\mu \varphi)^4 + H_2[\varphi](\partial_\mu \partial^\mu \varphi)^2 + \cdots$$

In today talk, we expand the action up to second order in derivative and constraint it $\mathcal{N}$=2 supersymmetry.

N-components Lagrangian $\mathcal{L} \sim g_{ij}[\varphi](\partial_\mu \varphi)^i (\partial^\mu \varphi)^j .$

$g_{ij}[\varphi]$: the metric of target spaces

:functional of field variables and coupling constants

The point of view of non-linear sigma model:

Two-dimensional case

In perturbative analysis, the 1-loop beta-function for 2-dimensional non-linear sigma model proportional to Ricci tensor of target spaces.

Alvarez-Gaume, Freedman and Mukhi Ann. of Phys. 134 (1982) 392

The perturbative results

$$\beta(g_{i\bar{j}}) = \frac{1}{2\pi} R_{i\bar{j}}$$

⇨ Ricci flat (Calabi-Yau)

Non-perturbative?

D=2 (3) $\mathcal{N}$=2 supersymmetric non linear sigma model

$$S = \int d^2x d^2\theta d^2\bar{\theta} K[\Phi^i, \Phi^{\dagger\bar{i}}]$$

***i*=1～N : N is the dimensions of target spaces**

Where K is Kaehler potential and Φ is chiral superfield.

$$\begin{aligned}
\Phi^i(y) &= \varphi^i(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu\varphi^i(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial^\mu\partial_\mu\varphi^i(x) \\
&\quad + \sqrt{2}\theta\psi^i(x) - \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi^i(x)\sigma^\mu\bar{\theta} + \theta\theta F^i(x) \\
&\equiv \varphi^i(x) + \delta\Phi^i(x)
\end{aligned}$$

We expand the action around the scalar fields.

$$S=\int d^2x\left[g_{n\bar{m}}\left(\partial^\mu\varphi^n\partial_\mu\varphi^{*\bar{m}}+i\bar{\psi}^{\bar{m}}\sigma^\mu(D_\mu\psi)^n+\bar{F}^{\bar{m}}F^n\right)\right.$$

$$\left.-\frac{1}{2}K_{,nm\bar{l}}\bar{F}^{\bar{l}}\psi^n\psi^m-\frac{1}{2}K_{,n\bar{m}\bar{l}}F^n\bar{\psi}^{\bar{m}}\bar{\psi}^{\bar{l}}+\frac{1}{4}K_{,nm\bar{k}\bar{l}}(\bar{\psi}^{\bar{k}}\bar{\psi}^{\bar{l}})(\psi^n\psi^m)\right]$$

where

$$K_{,i}\equiv\frac{\delta K}{\delta\varphi^i}\qquad g_{i\bar{j}}=K_{,i\bar{j}}$$

: the metric of target spaces

From equation of motion, the auxiliary filed F can be vanished.

$$F^n=\frac{1}{2}g^{n\bar{m}}K_{,kl\bar{m}}\psi^k\psi^l$$

Considering only Kaehler potential term corresponds to second order to derivative for scalar field.

There is not local potential term.

The WRG equation for non linear sigma model

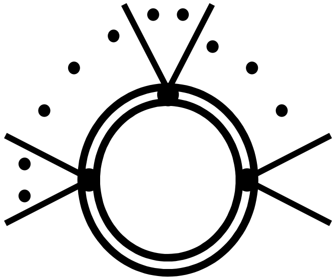
$$\frac{d}{dt}S[\Omega;t] = \frac{1}{2\delta t}\int_{p'} tr \ln \left(\frac{\delta^2 S}{\delta\Omega^i \delta\Omega^j}\right) - \frac{1}{2\delta t}\int_{p'}\int_{q'} \frac{\delta S}{\delta\Omega^i(p')}\left(\frac{\delta^2 S}{\delta\Omega^i(p')\delta\Omega^j(q')}\right)^{-1}\frac{\delta S}{\delta\Omega^j(q')} + \left[D - \sum_{\Omega_i}\int_p \widehat{\Omega}_i(p)\left(d_{\Omega_i} + \gamma_{\Omega_i} + \widehat{p}^\mu \frac{\partial}{\partial \widehat{p}^\mu}\right)\frac{\delta}{\delta\widehat{\Omega}_i(p)}\right]\widehat{S}$$

Consider the bosonic part of the action.

The second term of the right hand side vanishes in this approximation $O(\partial^2)$.

The first term of the right hand side

$$\frac{1}{2\delta t}\int_{p'} tr \ln \left(\frac{\delta^2 S}{\delta\Omega^i \delta\Omega^j}\right)$$



From the bosonic part of the action

$$\sim \frac{1}{4\pi}\ln\det g_{i\bar{j}} + \frac{1}{2\pi}R_{i\bar{j}}(\partial_\mu\varphi)^i(\partial^\mu\varphi^*)^{\bar{j}}$$

From the fermionic kinetic term

$$\sim -\frac{1}{4\pi}\ln\det g_{i\bar{j}}$$

Local potential term is cancelled.

Finally, we obtain the WRG eq. for bosonic part of the action as follow:

$$\frac{d}{dt}\int d^2x g_{i\bar{j}}(\partial_\mu\varphi)^i(\partial^\mu\varphi^*)^{\bar{j}}$$

$$= \int d^2x\Big[-\frac{1}{2\pi}R_{i\bar{j}} - \gamma[\varphi^k g_{i\bar{j},k} + \varphi^{*\bar{k}} g_{i\bar{j},\bar{k}} + 2g_{i\bar{j}}]\Big](\partial_\mu\varphi)^i(\partial^\mu\varphi^*)^{\bar{j}}.$$

The β function for the Kaehler metric is

$$\begin{aligned}\frac{d}{dt}g_{i\bar{j}} &= -\frac{1}{2\pi}R_{i\bar{j}} - \gamma[\varphi^k g_{i\bar{j},k} + \varphi^{*\bar{k}} g_{i\bar{j},\bar{k}} + 2g_{i\bar{j}}] \\ &\equiv -\beta(g_{i\bar{j}}).\end{aligned}$$

The perturbative results

$$\beta(g_{i\bar{j}}) = \frac{1}{2\pi}R_{i\bar{j}}$$

Alvarez-Gaume, Freedman and Mukhi Ann. of Phys. 134 (1982) 392

The other part of the action:

To keep supersymmetry, we derive the WRG eq. for the other part of the action from bosonic part.

Recall that the scalar part of the action is derived as follow:

$$\int dV\, K[\Phi, \Phi^\dagger] \sim \int d^2x\, g_{i\bar{j}} (\partial_\mu \varphi)^i (\partial^\mu \varphi^*)^{\bar{j}}.$$

And the one-loop correction term for scalar part is

$$\int dV\, \Delta K_1[\Phi, \Phi^\dagger] \sim \int d^2x\, R_{i\bar{j}} (\partial_\mu \varphi)^i (\partial^\mu \varphi^*)^{\bar{j}}.$$

In Kaehler manifold, the Ricci tensor is given from the metric as follow:

$$R_{i\bar{j}} = -\partial_i \partial_{\bar{j}} \ln \det g_{i\bar{j}}$$

The metric is given from the Kaehler potential

$$g_{i\bar{j}} = \partial_i \partial_{\bar{j}} K[\Phi, \Phi^\dagger]$$

Using this property, we can obtain the supersymmetric WRG eq. for Kaehler potential:

$$\frac{d}{dt}\int dV K[\Phi,\Phi^\dagger] = \frac{1}{2\pi}\int dV \ln\det g_{i\bar{j}}[\Phi,\Phi^\dagger] + \left[2-\sum_{\Omega^i}\int_p \hat{\Omega}^i(p)(d_{\Omega^i}+\gamma_{\Omega^i}+\hat{p}^\mu\frac{\partial}{\partial\hat{p}^\mu})\frac{\delta}{\delta\hat{\Omega}^i(p)}\right]\hat{S}$$

The fermionic part of the WRG eq. is

$$\frac{d}{dt}\int d^2x g_{i\bar{j}}\bar{\psi}^{\bar{j}}\sigma^\mu(D_\mu\psi)^i$$

$$= \int d^2x\left[-\frac{1}{2\pi}R_{i\bar{j}}-\gamma[\varphi^k g_{i\bar{j},k}+\varphi^{*\bar{k}}g_{i\bar{j},\bar{k}}+2g_{i\bar{j}}]\right]\bar{\psi}^{\bar{j}}\sigma^\mu(D_\mu\psi)^i$$

Fixed points with U(N) symmetry

The perturbative β function follows the Ricci-flat target manifolds.

$$\beta(g_{i\bar{j}}) = \frac{1}{2\pi} R_{i\bar{j}}$$

↳ Ricci-flat

We derive the action of the conformal field theory corresponding to the fixed point of the β function.

$$\beta[g_{i\bar{j}}] \equiv -\frac{d}{dt} g_{i\bar{j}}$$
$$= \frac{1}{2\pi} R_{i\bar{j}} + \gamma(\varphi^k g_{i\bar{j},k} + \varphi^{*\bar{k}} g_{i\bar{j},\bar{k}} + 2 g_{i\bar{j}})$$

To simplify, we assume U(N) symmetry for Kaehler potential.

$$K[\Phi, \Phi^\dagger] = \sum_{n=1}^{\infty} g_n x^n \equiv f(x) \quad \text{where} \quad x \equiv \vec{\Phi} \cdot \vec{\Phi}^\dagger$$

The function *f(x)* have infinite number of coupling constants.

$$f(x) = x + g_2 x^2 + g_3 x^3 + \cdots .$$

The Kaehler potential gives the Kaehler metric and Ricci tensor as follows:

$$\begin{aligned} g_{i\bar{j}} &= f'\delta_{i\bar{j}} + f''\varphi_i^*\varphi_{\bar{j}}, \\ R_{i\bar{j}} &= -[(N-1)\frac{f''}{f'} + \frac{2f''+f'''x}{f'+f''x}]\delta_{i\bar{j}} \\ & -[(N-1)(\frac{f^{(3)}}{f''} - \frac{(f'')^2}{(f')^2}) + \frac{3f^{(3)}+f^{(4)}x}{f'+f''x} - \frac{(2f''+f'''x)^2}{(f'+f''x)^2}]\varphi_i^*\varphi_{\bar{j}}, \end{aligned}$$

where

$$f' = \frac{df(x)}{dx}.$$

$$\beta[g_{i\bar{j}}] = \frac{1}{2\pi}R_{i\bar{j}} + \gamma(\varphi^k g_{i\bar{j},k} + \varphi^{*\bar{k}} g_{i\bar{j},\bar{k}} + 2g_{i\bar{j}}) = 0$$

⇓

$$\frac{\partial}{\partial t}f' = \frac{1}{2\pi}[(N-1)\frac{f''}{f'} + \frac{2f''+f'''x}{f'+f''x}] - 2\gamma(f'+f''x) = 0.$$

The solution of the $\beta = 0$ equation satisfies the following equation:

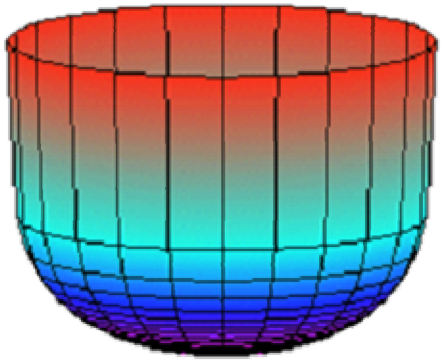
$$\frac{e^{axf'}}{a}\sum_{r=0}^{N-1}(-1)^r\frac{(N-1)!(xf')^{(N-1)-r}}{(N-1-r)!a^r} = \frac{1}{N}x^N + C_2.$$

Here we introduce a parameter which corresponds to the anomalous dimension of the scalar fields as follows:

$$a = -4\pi\gamma \Rightarrow \gamma = -\frac{a}{4\pi} = \frac{N+1}{2\pi}g_2$$

1. When $a > 0$, the anomalous dimension is negative.
   In $N=1$ case, the function $f(x)$ is given in closed form

$$f' = \frac{1}{ax}\ln(1+ax).$$



The target manifold takes the form of a semi-infinite cigar with radius $\sqrt{\frac{1}{a}}$.

It is embedded in 3-dimensional flat Euclidean spaces.

Witten Phys.Rev.D44 (1991) 314

This solution has been discussed in other context.

They consider the non-linear sigma model coupled with dilaton.

Witten Phys.Rev.D44 (1991) 314

Kiritsis, Kounnas and Lust Int.J.Mod.Phys.A9 (1994) 1361

Hori and Kapustin :JHEP 08 (2001) 045

$$I(r,\theta)=\frac{k}{4\pi}\int d^2x\sqrt{h}h^{ij}\partial_i u^\mu \partial_j u^\nu K_{\mu\nu}-\frac{1}{8\pi}\int d^2x\sqrt{h}\Phi(r,\theta)R.$$

In k>>1 region, we can use the perturbative renormalization method and obtain 1-loop β function:

$$\beta_{\mu\nu}=R_{\mu\nu}+2\nabla_\mu\nabla_\nu\Phi.$$

If one prefers to stay on a flat world-sheet, one may say that a non-trivial dilaton gradient in space-time is equivalent to assigning a non-trivial Weyl transformation law to target space coordinates.

Our parameter *a* (anomalous dim.) corresponds to *k* as follow.

$$a=\frac{2}{k}.$$

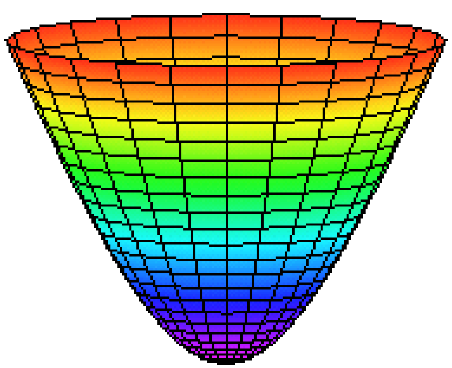
2. When a <0, the anomalous dimension is positive.

The metric and scalar curvature read

$$g_{i\bar{j}} = \frac{1}{1-|a|x}, \quad R = \frac{-|a|}{1-|a|x}.$$

The target space is embedded in 3-Minkowski spaces.

The vertical axis has negative signature.



In the asymptotic region, $\rho \to \infty$, the surface approach the lightcone.

Although the volume integral is divergent, the distance to the boundary is finite.

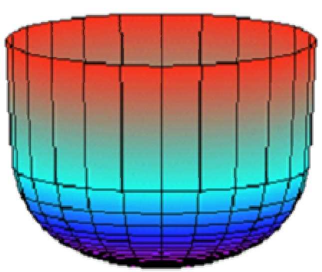
Two-dimensional fixed point theory

$$\beta[g_{i\bar{j}}] = \frac{1}{2\pi} R_{i\bar{j}} + \gamma(\varphi^k g_{i\bar{j},k} + \varphi^{*\bar{k}} g_{i\bar{j},\bar{k}} + 2g_{i\bar{j}}) = 0$$

$\gamma = 0$ ⇒ Ricci flat solution

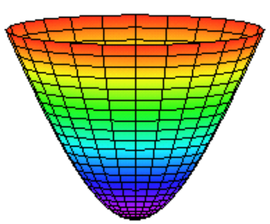
Free theory

$\gamma < 0$



Witten's Euclidean black hole solution

$\gamma > 0$



Minkowskian new solution

Three dimensional cases (renormalizability)

★The scalar field has nonzero canonical dimension.

$$dim[\varphi] = 1/2$$

$$\mathcal{L} = g_{ij}[\varphi, \varphi^*]\partial_\mu \varphi^i \partial^\mu \varphi^j$$

★ We need some nonperturbative renormalization methods.

WRG approach ← Our works

Large-N expansion

CP^{N-1} model

Inami, Saito and Yamamoto Prog. Theor. Phys. 103 (2000) 1283

Renormalization Group Flow

In 3-dimension, the beta-function for Kaehler metric is written:

$$\beta = \frac{1}{2\pi^2} R_{i\bar{j}} + \gamma[\varphi^k g_{i\bar{j},k} + \varphi^{*\bar{k}} g_{i\bar{j},\bar{k}} + 2g_{i\bar{j}}] + \frac{1}{2}[\varphi^k g_{i\bar{j},k} + \varphi^{*\bar{k}} g_{i\bar{j},\bar{k}}].$$

The C*P*^{N} model :SU(N+1)/[SU(N) × U(1)]

$$K[\Phi, \Phi^\dagger] = \frac{1}{\lambda^2} \ln(1 + \vec{\Phi}\vec{\Phi}^\dagger),$$

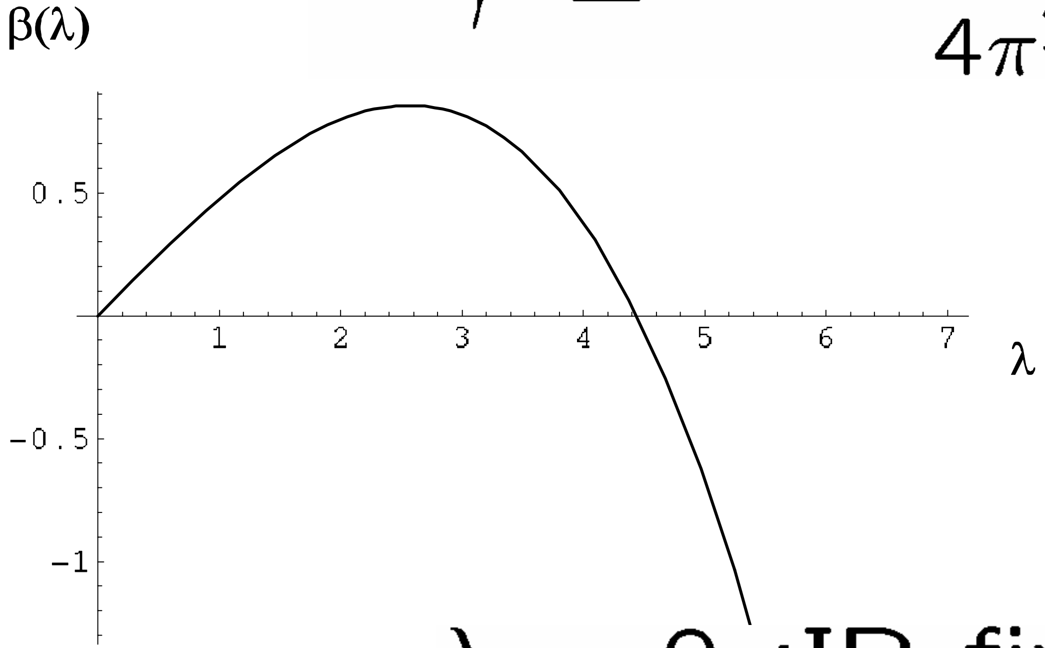
From this Kaehler potential, we derive the metric and Ricci tensor as follow:

$$g_{i\bar{j}} = \frac{\delta_{i\bar{j}}}{1 + \lambda^2 \varphi\varphi^*} - \frac{\lambda^2 \varphi_i^* \varphi_{\bar{j}}}{(1 + \lambda^2 \varphi\varphi^*)^2}$$

$$R_{i\bar{j}} = (N+1)\lambda^2 g_{i\bar{j}}$$

The β function and anomalous dimension of scalar field are given by

$$\beta(\lambda) = -\frac{(N+1)\lambda^3}{4\pi^2} + \frac{1}{2}\lambda,$$
$$\gamma = -\frac{(N+1)\lambda^2}{4\pi^2}.$$



There are two fixed points:

$$\lambda = 0 \quad \text{:IR fixed point}$$
$$\lambda^2 = \frac{2\pi^2}{N+1} \quad \text{:UV fixed point}$$

Einstein-Kaehler manifolds

The Einstein-Kaehler manifolds satisfy the condition

$$R_{i\bar{j}} = h\lambda^2 g_{i\bar{j}}.$$

If *h* is positive, the manifold is compact.

Using these metric and Ricci tensor, the β function can be rewritten

$$\begin{aligned} -\beta(g_{i\bar{j}}) &= \frac{\partial}{\partial t}\tilde{g}_{i\bar{j}}(\lambda\tilde{\varphi}, \lambda\tilde{\varphi}^*) \\ &= -\frac{1}{2\pi^2}\tilde{R}_{i\bar{j}} - \gamma[\tilde{\varphi}^k\tilde{g}_{i\bar{j},k} + \tilde{\varphi}^{*\bar{k}}\tilde{g}_{i\bar{j},\bar{k}} + 2\tilde{g}_{i\bar{j}}] \\ &\quad -\frac{1}{2}[\tilde{\varphi}^k\tilde{g}_{i\bar{j},k} + \tilde{\varphi}^{*\bar{k}}\tilde{g}_{i\bar{j},\bar{k}}]. \end{aligned}$$

The value of *h* for hermitian symmetric spaces.

G/H	Dimensions(complex)	***h***
SU(N+1)/[SU(N) × U(1)]	N	N+1
SU(N)/SU(N-M) × U(M)	M(N-M)	N
SO(N+2)/SO(N) × U(1)	N	N
Sp(N)/U(N)	N(N+1)/2	N+1
SO(2N)/U(N)	N(N+1)/2	N-1
E_6/[SO(10) × U(1)]	16	12
E_7/[E_6 × U(1)]	27	18

Because only λ depends on *t*, the WRG eq. can be rewritten

$$\frac{\dot{\lambda}}{\lambda}\tilde{\varphi}^k\tilde{g}_{i\bar{j},k}+\frac{\dot{\lambda}}{\lambda}\tilde{\varphi}^{*\bar{k}}\tilde{g}_{i\bar{j},\bar{k}}$$

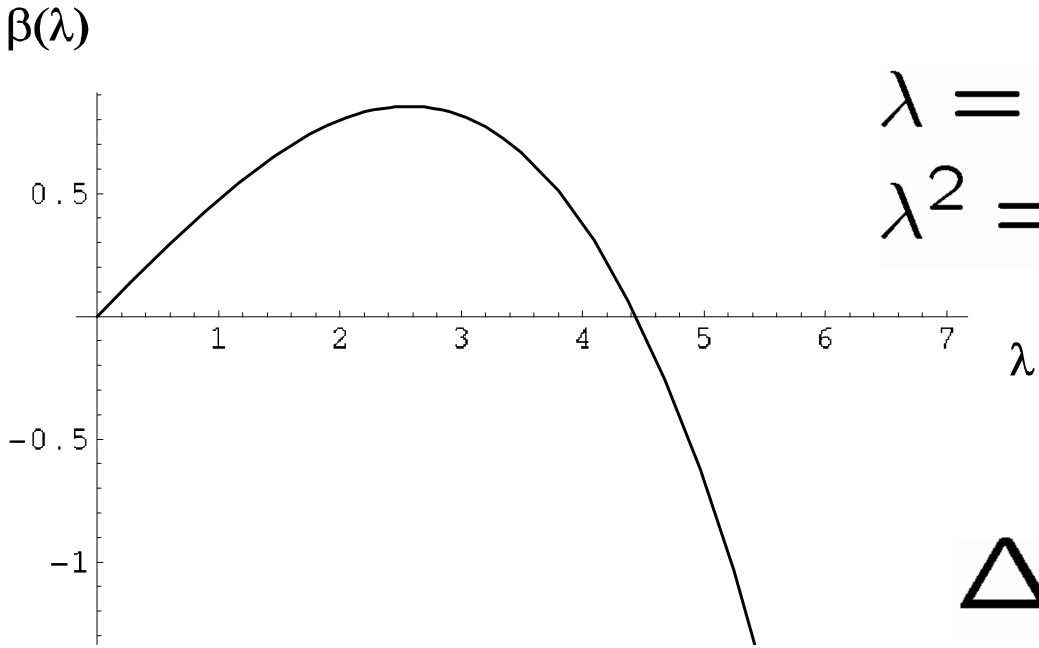
$$= -\left(\frac{h\lambda^2}{2\pi}+2\gamma\right)\tilde{g}_{i\bar{j}}-(\gamma+\frac{1}{2})[\tilde{\varphi}^k\tilde{g}_{i\bar{j},k}+\tilde{\varphi}^{*\bar{k}}\tilde{g}_{i\bar{j},\bar{k}}].$$

We obtain the anomalous dimension and β function of λ:

$$\gamma = -\frac{h\lambda^2}{4\pi^2}$$

$$\beta(\lambda)\equiv -\frac{d\lambda}{dt}=-\frac{h}{4\pi^2}\lambda^3+\frac{1}{2}\lambda.$$

The constant *h* is positive (compact E-K) **Renormalizable**



$$\lambda = 0 \text{ :IR fixed point}$$

$$\lambda^2 = \frac{2\pi^2}{h} \text{ :UV fixed point}$$

At UV fixed point

$$\gamma_c = -\frac{1}{2}$$

$$\Delta_\varphi \equiv d_\varphi + \gamma_\varphi = 0$$

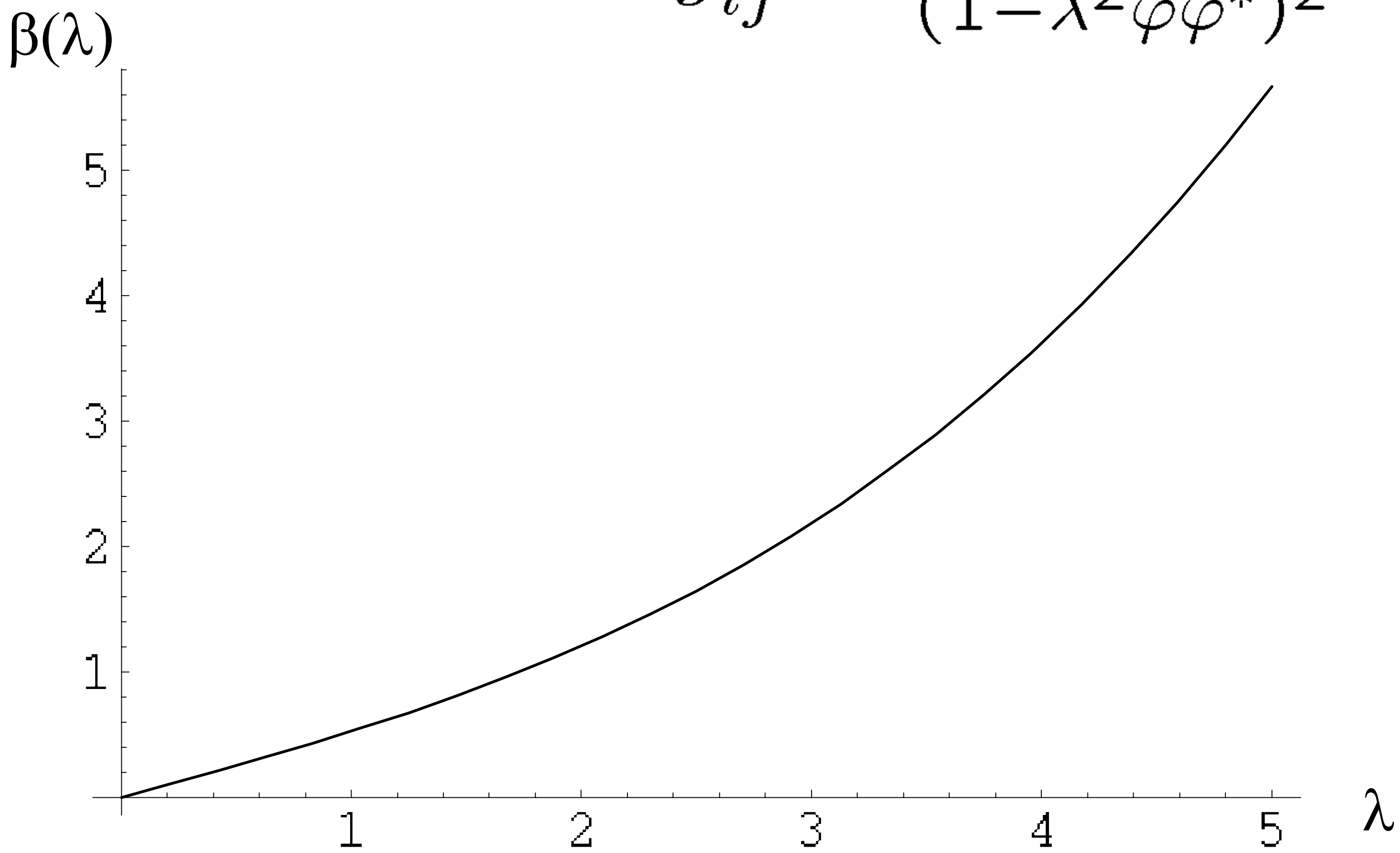
If the constant *h* is positive, it is possible to take the continuum limit by choosing the cutoff dependence of the bare coupling constant as

$$\lambda(\Lambda) \to \lambda_c - \frac{M}{\Lambda}.$$

M is a finite mass scale.

The constant *h* is negative (example Disc with Poincare metric)

$$g_{i\bar{j}} = \frac{\delta_{i\bar{j}}}{(1-\lambda^2\varphi\varphi^*)^2} \quad i,j=1$$



We have only IR fixed point at λ=0.

nonrenormalizable

SU(N) symmetric solution of WRG equatrion

We derive the action of the conformal field theory corresponding to the fixed point of the β function.

$$
\begin{aligned}
\beta(g_{i\bar{j}}) &= \frac{1}{2\pi^2} R_{i\bar{j}} \\
&+ \gamma[\varphi^k g_{i\bar{j},k} + \varphi^{*\bar{k}} g_{i\bar{j},\bar{k}} + 2g_{i\bar{j}}] \\
&+ \frac{1}{2}[\varphi^k g_{i\bar{j},k} + \varphi^{*\bar{k}} g_{i\bar{j},\bar{k}}] \\
&= 0.
\end{aligned}
$$

To simplify, we assume SU(N) symmetry for Kaehler potential.

$$K[\Phi, \Phi^\dagger] = \sum_{n=1}^{\infty} g_n x^n \equiv f(x)$$

Where,

$$x \equiv \vec{\Phi} \cdot \vec{\Phi}^\dagger$$

The function *f(x)* have infinite number of coupling constants.

$$f(x) = x + g_2 x^2 + g_3 x^3 + \cdots.$$

The β=0 can be written

$$\begin{aligned} \frac{\partial}{\partial t} f' &= \frac{1}{2\pi^2}[2(N+1)g_2 + (6(N+2)g_3 - 4(N+3)g_2^2)x \\ &\quad -(18(N+7)g_2 g_3 - 8(N+7)g_2^3 - 12(N+2)g_4)x^2] \\ &\quad -2\gamma(1 + 4g_2 x + 9g_3 x^2) - (2g_2 x + 6g_3 x^2) + O(x^3) \\ &= 0. \end{aligned}$$

We choose the coupling constants and anomalous dimension, which satisfy this equation.

$$\begin{aligned}
\gamma &= \frac{N+1}{2\pi^2} g_2, \\
g_3 &= \frac{2(3N+5)}{3(N+2)} g_2^2 + \frac{2\pi^2}{3(N+2)} g_2, \\
g_4 &= 3 g_2 g_3 - \frac{2(N+7)}{3(N+3)} g_2^3 + \frac{\pi^2}{N+3} g_3 \\
&= \frac{1}{3(N+2)(N+3)} ((16N^2 + 66N + 62) g_2^3 \\
&\quad +2\pi^2(6N+14) g_2^2 + 2\pi^4 g_2).
\end{aligned}$$

Similarly, we can fix all coupling constant g_n using g_2 order by order.

The following function satisfies β=0 for any values of parameter g_2

$$f' = 1 + 2g_2 x + [\frac{2(3N+5)}{N+2} g_2^2 + \frac{2\pi^2}{N+2} g_2] x^2 + \frac{4}{3(N+2)(N+3)} [(16N^2 + 66N + 62) g_2^3 + 2\pi^2 (6N+14) g_2^2 + 2\pi^4 g_2] x^4 + \cdots .$$

If we fix the value of g_2 , we obtain a conformal field theory.

We take the specific values of the parameter, the function takes simple form.

$$\boxed{g_2 = 0}$$

$$f(x) = x$$

This theory is equal to IR fixed point of CP^N model

$$\boxed{g_2 = -\frac{1}{2} \cdot \frac{2\pi^2}{N+1} \equiv -\frac{1}{2} b} \quad (\gamma = -1/2)$$

$$f(x) = \frac{1}{b} \ln(1 + bx)$$

This theory is equal to UV fixed point of CP^N model.

Then the parameter describes a marginal deformation from the IR to UV fixed points of the CP^N model in the theory spaces.

The target manifolds of the conformal sigma models

◆Two dimensional fixed point target space for $\gamma \neq -\frac{1}{2}$

- The line element of target space

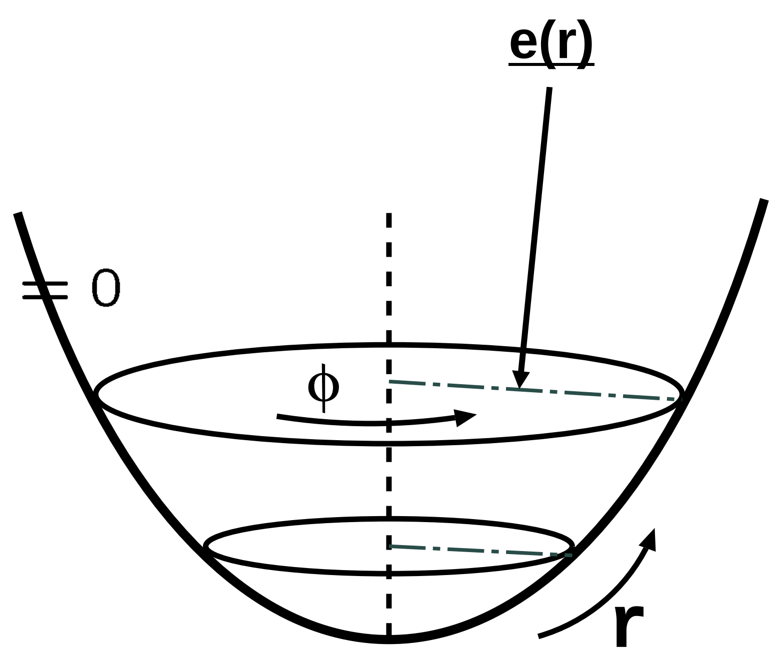
$$ds^2 = dr^2 + e(r)^2 d\phi^2$$

- RG equation for fixed point

$$\frac{1}{2\pi^2}\frac{\partial^2 e(r)}{\partial r^2} + e(r) + 2d_\varphi e(r)\frac{\partial e(r)}{\partial r} = 0$$

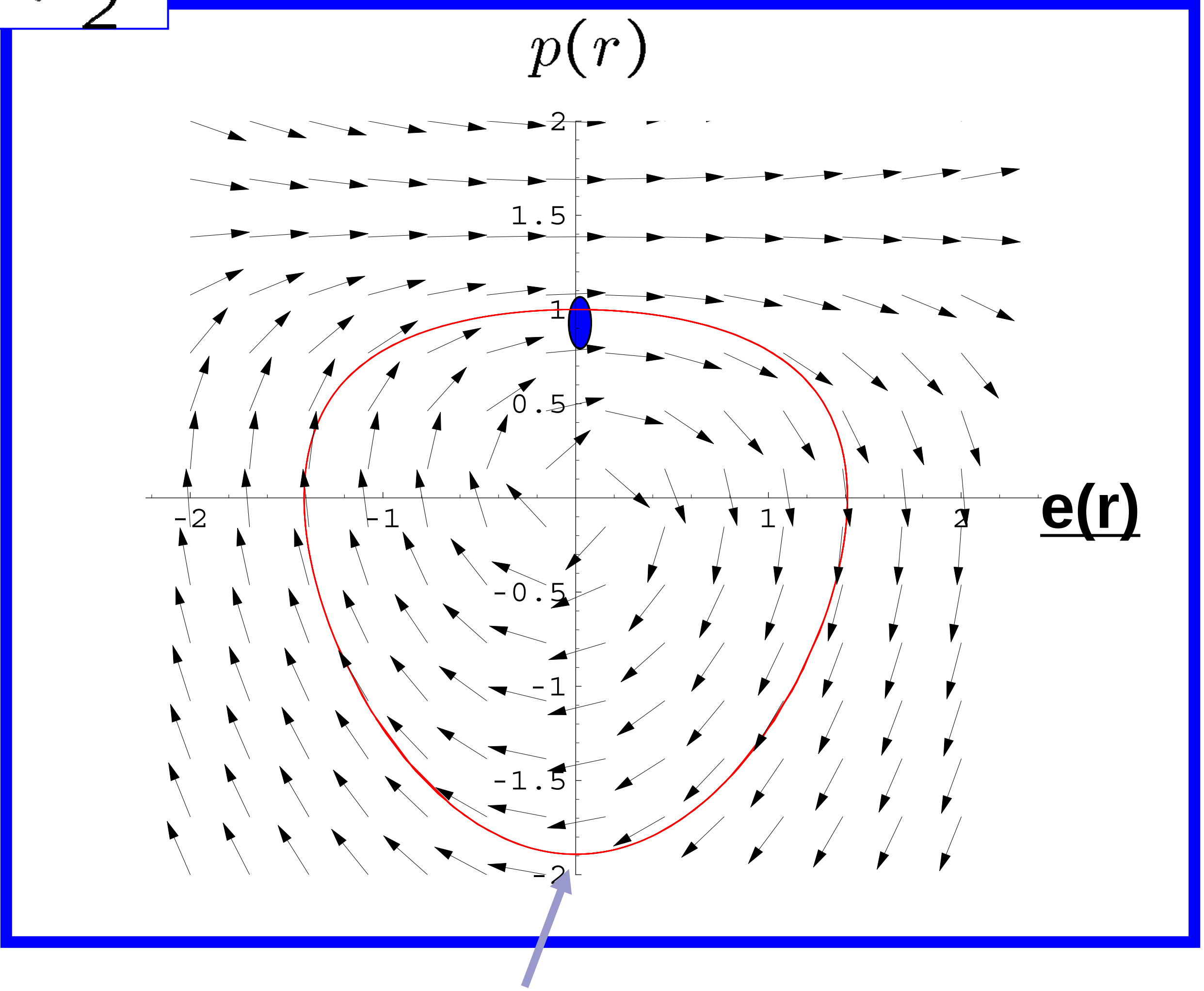
$$(d_\varphi = \frac{1}{2} + \gamma)$$

$$\begin{cases} e'(r) = p(r) \\ p'(r) = -2\pi^2 e(r)(1 - 2d_\varphi p(r)) \end{cases}$$



$$0 < d_\varphi < \frac{1}{2}$$

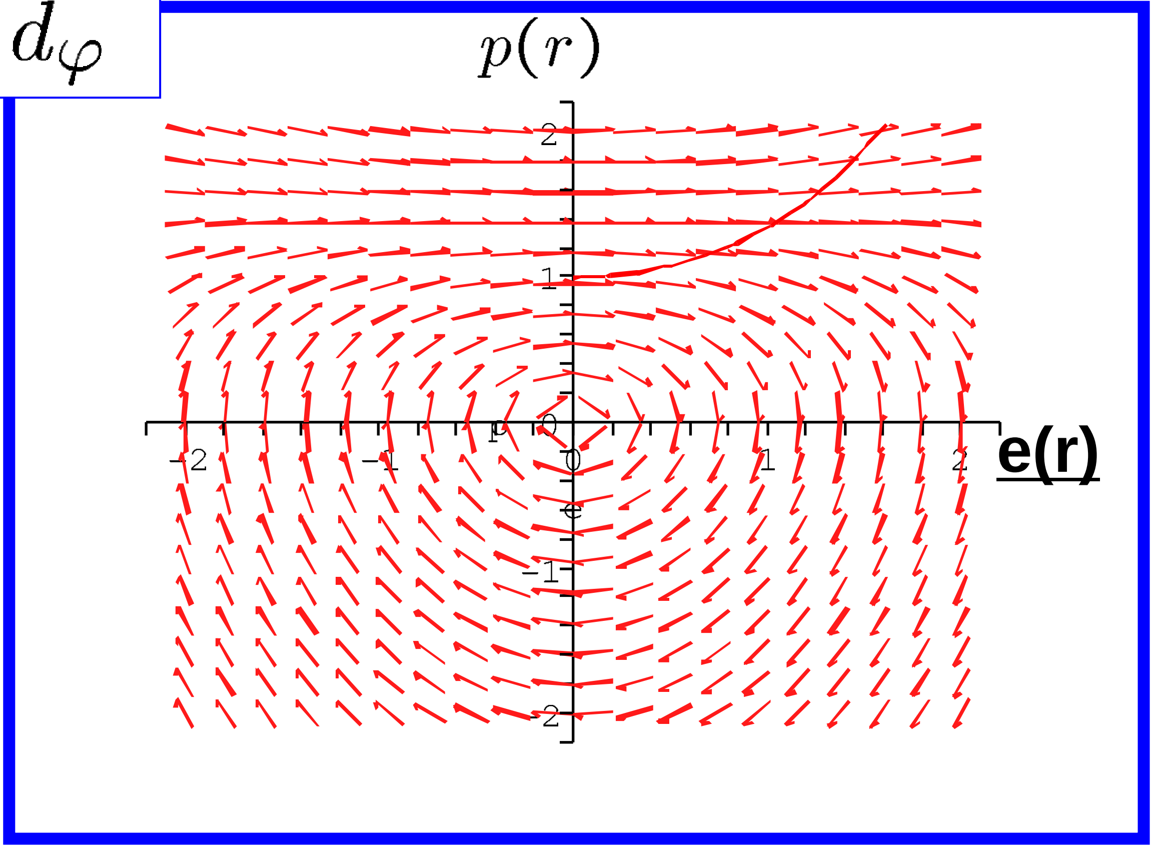
“Deformed” sphere



At the point, the target mfd. has conical singularity.

It has deficit angle. Euler number is equal to $\mathbf{S^2}$

$$\frac{1}{2} < d_\varphi$$



Non-compact manifold

Summary of the geometry of the conformal sigma model

$d_\varphi = 0$	**: Sphere S^2 (CP^1)**
$0 < d_\varphi < \frac{1}{2}$	**: Deformed sphere**
$d_\varphi = \frac{1}{2}$	**: Flat R^2**
$\frac{1}{2} < d_\varphi$	**: Non-compact**

Summary and Discussions

Using the WRG, we can discuss broad class of the NLsigmaM.

Two-dimensional nonlinear sigma models

We construct a class of fixed point theory for 2-dimensional supersymmetric NLsigmaM which vanishes the nonperturbative beta-function.

These theory has one free parameter which corresponds to the anomalous dimension of the scalar fields.

In the 2-dimensional case, these theory coincide with perturbative 1-loop beta-function solution for NLσM coupled with dilaton.

Three-dimensional nonlinear sigma models

We found the NLσMs on Einstein-Kaehler manifolds with positive scalar curvature are renormalizable in three dimensions.

We constructed the three dimensional conformal NLσMs.

$$d_\varphi = 0$$ **: Sphere S^2 (CP^1)**

$$0 < d_\varphi < \frac{1}{2}$$ **: Deformed sphere**

$$d_\varphi = \frac{1}{2}$$ **: Flat R^2**

$$\frac{1}{2} < d_\varphi$$ **: Non-compact**

Future problems:

RG flow around the novel CFT: stability, marginal operator…

The properties of a 3-dimensional CFT