A TE$_{21}$ second-harmonic gyrotron backward-wave oscillator with slotted structure

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Second-harmonic gyrotron backward-wave oscillator (gyro-BWO) with a reduced magnetic field strength is a tunable source in the millimeter wave regime, but it has long been impeded by the severe mode competition as a result of low efficiency and narrow bandwidth. This study employs a slotted structure functioning as a mode selective circuit to suppress the lower order transverse modes. In addition, a two-step tapered waveguide is adopted to stabilize the higher-order transverse modes and axial modes. Some important characteristics of the slotted gyro-BWO will be analyzed and discussed. As a calculated result, the interaction efficiency is improved and the stable tuning range is broadened. A stable, Ka-band, slotted second-harmonic gyro-BWO is capable of producing an efficiency of 23% with a 3 dB tuning bandwidth of 9% at 5 A and 100 kV.

I. INTRODUCTION

In recent years, the tunable high-power coherent millimeter wave has been an important source for various applications such as plasma diagnostics, fusion, advanced communication, high resolution radar, electron spin resonant spectroscopy, and material processing. The gyrotron backward-wave oscillator (gyro-BWO), based on electron cyclotron maser interaction, is a promising candidate for frequency tunable source. Theoretical study of gyro-BWO was first analyzed in the mid-1960s (reviewed in Ref. 3). Starting behavior and nonlinear behavior have been investigated by simulations. The experimental study of gyro-BWO has been carried out since early 1990s.11–13 Gyrotron is known to operate close to the Doppler shifted cyclotron resonance frequency (or its harmonics). To pursue high-frequency operation, strong magnetic field is required and generally provided by bulky superconducting magnet. Harmonic operation of gyrotron, effectively reducing the strength of the magnetic field, has been of growing interest. However, it is subject to severe mode competition as well as high oscillation threshold.19–23

Figure 1(a) shows the frequency-$k_z$ diagram of a Ka-band TE$_{21}$ second-harmonic gyro-BWO. The oscillations occur at the intersection of beam-wave resonant line and waveguide mode. Point 1 is the intersection of TE$_{21}$ waveguide mode with second cyclotron harmonic ($s=2$), denoted as TE$_{21}^{(2)}$. The major competing modes are TE$_{11}$ with fundamental cyclotron harmonic (TE$_{11}^{(1)}$) and TE$_{31}$ with third cyclotron harmonic (TE$_{31}^{(3)}$), as shown in points 2 and 3, respectively. The low-order harmonic oscillation TE$_{21}^{(2)}$, generally having a much stronger interaction strength, spoils severely the tuning property of the operating mode (TE$_{21}^{(2)}$). To suppress these unwanted oscillations, several circuits have been developed. A sliced waveguide, which interrupts the wall current of unfavorable mode, was developed at UC Davis for TE$_{21}$ second-harmonic gyrotron traveling-wave-tube amplifier (gyro-TWT) and provided a very high output power (207 kW at 15.7 GHz).20 A Russian and United Kingdom group has designed a helical corrugated structure that changes the dispersion of the waveguide and reported a broad bandwidth (10% frequency bandwidth for X-band gyro-TWT25 and 17% tuning bandwidth for X-band gyro-BWO26,27). In addition, slotted structure was found to greatly lower the requisite beam power for a harmonic gyrotron.28,29 This cross-sectional geometry was first applied to a magnetron in 1948 and then to a gyrotron in the 1980s. The experimental research of slotted gyrotron was developed and demonstrated on a third-harmonic gyro-TWT amplifier at UC Davis.31–33 However, no research has been done on the slotted harmonic gyro-BWO.

This study discusses the effect of slotted radius ratio on the dispersion of waveguide modes which avoids the major competing mode, i.e., TE$_{11}^{(1)}$. A well-designed taper structure was adopted not only to enhance the efficiency, but also to prohibit other competing transverse mode TE$_{31}^{(3)}$ and higher-order axial modes ($l=2,3,...$) of TE$_{21}^{(2)}$ (axial mode competition of gyro-BWO with smooth structure; see Ref. 34). Finally, a design of a high-efficiency, broadband, second-harmonic TE$_{21}$ gyro-BWO will be presented.

II. NUMERICAL MODEL

A steady-state, particle-tracing, self-consistent code is developed for the simulation, according to the analysis in Refs. 29, 31, and 35. A representative cross-sectional view of slotted structure and the electron trajectory are shown in Fig. 2. The temporary position of an electron and its guiding center are specified in polar coordinates ($r$, $\theta$) and ($r_e$, $\phi$). The Larmor radius of the gyrating electron is denoted as $r_L$, and the angular position in guiding center coordinates is $\phi$. The $B_r$ of the interaction region ($r < a$) is given by summing up all the azimuthal components:
The slot region (II: $a < r < b$) is approximated by the lowest order of Bessel functions without azimuthal dependence. $B_z$ in the $p$th slot is given by

$$B_z^p = \frac{\pi}{N} F(kr)f(z)e^{ikz}e^{i(2\pi m/N)p-a\theta},$$

(3)

where

$$F(kr) = J_0(kr)Y_0(kb) - J_0(kb)Y_0(kr),$$

$$J_0(kr)Y_0(kb) - J_0(kb)Y_0(kr).$$

$J_0$ and $Y_0$ are zero-order Bessel functions of the first and second kind, respectively. All other transverse fields could be derived from $B_z$. The value $k$, similar to $k_c$ ($=x_m/a$) of the smooth-bore waveguide, will be discussed in greater detail in the next section. It is determined by matching the boundary condition between the interaction region and slot region (average value of $B_z^1$ with $B_z^p$ at $r=a$).

$$\frac{N\theta_0}{\pi} \sum_j \left( \frac{\sin \Gamma \theta_0}{\Gamma \theta_0} \right) \frac{J_j(ka)}{J_j'(ka)} = F(ka).$$

(4)

The slotted-waveguide mode is characterized by the phase difference between the adjacent slot ($2\pi m/N$). For example, in a structure of four slots, there are two linearly polarized waves (symmetry in azimuthal components ±γ): 2πmode ($m=0$, $\Gamma=\ldots,-4,0,4,\ldots$) and π mode ($m=2$, $\Gamma=\ldots,-6,-2,2,6,\ldots$); and two circularly polarized waves: π/2 mode ($m=1$, $\Gamma=\ldots,-3,1,5,\ldots$) and its oppositely polarized −π/2 mode ($m=-1$, $\Gamma=\ldots,-5,-1,3,\ldots$). However, there are still higher-order modes for this structure. For the reader’s convenience, each slotted-waveguide mode will be represented by the corresponding smooth-waveguide mode.
mode indexed $\text{TE}_{mn}$. The self-consistent field equation could be derived from Maxwell’s equations and are given by

$$\left( \frac{d^2}{dz^2} + k_0^2 \right)f(z) = -\frac{8k_0^2}{\zeta \cdot \alpha} \frac{1}{f(z)} \int \langle \mathbf{J} \cdot \mathbf{E}^* \rangle da,$$

(5)

where $\zeta$ is an integration factor ($fB_x \cdot B_z^* \, rdrd\theta$). The beam-wave interacting strength contributes to the right-hand side of Eq. (5). In smooth-bore waveguide, $\zeta$ simply equals $r_w^2 J_m^2(r_w)/(1-m^2/k^2 r_w^2)$, where $r_w$ is the wall radius. As for the slotted-bore waveguide, the integration factor should be calculated by a slotted cross section and the interaction strength comes from a series of azimuthal components. Thus the nonlinear field equation is modified as follows:

$$\left( \frac{d^2}{dz^2} + k_0^2 \right)f(z) = -\frac{8k_0^2 I_b}{a^2 D(k)} \sum_{q=-\infty}^{Q} \sum_{s=-\infty}^{Q} d_q W_q p_{eq} \times J_{-1}(kr_{eq}) \int J_q(kr_{eq}) e^{i\theta_{eq}} d\theta,$$

(6)

where $A_q = \omega t q_s + s \varphi_q + (s-\Gamma)\varphi_q + (s-\Gamma/2)\varpi_t$, $I_b$ is the operating beam current and the weighting function of $q$th electron is denoted by $W_q$. $a^2 D(k)$ is the integral factor of the slotted case and $D(k)$ is given as

$$D(k) = \sum_{\Gamma} D_1^k(k) + \frac{\pi \theta_n}{N} \left[ \frac{b^2}{a^2} G^2(kb) - G^2(ka) - 1 + \frac{1}{k^2 a^2} \right],$$

(7)

and

$$G(kr) = F''(kr).$$

The electrons gyrate in a static uniformly applied magnetic field and interact with rf fields. The equations for both motion and trajectory are determined with the guiding center coordinate, which could be simply developed from the smooth-bore case by summing up all the azimuthal component interactions of a mode. The space charge effect is assumed negligible. The structure is assumed to be a weak nonuniform conductor without Ohmic loss. Outgoing wave boundary condition is imposed at both ends. The starting threshold calculation of the nonlinear self-consistent code is validated using the linear code.32

For stability consideration, the start currents ($I_m$) of all other competing transverse modes should be greater than the operating current. In the following two sections, suppression of lower-order transverse modes will be discussed using the characteristics of the slotted structure. The rest of the competing modes could be suppressed by properly tapering the axial structure.

III. CHARACTERISTICS OF SLOTTED INTERACTION STRUCTURE

To suppress the competing transverse modes, let us start with the intrinsic nature of the slotted waveguide. Figure 3 shows the normalized $ka$ value ($x_{mn}$) of four transverse modes versus the radius ratio $(b/a)$ for four-vane, six-vane, and eight-vane slotted waveguides. The normalized $x_{mn}$ values, generally speaking, are decreased with the increase in radius ratio. Noteworthily, the $\pi$ mode in each slotted structure decreases the fastest among all modes. To examine the change of the dispersion curves, Fig. 1(b) shows the frequency-$k_\perp$ diagram for a four-vane slotted-bore waveguide of radius ratio 1.5. The cutoff frequency of the operating $\text{TE}_{21}$ mode is maintained at 31 GHz. The competing transverse modes shift up from broken lines (smooth-bore) to solid lines (slotted-bore). The grazing magnetic field ($B_g$) for the most competitive $\text{TE}_{21}^{(1)}$ mode moved up to 8 kG when $b/a$ is greater than 1.15. This means that by selecting a proper radius ratio above this value, the troublesome low-order harmonic oscillation could be effectively suppressed.

In addition to the dispersive property, some important features of slotted waveguide are to be discussed here. The threshold beam power for slotted gyrotron was reported in the small signal theorem.29 Here we will provide a more thorough explanation using the concept of the interaction coefficient. The threshold beam power $P_b^{th}$ is inversely proportional to the interaction coefficient $F$ defined as a structure factor $a^2 D(k)$ multiplied by a beam-wave coupling factor $H_{\perp}$,

$$F = \left( \frac{a^2}{a^2 D(k)} H_{\perp} \right)^2 \approx \frac{1}{P_b^{th}}.$$  

(8)

A slotted-waveguide mode is composed of a series of azimuthal components. The beam interacts independently
with each component in the small signal regime. The coupling coefficient of 
harmonic interacting with azimuthal component \( \Gamma \) is defined by

\[
H_{d} = J_{2}^{2}(kr_{c})J_{2}^{2}(kr_{l}).
\]

(9)

The magnitude is decreased by the order of Bessel function 
and thus stronger coupling occurs when \( \Gamma = s \) or \( \Gamma = s \pm 1 \). Furthermore, as the 
cutoff wave number \( k \) increases (i.e., greater radius ratio), the coupling coefficient decreases 
for \( \Gamma = s \) interaction and increases for \( \Gamma = s \pm 1 \) interaction. The interaction 
coefficient takes into consideration both the strength of interacting component \( [a_{d}^{2}/D(k)] \) 
and field energy density per unit length \( [dU/dz \approx a_{d}^{2}D(k)] \). With slight slotted 
boundary, the strength of main component \( (j=0) \) would be 
dominant and side components \( (j \neq 0) \) are vanishingly small. 
As the radius ratio increases, the strength of side components 
goes up and the slotted-waveguide mode could be excited.

As seen in Fig. 1(b), it is speculated that possible 
competing modes would be \( \pi/2 \) mode second-harmonic (point 2; 
TE\(_{11}\) for left-hand circularly polarized wave and TE\(_{11}\) for 
right-hand circularly polarized wave), \( \pi/2 \) mode third-harmonic (point 3; TE\(_{31}\) and 
2\( \pi \) mode third-harmonic (point 4; TE\(_{01}\) oscillations.

Figure 4(a) shows the variation of the interaction 
coefficient over slot radius ratio at \( B_{0} = 1.01 B_{g} \), where \( B_{g} \) is the 
grazing magnetic field of the respective mode. The interaction 
coefficient \( F \) of the operating mode TE\(_{21}\) could benefit 
from slotted structure about four times as \( b/a \) increases from 
1.0 to 1.5. Nevertheless, the side component interaction 
TE\(_{21}\) and TE\(_{01}\) modes also grow rapidly to a risky level. 
The former which has an equal harmonic interaction to op-
erating mode was found to be the most competitive mode on 
slotted third-harmonic gyro-TWT experiment.

To demonstrate the competition in slotted gyro-BWO, a 
uniform 10 cm slotted waveguide is adopted for the prelimi-
nary analysis. The starting currents for the interested modes 
versus magnetic field at \( b/a = 1.5 \) and \( a = 0.233 \) cm are shown 
in Fig. 4(b). The oscillation threshold of the scrupled second-
harmonic oscillations (TE\(_{21}\) and TE\(_{41}\)) are well above the 
operating mode due to the detuning of the dispersive curves. 
This means that the major threats are clear. The remaining 
competing modes, i.e., TE\(_{01}\) and TE\(_{31}\), can be further sup-
pressed by properly tapering the axial geometry, as discussed 
in the next section.

Figure 5 shows the variation in oscillation threshold over 
the initial guiding center position \( (r_{c}) \). The coupling strength of 
\( \Gamma_{r} = s \pm 1 \) is maximum when \( kr_{c} \) equals 1.841 [the first root of 
\( J_{2}(x) = 0 \)] and becomes zero as \( r_{c} = 0 \). The starting currents 
for both oscillations tend to increase as \( r_{c} \) decreases to 0. 
Nevertheless, the interaction \( \Gamma_{r} = s + 1 \) of TE\(_{21}\) and TE\(_{01}\) 
present the characteristics of peniotron interaction as the 
injected electron approach to an axis-encircling beam, mak-
ing it possible to be excited. As for the interaction \( \Gamma_{r} = s - 1 \), 
peniotron interaction leads to the absorption, and thus the
starting current of $\text{TE}_{11}^{(2)}$ grows exponentially as $r_c$ approaches to zero. As in Fig. 4(b), the spurious oscillations of the uniform slotted configuration are $\text{TE}_{01}^{(3)}$ and $\text{TE}_{31}^{(3)}$. The beam wave coupling of these high-order modes is weak. However, the competing modes traveling with low propagation constant ($k_z$) in uniform structure have the advantage of longer beam-wave interaction and probably constitute the competition. In a down-tapered structure, these low-$k_z$ oscillations would have a higher degree of reduction in effective interaction length than the relative high-$k_z$ operating mode. Moreover, the geometry tapering might enhance the interaction efficiency, but it might induce the mode competition. Thus, it is important to optimize the design by compromising among transverse mode competition, axial mode competition, and efficiency enhancement.

IV. TAPERING AXIAL GEOMETRY

The high-$k_z$ oscillation possesses a longer effective length than the low-$k_z$ oscillation in a down-taper structure because of deeper penetration. Tapering the waveguide geometry can suppress the low-$k_z$ oscillations (relative to $\text{TE}_{21}^{(2)}$, i.e., $\text{TE}_{01}^{(3)}$ and $\text{TE}_{31}^{(3)}$). To start with, the 10 cm uniform slotted waveguide is replaced by a 4 cm uniform section followed by a 6 cm taper section, as depicted in Fig. 6(a). The inner wall radius $a$ is the function of axial position and the outer radius $b$ is 1.5 times that of $a$ throughout the waveguide. Figure 6(b) shows the starting current versus the operating magnetic field for several transverse modes. The oscillation threshold of competing modes ($\text{TE}_{01}^{(3)}$ and $\text{TE}_{31}^{(3)}$) at respective low-$k_z$ regions (6.27 and 7.6 kG) becomes higher than that of the operating mode (in comparison with Fig. 4) due to a much reduced effective interaction length. The starting currents of all competing transverse modes are thus above the operating beam current ($I_{op}=5$ A).

The higher-order axial mode ($l=2$) always possesses a larger propagation constant and leads to longer effective interaction length than the fundamental axial mode ($l=1$) in the taper structure. Nevertheless, the interaction strength of the fundamental axial mode is generally stronger than that of the high-order axial modes. Thus, a sufficient interaction length like the uniform first section in Fig. 6(a) could assure that the $l=1$ mode gets the superiority.

In addition, the geometry of a taper structure also plays an important role in efficiency enhancement. The steep-tapered second section in Fig. 6(a) enhances the high-$k_z$ operating region ($B_0=7–8$ kG). To go a step further, the first uniform section is prolonged to 6 cm and gradually tapered for enhancement of the low-$k_z$ operating region ($B_0=6.2–7$ kG). Noticeably, this action might probably cause the axial mode competition; therefore, the slope is critical.
Figure 7 compares the starting currents between the high-order axial mode ($l=2$) and fundamental axial mode ($l=1$) for three slopes of two-step tapered axial geometries. The inner radii of the upstream end and downstream end are fixed at $a_1=0.233$ and $a_2=0.218$ cm, respectively. Three values of $a_2$ (0.228, 0.226, and 0.223 cm) are examined. For a properly tapered first section ($a_2=0.228$ cm), the starting current of $l=1$ mode is always lower than that of $l=2$ mode in entire magnetic tuning. As $a_2$ decreases to 0.226 cm, the axial mode competition occurs at the region $B_0<6.85$ kG. The competition region expanded to $B_0<6.95$ kG for the first section of even steepness ($a_2=0.223$ cm).

V. AN OPTIMIZED DESIGN WITH NOVEL MODE CONVERTER

The mode converter is essential and critical for gyro-BWO devices, because the rf field energy is extracted at the beam entrance. A circularly polarized TE$_{21}$ mode converter had been developed for smooth waveguide. Here, we adopt similar design principles to design a novel Y-type mode converter, as described in Fig. 8(a). Figure 8(b) shows the calculated transmission loss and reflection loss. The $-1$ dB transmission loss is greater than 5 GHz with the mode purity of 99.999%.

To summarize, a series of studies on solving the competing transverse modes as well as high-order axial modes are examined. A design for overall consideration is to be presented. Figure 9(a) shows the starting curves of the optimized structure which consider the extra length of the mode converter (1.7 cm, added to the front of the interaction section). The $I_{st}$ of TE$_{01}^{(3)}$ and TE$_{31}^{(3)}$ are far above the operating current due to the tapered first interaction section. Noticeably, we did not discuss the TE$_{−11}^{(2)}$ mode in the variation of axial geometry, since its oscillation threshold is much higher than the operating mode (in the discussion of Fig. 4). The taper structure would also enhance the high-$k_z$ TE$_{−11}$ oscillation.

FIG. 8. (Color online) (a) A Y-type mode converter for the four-vave slotted waveguide. (b) Calculated transmission (solid line with empty triangles) and reflection (solid line with filled circles) coefficient.

FIG. 9. (a) Starting current vs magnetic field for all interested modes. (b) Calculated efficiency and frequency vs magnetic field for three beam currents: $I_b=3$ (solid line with filled circles), $I_b=4$ (solid line with empty triangles), and $I_b=5$ (solid line with crosses). The beam velocity spread $\Delta p_z/p_{z0}$ is set to be 5%.
tion and thus the starting threshold is much closer to the operating current than that in the uniform case. This configuration is free of transverse mode competition at $I_b = 5$ A and free of axial mode competition at the operating region $B_z = 6.3–8$ kG. Figure 9(b) shows the efficiency and tuning bandwidth for 3, 4, and 5 A using a velocity spread of 5%. The slotted $TE_{21}$ second-harmonic gyro-BWO shows a peak efficiency of 23% at $I_b = 5$ A and the extremely wide bandwidth of 9% at $I_b = 4–5$ A.

VI. CONCLUSIONS

We have reported the characteristics and the design of a $TE_{21}$ Second-harmonic slotted gyro-BWO. The slotted structure is found to be able to shift the dispersion curves. The $\lambda_{mn}$ of a π mode in slotted waveguide is more sensitive to the radius ratio than any other modes, which was employed to suppress the lower order transverse modes. The increase in radius ratio, on the other hand, might also excite the unwanted transverse modes due to the enhancement of side-component interaction. These induced higher-order transverse modes are remedied using a two-step taper structure which could skillfully avoid the high-order axial modes as well.

As a simulated result, the slotted $TE_{21}$ second-harmonic gyro-BWO appears to be stable (free of competition below 5 A), high efficiency (23%), and have a broad bandwidth (a 3 dB bandwidth of 9%). The underlying physics shown here is extendable to higher-harmonic gyro-BWO such as $TE_{41}$ mode.

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