From AdS/CFT to Cosmology

Chen-Pin Yeh
Stanford University

Base on the work hep-th/0606204
"Holographic framework for eternal inflation"

NTHU, September 14

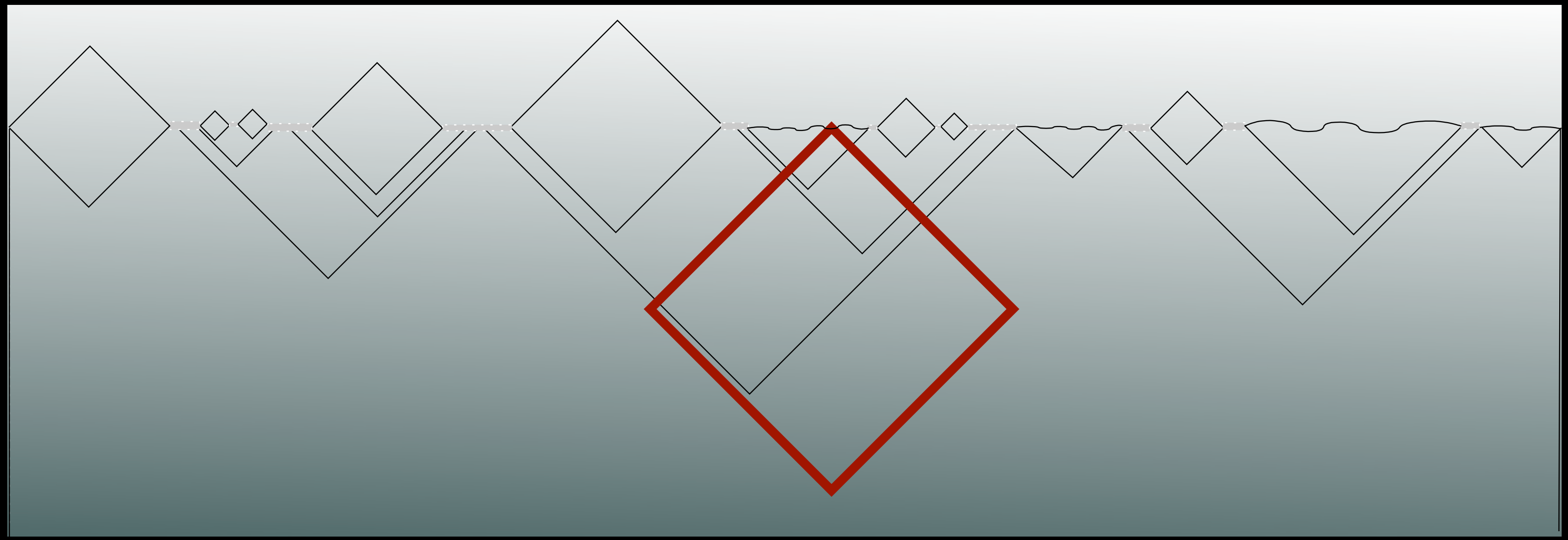
Outline

- Motivation
- Introduction to AdS/CFT
- Universe created by tunneling
- Correlator in CDL background
- Future works

The questions

1. Can we understand the universe we observe

2. If it exists the landscape, can we say anything about why we live in this specific vacuum



False vacuum eternal inflation

from hep-th/0606114

3. What is the right question to ask

4. Does it have the consistent math framework for understanding the eternal inflation

5. Does it have observational consequence of the landscape(initial condition, bubble collision...)

....

AdS/CFT basic

$\mathcal{N}$=4 SU(N) SYM in M_4 is equivalent to IIB string theory in $AdS_5 \times S_5$

- Couplings

$$\frac{R^4}{\alpha'^2} = \lambda \qquad g_s = \frac{\lambda}{N} \qquad \lambda \equiv g_{\rm ym}^2 N$$

- Global symmetry

Conformal symmetry = Isometry = SO(2,4)

- Correlation function

$$\left\langle \exp \int_{\partial B} \phi_0 \mathcal{O} \right\rangle_{CFT} = Z_B(\phi_0)$$

In the classical supergravity limit

$$Z_B(\phi_0) = e^{-S(\phi_0)}$$

where $S(\phi_0)$ is the classical action

Massless scalar example

Euclidean AdS in Poincare coordinate

$$ds^2 = \frac{1}{x_0^2} \sum_{i=0}^{d} (dx_i)^2$$

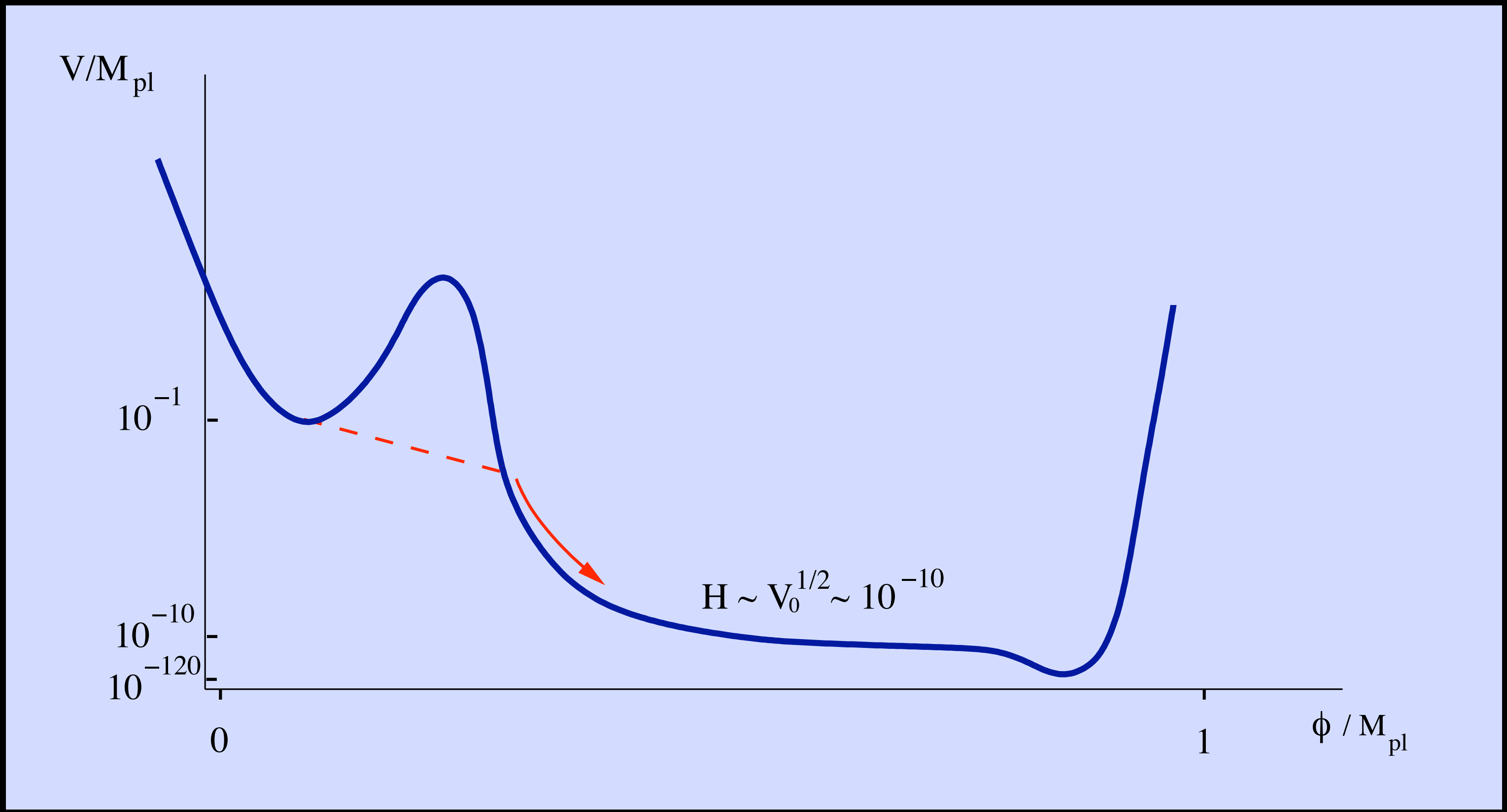
Relation between bulk and boundary correlator

$$\lim_{x_0, x_0' \to 0} x_0'^{-d} x_0^{-d} G_B \left((x_0', \vec{x}'), (x_0, \vec{x}) \right) = \frac{1}{|\vec{x}' - \vec{x}|^{2d}}$$

Open universe from bubble nucleation

Coleman-De Luccia instanton in Euclidean 4D

Considering the scalar couple to gravity with the following potential,



Assuming the SO(4) symmetry

$$ds^2 = dt^2 + a(t)^2 d\Omega_3^2 \qquad = a(X)^2(dX^2 + d\Omega_3^2)$$

where $d\Omega_3^2 = d\theta^2 + \sin^2\theta d\Omega_2^2$

The equation of motion is

$$(a')^2 = \frac{1}{2}a^2(\Phi')^2 - a^4 V(\Phi) + a^2$$

$$\Phi'' + \frac{2a'\Phi'}{a} = a^2 \frac{\partial V}{\partial \Phi}$$

- Continuation to Lorentzian signature

$$X \to T + \frac{\pi}{2} i \qquad \theta \to iR$$

$$ds^2 = a(T)^2(-dT^2 + dR^2 + \sinh^2 R \; d\Omega_2^2)$$

$$= a(T)^2(-dT^2 + d\mathcal{H}_3^2)$$

region I

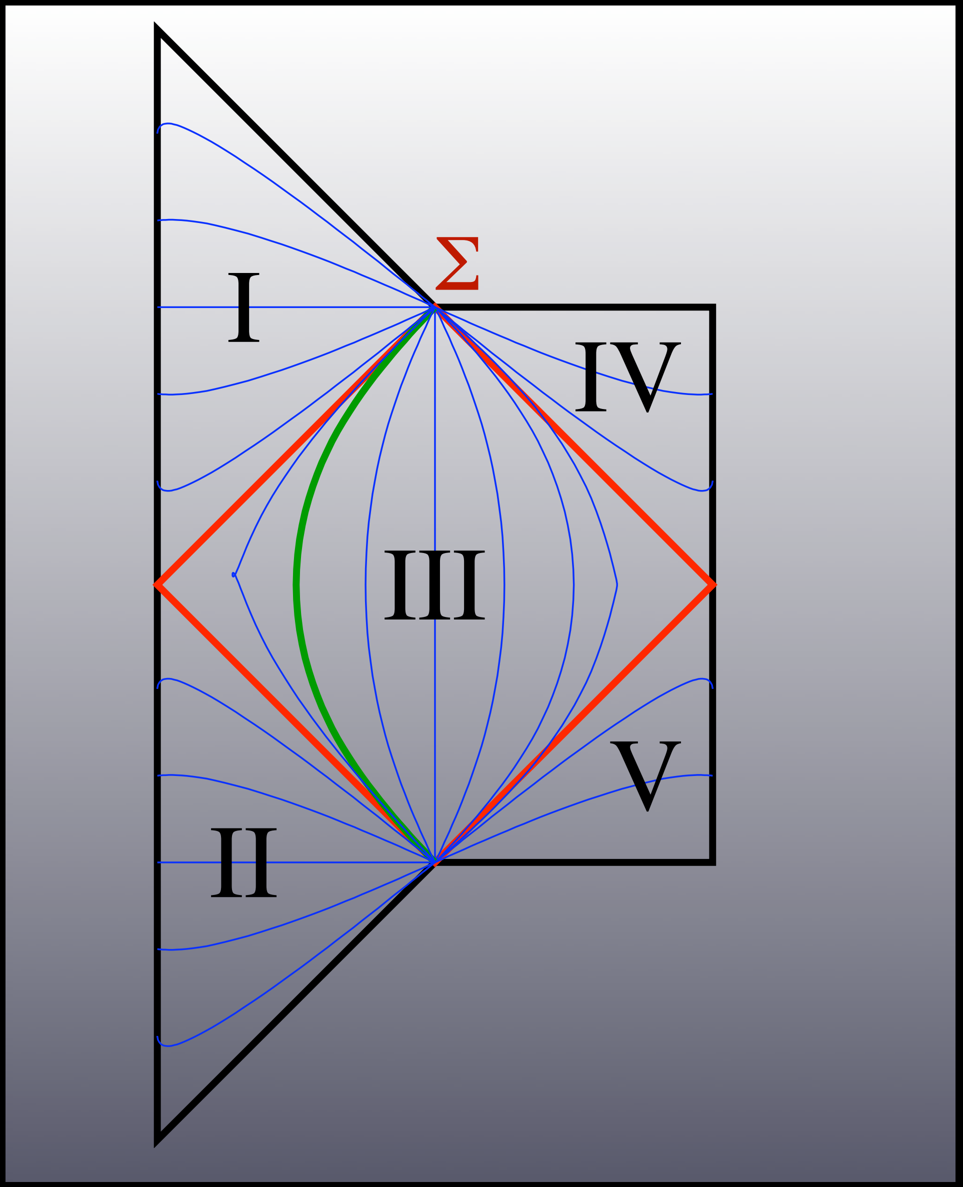
(open FRW with SO(1,3))

$$\theta \to \frac{\pi}{2} - it$$

$$ds^2 = a(X)^2(dX^2 - dt^2 + \cosh^2 t \; d\Omega_2^2)$$

region III

(Bubble wall region)



Penrose diagram for the Lorentzian continuation of the CDL instanton

Two point correlator in CDL background
(for massless scalar χ)

$$\hat{G}(X_1, X_2; \theta) = a(X_1)a(X_2)\langle\chi(X_1, 0)\chi(X_2, \theta)\rangle$$

The rescaled two point function satisfy

$$\left[-\partial_{X_1}^2 + U(X_1) - \nabla^2\right] \hat{G}(X_1, X_2; \theta) = \delta(X_1 - X_2)\frac{\delta(\theta)}{\sin^2\theta}$$

where

$$U(X) \equiv a''(X)/a(X) = 1 - \left(\frac{1}{2}(\Phi')^2 + 2a^2V(\Phi)\right)$$

and $U(X) \to 1 \text{ as } X \to \pm\infty$

Expanding in the complete set

$$\delta(X_1 - X_2) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} u_k(X_1) u_k^*(X_2) + u_B(X_1) u_B^*(X_2)$$

where

$$[-\partial_X^2 + U(X)] u_k(X) = (k^2 + 1) u_k(X)$$

$$[-\partial_X^2 + U(X)] u_B(X) = 0$$

then

$$\hat{G}(X_1, X_2; \theta) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} u_k(X_1) u_k^*(X_2) G_k(\theta) + u_B(X_1) u_B^*(X_2) G_B(\theta)$$

$G_k(\theta)$ and $G_B(\theta)$ are the green function on S^3

$G_k(\theta)$ and $G_B(\theta)$ satisfy the following equations separately

$$[-\nabla^2 + (k^2+1)]G_k(\theta) = \frac{\delta(\theta)}{\sin^2\theta}$$

$$-\nabla^2 G_B(\theta) = \frac{\delta(\theta)}{\sin^2\theta}$$

The solutions are

$$G_k(\theta) = \frac{\sinh k(\pi-\theta)}{\sin\theta \sinh k\pi}$$

$$G_B(\theta) = \lim_{k\to i}\left(G_k(\theta) - \frac{2}{\pi}\frac{1}{k^2+1}\right) + \frac{1}{2\pi} = \frac{\cos\theta}{\sin\theta}\left(1-\frac{\theta}{\pi}\right)$$

The complete set of orthonormal modes $u_k(X)$ with $k > 0$ is give by the waves coming in from the left and those coming from the right

The modes from the left

$$u_k(X) \to e^{ikX} + \mathcal{R}(k)e^{-ikX} \quad (X \to -\infty)$$

$$u_k(X) \to \mathcal{T}(k)e^{ikX} \quad (X \to \infty)$$

The modes from the right

$$u_{-k}(X) \to \mathcal{T}_r(k)e^{-ikX} \quad (X \to -\infty)$$

$$u_{-k}(X) \to e^{-ikX} + \mathcal{R}_r(k)e^{ikX} \quad (X \to \infty)$$

Taking the $X, X' \to -\infty$ limit and using the relations $\mathcal{R}(k)^* = \mathcal{R}(-k)$, $\mathcal{R}(k)\mathcal{R}(k)^* + \mathcal{T}(k)\mathcal{T}(k)^* = 1$, we get

$$\hat{G}_c(X_1, X_2; \theta) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} \left(e^{ik\delta X} + \mathcal{R}(k) e^{-ik\bar{X}} \right) G_k(\theta)$$

where $\delta X = X_1 - X_2$ and $\bar{X} = X_1 + X_2$

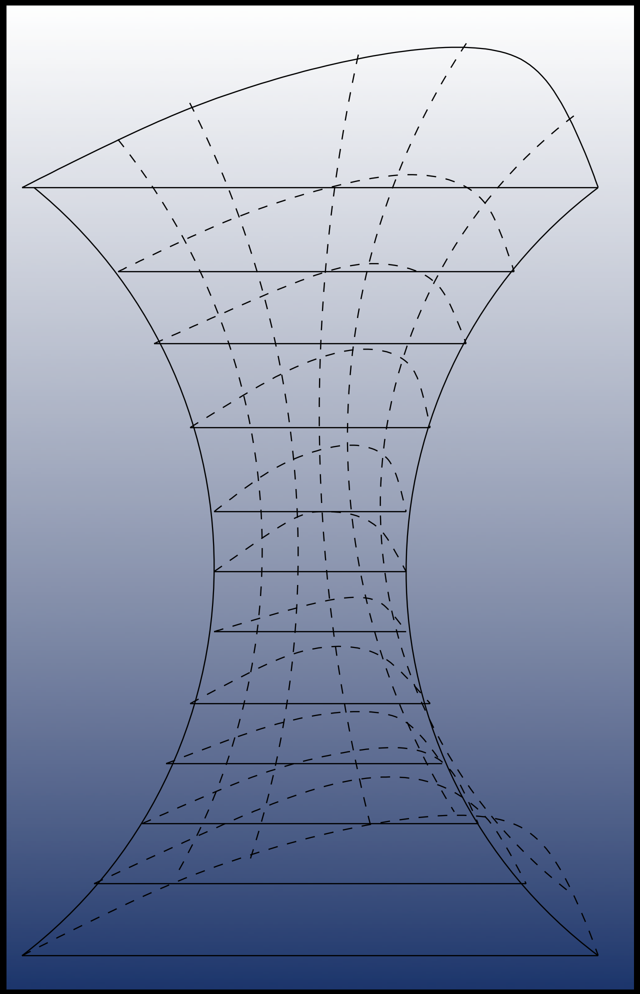
Thus for a given instanton geometry, we can get the Euclidean two point function by calculating the corresponding potential $U = a''/a$, then solving the scattering problem to get the reflection coefficient $\mathcal{R}$
And analytically continue to Lorentzian FRW by

$$X \to T + \pi i/2, \qquad \theta \to iR$$

Correlation function (a thin-wall example)

$$a(X) = e^X \quad (X \leq 0)$$

$$a(X) = \frac{1}{\cosh X} \quad (X \geq 0)$$



The potential is

$$U(X) = 1 - \frac{2}{\cosh^2 X}\Theta(X) - \delta(X)$$

and the reflection coefficient is

$$\mathcal{R}(k) = \frac{i(k+i)}{(2k+i)(k-i)}$$

Continuous modes

$$\hat{G}_c^{(\bar{X})}(X_1, X_2; \theta) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{i(k+i)}{(2k+i)(k-i)} e^{-ik\bar{X}} \frac{\sinh k(\pi - \theta)}{\sin\theta \sinh k\pi}$$

$$= \frac{1}{2\sin\theta} \frac{2\pi i}{2\pi} \sum_{n=2}^{\infty} e^{n\overline{X}} \frac{n+1}{(2n+1)(n-1)} \frac{1}{\pi} \left(e^{-in\theta} - e^{in\theta}\right)$$

$$+ \frac{1}{2\sin\theta} \frac{2\pi i}{2\pi} \frac{(-i)}{\pi} \partial_k \left\{ \frac{k+i}{2k+i} e^{-ik\bar{X}} \left(e^{k\pi - k\theta} - e^{-k\pi + k\theta}\right) \right\} \Big|_{k=i}$$

$$= \frac{1}{12\pi\sin\theta} \Bigg[-ie^{(-\bar{X}+i\theta)/2} \log \frac{1 + e^{(\bar{X}-i\theta)/2}}{1 - e^{(\bar{X}-i\theta)/2}} + ie^{-(\bar{X}+i\theta)/2} \log \frac{1 + e^{(\bar{X}+i\theta)/2}}{1 - e^{(\bar{X}+i\theta)/2}}$$

$$-4ie^{\bar{X}-i\theta} \log(1 - e^{\bar{X}-i\theta}) + 4ie^{\bar{X}+i\theta} \log(1 - e^{\bar{X}+i\theta}) + \frac{2}{3} ie^{\bar{X}-i\theta} - \frac{2}{3} ie^{\bar{X}+i\theta}$$

$$+ \frac{1}{2\pi\sin\theta} \Bigg[-\frac{i}{9} e^{\bar{X}} \left(e^{-i\theta} - e^{i\theta}\right) - \frac{2}{3}(-i\bar{X} + \pi - \theta) e^{\bar{X}-i\theta} + \frac{2}{3}(-i\bar{X} - \pi + \theta) e^{\bar{X}+i\theta}$$

Bound state

$$\hat{G}_B(X_1, X_2; \theta) = u_B(X_1) u_B(X_2) G_B(\theta) = \frac{2}{3} e^{\bar{X}} \frac{\cos\theta}{\sin\theta} \left(1 - \frac{\theta}{\pi}\right)$$

Analytically continue to Lorentzian space by $X \to T + \pi i/2$, $\theta \to iR$

and define $\delta T \equiv T_1 - T_2$, $\bar{T} \equiv T_1 + T_2$

we get the following $\bar{T}$-dependent part of the correlator $\langle \chi(T,0)\chi(T',R)\rangle^{(\bar{T})}$

$$\langle \chi(T,0)\chi(T',R)\rangle^{(\bar{T})} = \frac{\hat{G}^{(\bar{T})}(X_1,X_2;\theta)}{a(T_1)a(T_2)}$$

$$= -\frac{2}{3}i + \frac{Re^R}{3\pi \sinh R} + \frac{e^{-\bar{T}}}{12\pi \sinh R}\Bigg[ie^{-(\bar{T}+R)/2}\log\frac{1+ie^{(\bar{T}+R)/2}}{1-ie^{(\bar{T}+R)/2}} - ie^{-(\bar{T}-R)/2}\log\frac{1+ie^{(\bar{T}-R)/2}}{1-ie^{(\bar{T}-R)/2}}$$
$$+4e^{(\bar{T}+R)}\log(1+e^{-(\bar{T}+R)}) - 4e^{(\bar{T}-R)}\log(1+e^{(\bar{T}-R)}) + 4\bar{T}e^{(\bar{T}-R)}\Bigg]$$

The $\delta T = T_1 - T_2$ dependent part is

$$\langle \chi(T_1,0)\chi(T_2,R)\rangle^{(\delta T)} = -\frac{1}{2\pi}\frac{e^{-\bar{T}}}{\cosh\delta T - \cosh R}$$

Large separation behavior

The geodesic distance ℓ between the two points on $\mathcal{H}_3$ located at $(R_1, 0)$ and (R_2, α), where α is an angle in S^2, is given by

$$\cosh \ell = \cosh R_1 \cosh R_2 - \sinh R_1 \sinh R_2 \cos \alpha$$

In the large separation

$$\ell \sim R_1 + R_2 + \log(1 - \cos \alpha)$$

We take the late time limit as well as the large distance limit. Namely, we first take the $\bar{T} + \ell \to \infty$ limit and drop the terms with positive powers of $e^{-(\bar{T}+\ell)}$. The rest of the terms are given as an expansion in powers of $e^{\bar{T}-\ell}$

Two interesting things coming out of this limit

1. Discrete sum

$$G_1 = \sum_{\Delta=2}^{\infty} G_\Delta e^{(\Delta-2)(T_1+T_2)} \frac{e^{-(\Delta-1)\ell}}{\sinh \ell}$$

$$e^{(\Delta-2)(T_1+T_2)} \frac{e^{-(\Delta-1)\ell}}{\sinh \ell} \sim e^{(\Delta-2)(T_1+T_2)} e^{-\Delta R_1} e^{-\Delta R_2} (1-\cos\alpha)^{-\Delta}$$

2. Nonnormalizable mode

$$G_2 = \frac{\ell e^{\ell}}{3\pi \sinh \ell} \sim R_1 + R_2 + \log(1-\cos\alpha)$$

Summery

- AdS/CFT gives us the concrete holographic framework to describe the gravity in AdS

- It's likely that our universe is described by eternal inflation. In particular the universe was tunneling from false vacuum with higher cosmological constant

- We gave a circumstantial evidence for the existence of the holographic dual of this tunneling backgound in the spirit of AdS/CFT

Future works

- What is the boundary theory ?
- If there is the conformal theory, what is the central charge?
- What does the nonnormalizable mode imply? Liouville theory?
- How to include bubble collision in the picture?
- Does it have observational consequence of the tunneling event?
- It will be interesting to know if boundary theory can tell us the measure of the eternal inflation