Modified D-term Inflation and Hilltop Inflation Models

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Contents

Basic ideas about inflation cosmology,
focus on (single field) slow-roll models.

Right-Handed sneutrino modified D-term Inflation
Chia-Min Lin, John McDondald
Phys. Rev. D74:063510, 2006 hep-ph/0604245

Chia-Min Lin, John McDonald
Phys. Rev. D77:063529, 2008 0710.4273 [hep-ph]

Hilltop Inflation

Kazunori Kohri, Chia-Min Lin, David H. Lyth
JCAP 0712: 004, 2007 0707.3826 [hep-ph]

Cosmological Principle

The Universe is Homogeneous and Isotropic on large scales (and therefore is described by Robertson-Walker metric)

Robertson-Walker metric

$$ds^2 = -dt^2 + a(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

$k = 0,$ flat $\quad k < 0,$ open $\quad k > 0,$ closed

a(t) is the scale factor

Put this metric into Einstein Equations of General Relativity:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

We obtain

Friedmann Equation

$$H^2 = \frac{\rho}{3M_P} - \frac{k}{a^2}$$

$$H \equiv \frac{\dot{a}}{a} \quad \text{(Hubble parameter)}$$

Raychaudhuri Equation

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_P}(\rho + 3P)$$

$$M_P \approx 2.44 \times 10^{18}\,\mathrm{GeV}$$

The fluid Equation

$$\dot{\rho} + 3H(\rho + P) = 0$$

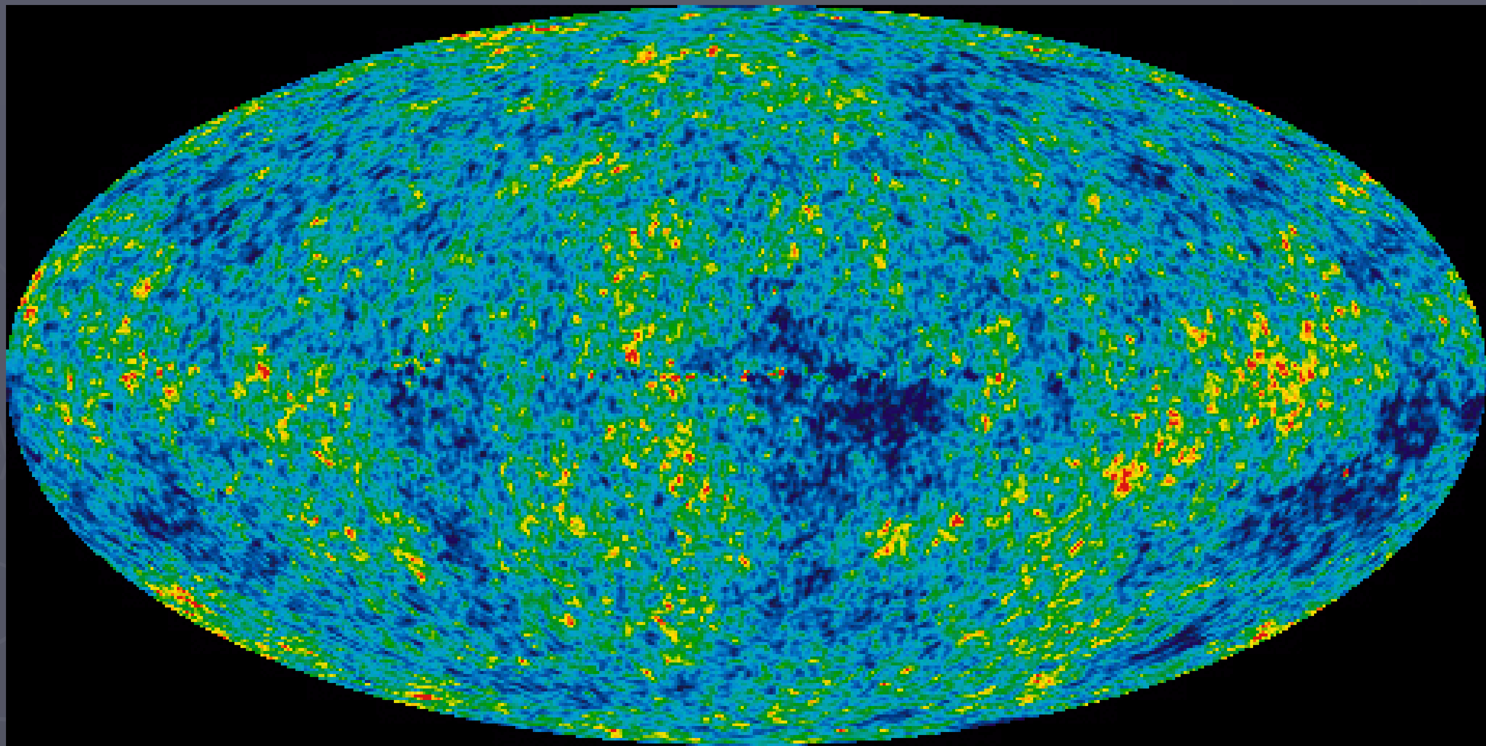
Matter Domination:

$$P = 0, \quad a \propto t^{2/3}, \quad \rho \propto a^{-3}$$

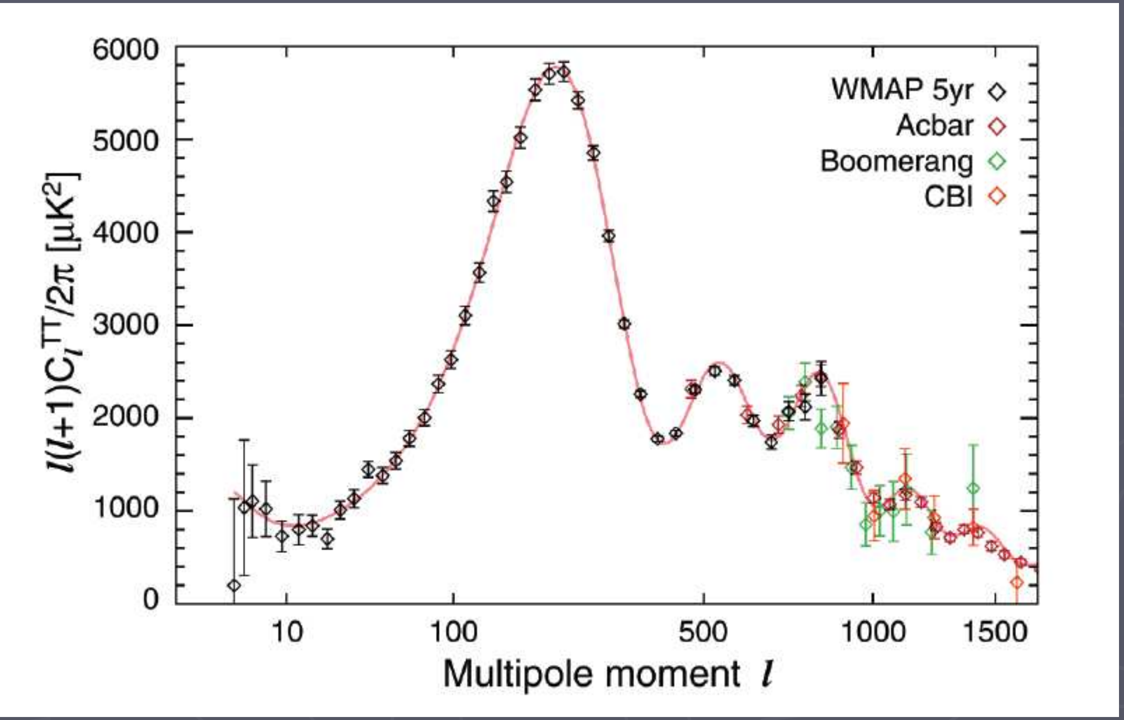
Radiation Domination:

$$P = \frac{\rho}{3}, \quad a \propto t^{1/2}, \quad \rho \propto a^{-4}$$

WMAP Five Year Microwave Sky



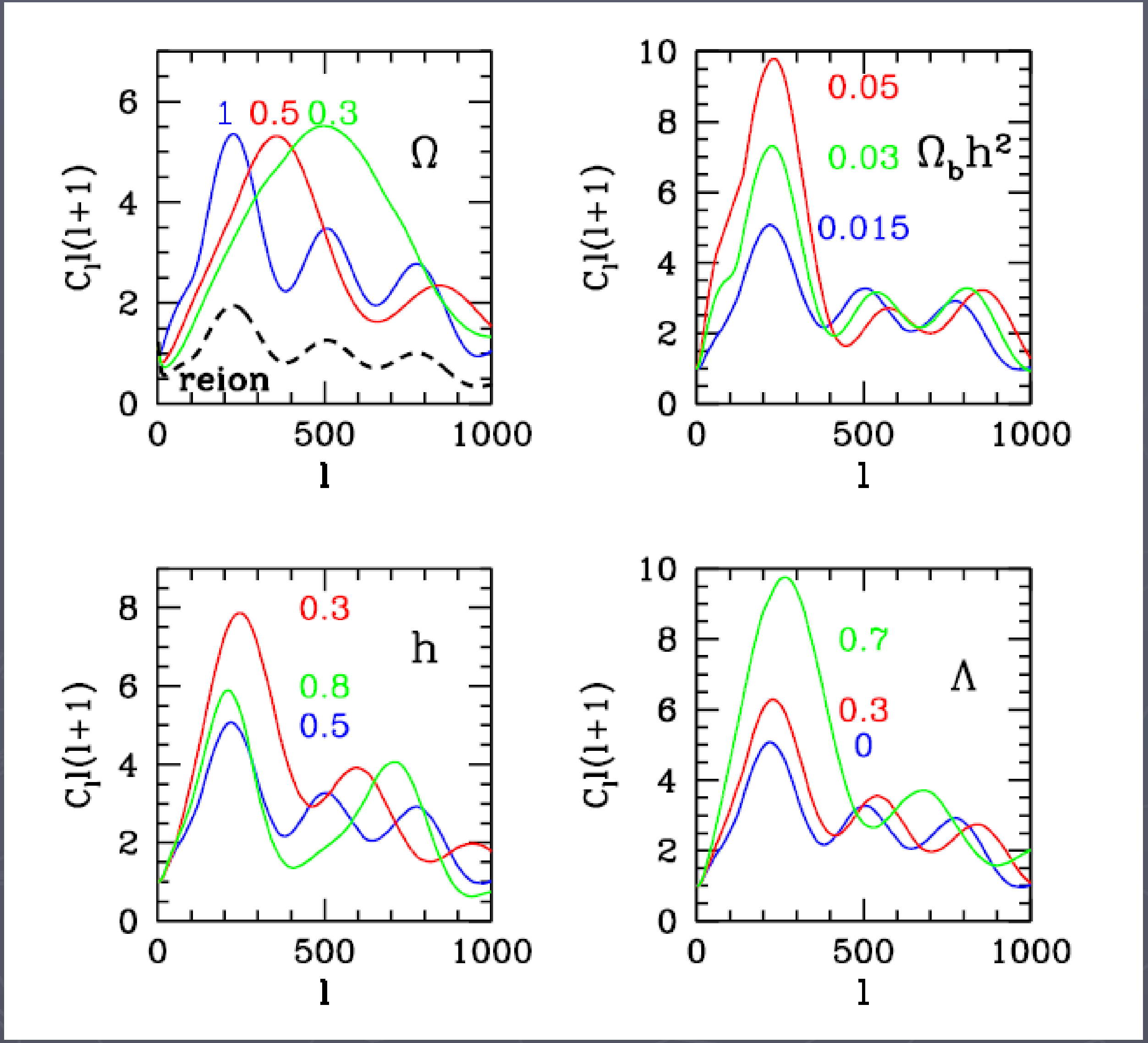
CMB Power Spectrum



$$\frac{\Delta T(\theta,\phi)}{T} = \sum_{l,m} a_{lm} Y_{lm}(\theta,\phi)$$

$$C_l \equiv \frac{1}{2l+1} \sum_{m=-l}^{l} |a_{lm}|^2$$

C_l **is called angular power spectrum**



Jungman G, Kamionkowski M, Kosowsky A, Spergel DN (1996)

Problems of The Big Bang Theory

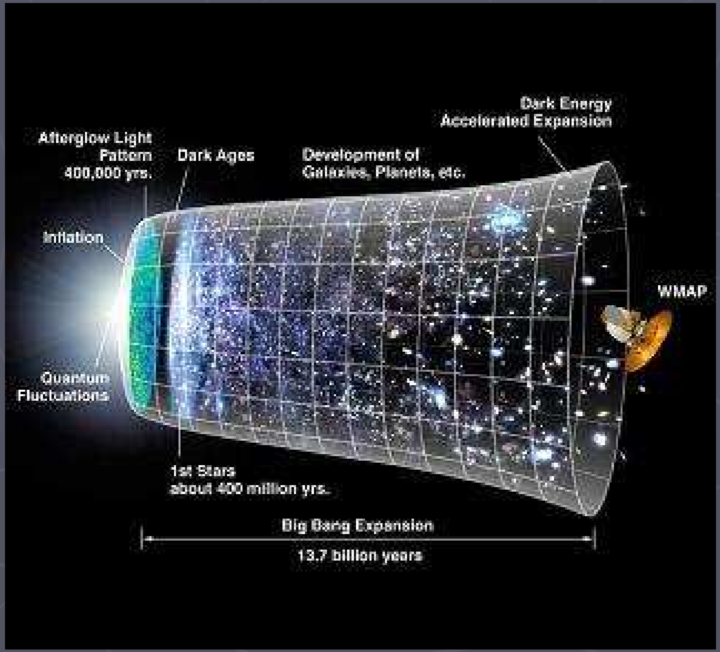
- The Homogeneity, isotropy problem
- The flatness problem
- Unwanted relics (e.g. monopoles)
- Etc.

In order to solve these problems, A. Guth (1981) introduced the idea of Cosmological Inflation.

What is Inflation?

Before the Hot Big Bang, there was a stage of accelerated expansion of the Universe when gravity acts as a repulsive force.

Inflation sets the initial conditions for the Big Bang, otherwise those fine-tuned initial condition should be put in by hand.



NASA/WMAP team

Inflation---how does it work?

Vacuum energy dominated universe:

From the fluid equation $\dot{\rho} + 3H(\rho + P) = 0$

$$P = -\rho \quad \Rightarrow \quad \dot{\rho} = 0$$

From Friedmann equation

$$\rho = 3H^2 M_P = const. \quad \Rightarrow \quad a \propto e^{Ht}$$

De Sitter phase

From Raychaudhuri equation

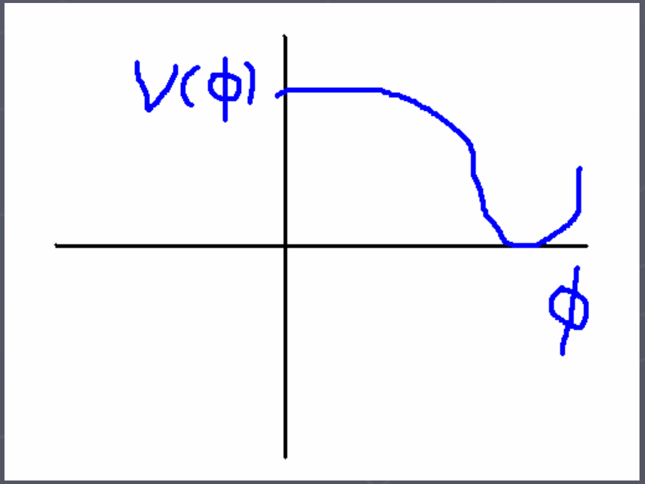
$$\frac{\ddot{a}}{a} = -\frac{1}{6M_P}(\rho + 3P) \quad \Rightarrow \quad \ddot{a} > 0$$

Accelerating

How to build an inflation model?

Inflation has to end at sometime, so the vacuum energy is actually time varying.

Traditionally, we introduce a scalar field with its potential.



The role of "vacuum energy" is played by the scalar potential.

A scalar field in a cosmological background

$$\rho = \frac{1}{2}\dot{\phi}^2 + V \qquad P = \frac{1}{2}\dot{\phi}^2 - V$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$H^2 = \frac{1}{3M_P^2}[V(\phi) + \frac{1}{2}\dot{\phi}^2]$$

Slow-roll approximation

$$\cancel{\ddot{\phi}} + 3H\dot{\phi} + V'(\phi) = 0$$

$$H^2 = \frac{1}{3M_P^2}[V(\phi) + \cancel{\frac{1}{2}\dot{\phi}^2}]$$

$$P = \cancel{\frac{1}{2}\dot{\phi}^2} - V \qquad \rho = \cancel{\frac{1}{2}\dot{\phi}^2} + V$$

$$P = -\rho$$

Equation of state of vacuum energy

Slow-roll parameters

$$\varepsilon \equiv \frac{1}{2} M_P^2 \left(\frac{V'}{V} \right)^2 \qquad \eta \equiv M_P^2 \frac{V''}{V}$$

Slow-roll implies

$$\varepsilon << 1$$

$$|\eta| << 1$$

This means the potential should be very flat with small curvature.

Number of e-folds

$$N \equiv \int_{t}^{t_2} H(t)dt \qquad a \propto e^{-N}$$

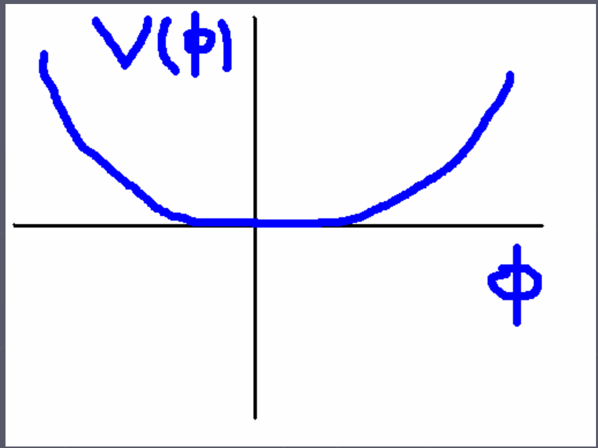
With slow-roll

$$N(\phi) = \int_{\phi_{end}}^{\phi} M_P^{-2} \frac{V}{V'} d\phi = \int_{\phi_{\mathrm{end}}}^{\phi} \frac{1}{\sqrt{2\varepsilon}} \mathrm{d}\phi$$

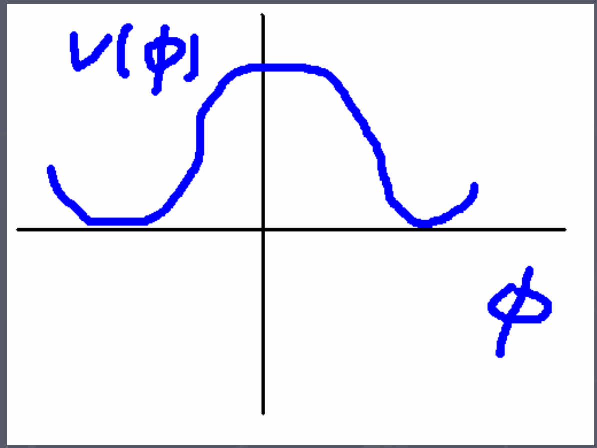
We are interested in $\mathrm{N} \simeq 60$

because that is the time when our universe leaves horizon.

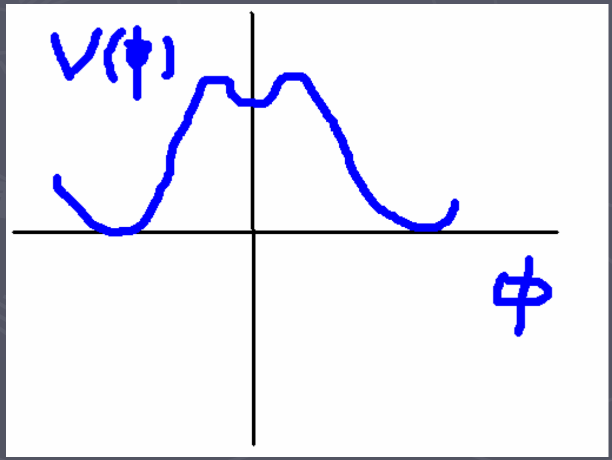
There are lots of inflation models



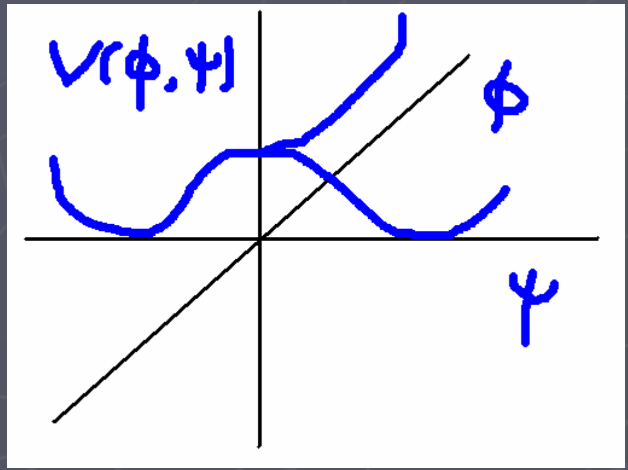
Chaotic Inflation



New Inflation



Old Inflation (not slow-roll)



Hybrid Inflation

Even More…!

Double Inflation

Assisted Inflation

Brane Inflation

Supernatural Inflation

A-term Inflation

Warm Inflation

F-term Inflation

D-term Inflation

P-term Inflation

DBI Inflation

Racetrack Inflation

…etc.

A Bonus of Inflation

Density perturbations arise due to quantum fluctuations in the scalar field.

All of the structure we see in the universe is a result of quantum fluctuations during inflation!

The structure we see today is formed from primordial density perturbation by gravitational instability.

The Spectrum

$$g(\vec{k}) = \frac{1}{(2\pi)^{3/2}} \int g(\vec{x}) e^{-i\vec{k}\cdot\vec{x}} d^3x$$

$$\left\langle g^*(\vec{k}) g(\vec{k}') \right\rangle = \delta^3(\vec{k}-\vec{k}') \frac{2\pi^2}{k^3} P_g(k)$$

The Spectrum

From Quantum to Classical

During Inflation from the event horizon 1/H, quantum fluctuations can be converted into classical perturbations, a process similar to Hawking radiation:

Hawking temperature:

$$T_H = \frac{H}{2\pi}$$

During Inflation:

$$P_\phi = \left(\frac{H}{2\pi}\right)^2 \Bigg|_{k=aH}$$

Me and Prof. Hawking

How to compare an inflation model with observation?

The spectrum of scalar curvature perturbations $R=-\left[\frac{H}{\dot{\phi}}\delta\phi\right]$ **is given by**

$$P_R=\left[\left(\frac{H}{\dot{\phi}}\right)\left(\frac{H}{2\pi}\right)\right]^2=\frac{1}{12\pi^2 M_P^6}\frac{V^3}{V'^2}=\frac{1}{24\pi^2 M_P^4}\frac{V}{\varepsilon}$$

$$\frac{\delta T}{T}\propto R$$

Sachs-Wolfe effect

From WMAP $P_R^{1/2}\approx 5\times 10^{-5}$ **at** $N\approx 60$

We call this CMB normalization.

We can see from here, the lower the scale, the flatter the potential must be.

The spectral index

$$n_s - 1 \equiv \frac{d \ln P_R}{d \ln k}$$

$$n_s = 1 + 2\eta - 6\varepsilon$$

The latest five-year WMAP results indicate

$$0.948 < n_s < 0.977$$

Some other parameters

The tensor-to-scalar ratio

$$r \equiv \frac{P_T}{P_R} = 16\varepsilon = \left(\frac{V_0^{1/4}}{3.3 \times 10^{16} \mathrm{GeV}} \right)^4$$

A measure of gravity waves

Detection of r may fix the inflation scale

The running spectral index

$$\alpha \equiv \frac{dn_s}{d \ln k} = 16\varepsilon\eta - 24\varepsilon^2 - 2\xi^2$$

where

$$\xi^2 \equiv M_P^4 \frac{V'V'''}{V^2}$$

In our models, both are too small to be relevant.

After Inflation

The energy stored in the vacuum-like state is then transformed into thermal energy (a process called reheating), and the Universe becomes extremely hot. From that point, its evolution is described by the standard hot universe theory.

Inflation can be regarded as a theory of the origin of all the matter.

How to tell that whether an inflation model is good or bad?

Beside fitting the observed parameters:

Very small parameters tends to weaken the credibility of a theory.

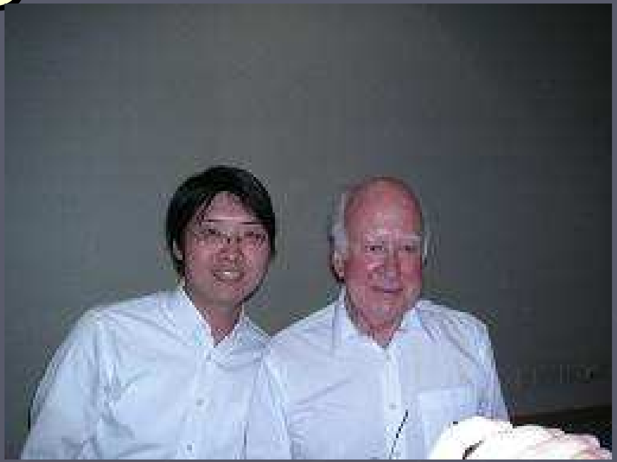
Physical motivation of the scalar potential?

There are also issues about reheating, baryogenesis, cosmic string problems…..

Cosmic Strings

Abelian Higgs model

$$L=-\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu}+\left(D_\mu\phi\right)^*\left(D^\mu\phi\right)-\frac{\lambda}{4}\left(|\phi|^2-\eta^2\right)^2$$

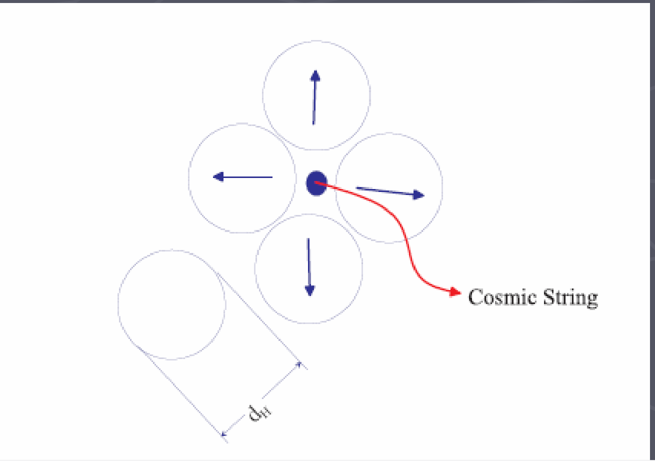


Me & Prof. Higgs

The energy per unit length of the local string is

$$\mu \simeq 2\pi\eta^2$$

After inflation, cosmic strings may form via Kibble Mechanism.



Why do we consider Supersymmetry in inflation models?

It is difficult to see how flat direction fields can be present in a nonsupersymmetric theory given the effect of radiative corrections.

The Scalar Potential in SUSY

In a supersymmetric theory, the tree-level potential for a scalar field is the sum of a F-term and a D-term.

$$V = V_F + V_D$$

$$V_F \equiv \sum_n |F_n|^2$$

$$F_n = -\left(\frac{\partial W}{\partial \phi_n}\right)^*$$

$$V_D \equiv \frac{1}{2} g^2 \left(\sum_n q_n |\phi_n|^2 + \xi \right)^2$$

Fayet-Iliopolous term

Hybrid Inflation

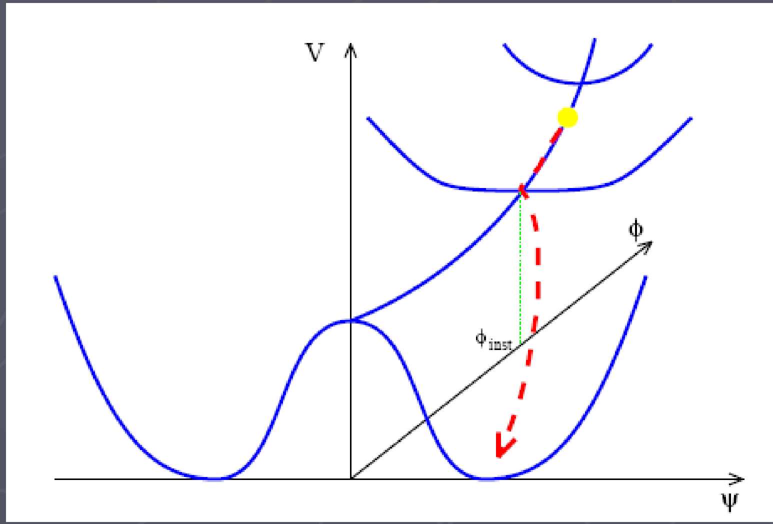
Linde 1991

$$V(\phi,\psi)=\kappa^2\left(M^2-\frac{\psi^2}{4}\right)^2+\frac{\lambda^2\psi^2\phi^2}{4}+\frac{m^2\phi^2}{2}$$

The vacua lie at $\langle\psi\rangle=\pm 2M$ $\quad\langle\phi\rangle=0$

There is a valley of minima for

$|\phi|>\phi_c=\frac{\sqrt{2}\kappa M}{\lambda}$ **where** $\psi=0$



In this regime

$$V(\phi)=V_0+\frac{m^2}{2}\phi^2 \qquad V_0=\kappa^2M^4$$

Why is Hybrid Inflation interesting?

Consider

$$V(\phi)=V_0+\frac{m^2}{2}\phi^2$$

if $V_0=0$ ⇒ $\phi > M_P$

Large field model (chaotic inflation)

More difficult to realize from a effective field theory point of view.

Parameters in Hybird Inflation is not very small.

Hybrid Inflation is easy to construct from SUSY

F-term Inflation

The superpotential for F-term Inflation is

$$W=\kappa S(\phi_+\phi_- - M^2)$$

The scalar potential is given by

$$V=\kappa^2|\phi_+\phi_- -M^2|^2+\kappa^2|S|^2(|\phi_+|^2+|\phi_-|^2)$$

For $|S|>S_c=M$ ⇒ $\phi_-=\phi_+=0$ ⇒ $V_0=\kappa^2 M^4$

Including 1-loop correction, we obtain

$$V(S)\simeq V_0+\frac{\kappa^4 M^4}{16\pi^2}\ln\left(\frac{\kappa^2|S|^2}{\lambda^2}+\frac{3}{2}\right)$$

Predictions of F-term Inflation

$$n_s \simeq 1 - \frac{1}{N} = 0.983$$

for N=60

Also from constraint of cosmic string study

$$M \leq 2 \times 10^{15} \mathrm{GeV}$$

$$\kappa \leq 7 \times 10^{-7}$$

Problem of F-term Inflation

Supergravity correction to the F-term scalar potential destroys the required flatness.

Results in $|\eta| \approx 1$

Called η problem

There is no such a problem in D-term Inflation.

D-term Inflation

$$W_D = \lambda S \Phi_+ \Phi_-$$

$$V(S,\Phi_+,\Phi_-) = \lambda^2 \left[|S|^2 \left(|\Phi_+|^2 + |\Phi_-|^2 \right) + |\Phi_+|^2 |\Phi_-|^2 \right] + \frac{g^2}{2} \left(|\Phi_+|^2 - |\Phi_-|^2 + \xi \right)^2$$

g **is the** $U(1)_{FI}$ **gauge coupling**

The true vacuum is given by $\langle S \rangle = 0,\ \langle \Phi_+ \rangle = 0,\ \langle \Phi_- \rangle = \sqrt{\xi}$

when $|S| > |S|_c = \frac{g\xi^{1/2}}{\lambda}$ $\quad \Phi_+ = \Phi_- = 0$

The 1-loop potential is

$$V(S) \simeq V_0 + \frac{g^4 \xi^2}{16\pi^2} \ln\left(\frac{|S|^2}{\Lambda^2} \right) \qquad V_0 = \frac{g^2 \xi^2}{2}$$

Predictions of D-term Inflation

$$n_s \simeq 1 - \frac{1}{N} = 0.983$$ **for N=60**

After imposing CMB normalization:

$$\xi^{1/2} = 7.9 \times 10^{15} \left(\frac{60}{N} \right)^{1/4} \text{GeV}$$

However, after D-term inflation, the U(1) symmetry is broken and cosmic strings are produced, there is a cosmic string bound:

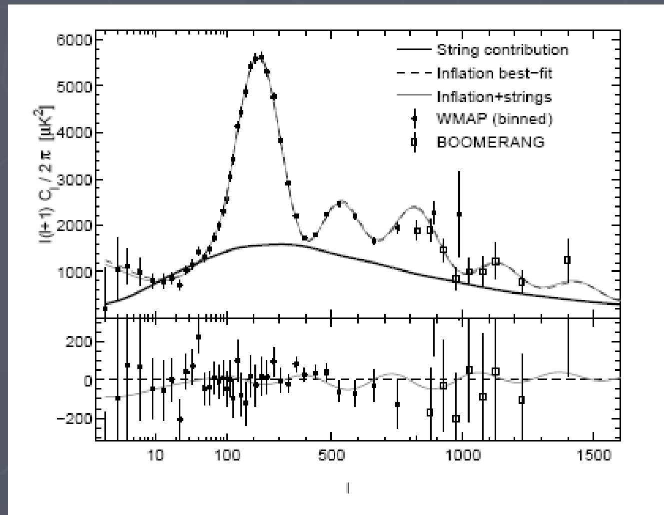
$$\xi^{1/2} < 3.9 \times 10^{15} \text{GeV}$$

Cosmic String Problem

A cosmic string network will modify the CMB Angular power spectrum:

$$C_l = C_l^{\mathrm{inf.}} + C_l^{str.}$$

$$C_l^{\mathrm{str.}} \propto \mu^2 \propto \xi^2$$



Cosmic string contribution to C_l **must be less than 10%**

Bevis et al. 2008

$$\Rightarrow \quad \xi^{1/2} < 3.9 \times 10^{15}\,\mathrm{GeV}$$

Bevis et al. 2007

However D-term inflation predicts

$$\xi^{1/2} = 7.9 \times 10^{15} \left(\frac{60}{N}\right)^{1/4} \mathrm{GeV}$$

Supergravity Modified D-term Inflation

$$W_\nu = \lambda_\nu \Phi H_u L + \frac{m_\Phi}{2}\Phi^2$$

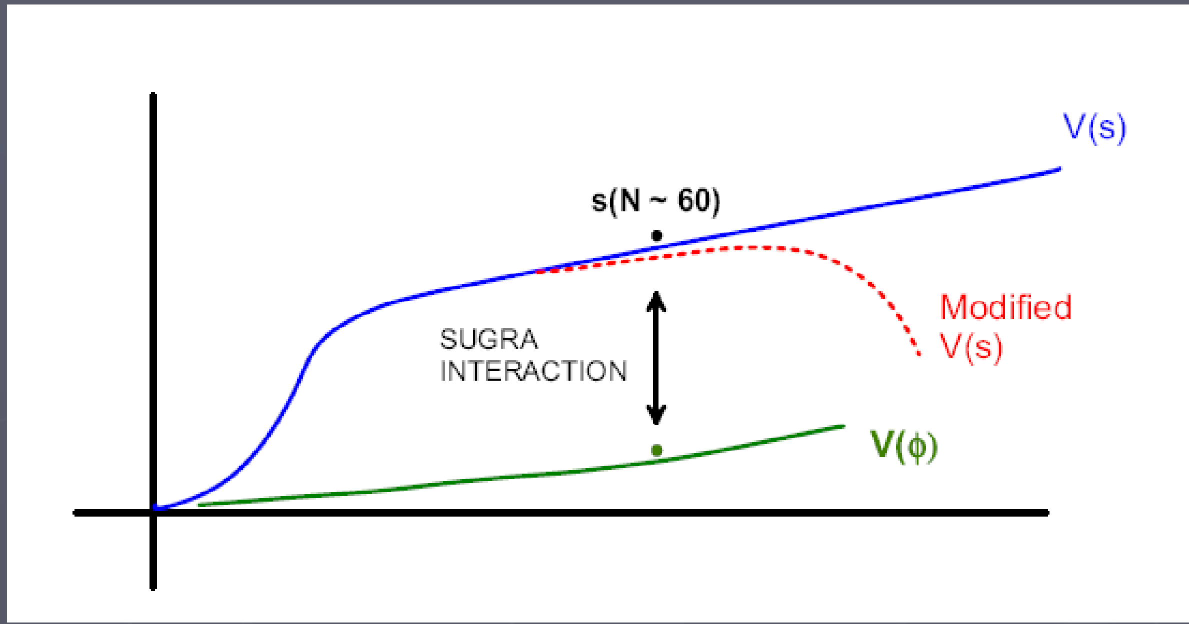
Φ **is the Right-Handed neutrino superfield**

Consider a Kahler potential:

$$K = |S|^2 + |\Phi|^2 + \frac{c|S|^2|\Phi|^2}{M_P^2}$$

We obtain

$$\Delta V = -\frac{(c-1)m_\Phi^2|\Phi|^2|S|^2}{M_P^2}$$



$$n_s = 1 + 2\eta - 6\varepsilon$$

We reduce the spectral index by increasing the curvature of the potential.

$$P_R = \frac{1}{12\pi^2 M_P^6}\frac{V^3}{V'^2} = \frac{1}{24\pi^2 M_P^4}\frac{V}{\varepsilon}$$

We reduce the scale of inflation by reducing the slope of the potential.

Results

$$n_s = 1 - \frac{\gamma}{N}\left(\frac{1}{(1-e^{-\gamma})} + 1\right)$$

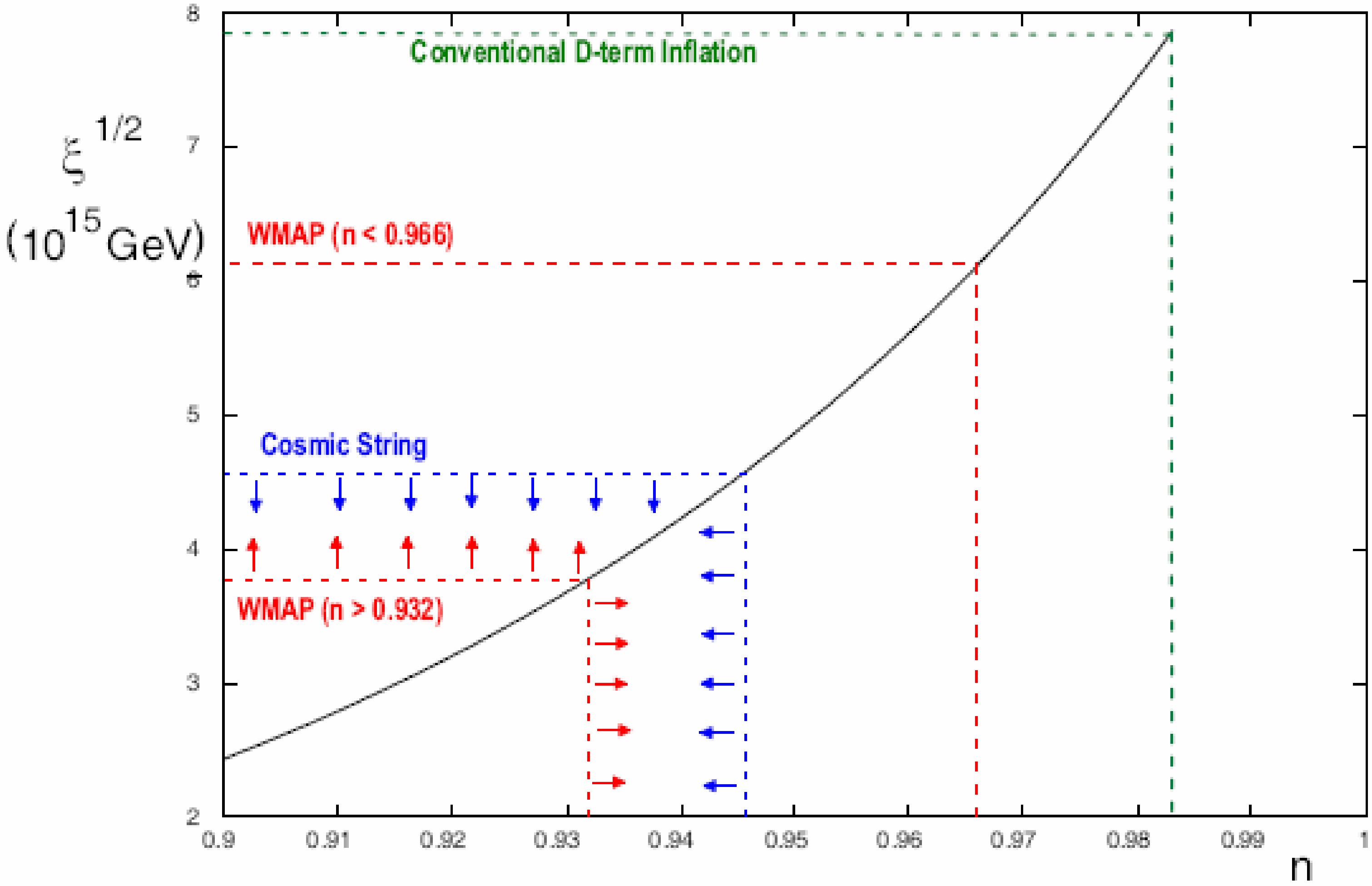
$$\gamma \equiv \kappa \frac{\phi^2 N}{3H^2}$$

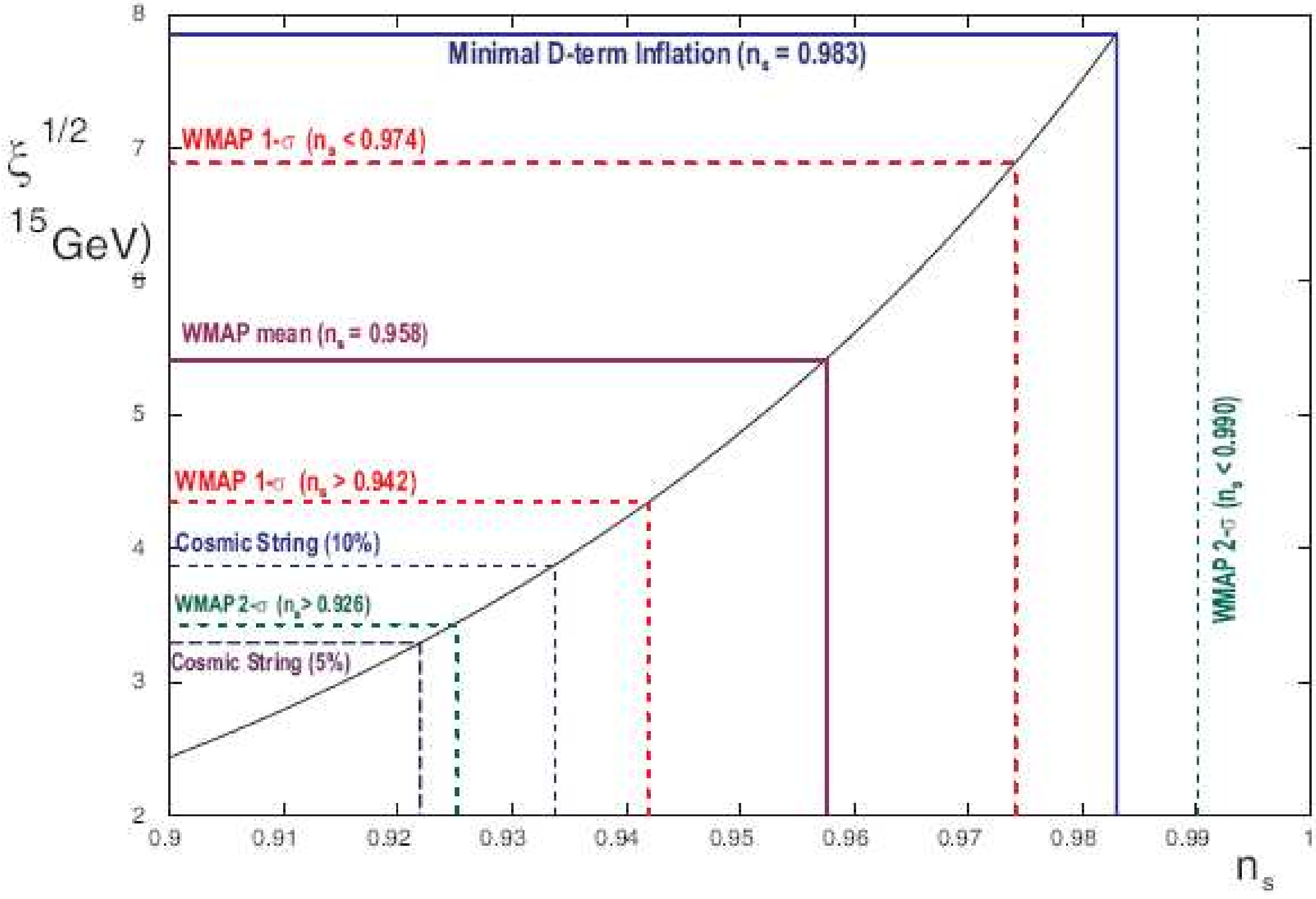
$$P_\xi = \frac{N\xi^2}{3M_P^4}\frac{1}{\Gamma}$$

$$\kappa \equiv \frac{(c-1)m_\Phi^2}{M_P^2}$$

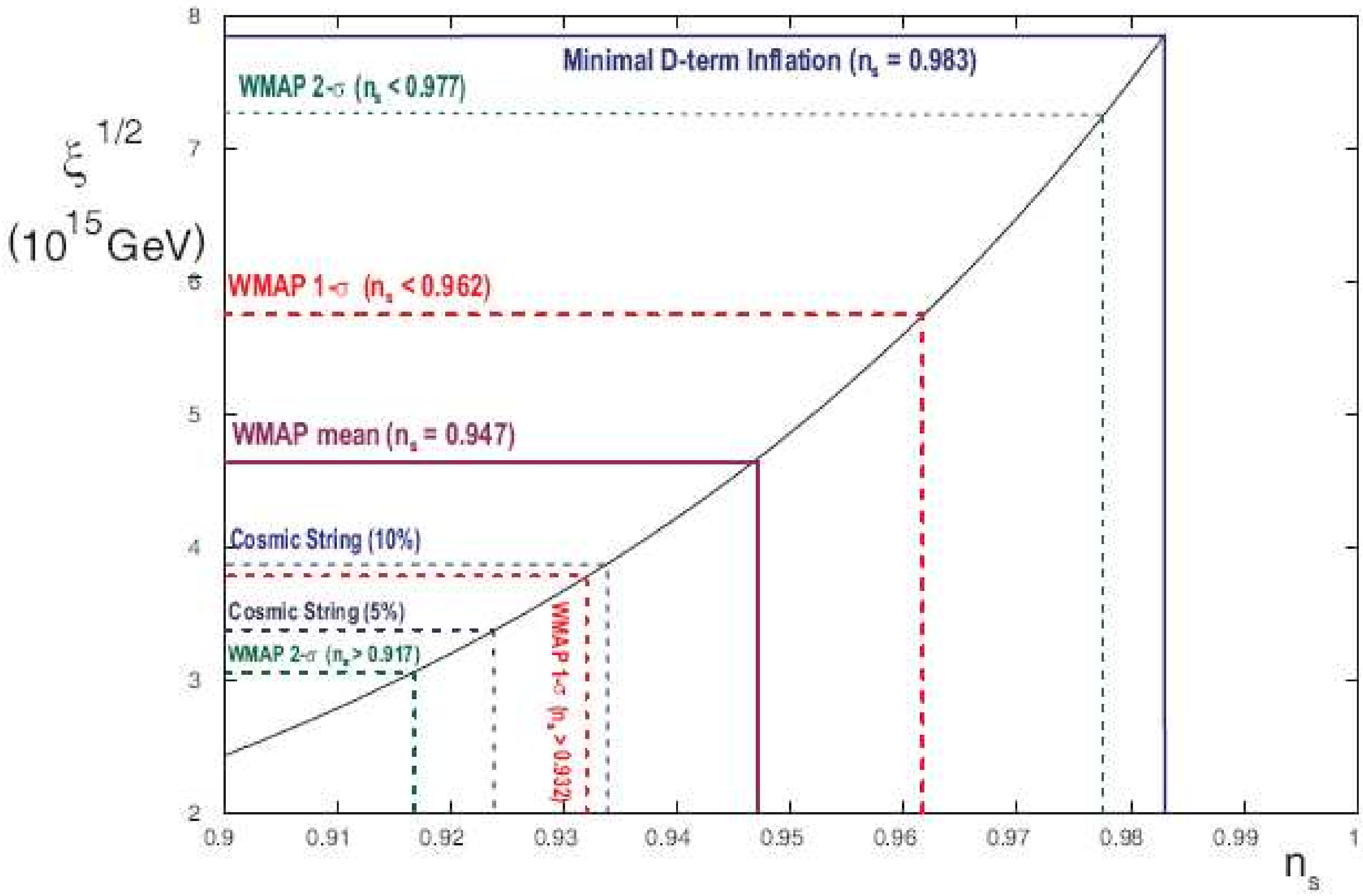
$$\Gamma \equiv \frac{\gamma e^{-2N}}{(1-e^{-\gamma})}$$

C.M. Lin and J. McDonald 2006

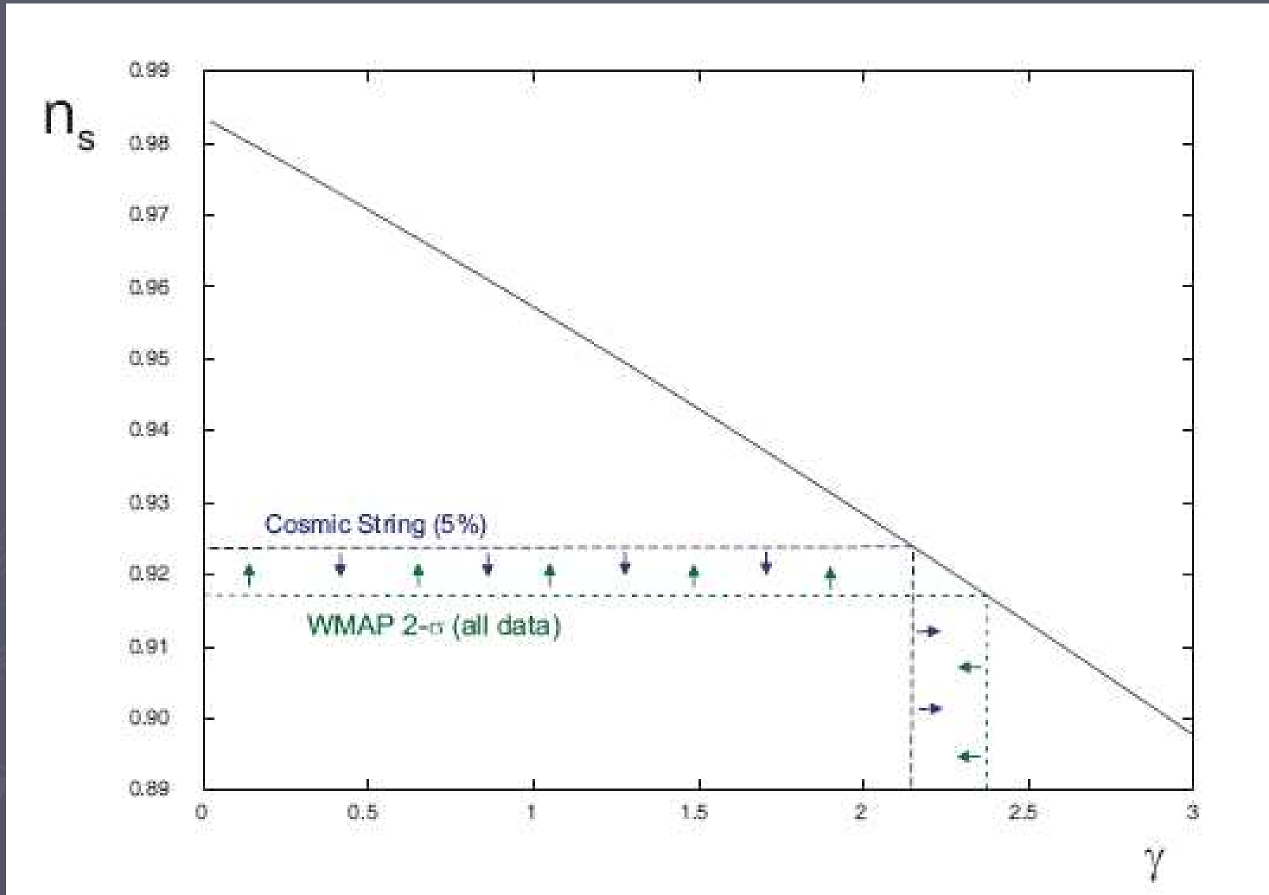




(a) WMAP data only.



(b) WMAP + ALL data.



n_s
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0.92
0.91
0.90
0.89
Cosmic String (5%)
WMAP 2-σ (all data)
0
0.5
1
1.5
2
2.5
3
γ

Additional SUGRA corrections

We consider the Kahler potential

$$K = |S|^2 + |\Phi_+|^2 + |\Phi_-|^2 + \frac{c|S|^2|\Phi|^2}{M_P^2} + f_+\left(\frac{|S|^2}{M_P^2}\right)|\Phi_+|^2 + f_-\left(\frac{|S|^2}{M_P^2}\right)|\Phi_-|^2 + b\frac{|S|^4}{M_P^2}$$

with $$f_\pm\left(\frac{|S|^2}{M_P^2}\right) = \frac{c_\pm |S|^2}{M_P^2} \equiv \frac{c_\pm s^2}{2}$$

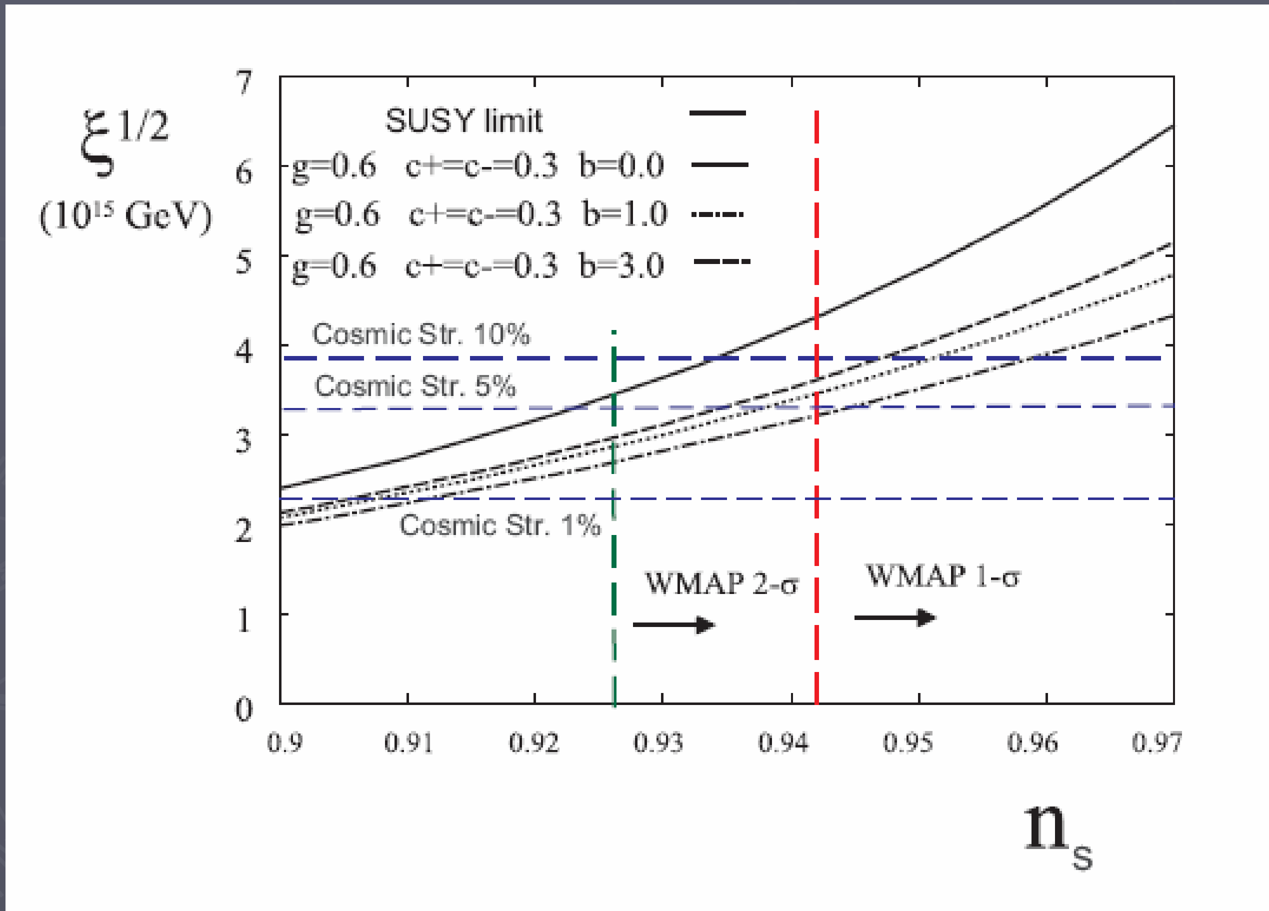
The scalar potential is

$$V(s,\phi) = \frac{g^2\xi^2}{2}\left\{1 + \frac{g^2}{16\pi^2}\left[2\ln\left(\frac{g^2\xi z}{\Lambda^2}\right) + f_V(z)\right]\right\} - \frac{\kappa m_\phi^2 s^2 \phi^2}{4} + \frac{m_\phi^2 \phi}{2}$$

$$z \equiv \frac{\lambda^2 s^2}{2g^2\xi}\exp\left(\frac{s^2}{2M_P^2} + b\frac{s^4}{4M_P^4}\right)\frac{1}{(1+f_+)(1+f_-)}$$

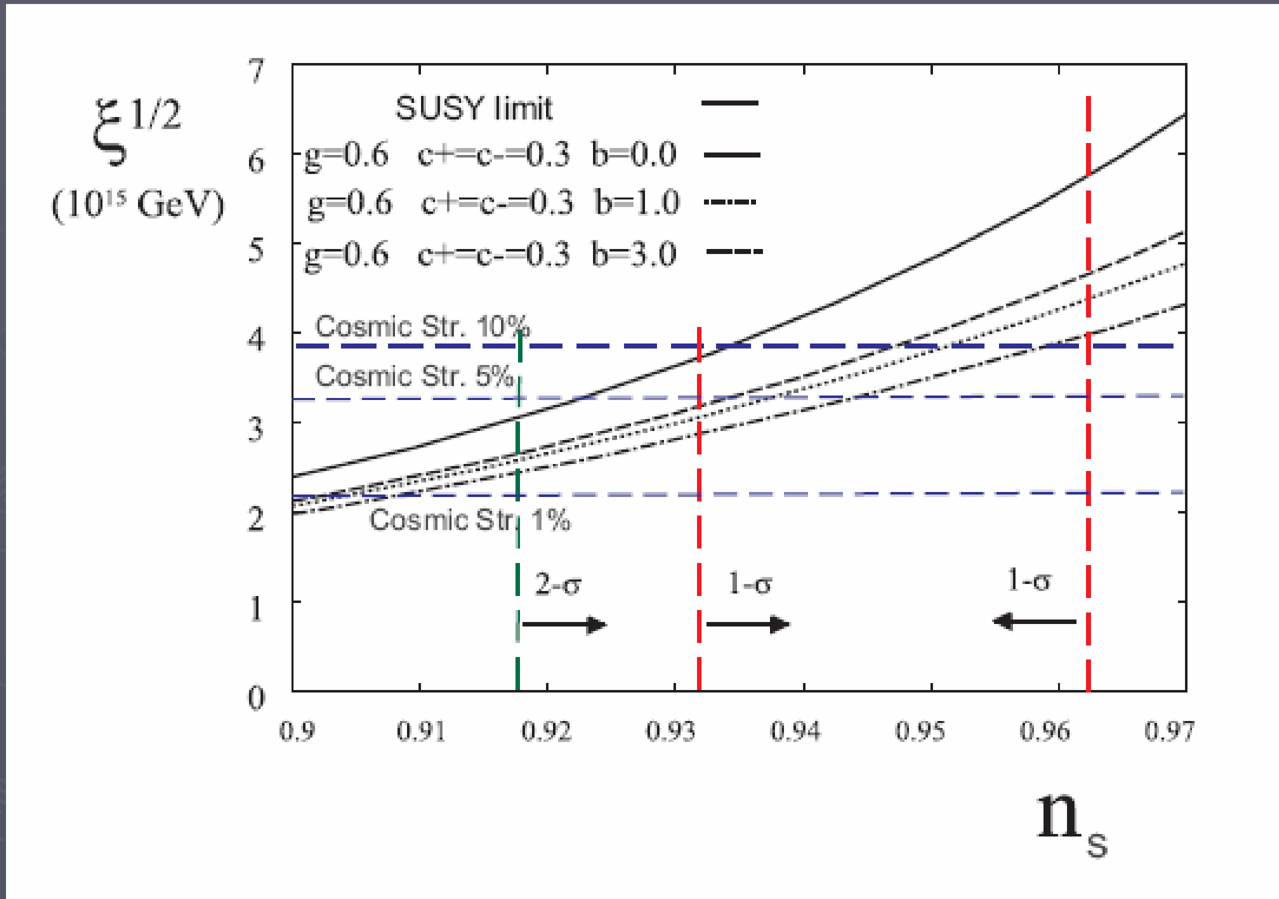
$$f_V(z) = (z+1)^2\ln\left(1+\frac{1}{z}\right) + (z-1)^2\ln\left(1-\frac{1}{z}\right)$$

WMAP



$\xi^{1/2}$
(10^{15} GeV)
SUSY limit
g=0.6 c+=c-=0.3 b=0.0
g=0.6 c+=c-=0.3 b=1.0
g=0.6 c+=c-=0.3 b=3.0
Cosmic Str. 10%
Cosmic Str. 5%
Cosmic Str. 1%
WMAP 2-σ
WMAP 1-σ
0
1
2
3
4
5
6
7
0.9
0.91
0.92
0.93
0.94
0.95
0.96
0.97
n_s

WMAP+ALL



$\xi^{1/2}$
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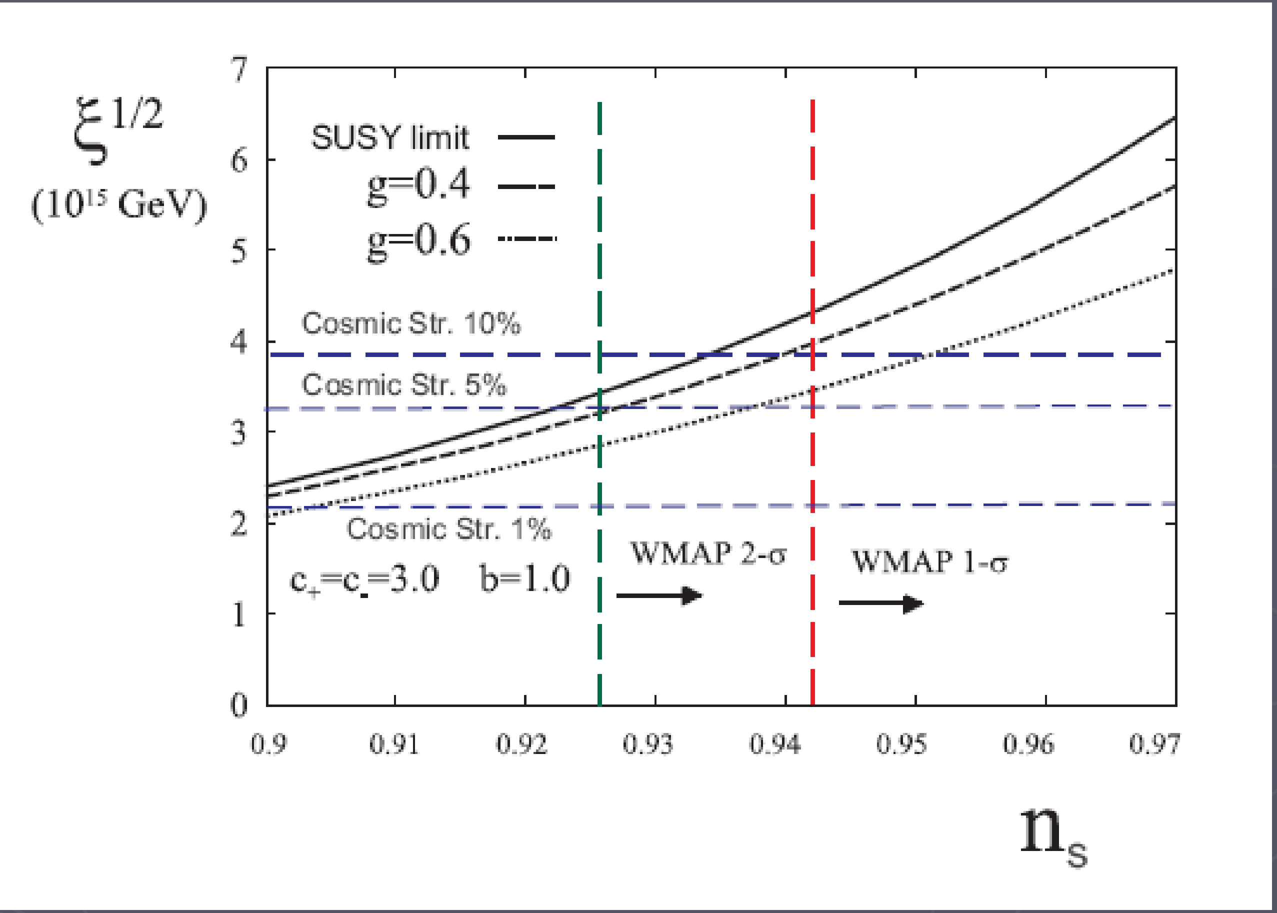
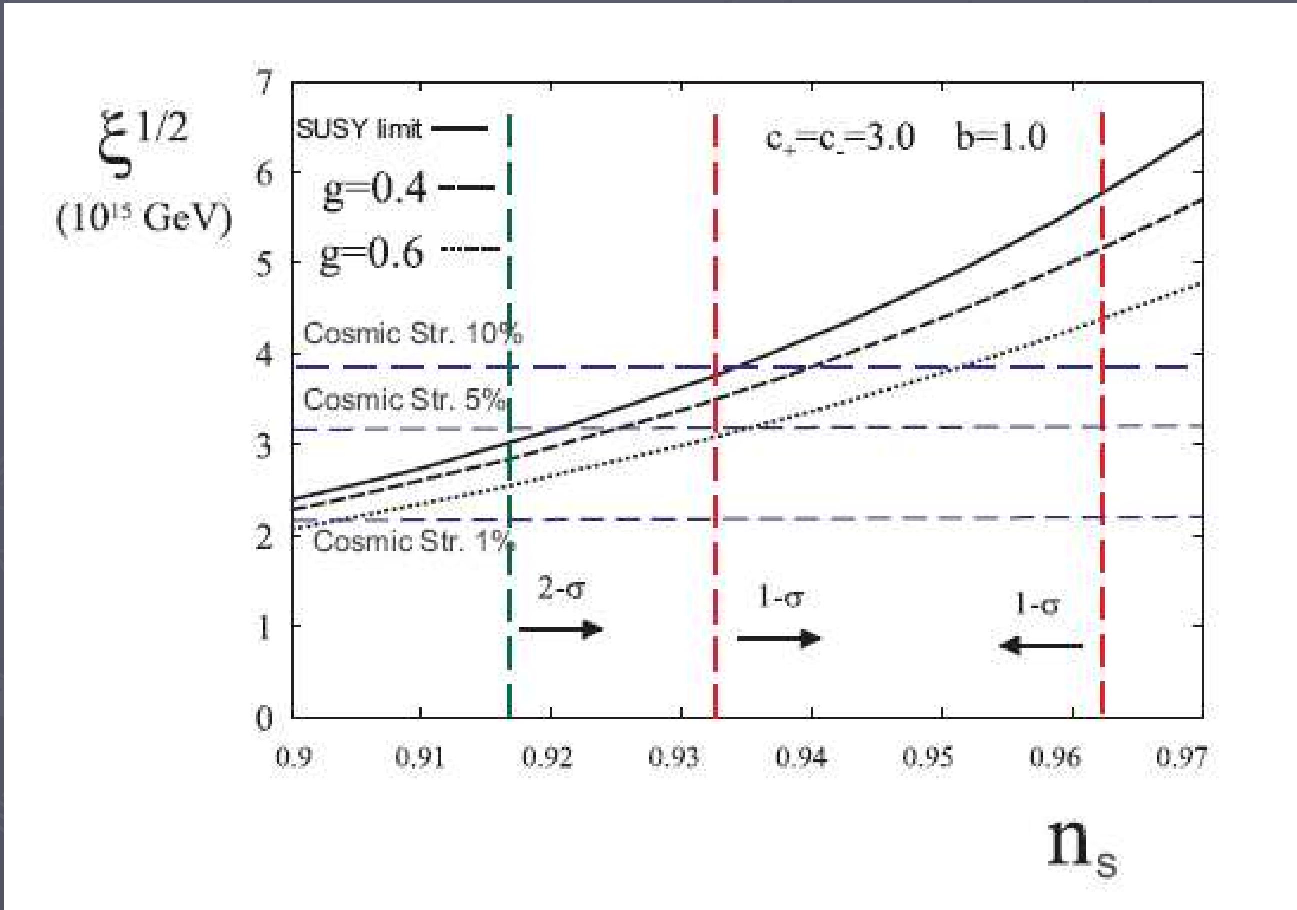
SUSY limit
g=0.6 c+=c-=0.3 b=0.0
g=0.6 c+=c-=0.3 b=1.0
g=0.6 c+=c-=0.3 b=3.0
Cosmic Str. 10%
Cosmic Str. 5%
Cosmic Str. 1%
2-σ
1-σ
1-σ
0
1
2
3
4
5
6
7
0.9
0.91
0.92
0.93
0.94
0.95
0.96
0.97
n_s

WMAP



$\xi^{1/2}$
(10^{15} GeV)
SUSY limit
g=0.4
g=0.6
Cosmic Str. 10%
Cosmic Str. 5%
Cosmic Str. 1%
$c_+=c_-=3.0$ b=1.0
WMAP 2-σ
WMAP 1-σ
0
1
2
3
4
5
6
7
0.9
0.91
0.92
0.93
0.94
0.95
0.96
0.97
n_s

WMAP+ALL



$\xi^{1/2}$
(10^{15} GeV)
SUSY limit
g=0.4
g=0.6
$c_+=c_-=3.0$ b=1.0
Cosmic Str. 10%
Cosmic Str. 5%
Cosmic Str. 1%
2-σ
1-σ
1-σ
0
1
2
3
4
5
6
7
0.9
0.91
0.92
0.93
0.94
0.95
0.96
0.97
n_s

Hilltop Inflation

Proposed by L. Boubekeur and D. H. Lyth (2005)

Kazunori Kohri, Chia-Min Lin, David H. Lyth (2007) considered the following parameterization of Hilltop Inflation models:

$$V(\phi)=V_0 \pm \frac{1}{2} m^2 \phi^2 - \lambda \frac{\phi^p}{M_P^{p-4}} + \cdots$$

$$\equiv V_0 \left(1 + \frac{1}{2} \eta_0 \frac{\phi^2}{M_P^2} \right) - \lambda \frac{\phi^p}{M_P^{p-4}} + \cdots$$

with $$\eta_0 = \pm \frac{m^2 M_P^2}{V_0}$$

Lin, Lyth, Kohri

Results

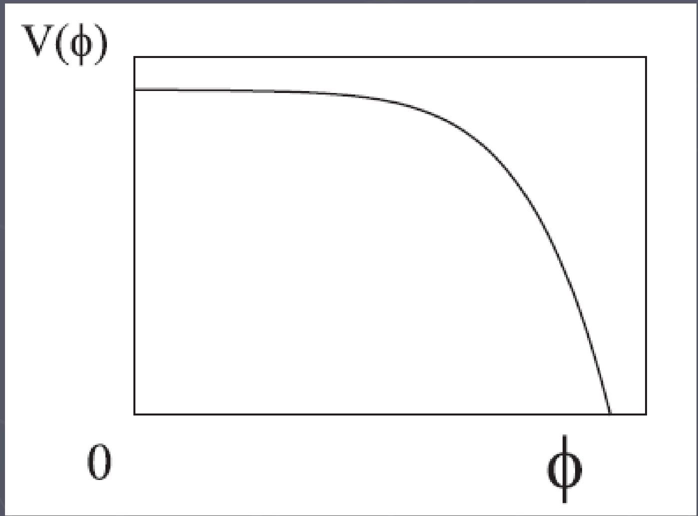
$$P_{\varsigma} = \frac{1}{12\pi^2}\left(\frac{V_0}{M_P^4}\right)^{\frac{p-4}{p-2}} e^{-2\eta_0 N} \frac{\left[p\lambda(e^{(p-2)\eta_0 N}-1)+\eta_0 x\right]^{\frac{2p-2}{p-2}}}{\eta_0^{\frac{2p-2}{p-2}}(\eta_0 x - p\lambda)^2}$$

$$n_s = 1 + 2\eta_0\left[1 - \frac{\lambda p(p-1)e^{(p-2)\eta_0 N}}{\eta_0 x + p\lambda(e^{(p-2)\eta_0 N}-1)}\right]$$ **where** $x = \frac{p(p-1)\lambda}{1+\eta_0}$

$$\alpha = 2\eta_0^2\lambda p(p-1)(p-2)\frac{e^{(p-2)\eta_0 N}(\eta_0 x - p\lambda)}{[\eta_0 x + p\lambda(e^{(p-2)\eta_0 N}-1)]^2}$$

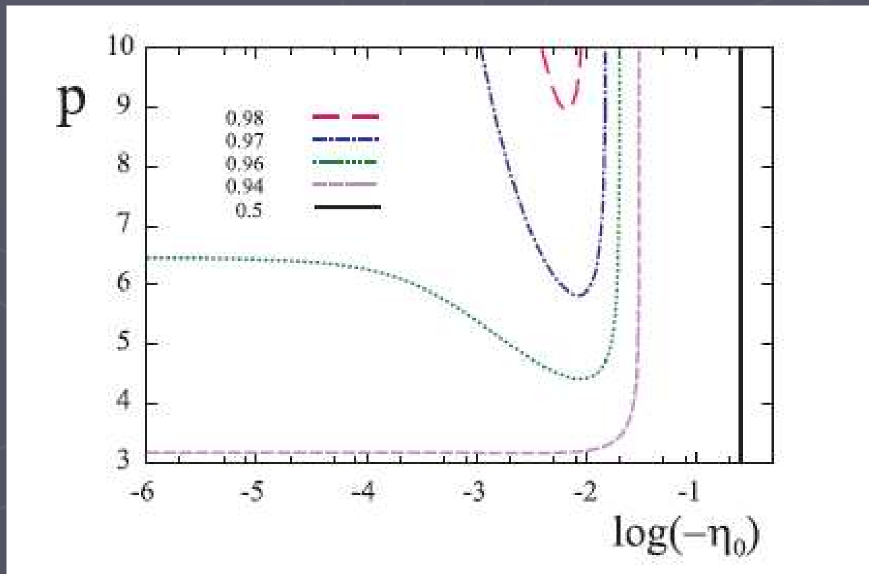
Model 1

$p > 2 \qquad \eta_0 \leq 0$



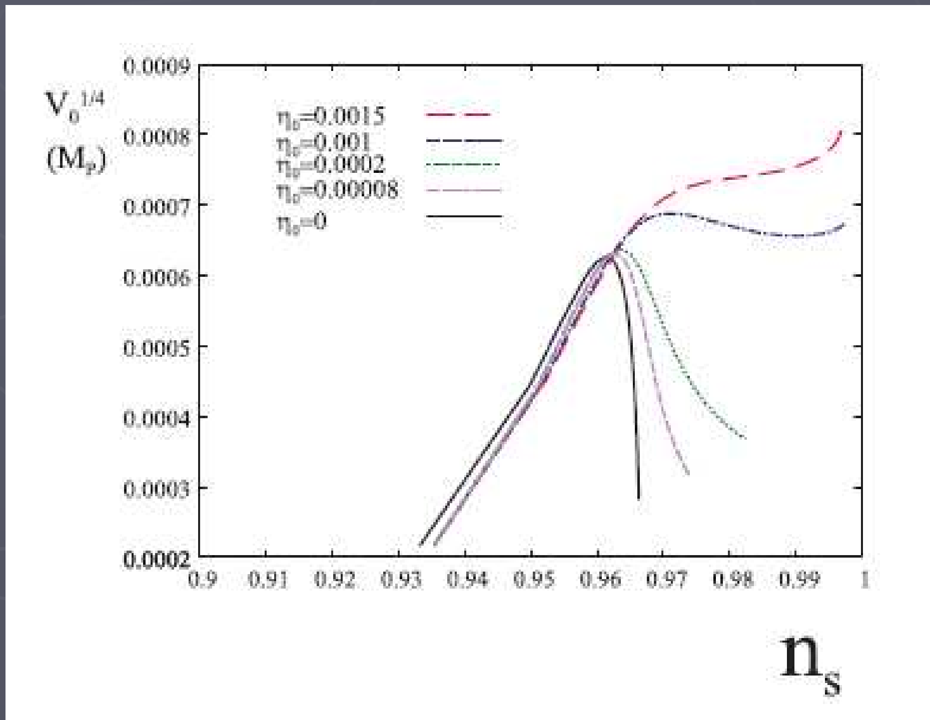
New Inflation

Modular Inflation



Model 1

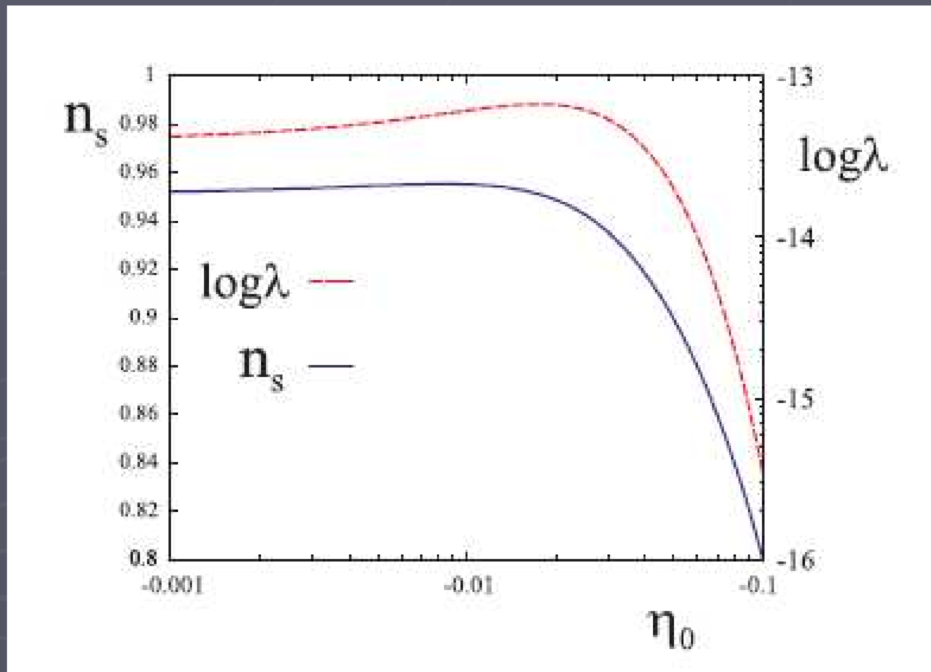
If $\lambda = \frac{V_0}{M_P^4}$ ⇒ **Modular Inflation**



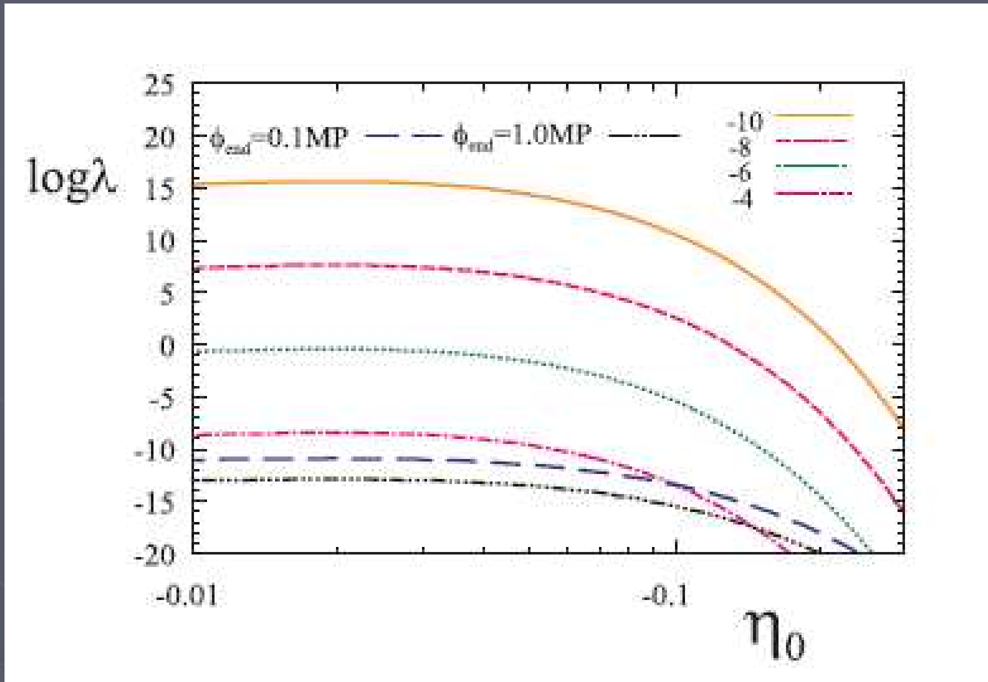
3<p<100

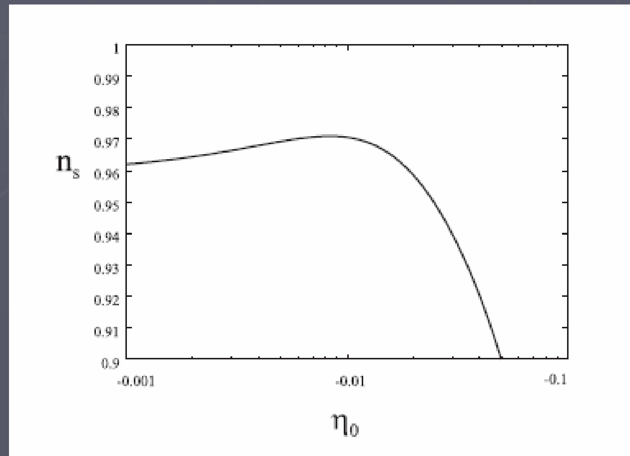
Model One, p=4

New Inflation

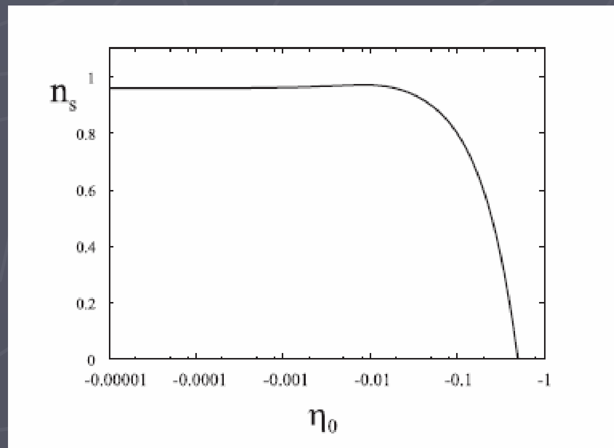


Model 1, p=6



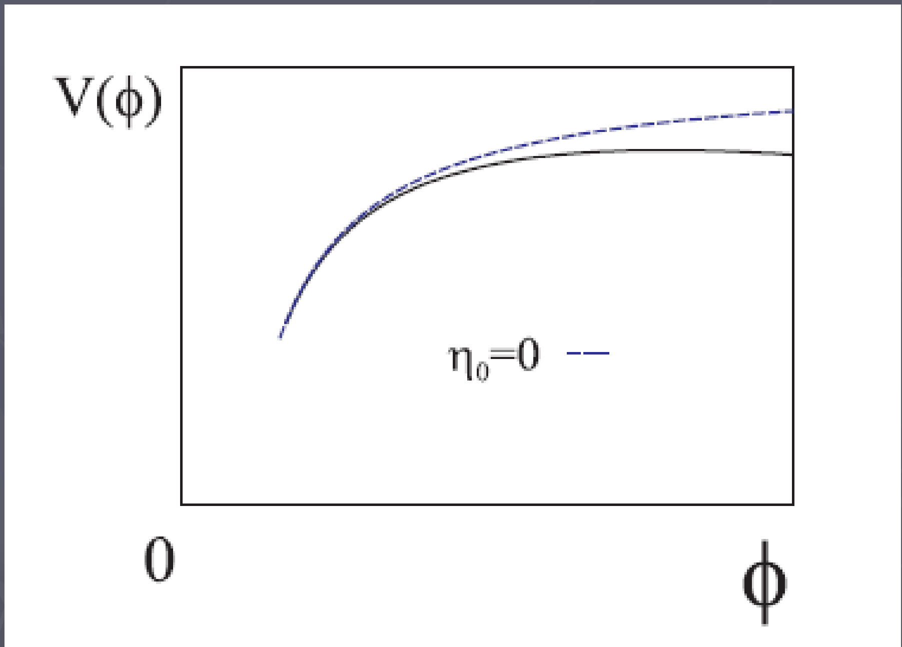


Contours of $\log(V_0^{1/4}/M_P)$



Model 2

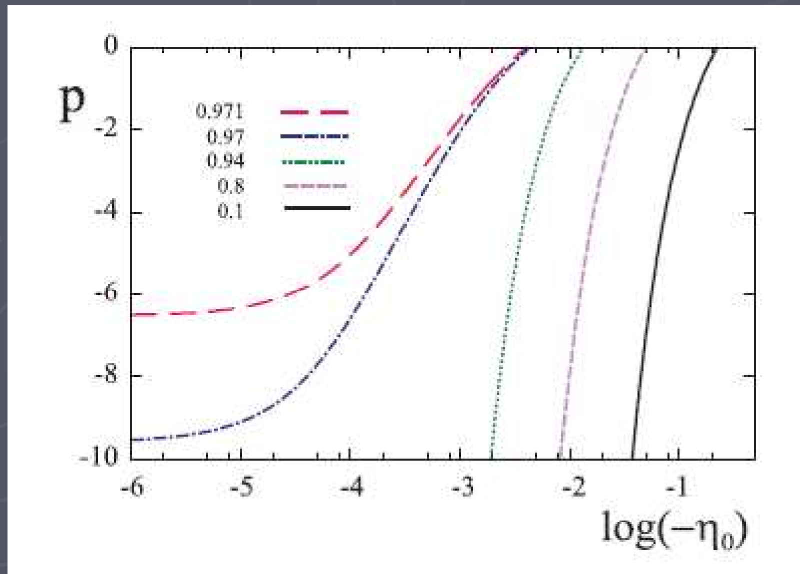
$\eta_0 < 0 \quad p < 0$



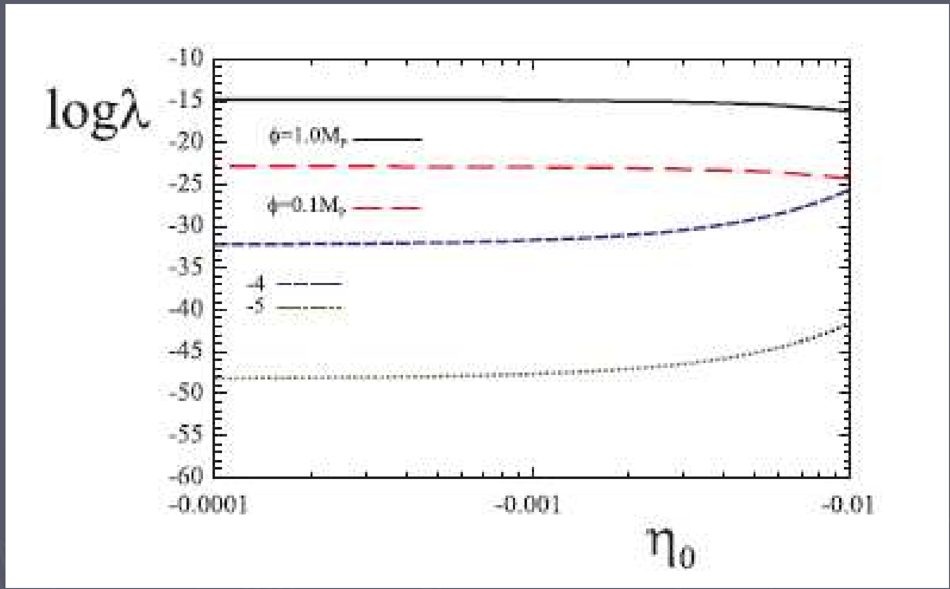
Mutated Hybrid Inflation

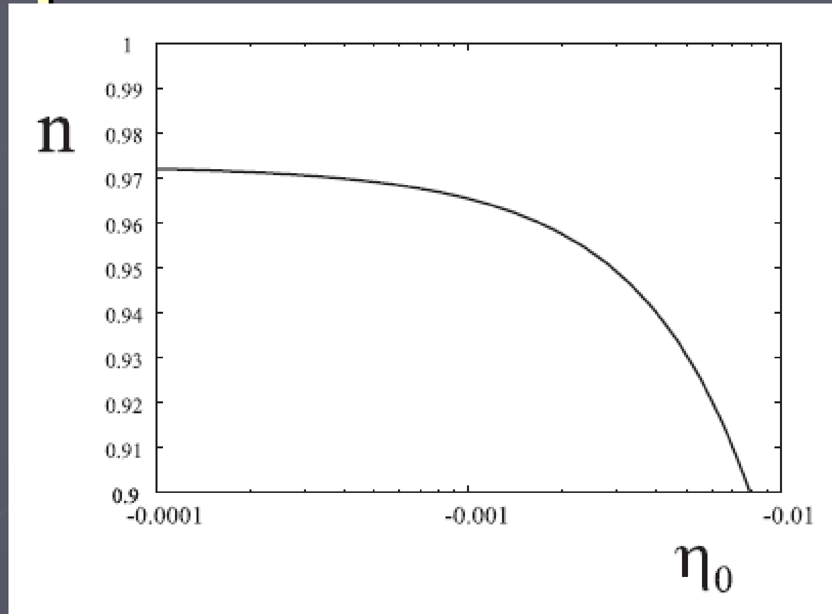
Smooth Hybrid Inflation

F- and D-term Inflation

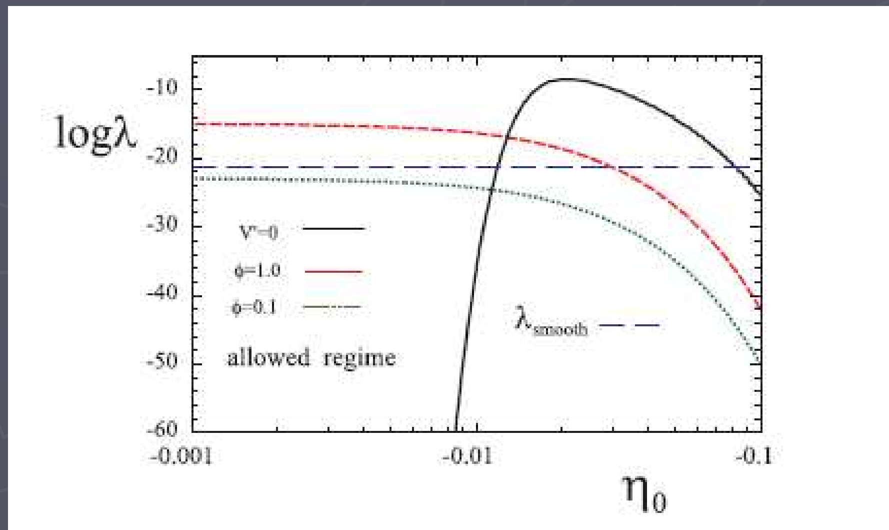


Model 2, p=-4





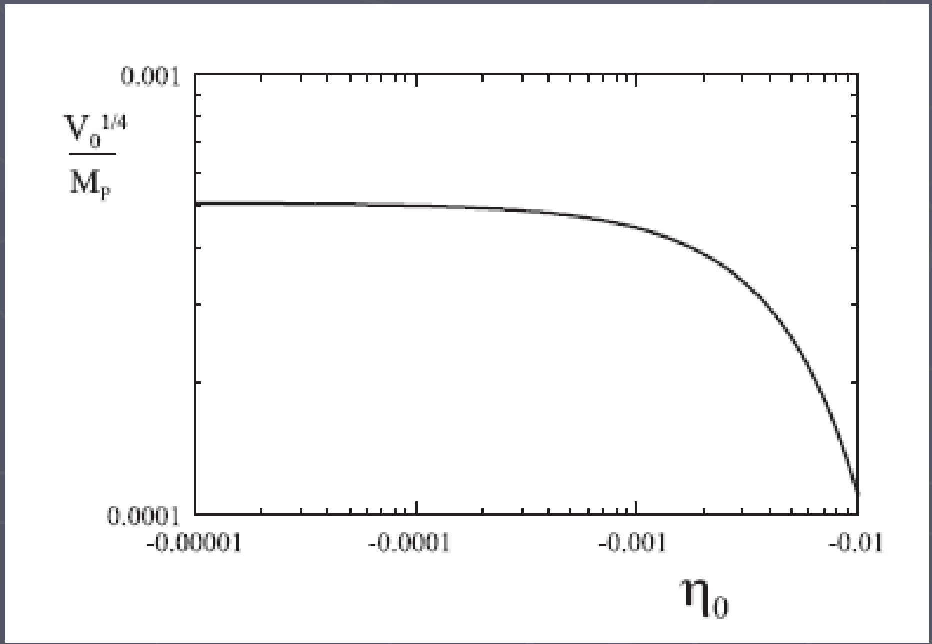
$$\log(V_0^{1/4} / M_P)$$



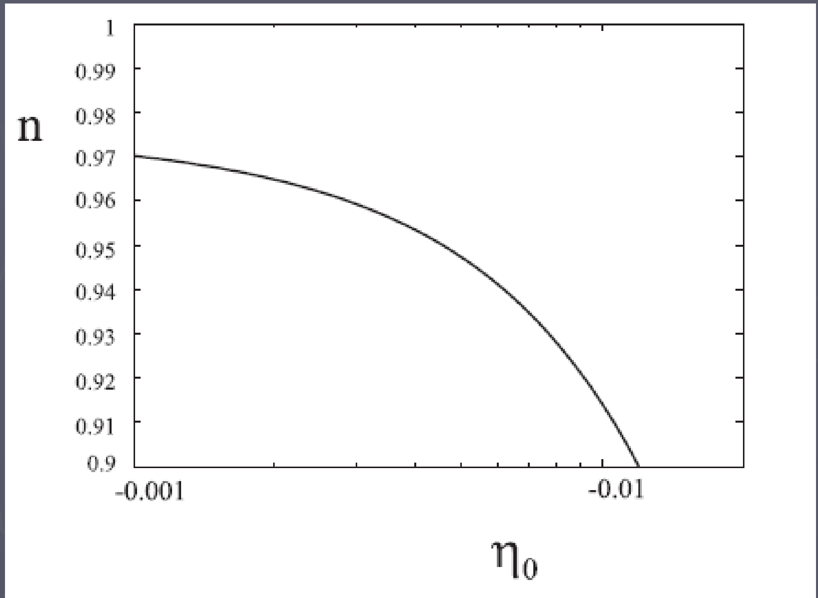
Model 2, p=-4

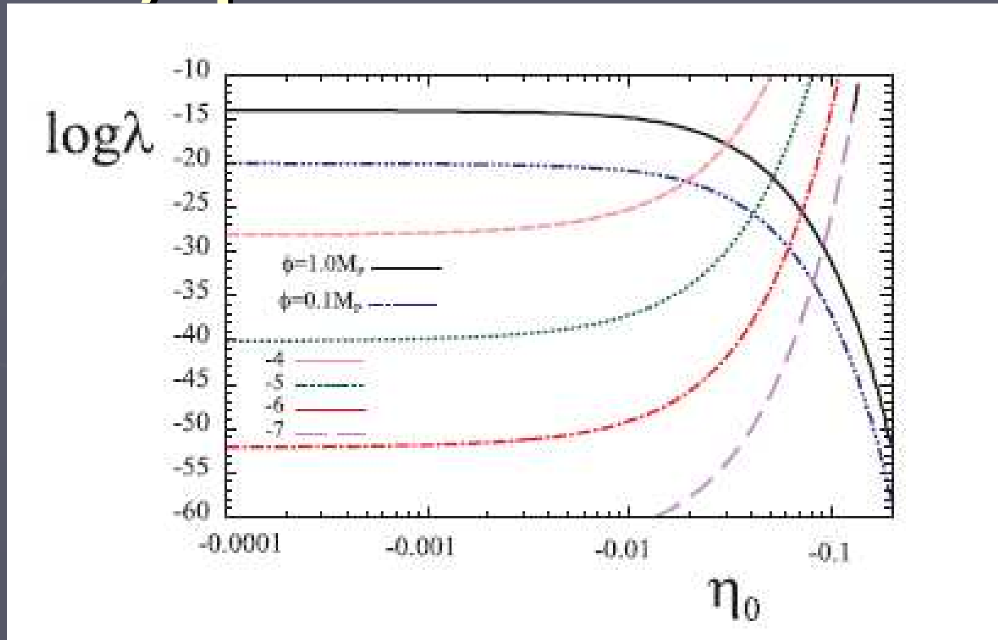
$$\lambda = \frac{2}{27}\left(\frac{V_0^{1/4}}{M_P}\right)^6$$

Smooth hybrid inflation



Model 2, p=-2



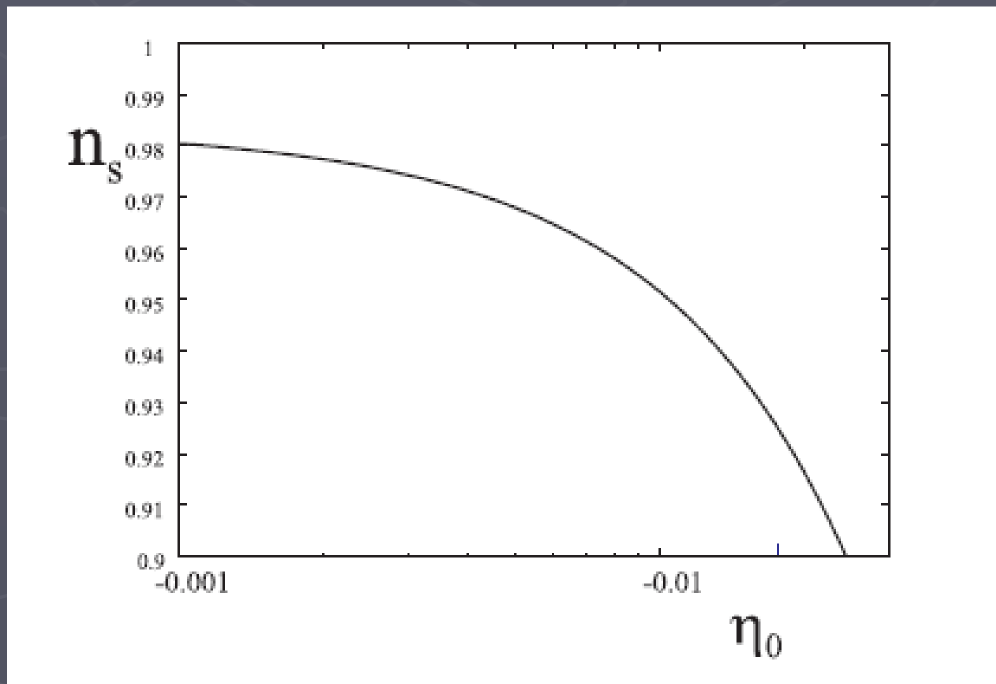


Contours of $\log(V_0^{1/4} / M_P)$

Model 2, p=0

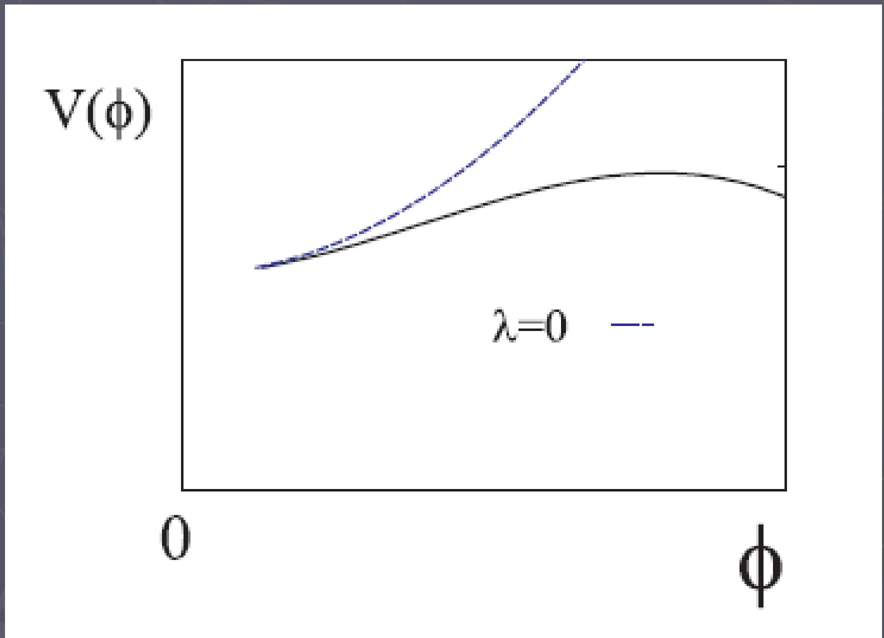
Take $p \to 0$ **with** $\lambda p = -\frac{V_0}{M_P^4}\frac{g^2}{4\pi^2}$

Modified F- and D-term Inflation



Model Three

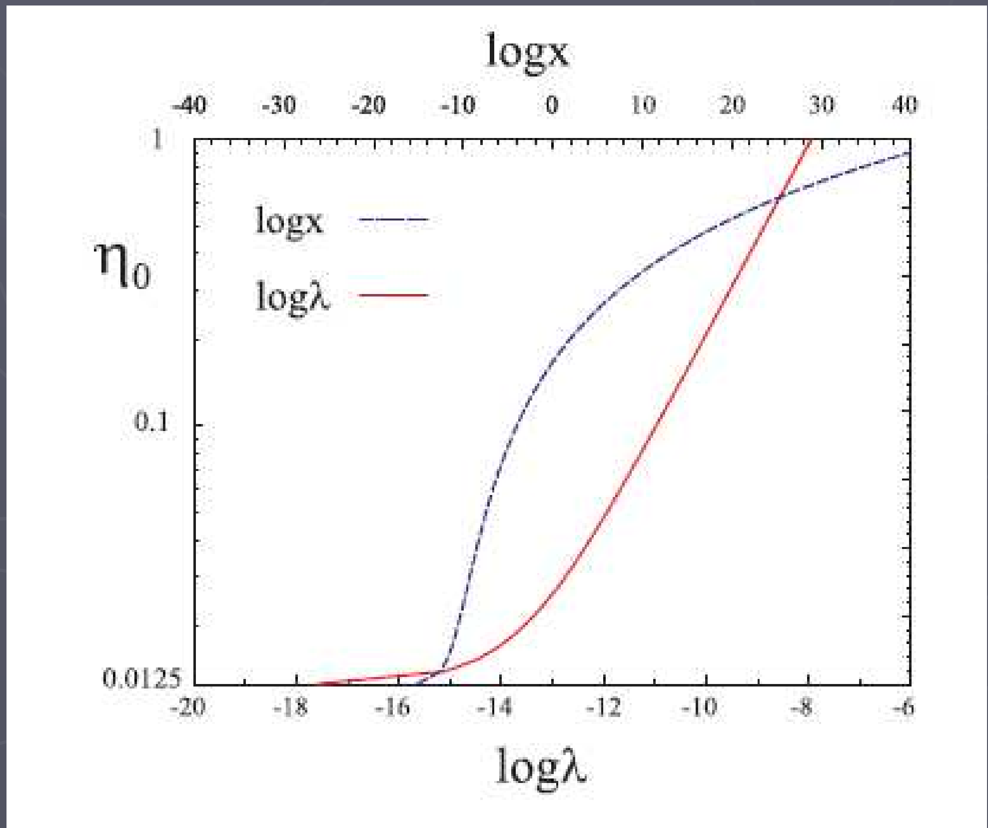
$$\eta_0 > 0 \qquad p > 2$$



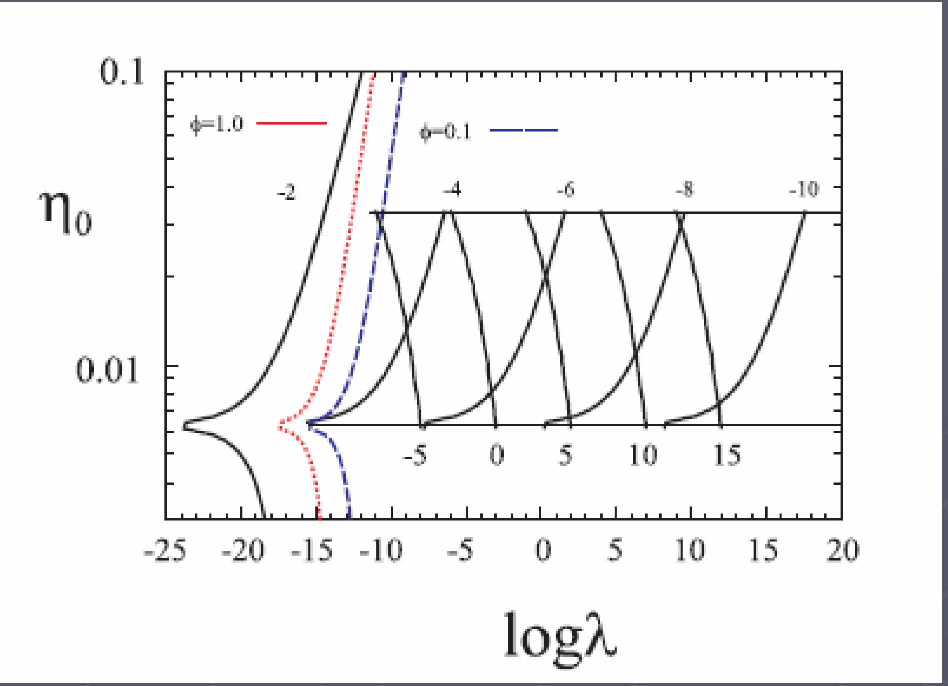
Hilltop Tree level Inflation

Model 3, p=4

We fix n=0.95 in this case.



Model 3, p=6



$$\log(V_0^{1/4}/M_P) = -2, -4, -6, -8, -10$$

$$\log(x) = -5, 0, 5, 10, 15$$

Conclusion

- SUGRA corrections due to a RH sneutrino can modify the D-term hybrid inflation scalar potential in such a way that the energy density during inflation is consistent with non-observation of cosmic strings in the CMB and the spectral index is within the limits favored by WMAP data.
- Hilltop Inflation is an generic possibility for both hybrid and non-hybrid inflation models.
- Hilltop Inflation allows a whole range of models to fit observation.