M-Theory Superalgebra From Multiple Membranes

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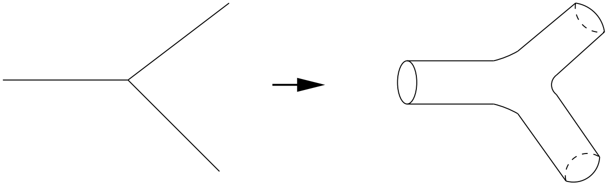
Perturbative String

Let me start by briefly recalling what string theory has achieved.

String theory is a natural extension of quantum field theory based on **particles** to **strings**.



This simple-looking extension brings us amazingly **rich** consequences ...

- Various particles in the nature are regarded as different excitations of single string – "**Unification**"
  massless excitations contain **graviton** and **gauge fields**.
- **UV finite** ($\leftrightarrow$ non-renormalizability of gravity).
- Conformal symmetry on the worldsheet
  $\rightarrow$ eq. of motion in the target space.

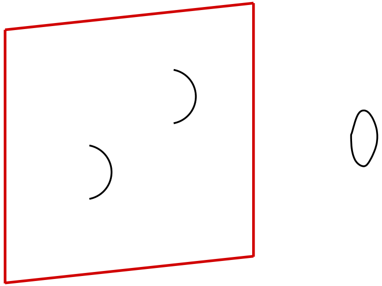
These features make string theory a very promising framework for describing quantum gravity, and why our world is like as we observe. (And more reasons, as we will see.)

Strong Constraints from the **Consistency**:

Space-time dimension, particle contents, gauge group, ...
5 types of superstring theory in 10D flat space.

D-branes

D-branes (Dirichlet-branes) are hyper-surfaces on which the end points of open strings are constrained.



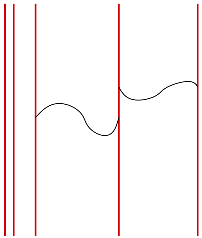
D-branes are **solitonic** objects in string theory and play crucial roles in the understanding of **non-perturbative** aspects of string theory.

Remarks:

- This is a tool to incorporate non-perturbative effects using perturbative string techniques (like perturbation around instanton background). Then one can extrapolate.
- Field theory of strings?

  Little complicated, though it has a very beautiful structure.
  Supersymmetric formulation is still not complete.
  Thus D-branes have been described as solitons mostly in the low energy supegravity or gauge theory approximation so far.
  (However, there has been a rapid progress which is still going on in the construction of classical solutions in open string field theory.)

Gauge Symmetry on Multiple D-brane Worldvolume



The low energy effective actions of **multiple** D-branes are given by dimensional reductions of 10D super Yang-Mills theory. The gauge symmetry based on **Lie algebra** was very characteristic to D-brane physics which led us to ...

- AdS-CFT
  Yang-Mills theory dynamics at strong coupling, black hole thermodynamics, singularity resolution, ...
- Non-commutative geometry
  Field theories on non-commutative spaces, non-commutative solitons as D-branes, ...
- **Matrix Model for M-theory**, IIB Matrix Model, ...
- D-brane interpretation of gauge theory solitons.
- Geometric description for solutions of supersymmetric gauge theories.
- Standard model/GUTs model building in string theory.

etc. etc.

M-Theory

Hypothetical 11D theory which contains 5 superstring theories in 10D in the corners of its moduli space.

- Reduces to 11D supergravity at low energy.
- Membranes and five-branes are expected to be fundamental objects. **However, fundamental formulation (definition) of M-theory is still lacking.**

Related to type IIA superstring theory by S^1 compactification:

	wrap S^1	transverse to S^1
M2	F1	D2
M5	D4	NS5

$$g_s \ell_s^3 = \ell_M^3, \; \ell_s^2 = \ell_M^3 / R_{11}$$

$\rightarrow g_s = (R_{11}/\ell_M)^{3/2}$: Strong string coupling = Large S^1 radius

Big Question:

What is the fundamental (characteristic) degrees of freedom in M-theory?

If there is an **algebraic structure** which describes multiple M-theory membranes, like Lie algebra for multiple D-branes, it will be a big clue !

Let me explain a candidate structure which is a natural extension of Lie algebra.

Lie Algebra

An algebra with an anti-symmetric product that satisfies the **Jacobi Identity**:

$$[A, [B, C]] = [[A, B], C] + [B, [A, C]]$$

Lie 3-Algebra

Introduce a multi-linear map which we call **3-bracket** $[*, *, *] : \mathcal{V}^{\otimes 3} \to \mathcal{V}$ $(\mathcal{V} = \sum_{a=1}^{\dim \mathcal{V}} v_a T^a; v_a \in \mathbb{C})$ satisfying following properties:

1. Skew-Symmetry:

$$[A_{\sigma(1)}, A_{\sigma(2)}, A_{\sigma(3)}] = (-1)^{|\sigma|}[A_1, A_2, A_3]$$

2. **Fundamental Identity**:

$$\begin{aligned} & [A_1, A_2, [B_1, B_2, B_3]] \\ = \; & [[A_1, A_2, B_1], B_2, B_3] + [B_1, [A_1, A_2, B_2], B_3] + [B_1, B_2, [A_1, A_2, B_3]] \end{aligned}$$

Lie 3-algebra can be expressed in terms of the basis T^a and the structure constants ${f^{abc}}_d$:

$$[T^a, T^b, T^c] = {f^{abc}}_d T^d$$

An element $T^a \in \mathcal{A}$ is called a **center** if $[T^a, T^b, T^c] = 0$, $\forall\, T^b, T^c \in \mathcal{A}$. ${f^{abc}}_d = 0$ in this case. Center elements play crucial roles in our work.

Trace (Inner Product)

We assume the structure (FK-Shih-Takimi)

$$\mathcal{V} = \mathcal{V}_{tr} \oplus \mathcal{V}_{ntr}$$

$\mathcal{V}_{tr}$: elements with inner product (**trace elements**)
$\mathcal{V}_{ntr}$: do not have invariant inner product (**non-trace elements**)

By definition, elements $T^a, T^b \in \mathcal{V}_{tr}$ have inner product $\langle *, * \rangle$: $\mathcal{V}_{tr} \otimes \mathcal{V}_{tr} \to \mathbb{C}$:

$$\langle T^a, T^b \rangle = h^{ab}$$

We require following **invariance of the inner product** which will be important for the gauge invariance of the Bagger-Lambert action which we will discuss later:

$$\langle [T^a, T^b, T^c], T^d \rangle + \langle T^c, [T^a, T^b, T^d] \rangle = 0$$

Together with the skew-symmetry property of the 3-bracket, the invariance of the metric requires the indices of structure constants $f^{abcd} \equiv f^{abc}{}_e h^{ed}$ to be totally anti-symmetric:

$$f^{abcd} = \frac{1}{4!} f^{[abcd]}$$

Remember that this is guaranteed only for trace elements with invariant metric.

BLG Model for Multiple Membranes

2006 end $\sim$ 2007, Bagger-Lambert and Gustavsson constructed an interesting model with following promising properties as a candidate for multiple M2 model:

- Gauge symmetry based on **Lie 3-algebra**, which is a natural extension of Lie algebra.
  - The Basu-Harvey eq.* is BPS eq. of the model.
  - Seems to be able to explain (entropy) $\sim N^{3/2}$ which was expected from gravity side. (N: number of M2-branes.)
- Maximal $\mathcal{N} = 8$ rigid supersymmetry.
- Superconformal symmetry (classical, and expected to be quantum) consistent, in the context of AdS-CFT, with the near-horizon isometry of multiple-membrane solution of 11D supergravity.

* To be discussed later.

Mini-Revolution?

Multiple D2-brane action from the BLG model
(Mukhi-Papageorgakis, March '08)

Explicit relation to multiple D2-brane action in type IIA string. Their result supports the identification of the BLG model as multiple M-theory membrane action via the M-IIA relation.

* The procedure for the S^1 compactification was slightly mysterious at first (giving an expectation value to one of the scalars), but understood later for this model based on so called $\mathcal{A}_4$ algebra. This procedure is still little mysterious for the models with Lie 3-algebra which contains Lie algebra, found soon later (Gomis-Milanesi-Russo, Benvenuti-Rodriguez-Gomez-Tonni-Verlinde, Ho-Imamura-Matsuo '08).

Index Convention

Before explaining the detail of the BLG model, let me summarize our convention a bit.

$$
\begin{aligned}
\text{worldvolume coordinates} &: \quad \mu, \nu = 0, 1, 2 \\
\text{spatial worldvolume coordinates} &: \quad i, j = 1, 2 \\
\text{transverse space coordinates} &: \quad I, J = 3, \cdots, 10 \\
\text{all 11D coordinates} &: \quad m, n = 0, 1, \cdots, 10 \\
Spin(1,10) \text{ spinor indices} &: \quad \alpha, \beta = 1, \cdots, 32 \\
\text{basis of Lie 3-algebra } \mathcal{A} &: \quad a, b, \cdots, \dim \mathcal{A}
\end{aligned}
$$

The Bagger-Lambert Action

The action is given by $S = \int d^3x\, \mathcal{L}$, with

$$\mathcal{L} = -\frac{1}{2}\langle D^\mu X^I, D_\mu X^I\rangle + \frac{i}{2}\langle\bar{\Psi}, \Gamma^\mu D_\mu \Psi\rangle + \frac{i}{4}\langle\bar{\Psi}, \Gamma_{IJ}[X^I, X^J, \Psi]\rangle$$
$$-V(X) + \mathcal{L}_{CS}$$

X^I: 3-alg. valued $SO(8)$ vector
Ψ : 3-alg. valued Majorana spinors with $\Gamma\Psi = -\Psi$, $\Gamma \equiv \Gamma_{012}$
D_μ: covariant derivative

$$(D_\mu X^I(x))_a = \partial_\mu X^I_a(x) - \tilde{A}_\mu{}^b{}_a(x) X^I_b(x), \quad \tilde{A}_\mu{}^b{}_a \equiv A_{\mu cd} f^{cdb}{}_a$$

The potential

$$V(X) = \frac{1}{12}\langle[X^I, X^J, X^K], [X^I, X^J, X^K]\rangle$$

The Chern-Simons term

$$\mathcal{L}_{CS} = \frac{1}{2}\varepsilon^{\mu\nu\lambda}\left(f^{abcd}A_{\mu ab}\partial_\nu A_{\lambda cd} + \frac{2}{3}f^{cda}{}_g f^{efgb}A_{\mu ab}A_{\nu cd}A_{\lambda ef}\right)$$

* The choice of Lie 3-algebra turns out to include a choice of space-time.

Gauge Symmetry

Gauge transformations:

$$\begin{aligned}
\delta_\Lambda X_a^I &= \Lambda_{cd}[T^c, T^d, X^I]_a = \Lambda_{cd} f^{cde}{}_a X_e^I = \tilde{\Lambda}^e{}_a X_e^I \\
\delta_\Lambda \Psi_a &= \Lambda_{cd}[T^c, T^d, \Psi]_a = \Lambda_{cd} f^{cde}{}_a \Psi_e = \tilde{\Lambda}^e{}_a \Psi_e \\
\delta_\Lambda \tilde{A}_\mu{}^b{}_a &= \partial_\mu \tilde{\Lambda}_\mu{}^b{}_a - \tilde{\Lambda}^b{}_c \tilde{A}_\mu{}^c{}_a + \tilde{A}_\mu{}^b{}_c \tilde{\Lambda}^c{}_a, \quad \tilde{\Lambda}^b{}_a \equiv f^{cdb}{}_a \Lambda_{cd}
\end{aligned}$$

The **fundamental identity** leads to

$$\delta_\Lambda[\Phi(1), \Phi(2), \Phi(3)] = \Lambda_{cd}[T^c, T^d, [\Phi(1), \Phi(2), \Phi(3)]]$$

where Φ's collectively represent X^I and Ψ. Metric is not involved in this formula and it applies to both trace elements and non-trace elements.

On the other hand, the **invariance of the metric** leads to

$$\delta_\Lambda \langle Y, Z\rangle = \Lambda_{ab}\left(\langle [T^a, T^b, Y], Z\rangle + \langle Y, [T^a, T^b, Z]\rangle\right) = 0$$

for any **trace** elements Y, Z which transform as
$\delta_\Lambda Y = \Lambda_{cd}[T^c, T^d, Y]$, $\delta_\Lambda Z = \Lambda_{cd}[T^c, T^d, Z]$.

Those equalities are used to show the gauge invariance of the Bagger-Lambert action.

Worldvolume SUSY

SUSY transformations:

$$
\begin{aligned}
\delta_\epsilon X^I_a &= i\bar{\epsilon}\Gamma^I\Psi_a \\
\delta_\epsilon \Psi_a &= D_\mu X^I_a \Gamma^\mu \Gamma^I \epsilon - \frac{1}{6} X^I_b X^J_c X^K_d f^{bcd}{}_a \Gamma^{IJK}\epsilon \\
\delta_\epsilon \tilde{A}_\mu{}^b{}_a &= i\bar{\epsilon}\Gamma_\mu \Gamma_I X^I_c \Psi_d f^{cdb}{}_a
\end{aligned}
$$

where $\Gamma\epsilon = \epsilon$.
Corresponding Noether current:

$$
q^L = -\Gamma^\mu\Gamma^I\Gamma^0\langle D_\mu X^I, \Psi\rangle - \frac{1}{6}\Gamma^{IJK}\Gamma^0\langle [X^I, X^J, X^K], \Psi\rangle
$$

The Noether charge:

$$
Q^L = \int d^2x\, q^L
$$

Worldvolume SUSY Algebra

(Less ambitious part)

We calculated the Dirac bracket of super current and charge (c.f. Passerini '08):

$$
\begin{aligned}
i\{q^L, Q^L\}_D &= 2p_\mu \Gamma_+ \Gamma^\mu C + z_{IJ}\Gamma_+\Gamma^{IJ} C \\
&+ z_{0iIJKL}\Gamma_+\Gamma^{0iIJKL} C + z_{0IJKL}\Gamma_+\Gamma^{0IJKL} C
\end{aligned}
$$

$\Gamma_\pm \equiv \frac{1\pm\Gamma}{2}$, $\Gamma \equiv \Gamma_{012}$, C: 11D Charge Conjugation Matrix.

The Hamiltonian density (bosonic part):

$$\mathcal{H} = p_0 = \frac{1}{2}\langle D_0 X^I, D_0 X^I \rangle + \frac{1}{2}\langle D_i X^I, D_i X^I \rangle + V(X)$$

The momentum density:

$$p_i = \langle D_0 X^I, D_i X^I \rangle$$

Central Charges

$$z_{IJ} = \frac{1}{2}\left(-\varepsilon^{0ij}\langle D_i X^I, D_j X^J\rangle + \langle D_0 X^K, [X^K, X^I, X^J]\rangle\right)$$

$$z_{0iIJKL} = \frac{1}{6}\langle D_i X^I, [X^J, X^K, X^L]\rangle$$

$$z_{0IJKL} = -\frac{1}{8}\langle [X^M, X^I, X^J], [X^M, X^K, X^L]\rangle$$

Why Are We Interested In Central Charges?

The central charges of the superalgebra are charges of BPS (Bogomolny '75, Prasad-Sommerfield '75) objects satisfying

$$|\text{Mass}| = |\text{(Topological) Charge}|$$

This relation is **protected from quantum corrections** due to the representation theory of superalgebra (Witten-Olive '78). These objects gave indispensable clues when one needs information of a theory at strong coupling, e.g. in string duality, black hole micro-states counting etc. etc.

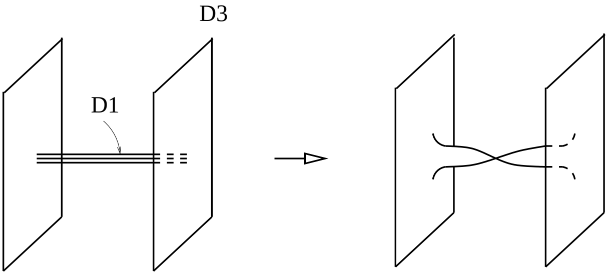
Meaning of the Central Charges

The central charges in the superalgebra of the BLG model have interpretation in terms of BPS branes. For example, z_{0iIJKL} corresponds to the charge of the solutions of the **Basu-Harvey equation** ('04) which describes M2-branes ending on M5-branes.

The Basu-Harvey equation actually gave one of the motivations for the construction of the BLG model.

Basu-Harvey Equation

The Basu-Harvey equation describes M2-branes ending on M5-branes. It is a natural extension of the **Nahm equation** ('79) which describes D1-branes ending on D3-branes.



Nahm Equation and D1-D3 System

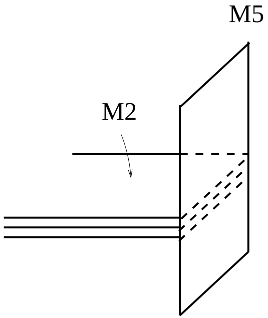
The Nahm Equation:

$$\frac{dX^I}{ds} \pm \frac{i}{2}\epsilon_{IJK}[X^J, X^K] = 0$$

X^I $(I = 1, 2, 3)$: $k \times k$ Hermitian matrix. k: number of monopoles.

With appropriate boundary conditions at $s : [-1, 1]$, the solutions of the Nahm equation is in one to one correspondence with solutions of the **BPS monopole equation**. Nahm equation describes D1-D3 system in terms of the field theory on D1-brane, whereas the monopole equation describes the same system in terms of the field theory on D3-branes. **The one to one correspondence simply means they are the same system looked from different view points.**

Basu-Harvey Equation



The Basu-Harvey Equation:

$$\frac{dX^I}{ds} \pm c\epsilon_{IJKL}[X^J, X^K, X^L] = 0$$

$(I = 1, \cdots, 4)$

Space-Time SUSY

(FK-Shih-Takimi ... More Ambitious)

Now, let us consider the case where there is a **central element** in the Lie 3-algebra. Then, the BLG model has an additional bosonic shift symmetry:

$$\delta_{\vec{a}} X^I_{\odot} = a^I \quad (a^I : \text{constant}),$$
$$\delta_{\vec{a}} \Psi_a = \delta_{\vec{a}} \tilde{A}_\mu{}^b{}_a = 0$$

as well as the SUSY partner fermionic shift symmetry:

$$\delta_\eta \Psi_a = \delta_{a\odot}\eta,$$
$$\delta_\eta X^I_a = \delta_\eta \tilde{A}_\mu{}^b{}_a = 0$$

$\odot$ denotes the central element.

We assume

$$h^{\odot a} = \delta^{\odot a}$$

so that $X^I_\odot$ can be interpreted as the center of mass coordinates in flat 11D space-time. We further assume that there is only one such central element.

The Noether charge density associated with the fermionic shift symmetry is

$$q^{NL} = -\Gamma^0 \Psi_{\odot}$$

and the Noether charge is

$$Q^{NL} = \int d^2x \, q^{NL}$$

Space-Time SUSY and Branes

(Hughes-Polchinski '86)

11D flat space has 11D super-Poincare symmetry.

When we put a BPS brane, it breaks half of the super-Poincare symmetry.

Massless fermions appear as **Goldstino** for the broken part of the super-Poincare symmetry, whereas the unbroken part of the super-Poincare symmetry becomes the **worldvolume supersymmetry**. Recall that $\Gamma\Psi = -\Psi$, $\Gamma\epsilon = +\epsilon$.

The Bagger-Lambert action is **NOT** written manifestly 11D super-Poincare covariant way. However, since we hope to identify it with multiple M-theory membranes, we'd like to expect that it keeps some sign of 11D super-Poincare symmetry.

Let Us Try !

The Dirac bracket of q^{NL} and Q^L are given as

$$
\begin{aligned}
& i\{q^{NL}, Q^L\}_D + i\{q^L, Q^{NL}\}_D \\
= \quad & p_I \Gamma^I C + \frac{1}{2} z_{iI} \Gamma^{iI} C + \frac{1}{2} z_{ijIJK} \Gamma^{ijIJK} C
\end{aligned}
$$

where

$$
p_I \equiv \partial_0 X^I_{\odot}
$$

is the momentum density in the direction transverse to the membranes.

We have obtained new central charges here. Note, in particular, we obtained the central charge z_{ijIJK} which describes **transverse M5-brane charge** (to be discussed later).

The Dirac bracket of q^{NL} and Q^{NL} is given by

$$i\{q^{NL}, Q^{NL}\}_D = \Gamma_- \Gamma^0 C = \frac{1-\Gamma}{2}\Gamma^0 C$$

Full Space-Time SUSY Algebra

Let us define

$$q = q^L + 2\sqrt{N} q^{NL}$$

$$Q = Q^L + 2\sqrt{N} Q^{NL}$$

N: "number" of membranes.

$$
\begin{aligned}
i\{q, Q\}_D &= 2(\Gamma^0 - \Gamma^{12}) CN + 2p_\mu \Gamma_+ \Gamma^\mu C + 2p_I \Gamma^I C\sqrt{N} \\
&+ z_{iI}\Gamma^{iI} C\sqrt{N} + z_{ijIJK}\Gamma^{ijIJK} C\sqrt{N} \\
&+ z_{IJ}\Gamma_+\Gamma^{IJ} C + z_{0iIJKL}\Gamma_+\Gamma^{0iIJKL} C \\
&+ z_{0IJKL}\Gamma_+\Gamma^{0IJKL} C
\end{aligned}
$$

This is almost the form of the 11D super-Poincare algebra ("M-theory Superalgebra" Townsend '95) !!!

(But we don't want Γ_+ for that ...)

M5-Brane Charges in the BLG Model

Historical Background

Soon after the M-theory proposal and the discovery of D-branes as RR charged objects, a matrix model was conjectured as a fundamental description of M-theory (Banks-Fischler-Shenker-Susskind '96).

Space-time SUSY algebra was calculated (Banks-Seiberg-Shenker '96), and there it was found that **the SUSY alg. does NOT contain transverse*** **5-brane charge**. This was not a good news for the matrix model as fundamental theory. It seemed it should be regarded as M-theory in a particular frame (infinite momentum frame) which drops off some information of the theory.

* Transverse to the S^1 which relates type IIA string and M-theory.

If the Space-time Superalgebra of the BLG model contains the transverse M5-brane charge, it will be a big advantage of the model over the matrix model for M-theory !

Quandary

The matrix model for M-theory can be regarded as a regularization of the covariant supermembrane action in the light-cone gauge. Since it can be related with a space-time covariant formulation, it was expected to have 11D super-Poincare algebra.

The BLG model has NOT been related to any covariant formulation so far. **Rather, we'd like to regard our result as a strong indication that the BLG model has a covariant origin**.

M5-Brane Charges in the BLG Model (Contin.)

An example of Lie 3-algebra is given by **Nambu-Poisson bracket** on an "internal" three-manifold (Ho-Matsuo '08). Let us consider a Nambu-Poisson bracket on T^3 given by

$$\{f, g, h\}_{\mathrm{NP}} = \sum_{\dot{\mu}\dot{\nu}\dot{\lambda}} \Omega \varepsilon^{\dot{\mu}\dot{\nu}\dot{\lambda}} \partial_{\dot{\mu}} f(y) \partial_{\dot{\nu}} g(y) \partial_{\dot{\lambda}} h(y)$$

$y^{\dot{\mu}}$ ($\dot{\mu} = \dot{1}, \dot{2}, \dot{3}$) are flat coordinates on T^3 with $y^{\dot{\mu}} \sim y^{\dot{\mu}} + 2\pi$ and Ω is a constant.

The invariant inner product can be defined by the integral over T^3:

$$\langle f, g \rangle \equiv \int_{T^3} d^3y\, f(y) g(y)$$

The trace elements of the Lie 3-algebra are given by square-integrable periodic functions on T^3. If we denote the basis of such functions on T^3 as $\chi^a(y)$ $(a = 1, 2, 3, \cdots)$, the Nambu-Poisson bracket can be written with structure constants:

$$\{\chi^a, \chi^b, \chi^c\}_{\mathrm{NP}} = \sum_d f^{abc}{}_d \chi^d$$

We normalize the basis as $\langle \chi^a, \chi^b \rangle = \delta^{ab}$; then the normalized central element is given as $T^{\odot} = 1/\sqrt{(2\pi)^3}$.

We would like to consider the case where the target space is also compactified on a T^3. By this we mean the identification:

$$X^I(y) \sim X^I(y) + 2\pi R^I$$

for say $I = 3, 4, 5$, where R^I is the compactification radius in the I-th direction.

Now let us consider a background configuration

$$X^I = R^I m_I y^{\dot{\mu}}, \quad \dot{\mu} = I - 2 \quad (I = 3, 4, 5)$$

The functions $y^{\dot{\mu}}$ $(\dot{\mu} = \dot{1}, \dot{2}, \dot{3})$ are not periodic functions on T^3. However, when the target space is also compactified, such jump can be undone for this wrapping configuration.

However, there is **NO** natural way to define invariant inner product for these elements. (Which should we choose, y or $y + 2\pi$?) Also,

$$\int_{T^3} d^3y \, \{y^{\dot{1}}, y^{\dot{2}}, 1\}_{NP} \cdot y^{\dot{3}} = 0$$

$$\neq \; -\int_{T^3} d^3y \, 1 \cdot \{y^{\dot{1}}, y^{\dot{2}}, y^{\dot{3}}\}_{NP} = \Omega(2\pi)^3$$

This means that the integration over T^3 does not provide an invariant metric for these new elements. Therefore, these elements should be included as non-trace elements.
But there is no problem that background field configuration takes values in such non-trace element.

The background wrapping configuration contributes to the five-brane charge as

$$z_{ijIJK}\sqrt{N} = -\frac{1}{3!}\varepsilon_{IJK}(2\pi)^3\varepsilon^0{}_{ij}\Omega R^3 R^4 R^5 m_3 m_4 m_5$$

This is interpreted as a charge of a five-brane wrapping the I-th direction for m_I times.

Note that the potential term can be rewritten as

$$\begin{aligned} & V(X) \\ = \; & \frac{1}{12}\Big(\langle [X^I, X^J, X^K] - W^{IJK} T^{\odot}, [X^I, X^J, X^K] - W^{IJK} T^{\odot}\rangle \\ & \quad +2W^{IJK}\langle [X^I, X^J, X^K], T^{\odot}\rangle - W^{IJK} W^{IJK}\Big) \\ = \; & \frac{1}{12}\langle [X^I, X^J, X^K] - W^{IJK} T^{\odot}, [X^I, X^J, X^K] - W^{IJK} T^{\odot}\rangle \\ & \quad -\frac{1}{2} W^{IJK} \varepsilon^{0ij} z_{ijIJK} - \frac{1}{12} W^{IJK} W^{IJK} \, , \end{aligned}$$

$$W^{IJK} = \varepsilon^{IJK} \Omega R^3 R^4 R^5 m_3 m_4 m_5$$

Thus the wrapping configuration saturates the minimal energy bound for a given winding number.

Why the Matrix Model for M-theory Could Not Describe Transverse Five-Brane?

From our result, we arrive at the view that **the Lie 3-algebra was essential for obtaining the transverse 5-brane charge**. To relate the BLG model to the matrix model, we must reduce the Lie 3-algebra to Lie algebra (a la Mukhi-Papageorgakis). However, this necessarily involves a wrapping of the M5-brane worldvolume. Thus, after the reduction to the matrix model, only D4-branes (or their T-dual) remain.

Summary and Future Direction

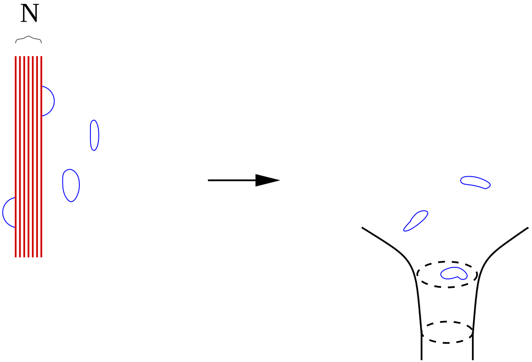
- By introducing **non-linearly realized part of the space-time SUSY**, we have constructed **11D super-Poincare algebra** with **new central charges** almost fully from the BLG model. This strongly indicates the existence of a space-time covariant formulation which reduces to the BLG model with appropriate gauge fixing and in some limit. It will be very interesting to find such covariant formulation (c.f. Carrollian limit of the BLG from covariant M5-brane action by Bandos-Townsend '08). Such formulation is very likely to be the covariant M5-brane action. This will also solve the problem of the unwanted Γ_+ projection.

- We introduced **non-trace elements** to the Lie 3-algebra as backgrounds. This was parallel to what has been done in the matrix models and hence it is a very natural thing to do. These backgrounds contribute to the central charges, typically to the **transverse M5-brane charge**.
- The **transverse M5-brane charge**, which was absent in the matrix model for M-theory, exists in the space-time superalgebra of the BLG model. It provides an explanation for why the matrix model could not describe transverse five-brane. Note that this central charge can be obtained only by introducing the **non-linearly realized part of the space-time SUSY**.

Appendix

AdS-CFT correspondence

Consider **large number (N)** of D-branes on top of each other.



Near-horizon decoupling limit

Open string theory → **non-gravitational** field theory.

Closed strings are on a curved background.

AdS-CFT correspondence

Conjecture (Maldacena '97)

The large N field theory and closed string theory on a higher dimensional space are equivalent (dual).

(Example: Conformal Field Theory $\leftrightarrow$ Anti-de Sitter space)

- Describing gravitational theory by non-gravitational theory.
- Concrete realization of the idea called **Holography**.