2017 Fall PHYS2310 電磁學 (Electromagnetism) Final Exam. (double sides) [Griffiths Chs. 4-7.1] 2018/01/11, 10:10am – 12:00am, 教師:張存續

- $\diamond$  Answer the questions as complete as possible.
- 1. (20%) Two long coaxial cylindrical metal tubes (inner radius *a*, outer radius *b*) stand vertically in a tank of dielectric oil (susceptibility  $\chi_e$ , mass density  $\rho$ ). The inner one is maintained at potential *V*, and the outer one is grounded (See the figure).
- (a) Find the electric field E in the air part and the oil part? (8%)
- (b) Find the capacitance? (6%)
- (c) Find the height (h) that the oil rises in the space between the tubes? (6%)

2. (20%) Consider a uniformly polarized dielectric sphere of radius R.  $\mathbf{P} = P_0 \hat{\mathbf{z}}$ 

(a) Find the surface bound charge density  $\sigma_b$  and the volume bound charge density  $\rho_b$ . (10%) (b) Find the potential V of the dipole sphere for  $r \ge R$ . (10%)

[Hint: Use the dipole approximation, or  $V = \frac{1}{4\pi\varepsilon_0} \oint_{S} \frac{\sigma_b}{c} da' + \frac{1}{4\pi\varepsilon_0} \int_{v} \frac{\rho_b}{c} d\tau'$ ].



- 3. (20%) Boundary conditions and applications.
- (a)  $\nabla \cdot \mathbf{D} = \rho_f$ . Find the boundary condition for the normal component of  $\mathbf{D}$ ,  $D^{\perp}$ . (6%)
- (b)  $\nabla \times \mathbf{H} = \mathbf{J}_{f}$ . Find the boundary condition for the tangential component of  $\mathbf{H}$ ,  $\mathbf{H}^{\prime\prime}$ . (6%)
- (c) Consider the interface between two dielectric materials with  $\varepsilon_1$  and  $\varepsilon_2$  as shown in the figure. Find the relations between the normal and the tangential components of the electric fields. Assume that there is no surface charge, i.e.,  $\sigma_f = 0.$  (8%)

$$\varepsilon_1, E_1^{\perp}, E_1^{\#}$$
  
 $\varepsilon_2, E_2^{\perp}, E_2^{\#}$ 

- 1 -



- 4. (20%) An infinitely long solenoid with air core having a radius *a* and *n* closely wound turns per unit length, as shown in the figure. The windings are slanted (傾斜) at an angle  $\theta$  and carry a current *I*.
- (a) Find the z-component of the magnetic flux density  $(B_z)$  both inside and outside the solenoid. (10%) [Hint: Use Ampere's law.]
- (b) Find the  $\phi$ -component of the magnetic flux density  $(B_{\phi})$  both inside and outside the solenoid. (10%) [Hint: Use cylindrical coordinates,  $r, \phi, z$ .]



- 5. (20%) A long cylinder of radius *R* carries a magnetization  $\mathbf{M}=M_0 \hat{\mathbf{z}}$ , where  $M_0$  is a constant.
- (a) Find  $\mathbf{J}_{b}$  within the material and  $\mathbf{K}_{b}$  on the surface of the material. (10%)
- (b) Find the magnetic field **B** due to **M** for points inside  $(r \le R)$  and outside the cylinder  $(r \ge R)$ . (10%)

