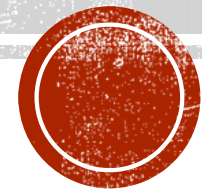


OTHER INTERESTING TOPICS ABOUT BLACK HOLES

Lecture 16, Introduction to Black Hole Astrophysics

NTHU, 6/15/2021



SYLLABUS

Week 1 (2/23): A brief history of black holes (BHs)

Week 2 (3/2): Einstein's theory of special relativity

Week 3 (3/9): Einstein's theory of general relativity

Week 4 (3/16): Schwarzschild BHs and Kerr BHs

Week 5 (3/23): Observational discovery of BHs

Week 6 (3/30): The birth of stellar mass BHs

Week 7 (4/13): BH accretion disks

Week 8 (4/20): Measuring BH masses and spins

Week 9 (4/27): Supermassive BHs (SMBHs) and Active Galactic Nuclei (AGN)

Week 10 (5/4): The SMBH at our Galactic center

Week 11 (5/11): AGN Jets

Week 12 (5/18): AGN feedback on galaxies

Week 13 (5/25): SMBHs origin and growth

Week 14 (6/1): Recent triumphs of GR – the first image of BH

Week 15 (6/8): Recent triumphs of GR – gravitational waves

Week 16 (6/15): Other interesting topics (primordial BHs, Hawking radiation, information paradox)





HAWKING RADIATION

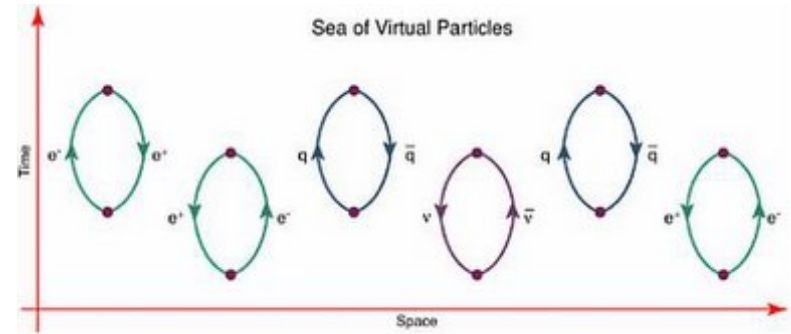


HAWKING RADIATION

- In 1974, Stephen Hawking made an attempt to put together Quantum Field Theory (QFT) and black holes as described by GR
- He discovered the weird fact that, strictly speaking, BHs are not totally black, but they can emit *blackbody radiation* – Hawking radiation
- The rigorous derivation requires full knowledge of the QFT; however, the simple picture we often see is enough to capture the essential points



VACUUM FLUCTUATIONS & VIRTUAL PARTICLES



- QFT predicts that vacuum is not exactly empty, but there are ***vacuum fluctuations***
- Due to the uncertainty principle,

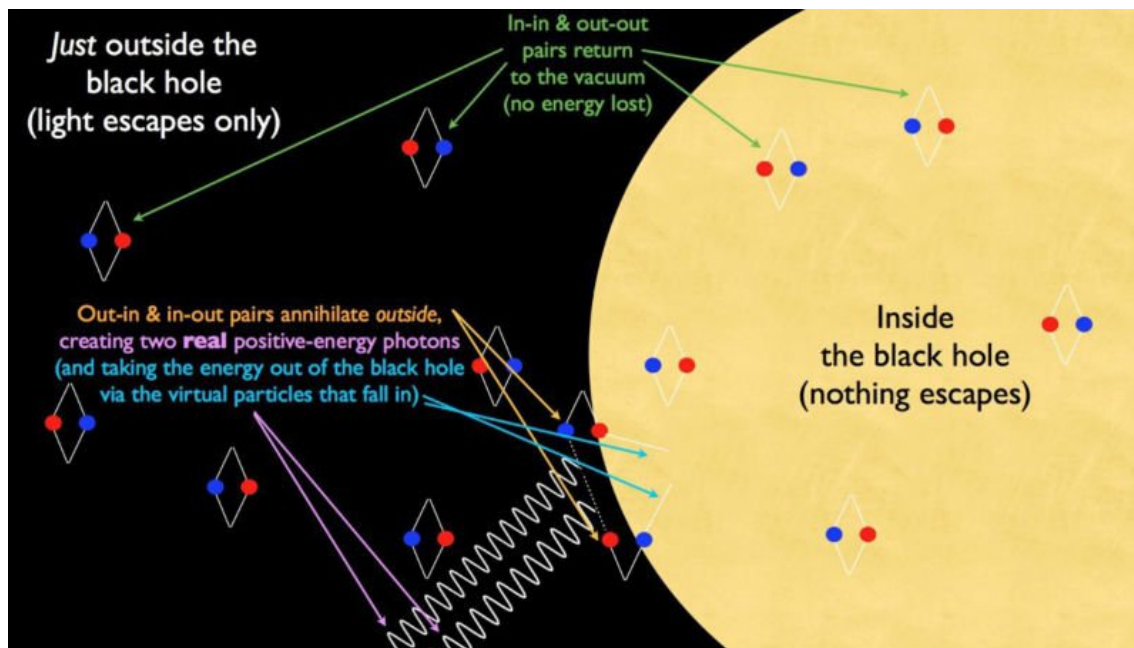
$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

for finite duration events, energy does not need to be strictly conserved as it could not be precisely defined

- Since $E=mc^2$, it means that even in the vacuum, a pair of ***virtual particles*** (a particle and an anti-particle) could be spontaneously created, as long as they annihilate within a short time



HAWKING RADIATION



- When the virtual particle pair is created near the event horizon, there would be chances where one virtual particle is inside and the other is outside the event horizon
- The virtual particle inside the event horizon could have **negative energy** due to curved spacetime, similar to what happens in the ergosphere during the Penrose process
- This virtual particle with negative energy would fall into the BH, reducing the mass-energy of the BH
- The other virtual particle could escape the BH, become real particles, annihilate with other particles near the horizon and emit photons



PROPERTIES OF THE HAWKING RADIATION

- The radiation has a blackbody form with a temperature of

$$T = \frac{\hbar c^3}{8\pi G M k_B} \approx 6 \times 10^{-8} \left(\frac{M}{M_{sun}} \right)^{-1} K$$

- For typical astrophysical black holes, $M > 3 M_{sun}$, T is extremely low and the process is extremely slow
- For mini black holes with $M < M_{sun}$ (e.g., primordial BHs), the radiation could be significant
- Because the radiated energy comes from the BH itself, the BH mass must decrease with time, i.e., BHs could **evaporate!**



BLACK HOLE EVAPORATION

- For blackbody radiation, the luminosity at temperature T can be expressed as

$$L = \sigma T^4 A = \left(\frac{\pi^2 k_B^4}{60 \hbar^3 c^2} \right) \left(\frac{\hbar c^3}{8\pi G M k_B} \right)^4 4\pi \left(\frac{2GM}{c^2} \right)^2$$

$$\Rightarrow \dot{M} c^2 = - \frac{\hbar c^6}{15360 \pi G^2 M^2}$$

$$\Rightarrow M^3(t) = M_0^3 - \frac{\hbar c^4}{5120 \pi G^2} t$$

$$\Rightarrow t_{evap} = 2.66 \times 10^{-33} M_0^3 \text{ yrs}$$

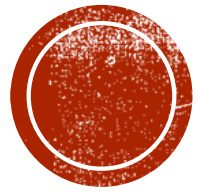


BLACK HOLE EVAPORATION

$$t_{evap} = 2.66 \times 10^{-33} M_0^3 \text{ yrs}$$

- Compare this **evaporation timescale** with the age of the universe, 13.8 Gyr, we can conclude that only mini BHs with mass $< 1.7 \times 10^{14}$ g will have had a chance to evaporate by now
- Because the radiative power (L) is proportional to M^{-2} , nearly all energy emitted is confined to the very last moments of the BH's life





PRIMORDIAL BLACK HOLES



PRIMORDIAL BLACK HOLES (原初黑洞)

- Mini BHs with $M < M_{\text{sun}}$ might be formed out of *primordial fluctuations*, if within some region of space density fluctuations are large enough to overcome pressure and collapse into primordial BHs (PBHs) due to gravity
- PBHs formed at time t after the Big Bang will have a mass

$$M \sim 10^{15} \left(\frac{t}{10^{-23} \text{s}} \right) g$$

- Plausible mass range of PBHs: 10^{-5} g ($t = \text{Planck time} = 10^{-43} \text{ s}$) to $10^5 M_{\text{sun}}$ ($t \sim 1 \text{ s}$)
- Why are PBHs interesting/important?
 - PBHs are non-baryonic, thus a plausible candidate for dark matter
 - PBHs, if existent, could contribute to the populations of massive smbhs or IMBHs



PRIMORDIAL BLACK HOLES (原初黑洞)

$$M \sim 10^{15} \left(\frac{t}{10^{-23} \text{s}} \right) g$$

- PBHs that formed before $t \sim 10^{-23} \text{ s}$ would have $M < 10^{15} \text{ g}$. These BHs would have evaporated already due to Hawking radiation
- These BHs should produce radiation around 100MeV, but observational limit on the gamma-ray background at this energy implies that their density could not exceed 10^{-8} of the average density of the universe -> PBHs of this mass range is not a significant component of the universe and cannot account for dark matter



OTHER CONSTRAINTS ON PBHS

- Though proposed ~50 years ago, there is still no evidence for PBHs. The recent LIGO detections of BHs with $> 30 M_{\text{sun}}$ reignited people's interests in PBHs
- Some of the available observational constraints on masses and abundance of PBHs:
 - Lensing of gamma-ray bursts
 - Capture of PBHs by NSs
 - Ignition of WDs
 - Microlensing of stars
- The exact mass ranges and limits on PBHs are still under debate. Current data generally suggests that PBHs could exist in some limited mass ranges, e.g., asteroid masses ($10^{16} \sim 10^{17}$ g), sub-lunar masses ($10^{20} \sim 10^{24}$ g) and smbhs/IMBHs ($10 \sim 10^3 M_{\text{sun}}$).
- It is an ongoing work to determine whether PBHs do exist and whether they can be a significant component of dark matter.





INFORMATION PARADOX



THE INFORMATION PARADOX

- The *no hair theorem* says that BHs only have three attributes: M, Q, and J, implying that all other information is lost to the outside world during the BH formation
- It wasn't a problem if the information is hidden within the event horizon forever
- However, the problem occurs if we consider a BH that could *evaporate* within the age of the universe due to Hawking radiation
- Hawking radiation, being blackbody radiation, does not encode information about the BH formation
- After the BH completely evaporates, the information about BH formation would be permanently lost in the universe!
- This violates a core concept in quantum physics – the value of a wave function of a physical system at one point in time should determine its value at any other time, i.e., the *information should be conserved*



HOW TO RESOLVE THE PARADOX

- Option 1: information is indeed permanently lost
 - Penrose: information may not be preserved in the presence of strong gravity
- Option 2: information is preserved
 - Stored in a Planck-sized remnant after evaporation
 - Holographic principle (from string theory): A volume of space can be encoded on a lower-dimensional boundary to that region. Therefore, information can be stored on the surface of the event horizon and emit with the Hawking radiation
- The solution to the information paradox is still an open question and could lead to deeper understanding of quantum gravity



SUMMARY

- Weird things happen when quantum effects are considered close to BHs
- **Hawking radiation:**
 - BHs emit blackbody radiation when virtual particles from vacuum fluctuations are created near the event horizon
 - Temperature of the Hawking radiation is inversely proportional to BH mass
 - Astrophysical BHs with $M > M_{\text{sun}}$ have no time to evaporate; mini BHs with $M < 10^{14}$ g could have evaporated

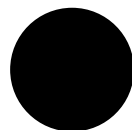


SUMMARY

- **Primordial BHs (PBHs):**
 - Density fluctuations in the early universe, if large enough, could collapse into PBHs with masses ranging from 10^{-5} g to $10^5 M_{\text{sun}}$
 - For PBHs $< \sim 10^{15}$ g, they should have evaporated and emit Hawking radiation in MeV ranges. Observations suggest that they don't have a significant contribution to the average density in the universe
 - PBHs in certain mass ranges are still being tested against observational data
- **Information paradox:**
 - Whether information is lost or not when things fall into the BH and the BH later evaporates
 - The solution remains an open question



BLACK HOLES



BLACK HOLES

