Baryon asymmetry from hypermagnetic helicity in inflationary cosmology

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I. INTRODUCTION

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I A. Baryon Asymmetry of the Universe (BAU)

< Observation >

- **There exists a net excess of baryons over anti-baryons in the Universe.**

$\longrightarrow$ **Baryon asymmetry of the Universe (BAU)**

$$\boxed{\frac{n_{\mathrm{B}}}{s} = 0.92 \times 10^{-10}}$$

(Spergel et al., Astrophys. J. Suppl. Ser. 148, 175 [2003])

n_{B} : Baryon number density, s : Entropy density

< Sakharov's three conditions > (Sakharov, JETP Lett. 5, 24 [1967])

(1) Baryon number nonconservation

(2) C and CP violation

(3) A departure from thermal equilibrium

< Models >

Reviews of the BAU: **(Dolgov Phys. Rep. 222, 309 [1992])** **No. I-3**
(Dine and Kusenko, Rev. Mod. Phys. 76, 1 [2004])

(1) GUT baryogenesis

← Decay of heavy particles $(M_{\rm GUT} \approx 10^{16}{\rm GeV})$

(Yoshimura, Phys. Rev. Lett. 41, 281 [1978]; 42, 746E [1978])

(2) Electro Weak (EW) baryogenesis

$\leftarrow \triangle(B+L) = N(\neq 0)$

$\triangle(B-L) = 0$

Anomalous process (Sphaleron process)

(Kuzmin, Rubakov, and Shaposhnikov, Phys. Lett. 155B, 36 [1985])

(3) Leptogenesis

$\longleftarrow \triangle(B-L) = \tilde{N}(\neq 0)$

B : Baryon number

L : Lepton number

(Fukugita and Yanagida, Phys. Lett. B 174, 45 [1986])

(4) Affleck-Dine baryogenesis

← Coherent motion of scalar fields in supersymmetric theory

(Affleck and Dine, Nucl. Phys. B249, 361 [1985])

(5) Spontaneous baryogenesis

$\longleftarrow \mathcal{L} \sim (\partial_\mu \theta) J_{\rm B}$

θ : Scalar field (the resultant Goldstone boson)

$J_{\rm B}$: Baryon current

(Cohen and Kaplan, Phys. Lett. B 199, 251 [1987]; Nucl. Phys. B308, 913 [1988])

(6) Black hole evaporation

(Zel'dovich, Pis'ma Zh. Eksp. Teor. Fiz. 24, 29 [1976])

(Dolgov, Zh. Eksp. Teor. Fiz. 79, 337 [1980]; Phys. Rev. D 24, 1042 [1981])

< Axial anomaly > (Funakubo, Prog. Theor. Phys. 96, 475 [1996])

$$\partial_\mu j^\mu_{B+L} = \frac{N_f}{16\pi^2}[g^2 \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) - g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}]$$

$$\partial_\mu j^\mu_{B-L} = 0$$

N_f : The number of the generations

$g\ (g')$ and $F_{\mu\nu}(B_{\mu\nu})$: The gauge coupling and the field strength of the $SU(2)_L\ (U(1)_Y)$ gauge field $A_\mu(x)\ (B_\mu(x))$, respectively

Integration

< Baryon number >

$$B(t_f) - B(t_i) = \frac{N_f}{32\pi^2}\int_{t_i}^{t_f} d^4x \left[g^2 \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) - g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}\right]$$

$$= N_f\,[N_{CS}(t_f) - N_{CS}(t_i)]$$

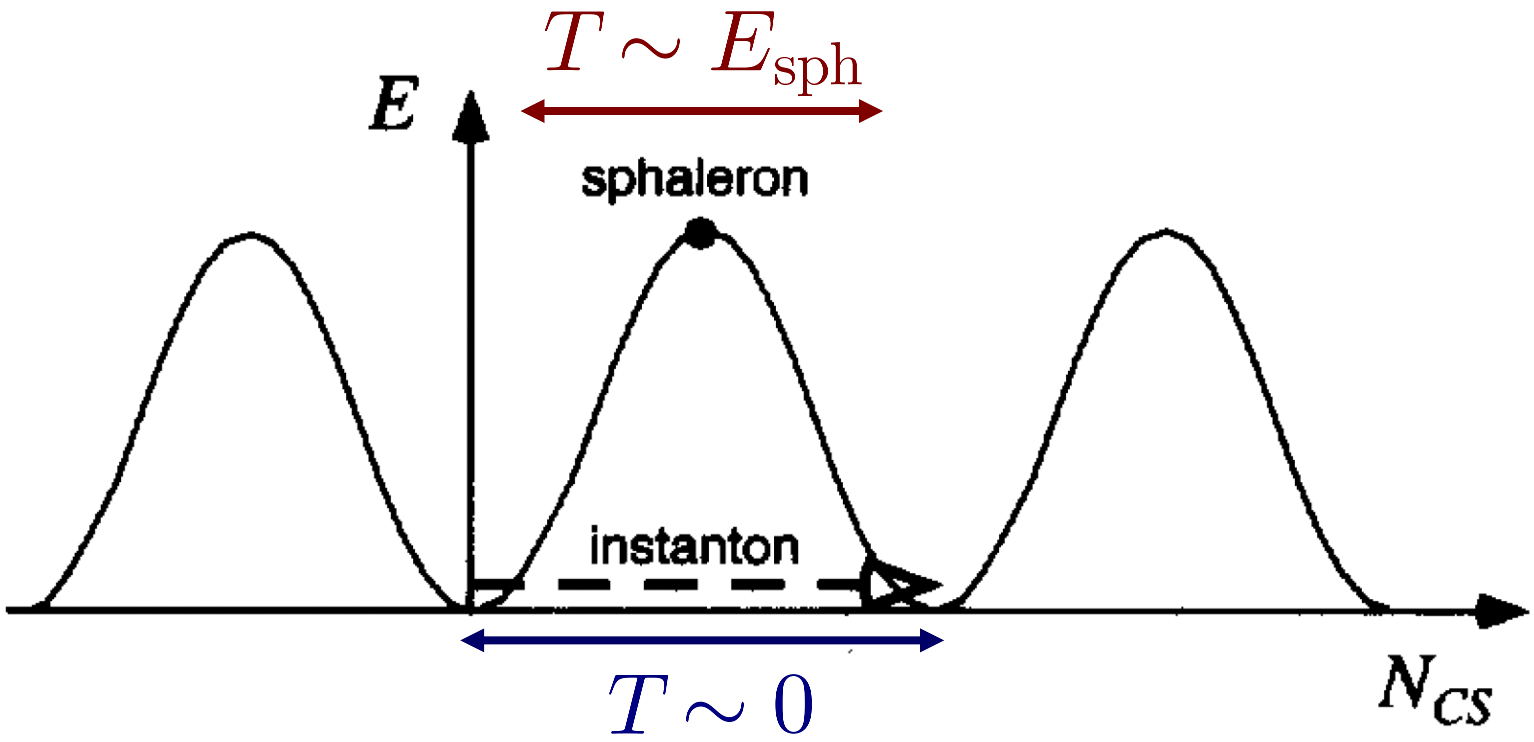
N_{CS} : Chern-Simons number

- Weyl gauge ($A_0 = 0$)

$$\rightarrow N_{CS}(t) = \frac{1}{32\pi^2}\int d^3x\,\epsilon_{ijk}\left[g^2 \mathrm{Tr}\left(F_{ij}A_k - \frac{2}{3}gA_iA_jA_k\right) - g'^2 B_{ij}B_k\right]_t$$

< Sphaleron process >

(Funakubo, Prog. Theor. Phys. 96, 475 [1996])



Schematic vacuum structure of the electroweak theory depicted along some direction in configuration space.

- **At zero temperature, the instanton transitions between vacua with different Chern-Simon numbers are suppressed.**
- **At finite temperature, these transitions can proceed via sphaleron.**

< I B. Inflationary cosmology >

Reviews: (Kolb and Turner, *The Early Universe* [1990]) (Riotto, arXiv:hep-ph/0210162)

< Einstein equation in general relativity >

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

: Matter and energy relate to the spacetime.

⟶ Spacetime itself varies in time.

$G_{\mu\nu} = R_{\mu\nu} - (1/2) R g_{\mu\nu}$: Einstein tensor, $T_{\mu\nu}$: Energy-momentum tensor

< Big Bang cosmology >

⟶ Big Bang theory can explain the following three observational facts simultaneously.

(1) Hubble's expansion low

(2) 2.7 K Cosmic Microwave Background (CMB) radiation

(3) Primordial nucleosynthesis

< Problems of big bang cosmology >

(1) Horizon problem (Homogeneity and Isotropy)

→ The problem that in the recombination epoch of electron and proton the Universe is homogeneous and isotropic over much lager scale than the Hubble horizon at that time.

(2) Flatness problem

< Friedmann-Robertson-Walker (FRW) metric >

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right] \quad (c = \hbar = 1)$$

$a(t)$: Scale factor, H : Hubble parameter

$K = +1$: **Closed Universe**

$K = 0$: **Flat Universe**

$K = -1$: **Open Universe**

▪ **Friedmann equation**

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{K}{a^2} \propto a^{-2}$$

Observationally, energy term is dominant.

$$\rho_{\mathrm{m}} \propto a^{-3},\ \rho_{\mathrm{r}} \propto a^{-4}$$

$\dot{} = \partial/\partial t$, H : Hubble parameter

→ The problem that in the Planck epoch the curvature term is tens of order of magnitude smaller than the energy term.

(3) Origin of primordial density perturbations

< Inflationary cosmology >

In the early Universe, the scale of the Universe grew exponentially in time when the potential energy of a scalar field, called an "inflaton", dominated.

(Sato, Mon. Not. R. Astron. Soc. 195, 467 [1981])
(Guth, Phys. Rev. D 23, 347 [1981])

→ Inflation accounts for the observed degree of homogeneity, isotropy, and flatness of the present Universe.

→ Inflation naturally produces effects on very large scales, larger than Hubble horizon, starting from microphysical processes operating on a causally connected volume.

- **Friedmann equation:** $\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho$

→ If the Universe is dominated by a matter whose energy density hardly decreases by the cosmic expansion (e.g., vacuum energy), exponential expansion is realized.

$$a(t) \propto e^{Ht}, \quad H = \sqrt{\frac{8\pi G}{3}\rho_{\rm inf}}, \quad \rho_{\rm inf} \sim {\rm const.}$$

$$\frac{K}{a^2} = (\Omega_{\rm tot} - 1)H^2, \quad \Omega_{\rm tot} \equiv \frac{8\pi G}{3H^2}\rho$$

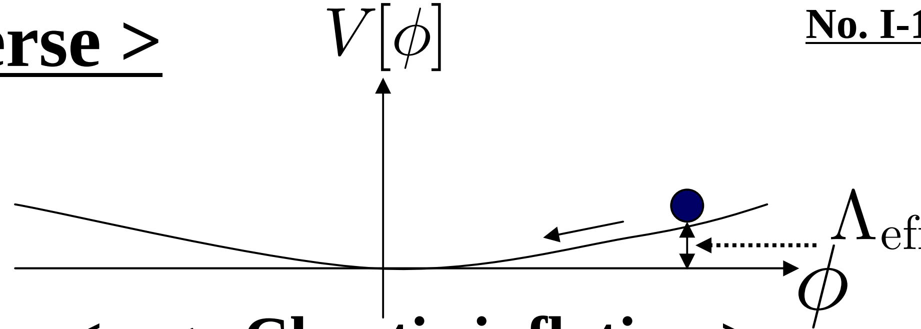
→ **If inflation is realized,** $\Omega_{\rm tot} \to 1$ **.**

→ In this case, the Hubble horizon expands exponentially and the curvature term decreases rapidly. Hence the flat Universe is realized.

⇩

The horizon and flatness problems are resolved simultaneously.

< Inflation in the early Universe >



→ **Inflation is realized by the potential energy of some slow-rolling scalar field, which is called "inflaton".**

< e.g., Chaotic inflation >
[Linde, Phys. Lett. B 129, 177 (1983)]

・**Friedmann equation** **(Case of the spatially flat Universe)**

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\left(\cancel{\frac{1}{2}\dot{\phi}^2} + V[\phi]\right) \approx \mathrm{const.} \Longrightarrow a(t) \propto e^{Ht}$$

↑ **Exponential expansion is realized.**

・**Equation of motion for ϕ**

$$\cancel{\ddot{\phi}} + 3H\dot{\phi} + \frac{dV[\phi]}{d\phi} = 0$$

→ As ϕ nears the potential minimum, the potential gradient becomes steep and the potential energy decreases rapidly, and consequently inflation ends.

⇓

ϕ begins to rapidly oscillate around the potential minimum. Then the oscillation energy is thermalized and it is released as radiation, and finally hot big-bang Universe is realized.

< History of the Universe in inflationary cosmology >

Time evolution [The temperature of the Universe]

- $O\left(10^{19}\mathrm{GeV}\right)$ ← **Planck epoch**
- ← **Inflation** ⇨ **Reheating**
- $O\left(10^{2}\mathrm{GeV}\right)$ ← **EWPT**
- $O\left(10^{2}\mathrm{MeV}\right)$ ← **QCDPT**
- $O\left(\mathrm{MeV}\right)$ ← **Primordial nucleosynthesis（He etc.）**
- $O\left(\mathrm{eV}\right)$ ← **Radiation dominated stage** ⇨ **Matter dominated stage**
- $\simeq 0.34\mathrm{eV}$ ← **Recombination of electron and proton**
- $2.3\times 10^{-4}\mathrm{eV}$ ← **Present time**

Generation of baryon number of the Universe

< I C. Generation of the BAU by large-scale hypermagnetic fields >

- The Friedmann-Robertson-Walker (FRW) spacetime is conformally flat.

⇨ **However, if the conformal invariance of the Maxwell theory is broken by some mechanism in the inflationary stage, electromagnetic quantum fluctuations can be generated.**

▪ Coupling of the dilaton to electromagnetic fields

(Ratra, Astrophys. J. 391, L1 [1992]) (KB & Yokoyama, Phys. Rev. D 69, 043507 [2004])

$$\mathcal{L} = -\frac{1}{4}e^{-\lambda\kappa\Phi}F_{\mu\nu}F^{\mu\nu}$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

: Electromagnetic field-strength tensor

Φ : Dilaton

A_μ : $U(1)$ gauge field

λ : Dimensionless constant, $\kappa = \sqrt{8\pi G}$, G : Newton's constant

→ **Breaking of the conformal invariance of the Maxwell theory**

⇩

Generation of large-scale magnetic fields in inflationary cosmology

↑

Before the the electroweak phase transition (EWPT), these fields are generated as hypermagnetic fields.

▪ Axion-like coupling of a pseudoscalar field to electromagnetic fields

(Brustein & Oaknin, Phys. Rev. Lett. 82, 2628 [1999]; Phys. Rev. D 60, 023508 [1999])

(Giovannini, Phys. Rev. D 61, 063004 [2000]; Phys. Rev. D 61, 063502 [2000])

$$\mathcal{L} = -\frac{1}{4} g_{\mathrm{ps}} \frac{\phi}{M} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

$\epsilon^{\mu\nu\rho\sigma}$: Levi-Civita tensor

ϕ : Pseudo scalar field, g_{ps} : Coupling constant, M : Mass scale

→ **Generation of the magnetic helicity, which is the volume integration of the inner product between a magnetic field and a vector potential.**

If the hypermagnetic helicity exists before the EWPT,

⇩

Generation of the BAU due to the Abelian anomaly

(Giovannini & Shaposhnikov, Phys. Rev. Lett. 80, 22 [1998]; Phys. Rev. D 57, 2186 [1998])

< This study >

- In addition to the existence of the dilaton coupled to hypercharge electromagnetic fields, we assume the existence of a pseudoscalar field with an axion-like coupling to hypercharge electromagnetic fields, and we consider the generation of the BAU.

- We consider the generation of the hypermagnetic helicity, i.e., the Chern-Simons number, through the coupling between a pseudoscalar field and hypercharge electromagnetic fields in the inflationary stage.

- The generated Chern-Simons number is converted into fermions at the EWPT owing to the Abelian anomaly.

II. MODEL

II A. Action

II B. Equation of motion

< II A. Action >

$$S = \int d^4x \sqrt{-g} \left[\mathcal{L}_{\text{inflaton}} + \mathcal{L}_{\text{dilaton}} + \mathcal{L}_{\text{ps}} + \mathcal{L}_{\text{HEM}} \right]$$

(1) Inflaton φ

$$\mathcal{L}_{\text{inflaton}} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - U[\varphi]$$

(2) Dilaton Φ

$$\mathcal{L}_{\text{dilaton}} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - V[\Phi]$$

(3) Pseudo scalar field ϕ

$$\mathcal{L}_{\text{ps}} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - W[\phi]$$

(4) $U(1)_Y$ **gauge field** Y_μ

$$\mathcal{L}_{\text{HEM}} = -\frac{1}{4} f(\Phi) \left(Y_{\mu\nu} Y^{\mu\nu} + g_{\text{ps}} \frac{\phi}{M} Y_{\mu\nu} \tilde{Y}^{\mu\nu} \right)$$

Breaking of the conformal invariance

$$V[\Phi] = \bar{V} \exp\left(-\tilde{\lambda}\kappa\Phi\right)$$

$\bar{V}$: Constant, $\kappa^2 \equiv 8\pi/M_{\text{Pl}}^2$, M_{Pl} : Planck mass

$\lambda,\ \tilde{\lambda}(>0)$: Dimensionless constants

$$W[\phi] = \frac{1}{2} m^2 \phi^2$$

m : Mass of a pseudo scalar field ϕ

$$f(\Phi) = \exp\left(-\lambda\kappa\Phi\right)$$

$g_{\text{ps}} = \bar{g}_{\text{ps}} \alpha'/(2\pi)$, $\alpha' = g'^2/(4\pi)$

$\bar{g}_{\text{ps}}$: Numerical factor, g' : $U(1)_Y$ gauge coupling constant

$Y_{\mu\nu} = \nabla_\mu Y_\nu - \nabla_\nu Y_\mu$: $U(1)_Y$ hypercharge field strength

$\tilde{Y}^{\mu\nu} = (1/2)\epsilon^{\mu\nu\rho\sigma} Y_{\rho\sigma}$

< II B. Equation of motion >

< Spatially flat Friedmann-Robertson-Walker (FRW) spacetime >

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) d\boldsymbol{x}^2$$

$a(t)$: Scale factor

< Equation of motion >

▪**Background homogeneous scalar fields** $\dot{} = \partial/\partial t$, H : Hubble parameter

(1) $\ddot{\varphi} + 3H\dot{\varphi} + \frac{dU[\varphi]}{d\varphi} = 0$, **(2)** $\ddot{\Phi} + 3H\dot{\Phi} + \frac{dV[\Phi]}{d\Phi} = 0$, **(3)** $\ddot{\phi} + 3H\dot{\phi} + \frac{dW[\phi]}{d\phi} = 0$

▪**Background Friedmann equation** **(Case of the slow-roll inflation)**

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{3}\left(\rho_\varphi + \rho_\Phi + \rho_\phi\right), \quad \rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + U[\varphi], \; \rho_\Phi = \frac{1}{2}\dot{\Phi}^2 + V[\Phi], \; \rho_\phi = \frac{1}{2}\dot{\phi}^2 + W[\phi]$$

$\rho_\varphi \gg \rho_\Phi, \; \rho_\varphi \gg \rho_\phi$ →

$$H^2 \approx \frac{\kappa^2}{3}\rho_\varphi \equiv {H_{\mathrm{inf}}}^2$$

H_{inf} : Hubble constant in the inflationary stage

▪ **$U(1)_Y$ gauge field**

By using the Coulomb gauge : $Y_0(t, \boldsymbol{x}) = 0$, $\partial_j Y^j(t, \boldsymbol{x}) = 0$,

(4) $$\ddot{Y}_i(t, \boldsymbol{x}) + \left(H + \frac{\dot{f}}{f}\right)\dot{Y}_i(t, \boldsymbol{x}) - \frac{1}{a^2}\partial_j\partial_j Y_i(t, \boldsymbol{x}) - \frac{g_{\mathrm{ps}}}{M}\frac{1}{af}\frac{d(f\phi)}{dt}\epsilon^{ijk}\partial_j Y_k(t, \boldsymbol{x}) = 0$$

▪ **Scale factor**

$$a(t) = a_1 \exp\left[H_{\rm inf}(t - t_1) \right]$$

t_1 : Time when a given comoving wavelength $2\pi/k$ of the $U(1)_Y$ gauge field first crosses outside the horizon during inflation, $k/(a_1 H_{\rm inf}) = 1$

▪ **Solution of the dilaton Φ** **(Case in which $\left| \ddot{\Phi}/(H_{\rm inf}\dot{\Phi}) \right| \ll 1$)**

$$\Phi = \frac{1}{\tilde{\lambda}\kappa} \ln\left[\tilde{\lambda}^2 w H_{\rm inf}\left(t - t_{\rm R}\right) + \exp\left(\tilde{\lambda}\kappa\Phi_{\rm R}\right) \right]$$

$$w \equiv \frac{\bar{V}}{3H_{\rm inf}^2/\kappa^2} \approx \frac{\bar{V}}{\rho_\varphi}$$

$t_{\rm R}$: End of inflation

$\Phi_{\rm R}$: Dilaton field amplitude at $t_{\rm R}$

We consider the case in which after inflation, the dilaton is stabilized by some mechanism that generates a potential minimum at $\Phi = \Phi_{\rm R} = 0$.

⟶ **After $t_{\rm R}$, $f = 1$, so that the standard Maxwell theory is recovered.**

▪ **Solution of a pseudoscalar field ϕ**

$$\phi = \phi_1 \exp\left\{ \frac{3}{2} \left[-1 \pm \sqrt{1 - \left(\frac{2m}{3H_{\rm inf}} \right)^2} \right] H_{\rm inf}\left(t - t_1\right) \right\}$$

III. GENERATION OF THE BAU

III A. Evolution of the $U(1)_Y$ gauge field

III B. Baryon asymmetry of the Universe (BAU)

< III A. Evolution of the $U(1)_Y$ gauge field >

< Quantization of $Y_\mu(t, \boldsymbol{x})$ >

▪**Canonical momenta** $\pi_0 = 0, \quad \pi_i = f(\Phi)a(t)\dot{Y}_i(t, \boldsymbol{x})$

▪**Canonical commutation relation**

$$[Y_i(t, \boldsymbol{x}), \pi_j(t, \boldsymbol{y})] = i \int \frac{d^3k}{(2\pi)^3} e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})} \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)$$

⇩

< Expression for $Y_i(t, \boldsymbol{x})$ >

$\boldsymbol{k}$: Comoving wave number, $k = |\boldsymbol{k}|$

$$Y_i(t, \boldsymbol{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[\hat{b}(\boldsymbol{k}) Y_i(t, \boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{b}^\dagger(\boldsymbol{k}) Y_i^*(t, \boldsymbol{k}) e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\right]$$

$$\left[\hat{b}(\boldsymbol{k}), \hat{b}^\dagger(\boldsymbol{k}')\right] = \delta^3(\boldsymbol{k} - \boldsymbol{k}'), \quad \left[\hat{b}(\boldsymbol{k}), \hat{b}(\boldsymbol{k}')\right] = \left[\hat{b}^\dagger(\boldsymbol{k}), \hat{b}^\dagger(\boldsymbol{k}')\right] = 0$$

$\hat{b}(\boldsymbol{k})$: Annihilation operator

$\hat{b}^\dagger(\boldsymbol{k})$: Creation operator

▪**Normalization condition** $Y_i(k,t)\dot{Y}_j^*(k,t) - \dot{Y}_j(k,t)Y_i^*(k,t) = \dfrac{i}{fa}\left(\delta_{ij} - \dfrac{k_i k_j}{k^2}\right)$

⟶ **In order to consider the components of circular polarizations, we choose x^3 axis to lie along the spatial momentum direction $\boldsymbol{k}$ and define $Y_\pm(k,t) \equiv Y_1(k,t) \pm iY_2(k,t)$.**

< Equation for $Y_\pm(k,t)$ >

$$\ddot{Y}_\pm(k,t) + \left(H_{\rm inf} + \frac{\dot{f}}{f}\right)\dot{Y}_\pm(k,t) + \left[\left(\frac{k}{a}\right)^2 \pm \frac{g_{\rm ps}}{M}\left(\frac{\dot{f}}{f}\phi + \dot{\phi}\right)\left(\frac{k}{a}\right)\right] Y_\pm(k,t) = 0$$

→ **We numerically solve this equation.**

- **During inflation** $(t_1 \le t \le t_{\rm R})$, $Y_\pm(k,t) = C_\pm(t) Y_\pm(k,t_1)$.

$$C_\pm(t_1) = 1$$

$$|Y_\pm(k,t_1)| \approx \frac{1}{\sqrt{2kf(t_1)}}$$

: Solution of the above equation in the short-wavelength limit.

- **We numerically solve the evolution of** $C_\pm(t)$.

< Numerical results of the evolution of $C_{\pm}(t)$ >

Fig. 1
[Case (ii)] $m < H_{\mathrm{inf}}$

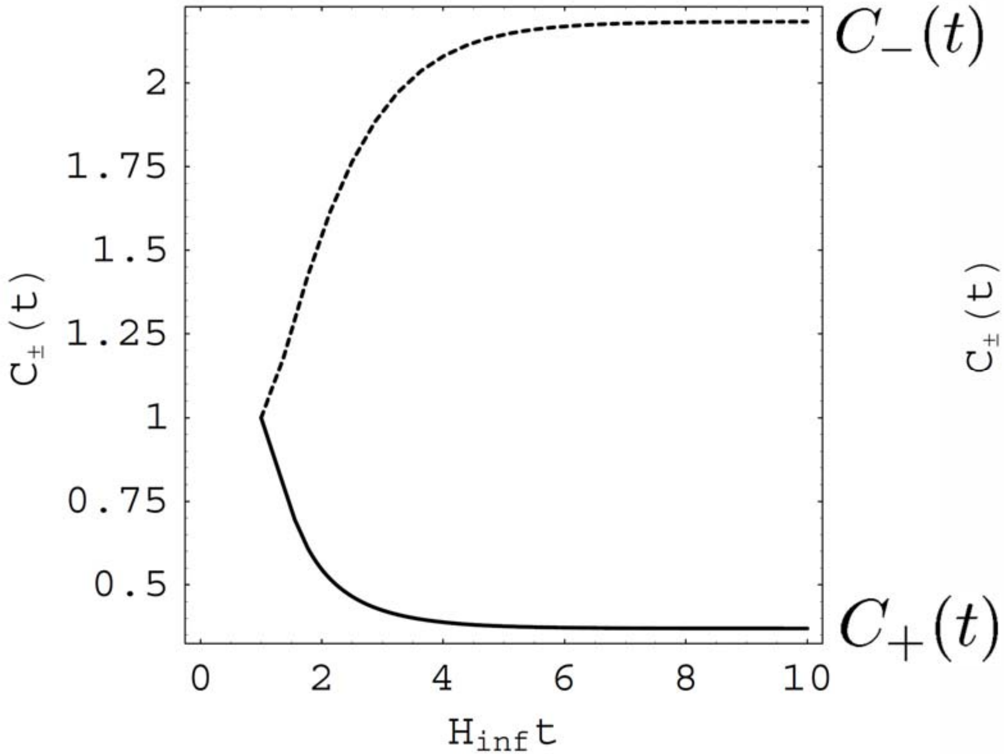
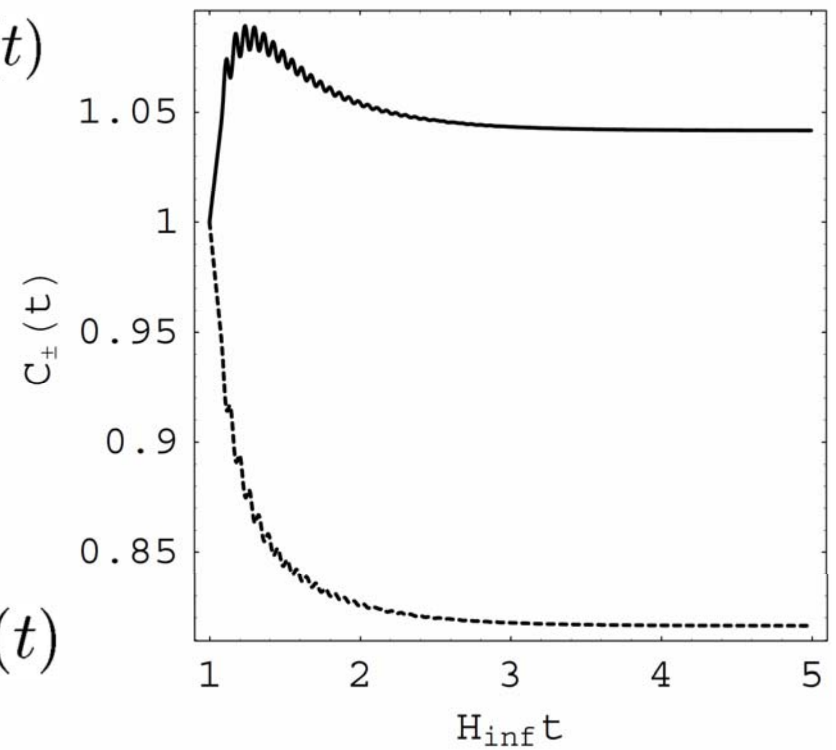


Fig. 2
[Case (iii)] $m \gg H_{\mathrm{inf}}$



- **The solid curves represents $C_+(t)$ and the dotted curves represents $C_-(t)$.**
- **After several Hubble expansion times, $C_{\pm}(t)$ becomes almost constant.**

< III B. Baryon asymmetry of the Universe (BAU) >

< Proper hyperelectric and hypermagnetic fields >

$$E_{Y\,i}^{\mathrm{proper}}(t, \boldsymbol{x}) = a^{-1} E_{Y\,i}(t, \boldsymbol{x}) = -a^{-1} \dot{Y}_i(t, \boldsymbol{x})$$

$$B_{Y\,i}^{\mathrm{proper}}(t, \boldsymbol{x}) = a^{-1} B_{Y\,i}(t, \boldsymbol{x}) = a^{-2} \epsilon_{ijk} \partial_j Y_k(t, \boldsymbol{x}) \quad (\epsilon_{123} = 1)$$

$E_{Y\,i}(t, \boldsymbol{x})$, $B_{Y\,i}(t, \boldsymbol{x})$: Comoving hyperelectric and hypermagnetic fields

- **The Chern-Simons number stored in the hypercharge electromagnetic fields, i.e., the hypermagnetic helicity, is converted into fermions at the EWPT owing to the Abelian anomaly.**

< Baryon number density n_{B} >

$$n_{\mathrm{B}} = \frac{n_{\mathrm{f}}\, g'^2}{8\pi^2} \int^t \boldsymbol{E}_Y \cdot \boldsymbol{B}_Y d\tilde{t}$$

n_{f} : Number of fermionic generations

(We use $n_{\mathrm{f}} = 3$.)

- **We assume that the EWPT is strongly first order.**

(e.g., Funakubo, Tao, and Toyoda, Prog. Theor. Phys. <u>114</u>, 369 [2005])

< $n_{\rm B}/s$ after the EWPT >

$$\frac{n_{\rm B}}{s} = n_{\rm f}\frac{g'^2}{8\pi^2}\frac{1}{k}\frac{\rho_B(L,t)}{s}\;\frac{|C_+(t_{\rm R})|^2-|C_-(t_{\rm R})|^2}{|C_+(t_{\rm R})|^2+|C_-(t_{\rm R})|^2}$$

$L = 2\pi/k$: Comoving scale

$$\rho_B(L,t) = \frac{1}{8\pi^2}\frac{1}{f(t_1)}\left(\frac{k}{a}\right)^4\left[|C_+(t_{\rm R})|^2+|C_-(t_{\rm R})|^2\right]$$

$$f(t_1) = \left(1-\tilde{\lambda}^2 wN\right)^{-X}$$

$$N = H_{\rm inf}\,(t_{\rm R}-t_1)$$

$$X \equiv \frac{\lambda}{\tilde{\lambda}}$$

< Estimates of the present value of $n_{\rm B}/s$ >

	$\lvert n_{\rm B}/s \rvert$	$B(H_0^{-1}, t_0)$ [G]	$H_{\rm inf}$ [GeV]	m [GeV]	$C_+(t_{\rm R})$	$C_-(t_{\rm R})$	X
(i)	3.6×10^{-10}	2.7×10^{-24}	1.0×10^{14}	1.0×10^{12}	0.367	2.23	-1.11×10^2
(ii)	1.0×10^{-10}	1.4×10^{-24}	1.0×10^{10}	1.0×10^{9}	0.369	2.18	-1.33×10^2
(iii)	1.5×10^{-10}	3.5×10^{-24}	1.0×10^{10}	1.0×10^{12}	1.04	0.816	-1.35×10^2
(iv)	0.96×10^{-10}	1.4×10^{-24}	1.0×10^{5}	1.0×10^{3}	0.368	2.17	-1.65×10^2
(v)	2.8×10^{-10}	4.7×10^{-24}	1.0×10^{5}	1.0×10^{7}	1.04	0.808	-1.68×10^2

Present field strength of the magnetic fields on Hubble horizon scale

($w = 1/(75)$, $g_{\rm ps} = 1.0$, $\tilde{\lambda} = 1.0$, $\phi_1 = M = m$,
H_0 : Hubble constant at the present time t_0)

$\longrightarrow$ **In this scenario, both large-scale magnetic fields and the BAU can be generated at the same time.**

IV. CONCLUSION

- We have studied the generation of the BAU from the hypermagnetic helicity in inflationary cosmology, taking into account the breaking of the conformal invariance of the hypercharge electromagnetic fields by introducing both a coupling with the dilaton and that with a pseudoscalar field.

→ **Owing to the coupling between the pseudoscalar field and the hypercharge electromagnetic fields, the hypermagnetic helicity (Chern-Simons number) is generated in the inflationary stage.**

→ **The Chern-Simons number is converted into fermions at the EWPT due to the Abelian anomaly, and at the same time the hypermagnetic fields are replaced by the ordinary magnetic fields.**

→ **The generated fermions can survive after the EWPT if the EWPT is strongly first order.**

⇩

- We have found that if the magnetic fields with sufficient strength on the horizon scale at the present time are generated, the resultant value of the ratio of the baryon number density to the entropy density, n_{B}/s, can be as large as 10^{-10}.

→ This scenario can explain both the origin of large-scale magnetic fields in galaxy and cluster of galaxies and that of the BAU simultaneously.

＜Appendix＞

< Sakharov's three conditions > (Detailed explanation)

(1) $\not{B}$ ← If this condition is not met, the BAU cannot be generated.

(2) $\not{C}$ **and** $\not{CP}$ ← If not, the baryon number cannot vary because the probability that baryon number increasing reactions occur is equal to the probability that baryon number decreasing reactions occur.

$$< n_{\mathrm{B}} >= \mathrm{Tr}[e^{-\beta H} n_B]$$
$$= \mathrm{Tr}[e^{-\beta H} C^{-1} n_B \mathrm{C}]$$
$$= -\mathrm{Tr}[e^{-\beta H} n_B] \Longrightarrow < n_{\mathrm{B}} >= 0$$

- $C^{-1} n_{\mathrm{B}} C = - n_{\mathrm{B}}$
- $(CP)^{-1} n_{\mathrm{B}} (CP) = - n_{\mathrm{B}}$

(3) A departure from thermal equilibrium

↳ If not, the baryon number cannot vary because the number of baryon number increasing reactions is equal to that of baryon number decreasing reactions.

→ The mass of particles is equal to that of anti-particles. Moreover, the chemical potential is zero in thermal equilibrium. $\Longrightarrow n_B = n_{\bar{B}}$

< Electroweak baryogenesis >

$$\triangle(B+L) = N(\neq 0)$$
$$\triangle(B-L) = 0$$

← **Anomalous process (Sphaleron process)**

- The condition that the EWPT is the first order and that after the EWPT, $\Gamma_{\rm sph} < H$, has to be met.

→ In $T > T_{\rm EW}$, the sphaleron process is in thermal equilibrium and hence $(B+L)$ can vary. However, if the state remains equilibrium, the third Sakharov's condition does not met. Thus, since the number of processes which increase $(B+L)$ is equal to that of processes which decrease $(B+L)$, $(B+L)$ is finally washed out.

- If the EWPT is the first order, the non-equilibrium state can be realized on the surface of the bubble of the phase in which the symmetry is broken. Hence, e.g., the processes which increase $(B+L)$ are dominant, and thus $(B+L)$ is finally generated.

- If the sphaleron process remains effective after the EWPT, the net $(B+L)$ again becomes zero. Hence, after the EWPT, the condition $\Gamma_{\mathrm{sph}} < H$ has to be met.

< Leptogenesis >

$$\triangle(B-L) = \tilde{N}\,(\neq 0)$$

During the stage in which $\Gamma_{\rm sph} \geq H$ is satisfied, $\triangle(B-L)$ is generated through lepton number non-conservation processes.

⇒ **In the anomalous process,** $(B-L)$ **is conserved. Moreover,** $(B+L)$ **is washed out. Hence, if** $(B-L)$ **is generated at the initial stage, the baryon number can be generated.**

< Axial anomaly > (Funakubo, Prog. Theor. Phys. 96, 475 [1996])

$$\partial_\mu j^\mu_{B+L} = \frac{N_f}{16\pi^2}[g^2 \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) - g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}]$$

$$\partial_\mu j^\mu_{B-L} = 0$$

N_f : The number of the generations

$g\ (g')$ and $F_{\mu\nu}(B_{\mu\nu})$: The gauge coupling and the field strength of the $SU(2)_L\ (U(1)_Y)$ gauge field $A_\mu(x)\ (B_\mu(x))$, respectively

$$B(t_f) - B(t_i) = \frac{N_f}{32\pi^2}\int_{t_i}^{t_f} d^4x\ \left[g^2 \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) - g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}\right]$$
$$= N_f\,[N_{CS}(t_f) - N_{CS}(t_i)]$$

- Weyl gauge ($A_0 = 0$)

$$\rightarrow N_{CS}(t) = \frac{1}{32\pi^2}\int d^3x\,\epsilon_{ijk}\left[g^2\mathrm{Tr}\left(F_{ij}A_k - \frac{2}{3}gA_iA_jA_k\right) - g'^2 B_{ij}B_k\right]_t$$

- **For a pure-gauge configuration,** $F_{ij} = B_{ij} = 0$**,**

  N_{CS} **takes an integer.**

- **The** $U(1)$ **contribution always vanishes for a pure gauge.**

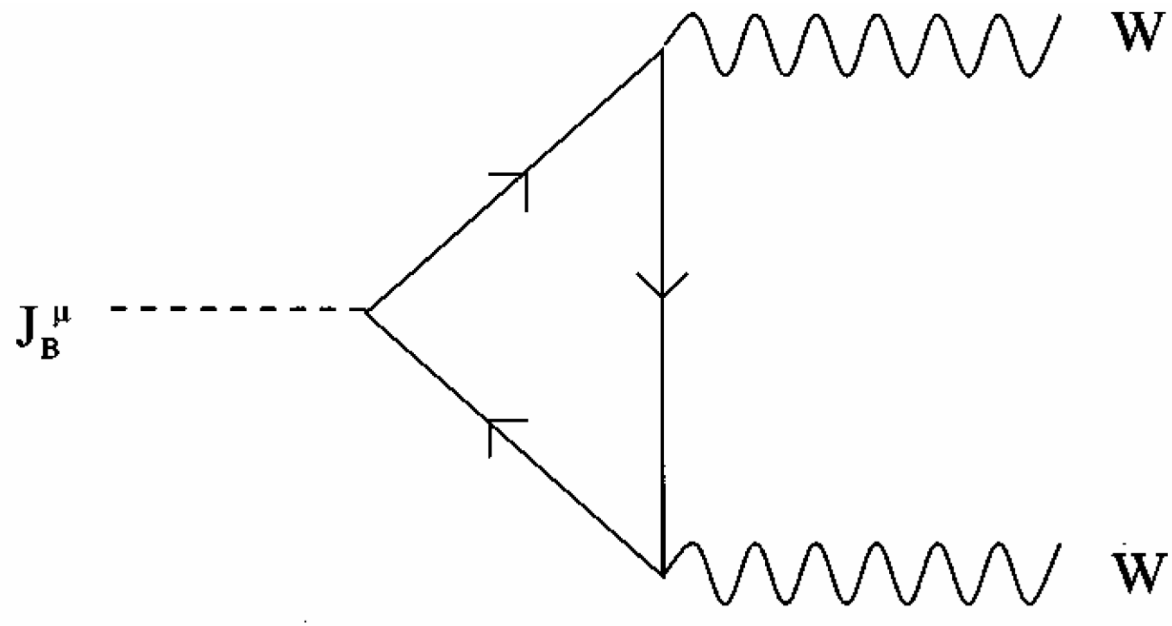
< This study >

- In the ordinary discussion of Chern-Simons number in terms of axial anomaly, in general, both the state before and after the anomalous process are vacuum. Hence, it is assumed that there do not exist finite electromagnetic fields ($SU(2)_L$ electromagnetic fields or $U(1)_Y$ electromagnetic fields).

- **However, if there exist external fields which have finite energy in the initial state, the baryon number can survive.**

In this study, we consider the case in which the external fields are generated as the quantum fluctuations of the electromagnetic fields.

(Trodden, Rev. Mod. Phys. 71, 1463 [1999])



The triangle diagram contributing to the anomaly in the baryon and lepton number current.

< Generation of primordial density perturbations from inflation >

No. B-1

- Since ϕ is a quantum field, its evolution is not homogeneous classical solution $\phi(t,x)=\phi_{\rm cl}(t)$, there exist its quantum fluctuations $\delta\phi(t,\vec{x})$.

$$\phi(t,\vec{x})=\phi_{\rm cl}(t)+\delta\phi(t,\vec{x})=\phi_{\rm cl}\left(t+\delta t(t,\vec{x})\right)$$

- **Quantum fluctuations influence the spacetime metric.**

$$a(t,\vec{x})\propto e^{H(t+\delta t)}\cong a(t)\left(1+H\delta t\right) \qquad \delta t(t,\vec{x})\equiv\frac{\delta\phi(t,\vec{x})}{\dot{\phi}_{\rm cl}}$$

$$\longrightarrow \quad ds^2=dt^2-a^2\left(t+\delta t(t,\vec{x})\right)d\vec{x}^2$$

$$\cong dt^2-a^2(t)\left(1+2\underline{H\delta t(t,\vec{x})}\right)d\vec{x}^2$$

← **Curvature perturbataions**

- Quantum fluctuations with the amplitude $\delta\phi=H/(2\pi)$ on the initial wavelength H^{-1} are generated per cosmic expansion time H^{-1}, and they are expanded by the cosmic expansion.

- **Curvature perturbations observed:** $H\delta t\cong\dfrac{H^2}{2\pi|\dot{\phi}_{\rm cl}|}$

→ During inflation both H and $\dot{\phi}_{\rm cl}$ varies very slowly. Hence quantum fluctuations with almost scale-invariant spectrum are generated.

< Baryon number density $n_{\rm B}$ >

$$n_{\rm B} = -\frac{n_{\rm f}}{2}\Delta n_{\rm CS}, \quad \Delta n_{\rm CS} = -\frac{g'^2}{4\pi^2}\int^t \boldsymbol{E}_Y \cdot \boldsymbol{B}_Y d\tilde{t}$$

$n_{\rm CS}$: Chern-Simons number density

- **Fourier modes:**

$$E_{Y\pm}^{\rm proper}(k,t) = E_{Y1}^{\rm proper}(k,t) \pm iE_{Y2}^{\rm proper}(k,t)$$

$$B_{Y\pm}^{\rm proper}(k,t) = B_{Y1}^{\rm proper}(k,t) \pm iB_{Y2}^{\rm proper}(k,t)$$

$$\longrightarrow \quad E_{Y\pm}^{\rm proper}(k,t)B_{Y\pm}^{\rm proper}(k,t) = \pm\frac{1}{2}\frac{1}{k}\frac{\partial\left[B_{Y\pm}^{\rm proper}(k,t)\right]^2}{\partial t}$$

< Energy density of hypermagnetic fields in real space >

$$\rho_{B_Y}(L,t) = \frac{k^3}{4\pi^2}\left[\left|B_{Y+}^{\rm proper}(k,t)\right|^2 + \left|B_{Y-}^{\rm proper}(k,t)\right|^2\right] f,$$

$$\left|B_{Y\pm}^{\rm proper}(k,t)\right|^2 = \frac{1}{a^2}\left(\frac{k}{a}\right)^2 |Y_\pm(k,t)|^2$$

< Chern-Simons number density during inflation >

$$\Delta n_{\rm CS} = -\frac{g'^2}{4\pi^2}\frac{1}{k}\frac{1}{f}\rho_{B_Y}(L,t)\mathcal{A}(t), \quad \mathcal{A}(t) = \frac{|C_+(t)|^2 - |C_-(t)|^2}{|C_+(t)|^2 + |C_-(t)|^2}$$

< Notes >

- **We consider the case in which the reheating stage is much before the EWPT ($T_{\rm EW} \sim 100{\rm GeV}$).**

- **The conductivity of the Universe $\sigma_{\rm c}$ is negligibly small during inflation, because there are few charged particles at that time. After reheating, however, a number of charged particles are produced, so that the conductivity immediately jumps to a large value: $\sigma_{\rm c} \gg H \ (t \geq t_{\rm R})$.**

→ **The proper hypermagnetic fields evolve in proportion to $a^{-2}(t)$ after reheating. Furthermore, the hypermagnetic helicity, i.e., the Chern-Simons number, is conserved.**

- **The Chern-Simons number will be released at the EWPT in the form of fermions, which will not be destroyed by the sphaleron processes if the EWPT is strongly first order.**

- **At the EWPT the hypermagnetic fields are replaced by the ordinary magnetic fields.**