

THE BIRTH OF STELLAR-MASS BLACK HOLES

Lecture 6, Introduction to Black Hole Astrophysics (PHYS480)

Hsiang-Yi Karen Yang, NTHU, 3/30/2021



M87 SMBH with magnetic field measured from polarization [[News link here](#)]



ANNOUNCEMENTS

- HW2 solutions will be posted on iLMS today. TA and me will provide answers to Q1 & Q5 and share them on iLMS
- ***HW3 will be posted on iLMS and course website today. It is due next Wednesday (4/7) at 13:30, online or directly to TA/me***
- Please start searching for black hole news for the oral presentation. Once you decide on the topic, please paste the news link here:

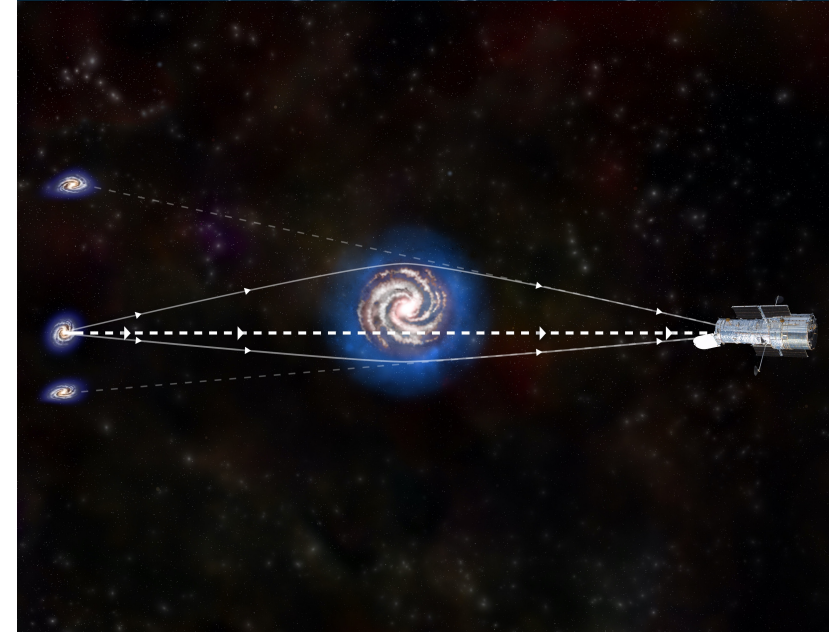
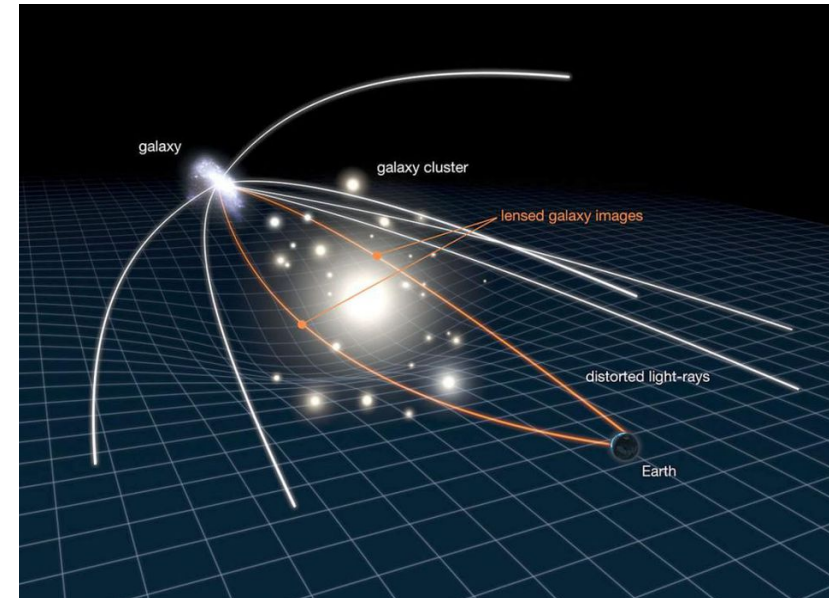
https://docs.google.com/spreadsheets/d/1_aYyMj1wf_uGheZ7zp_hvthmy4mdmPwlxFDdZOMG-nc/edit?usp=sharing

- The grading rubric for the oral presentation is available on iLMS. Please prepare your talk accordingly.
- Don't forget to tell the TA if you've asked questions during or after class to receive the participation points (10 points in total)!



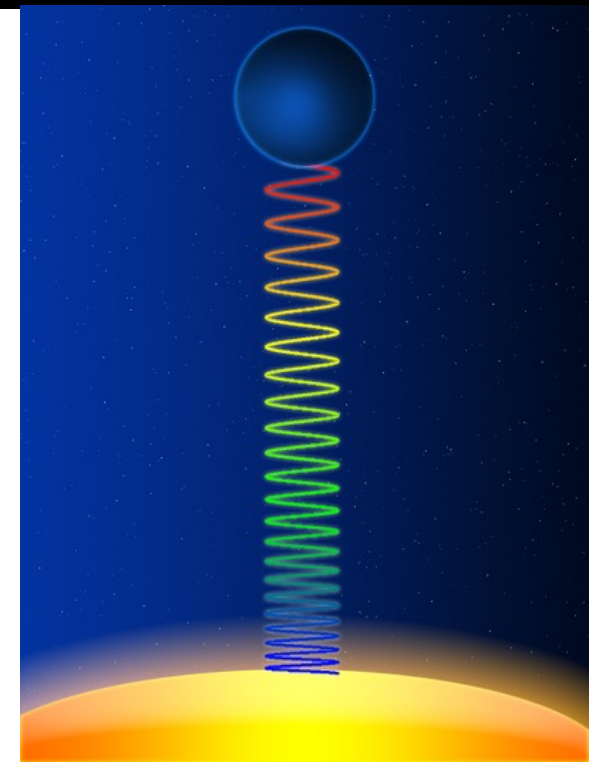
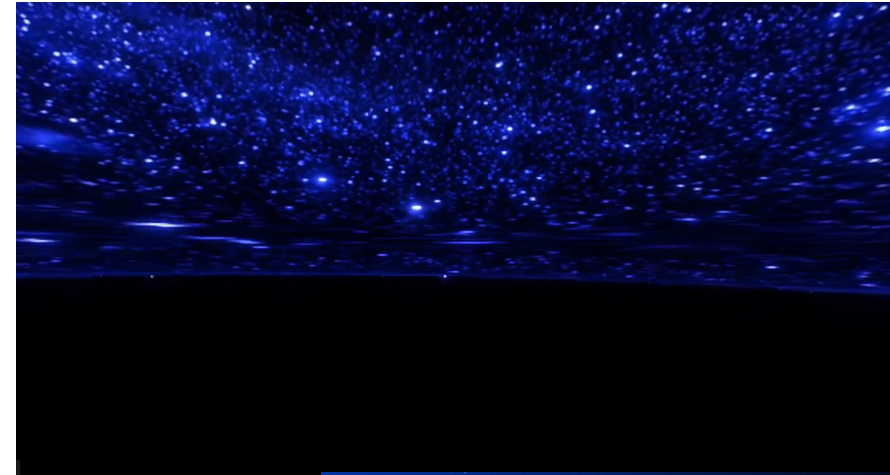
Q5 OF HW2 – MIRRORING EFFECT AROUND EINSTEIN RING?

- Recall that the Einstein ring comes from the **gravitational lensing** effect
- The “ring” is a special case when the background source is exactly behind the lens
- If the source has an offset w.r.t the axis, it will create images on opposite sides of the lens, one within the Einstein ring, the other outside the Einstein ring
- As the source moves, these two images would move accordingly, called “**mirroring effect**”



Q5 OF HW2 – WHY THE STARS APPEAR BLUE-SHIFTED?

- Recall **gravitational redshift** occurs because photons would lose energy when it climbs out of the curved spacetime of a massive object
- As we fall into the BH, the effect is inverted – photons emitted by distant galaxies gain energy as they approach the BH and would appear to be blue-shifted as viewed by us
- Note that in the video, Doppler's effect is ignored



REDSHIFT EFFECTS

- In reality, the amount of redshift/blueshift of light needs to account for all the following three effects, i.e., $z = z_g + z_d + z_c$

- 1) Gravitational redshift:
$$z_g = \frac{1}{\sqrt{1 - r_s/r}} - 1$$
 Steeply rising as $r \rightarrow r_s$
- 2) Doppler's effect:
$$z_d \cong \frac{v}{c}$$
 or
$$z_d^{rel} = \gamma \left(1 + \frac{v}{c} \right) - 1$$
 Steeply rising as $v \rightarrow c$
- 3) Cosmological expansion:
$$z_c \cong \frac{H_0 D}{c}$$
 or
$$z_c = \frac{a_{now}}{a_{past}} - 1$$
 Gradually rising



PREVIOUS LECTURE...

- ***Cygnus X-1*** is the first candidate of smbhs
 - Mass derived from orbit of the companion star
 - Size inferred from variability
 - X-ray properties different from accreting neutron stars (no pulses or X-ray bursts)
- So far dozens of smbhs (with dynamical masses) detected via X-ray, orbits, and gravitational waves
- X-ray binaries: accreting NSs or BHs, LMXB and HMXB, their distributions within the Milky Way and nearby galaxies
- Why study smbh? They are mini AGNs and they hold key for ***stellar evolution***



PREVIOUS LECTURE...

- **Quasar 3C 273** is the first candidate of SMBHs
 - Highly redshifted spectral lines -> distant and luminous
 - Variability and large luminosity consistent with a SMBH
- **Sgr A* at the Galactic center** is the first SMBH candidate with the accurate mass constraints
- **M87 SMBH** is the first confirmed BH with an event horizon
- Essentially every massive galaxy hosts a SMBH at the center, and they lie on the ***M- σ relation***
- SMBHs grow via mergers and accretion during the process of hierarchical galaxy formation
- However, quasars are more common in the early universe – “***downsizing***”
- Why study SMBHs?
 - Radiation and jet ***feedback*** of SMBHs are critical for understanding galaxy evolution
 - Their origin is still unknown!



THIS LECTURE

- A crash course on *stellar evolution*!!
- How low-mass and high-mass stars evolve
- The different fate of stars and their properties
 - White dwarfs (WDs)
 - Neutron stars (NSs)
 - Stellar-mass black holes (smbhs)
- Evolution of binary stars





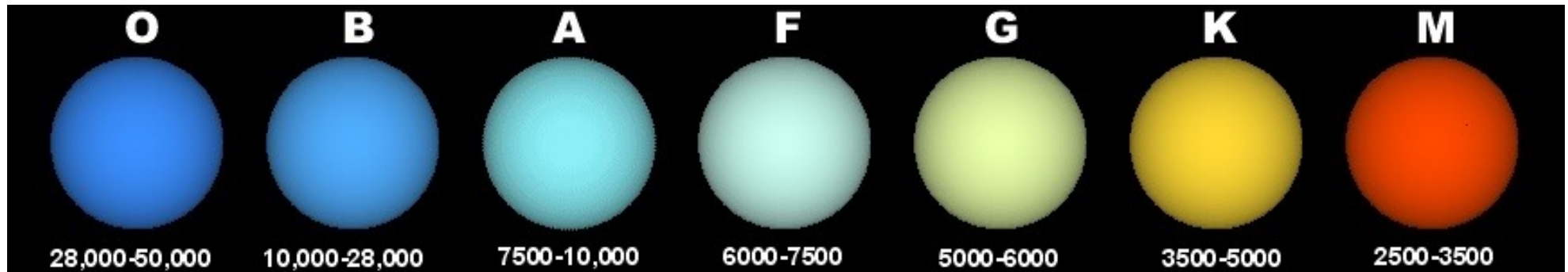
HOW STARS EVOLVE



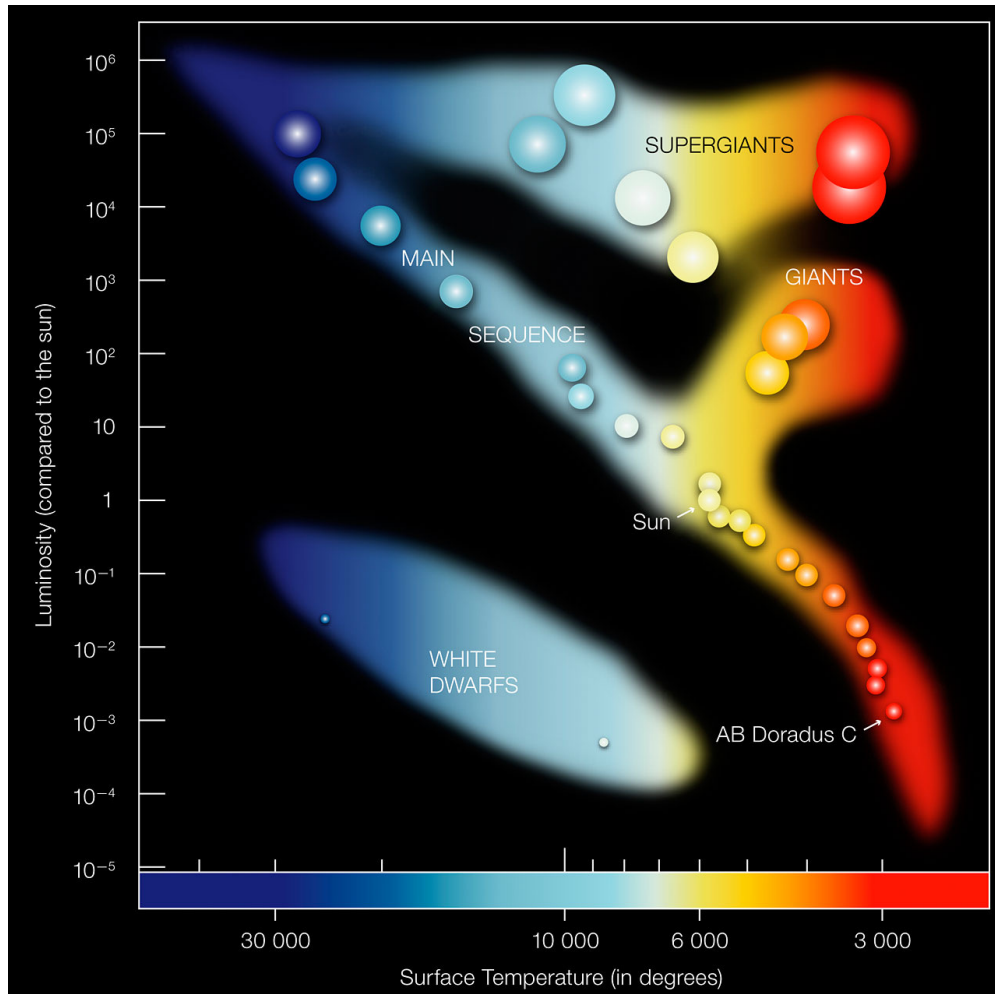


SOME REMINDERS ABOUT STARS

- Stars form in dense molecular gas (e.g., Orion nebula)
- Initial masses vary widely : $0.1-100 M_{\text{sun}}$; high-mass stars are rare
- Stars have variety of **colors** -> **temperature** ranging from 3000K-30000K -> **spectral types**: OBAFGKM
- Stars have variety of **luminosity** ($0.001 L_{\text{sun}} \sim 100,000 L_{\text{sun}}$)



HERTZSPRUNG-RUSSELL (H-R) DIAGRAM

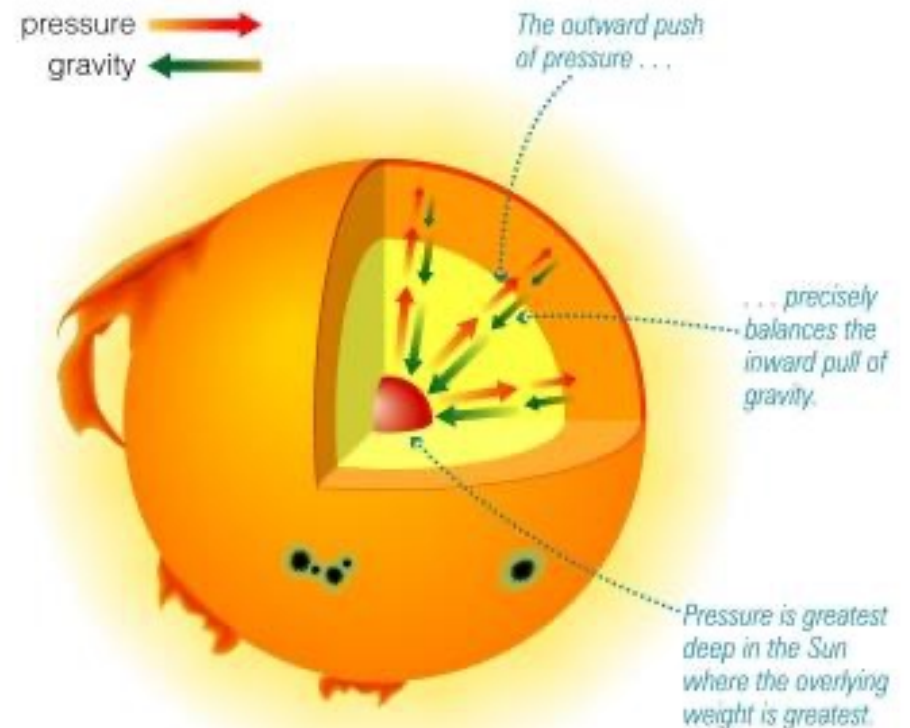


- Most stars lie on the **main sequence (MS/主序帶)** on the H-R diagram
 - Stars spend most of their lives on the MS
 - This is the $H \rightarrow He$ fusion phase
- Locations on the H-R diagram determined by mass and age
 - On the MS, high-mass = hot, low-mass = cool
 - When stars get old, they leave the MS and flit around the H-R diagram
- When it comes to stellar evolution, **mass** is the single most important parameter!



WHAT SUPPORTS THE MS STARS AGAINST GRAVITY?

- If nothing opposed gravity, Sun would collapse within the free-fall time of ~20 minutes!
- For MS stars, gravity is withheld by a **pressure gradient** -- “**hydrostatic equilibrium** (流體靜力平衡)”
- This equilibrium is **stable**
 - When gravity wins, core pressure grows, thereby increasing the pressure gradient that pushes outward



WHAT'S PROVIDING THE PRESSURE?

- Hydrogen burning during the MS: $4\text{H} \rightarrow \text{He}$
 - 0.7% of mass is converted to energy
 - $\sim 10^6$ times more efficient than chemical burning
- Eventually, the star runs out of hydrogen in its core. The time it takes until the star runs out of hydrogen is approximately

$$\tau \approx 1.0 \times 10^{10} \left(\frac{M}{M_{\odot}} \right)^{-2.5} \text{ yr}$$

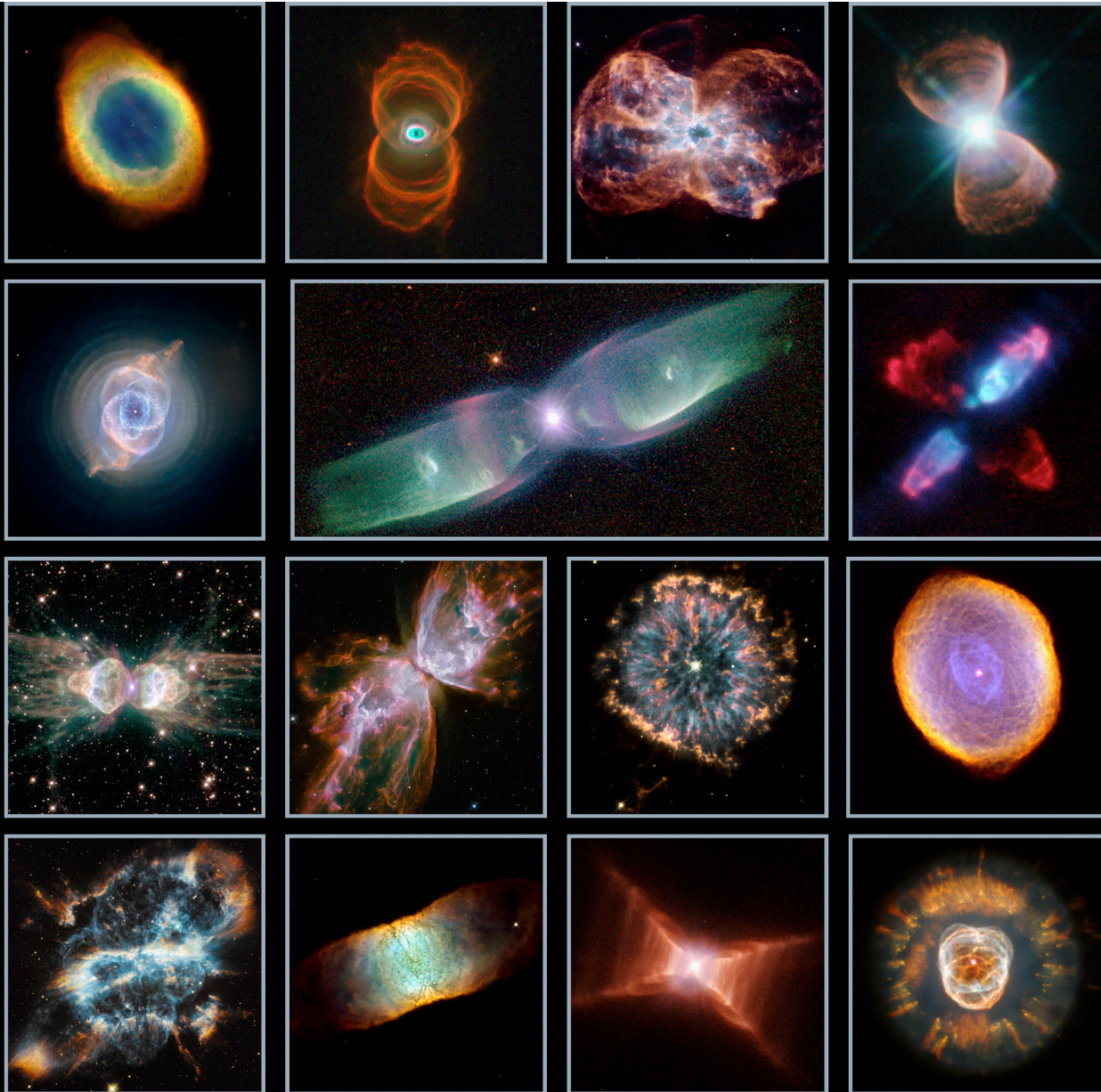
- Sun's lifetime on the MS is about 10 billion years (5 more billion years to go)
- Typical lifetime of a massive star ($\sim 40 M_{\text{sun}}$) is about Myr



POST-MS EVOLUTION OF LOW-MASS STARS

- Low-mass stars: $M < \sim 8 M_{sun}$
- Once hydrogen runs out in core...
 - Energy production stops and pressure gradient reduces
 - Core contracts due to gravity and heats up
 - Envelope of star expands and cool -> becoming a “**red giant (紅巨星)**”
 - Core keeps contracting and ignite helium fusion: $3\text{He} \rightarrow \text{C}$
 - Outer atmosphere is expelled as a “**planetary nebula (行星狀星雲)**”
 - When He runs out in the core, core contracts again but temperature not enough to trigger the next fusion process
 - Core collapses into a “**white dwarf (白矮星)**”



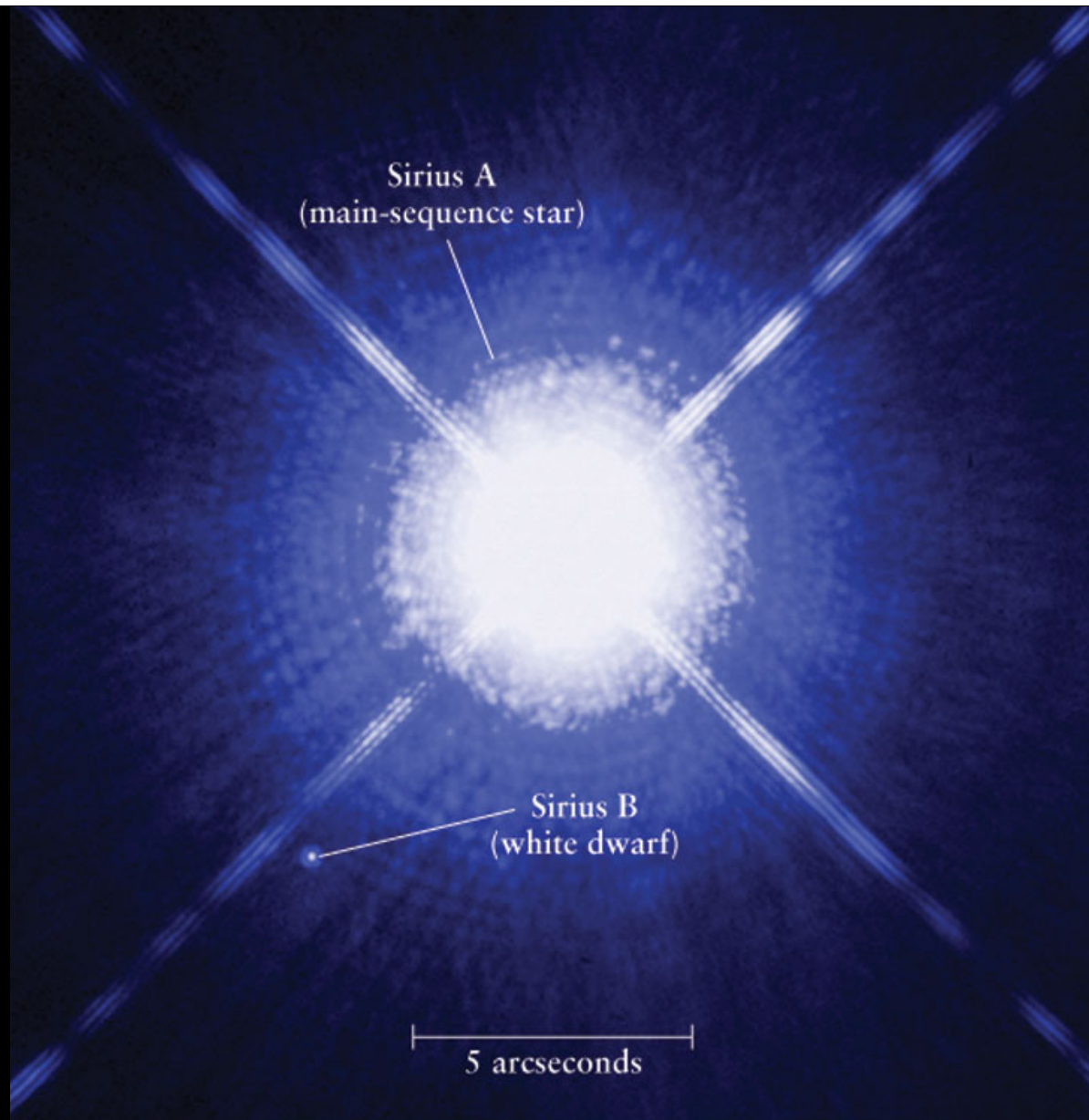


Hubble Views Of Planetary Nebula

Sirius A
(main-sequence star)

Sirius B
(white dwarf)

5 arcseconds



WHITE DWARFS

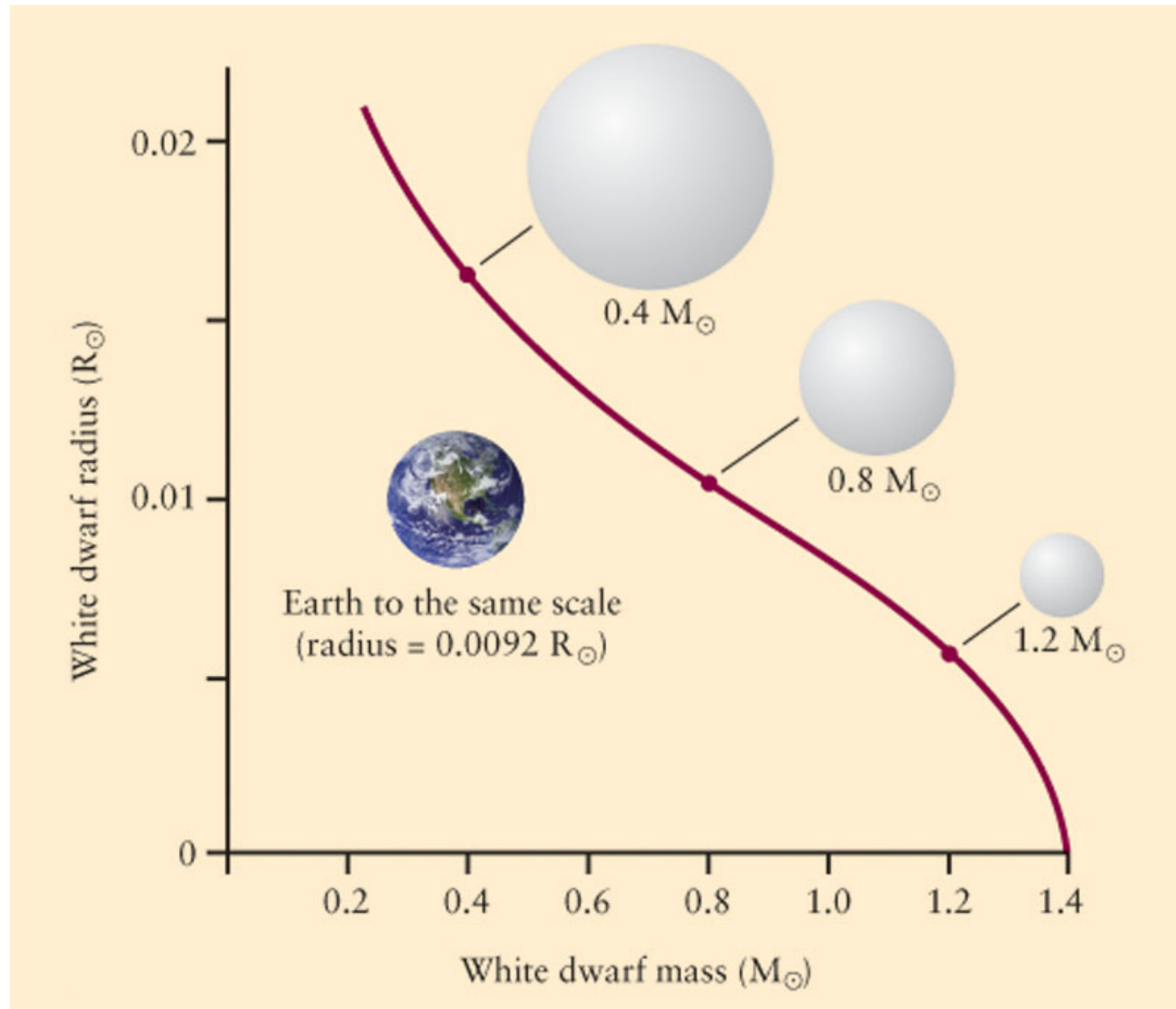
- A compact stellar remnant (“**compact object / 緻密天體**”) where the gravity is supported by “**electron degeneracy pressure**”
- The degeneracy pressure comes from the Pauli exclusion principle of fermions
 - As the electrons are squeezed into a more and more compact region, they can no longer stay at the ground state but have to occupy higher and higher energy levels
- For more and more massive stars, gravity would squeeze the electrons so that they become relativistic
- Since velocity of electrons has to be $< c$, there is a mass limit where the above mechanism could work
- In 1931, Subrahmanyan Chandrasekhar derived the “**Chandrasakhar limit**”: upper limit of mass of WDs $\sim 1.4 M_{sun}$



Subrahmanyan
Chandrasekhar,
Age 21



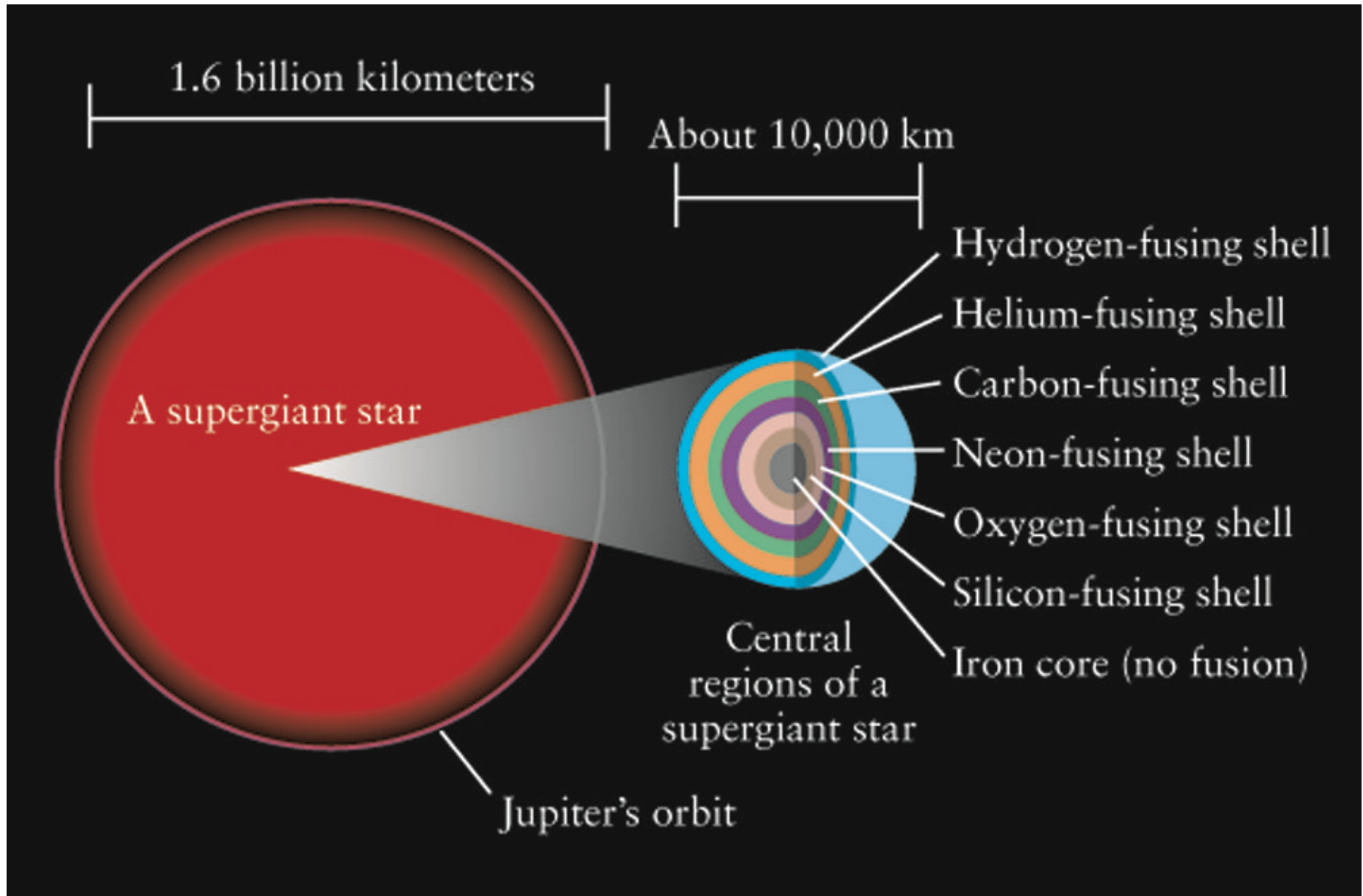
**WHITE DWARFS:
MASS ~ SUN,
RADIUS ~ EARTH**



EVOLUTION OF A HIGH-MASS STAR

- Stars with $M > \sim 8 M_{sun}$ take a different path
- Core gets hot enough to trigger nuclear fusion beyond Carbon
 - There is a sequence of reactions that go all the way from H to Fe (iron)
 - The reactions become less and less efficient, so mass must be consumed at a faster rate
 - Iron is the end product as it is the most stable nucleus (fusing it won't generate more energy)
 - End up with an onion-like structure

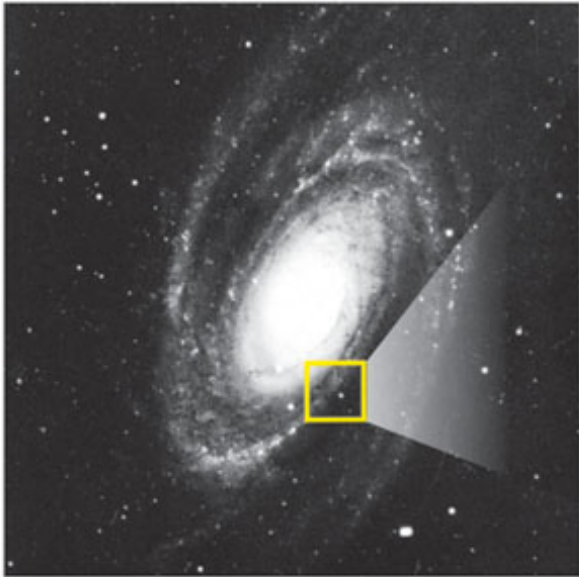




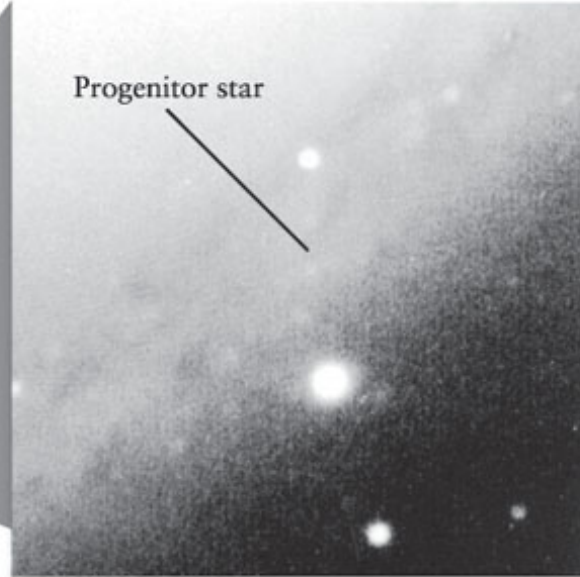
WHAT HAPPENS NEXT?

- Once iron is reached, fusion stops in core
- Without energy production, core is slowly crushed
- When core mass $> 1.4 M_{\text{sun}}$, pressure forces become incapable of supporting core, and the core undergoes catastrophic gravitational collapse in < 1 second
- Core collapse:
 - Releases about 10^{46} J
 - 99% emerge as neutrinos
 - 1% of energy (10^{44} J or 10^{51} ergs) emerges as radiation and kinetic energy
 - Star is blown apart – “**core collapse supernova**” or “**Type II supernova**”
- Supernova explosion is an important mechanism for distributing heavy elements synthesized in massive stars to the universe

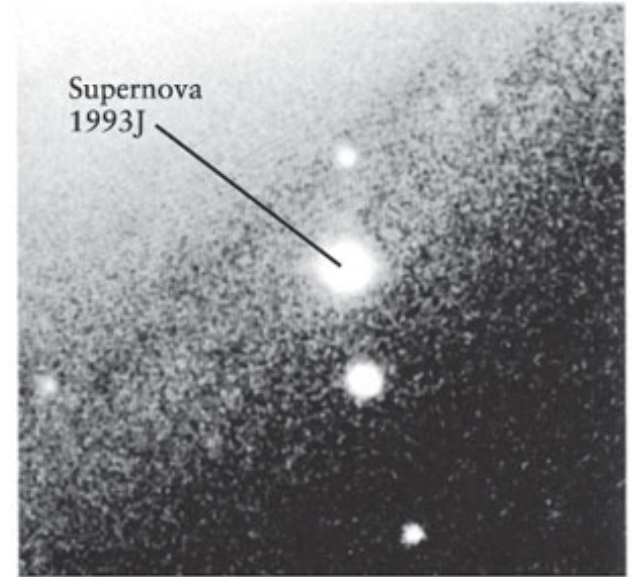




(a) Spiral galaxy M81

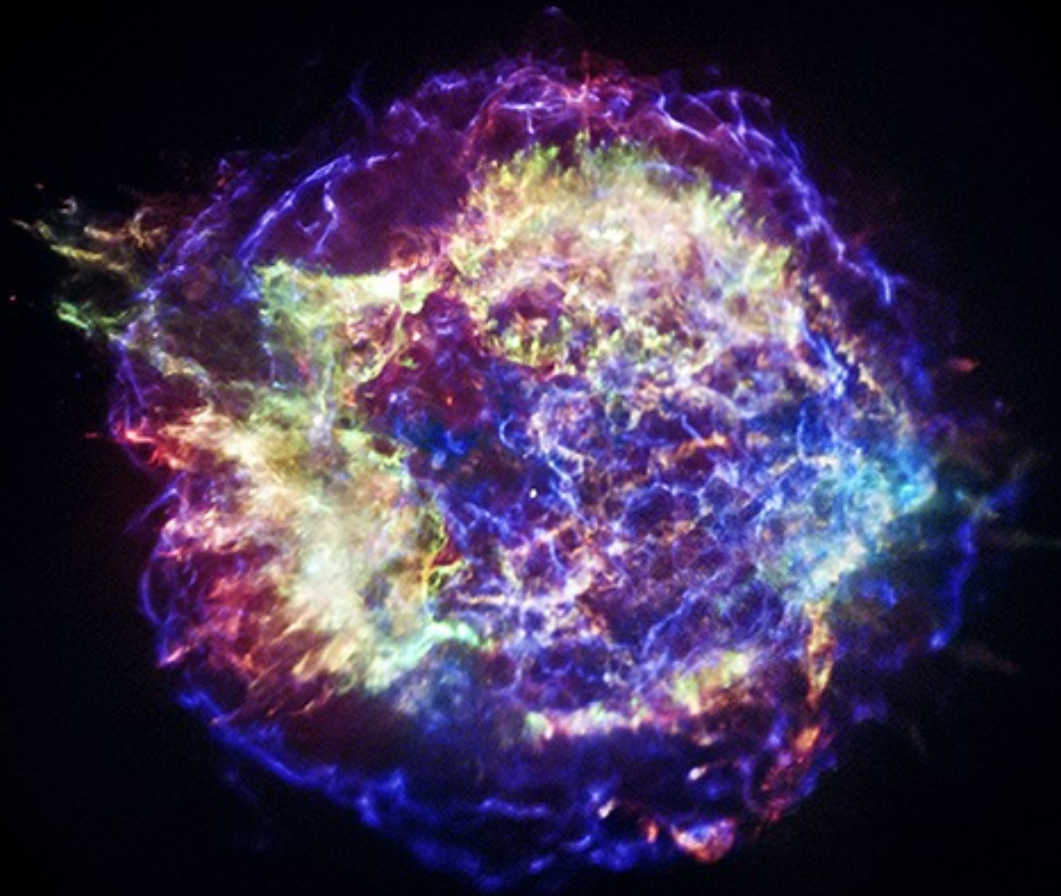


(b) Before the explosion



(c) After the explosion





Chandra X-ray View of Cassiopeia A Supernova Remnant

IN TERMS OF SIMULATING SUPERNOVAE...

- It had been a long standing problem in the simulation community to successfully make a star explode as a supernova
 - When core collapses, the bounced materials generate shock waves that propagate outward
 - But the shock waves are always stalled by the infalling materials at outer layers -> no explosions ☹️
- Improvements of modeling in recent years
 - 1D -> 2D -> 3D
 - Detailed modeling of interactions between neutrinos and matter
 - Instabilities (by neutrinos, turbulence, etc) are necessary to raise the shock and make the explosion happen
 - **Recent News: Secret Ingredient Found to Power Supernovas**





WHAT HAPPENS TO THE CORE?

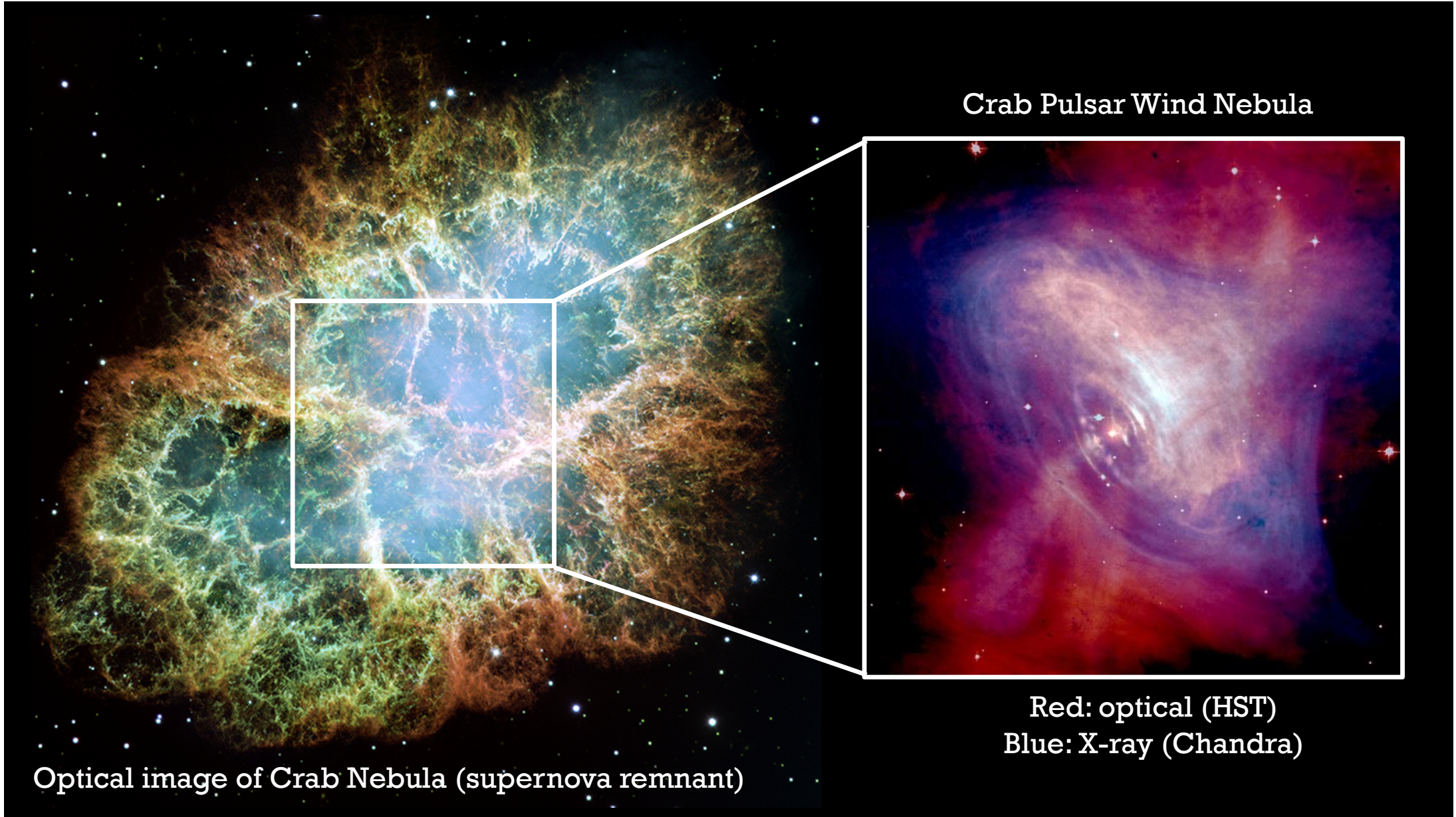
- If initial mass $M < 20 M_{\text{sun}}$, become a **neutron star**
 - Final mass $M \sim 1.5 - 3 M_{\text{sun}}$, $R \sim 10$ km

- Matter gets "neutronized":



- If initial mass $M > 20 M_{\text{sun}}$, core collapses all the way to a **black hole**
 - Final mass $M \sim 3 - 100 M_{\text{sun}}$, $r_{\text{S}} \sim 9 - 300$ km



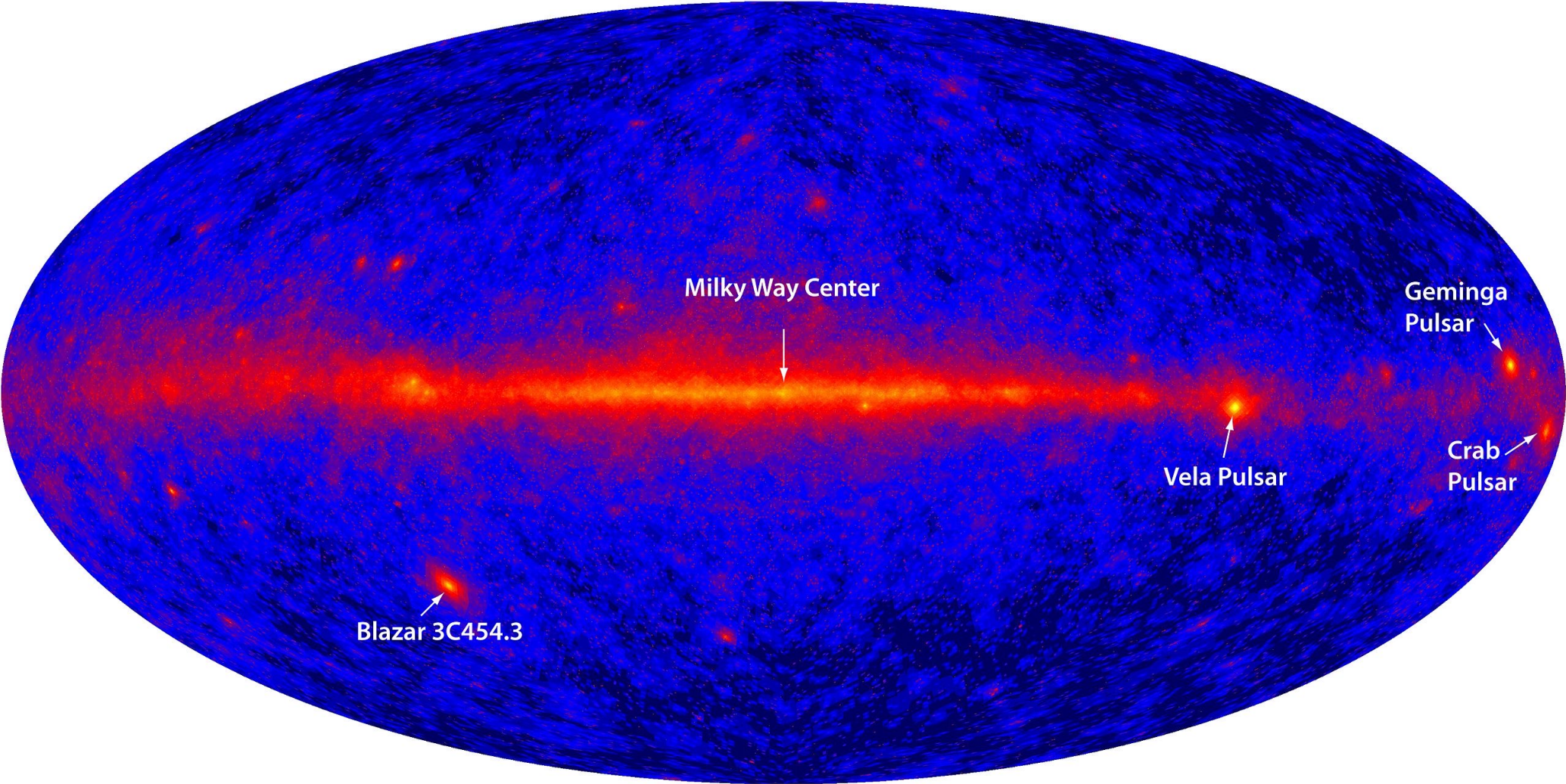


Crab Pulsar Wind Nebula

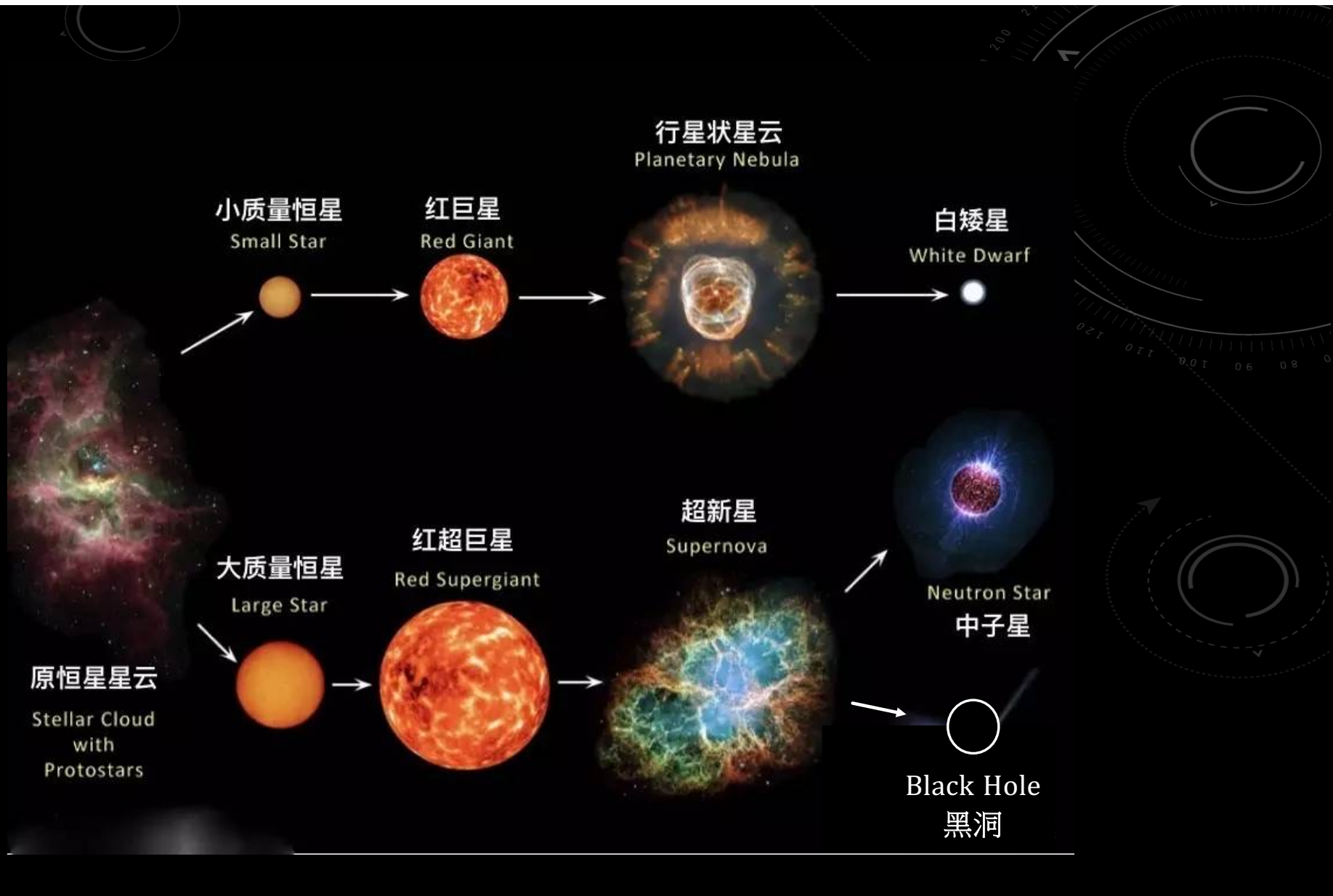
Optical image of Crab Nebula (supernova remnant)

Red: optical (HST)
Blue: X-ray (Chandra)

GAMMA-RAY ALL-SKY MAP BY FERMI TELESCOPE



行星状星云





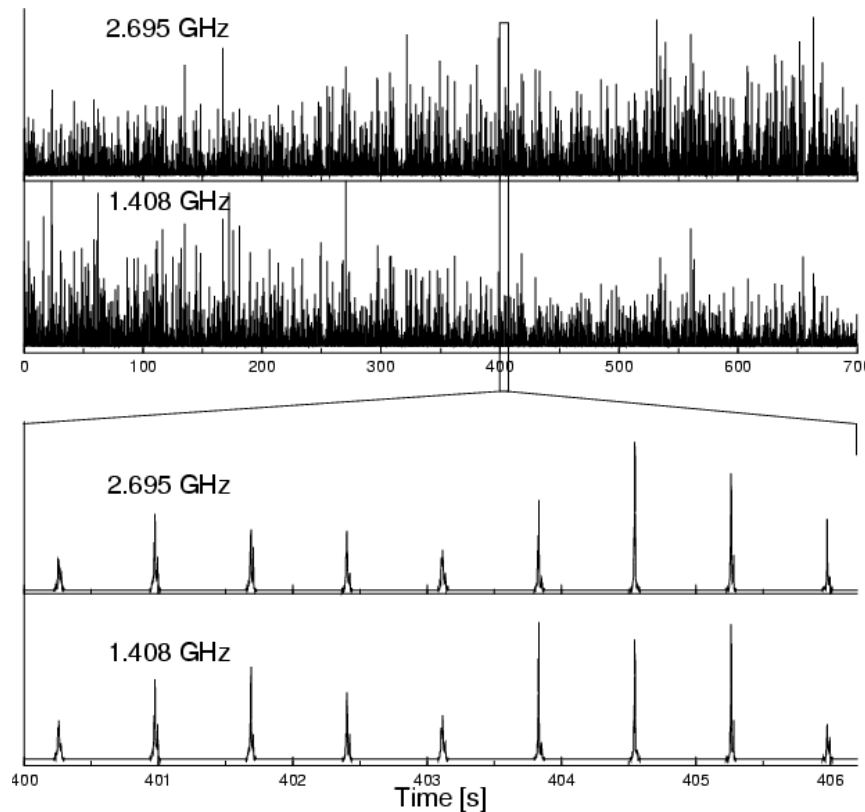
NEUTRON STARS



DISCOVERY OF PULSARS

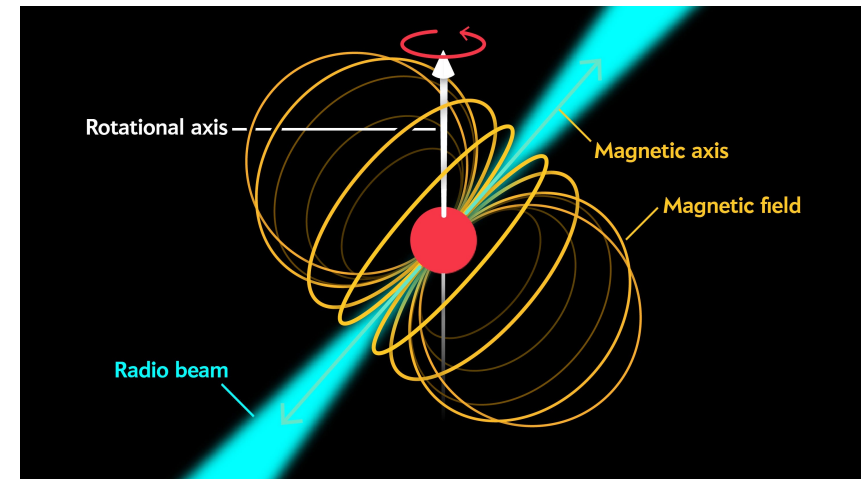
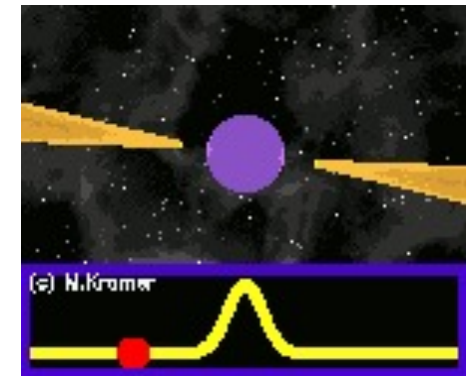
- Bell & Hewish (1967)
 - Constructed new type of radio telescope to study radio emission from quasars
 - Bell noticed a periodic signal pulsing every 0.71 seconds
- A rotating star will fly apart if the centrifugal force exceeds gravity
 - Many pulsars are rotating so rapidly that even a dense WD would fly apart
 - This led people to seriously consider the idea of a NS (first hypothesized by Zwicky in 1930s)

Jocelyn Bell
Burnell
(Age 24)



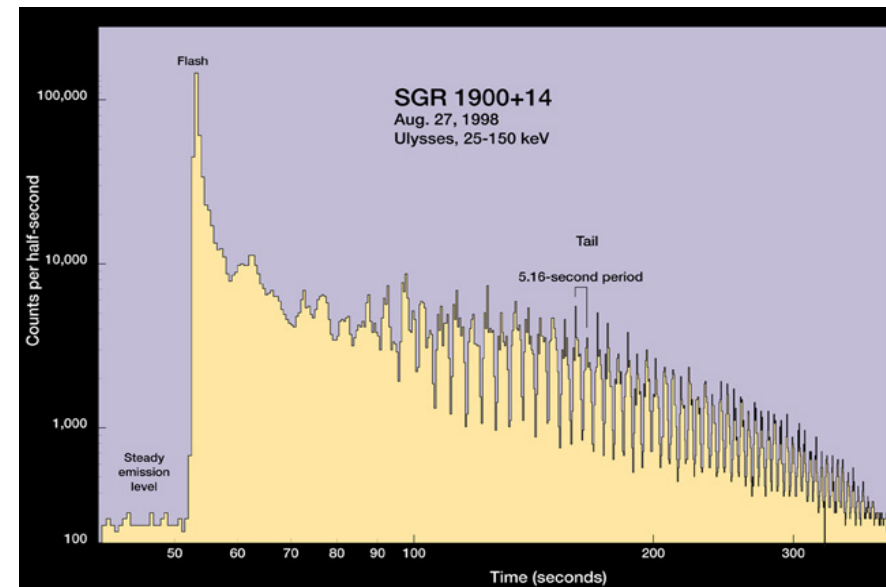
PULSARS

- Magnetized, rotating NSs
- At the magnetic poles, magnetic field is strong enough to produce high-energy particles, creating beams of light in radio and X-ray
- The beam can be seen from Earth as periodic pulses like a lighthouse
- Their pulses are precise clocks
 - Shapiro time delay to probe spacetime curvature
 - Pulsar navigation (similar to GPS)
- Rotational period ranges from milliseconds to seconds

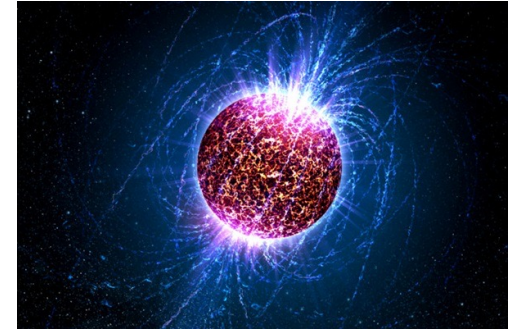


MAGNETARS

- A particular class of NSs that have extremely strong magnetic fields (up to 10^{11} Tesla!)
- Sometimes, these enormous magnetic fields “snap”, leading to intense explosions
 - A dramatic example occurred on 8/27/1998
 - During a 0.2s period of time, it produced 100x the total luminosity of our galaxy
 - It produced a major disturbance of our upper atmosphere (despite a distance at 20000 lyrs)
 - Every X-ray/gamma-ray satellite detected it
 - Good that it wasn't closer!



PROPERTIES OF NS

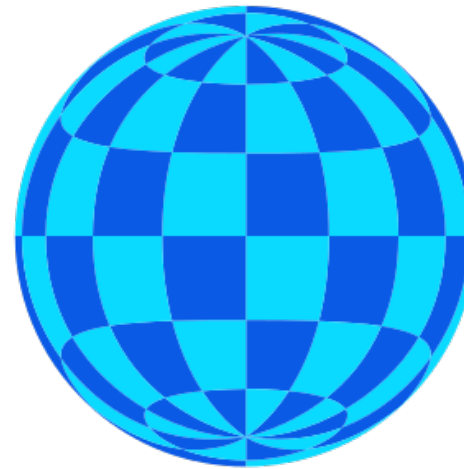
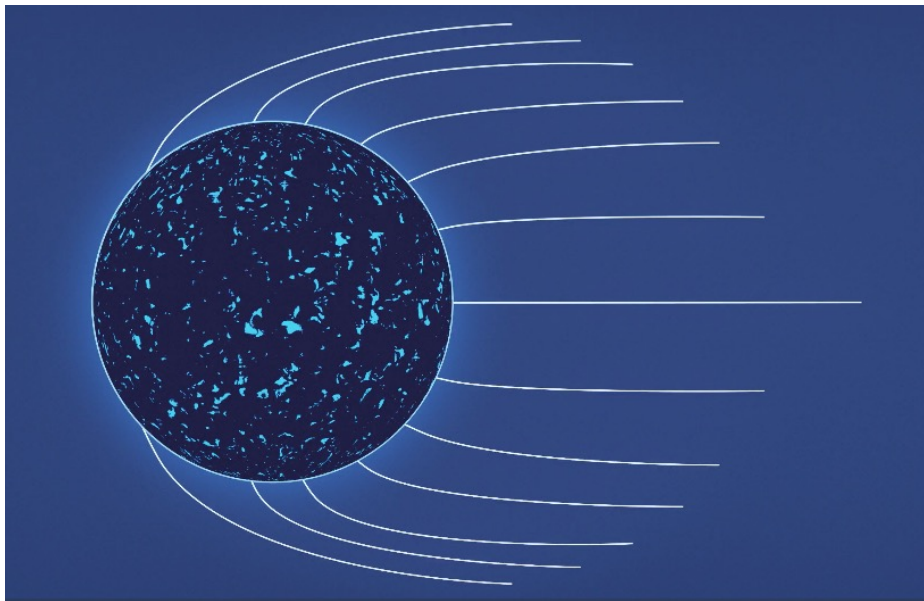


- A star made almost entirely out of neutrons $p + e^- \rightarrow n + \nu$
- Neutrons normally decay, but at high densities they can't
- Gravity supported by **neutron degeneracy pressure**
- It is close to being a black hole
 - Size close to the Schwarzschild radius; escape velocity close to c
- Surface gravity is so large that any mountain higher than a mm would crash!
 - Surface gravity $\sim 10^{13}$ of that on Earth!
 - Anything would be crashed onto the surface – let's call it “Lasagne effect”



GRAVITATIONAL LENSING OF NS

- Light bending effect is so strong that it is possible to see the back side of a NS!



Each patch represents 30 deg x 30 deg

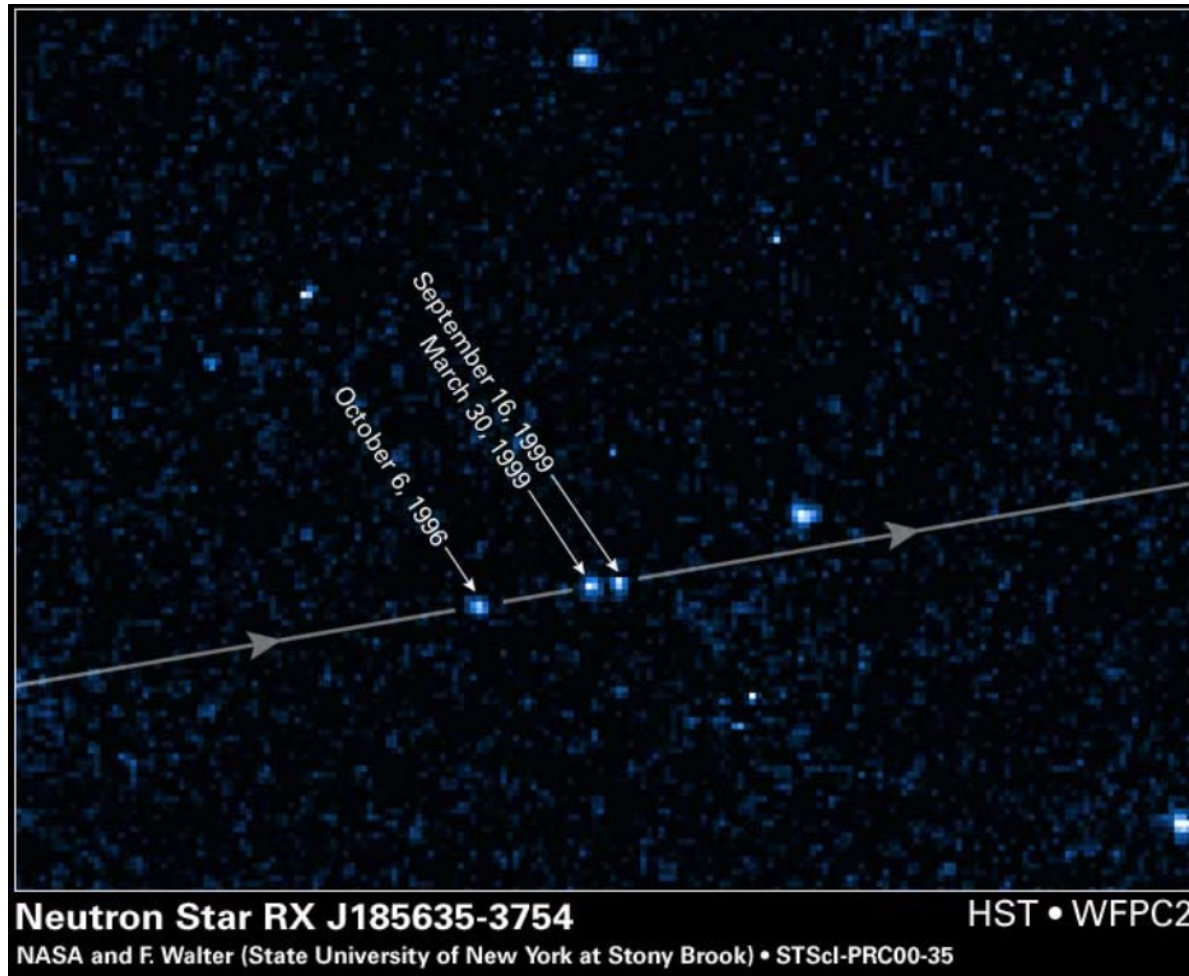


EXTREME CONDITIONS OF NS

- **Extreme gravity**
- **Extreme density**
 - $M \sim 2 M_{\text{sun}}$, $R \sim 10 \text{ km}$, density $\sim 5 \times 10^{17} \text{ kg/m}^3$ – as dense as an atomic nucleus
 - A teaspoon of NS material would weigh ~ 100 million tons!!
- **Extreme temperature**
 - $T \sim 10^{11-12} \text{ K}$ for a newly formed NS
 - As neutrinos take energy away, $T \sim 10^6 \text{ K}$, emitting in X-ray
- **Extreme rotation**
 - Record holder: PSR J1748-2446ad with 716 times per second, rotational velocity $\sim 0.24c$!
- **Extreme magnetic field**
 - Typical NSs: 10^{4-8} Tesla
 - Magnetars: 10^{8-11} Tesla
- These extreme conditions cannot be probed on Earth!

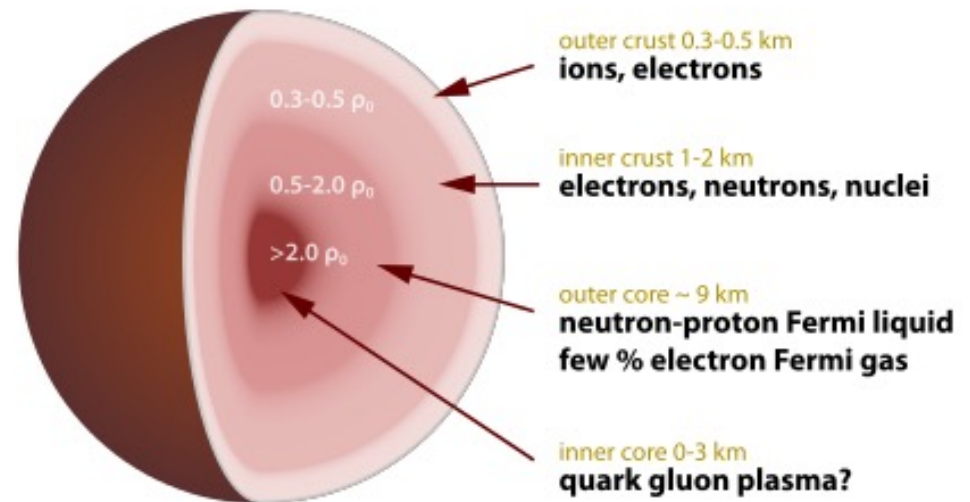


NS CAN BE SEEN IN X-RAY OR OPTICAL IF CLOSE ENOUGH



STRUCTURE OF NS

- State of matter at the core of a NS is still unknown
 - Cannot be probed in labs on Earth
 - May be new and exotic states of matter
- This is why it is crucial to measure the **"equation of state"** (EOS) of NSs
 - EOS is relationship between pressure and density
 - Different matter would have different levels of "squishiness", hence different EOS
 - Given an assumed EOS of any theoretical model, one can compute the expected **mass-radius relation**
 - Comparing with the observed mass-radius relation could constrain the EOS of NSs



NS INTERIOR COMPOSITION EXPLORER (NICER)

- NASA's X-ray instrument to probe the composition of NSs
- Installed on the ISS since 2017
- It has the capability to take rotation-resolved spectra in the soft (0.2-12keV) X-ray band
- It will measure the mass-radius relation for nearby pulsars and constrain their EOS

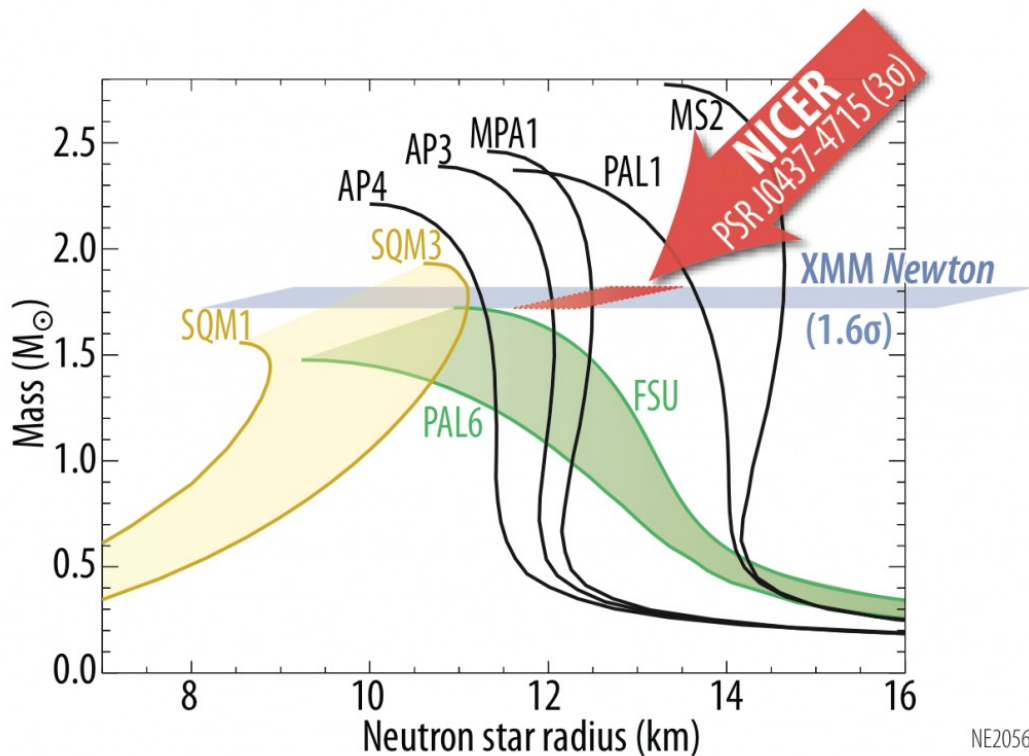


OBSERVING PULSAR J0030+0451

- Figure compares two models of the pulsar's hotspots
- Due to gravitational lensing, it is possible to see both hotspots of the pulsar
- The modeled locations, shapes, and numbers of hotspots are surprising, suggesting that magnetic field configurations are even more complex on NSs!
- **NASA's NICER Delivers Best-ever Pulsar Measurements, 1st Surface Map**



CONSTRAINING EOS IN THE FUTURE



- NICER will measure the mass and radius to high precisions, constraining the state of matter in the cores of NSs
- Left figure shows the **predicted** uncertainty levels of NICER measurements
- EOS constraints coming soon – stay tuned!!



UPPER LIMIT OF NS MASSES?

- Maximum mass of a NS is not known exactly
 - Limit could be derived in a similar way to the Chandrasakhar limit for WDs
 - But details depend on the unknown EOS
- In 1939, Robert Oppenheimer and others predicted the upper limit of NS masses to be $\sim 2.1 M_{\text{sun}}$ above which stars would collapse and no known forces could prevent it from collapsing into black holes (i.e., *the Tolman-Oppenheimer-Volkoff or TOV limit*)
- In 2018, using the NS-NS merger event GW170817 detected by LIGO, the upper limit is updated to $\sim 2.16 M_{\text{sun}}$
- Most massive NS found to date: $2.14 M_{\text{sun}}$ in 2019 [[Here's the news link](#)]
- Working assumption: $> 3 M_{\text{sun}}$ defined as stellar-mass BHs



WHY STUDY NEUTRON STARS?

- It allows us to probe physics in extreme conditions!
- Spacetime is almost as curved as BHs -> can be used to probe effects of GR
- NSs are governed by the same physics laws as BHs and can thus offer cross-checks of our understanding
- NSs have more "hair" and are more complex than BHs, it allows us to have more ways to study them
- For example, NS-NS mergers would have electromagnetic counterparts to the GWs, enabling multi-messenger astronomy!
- **Article -- "Golden Age of Neutron Stars"**

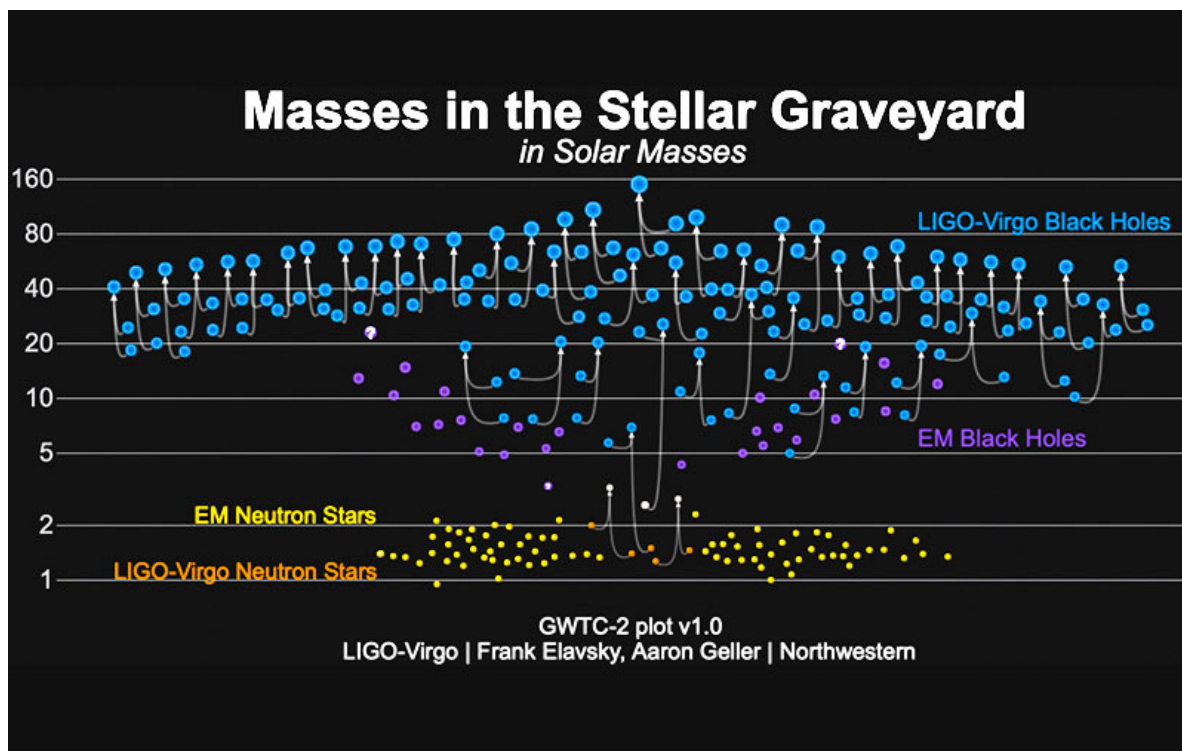




STELLAR-MASS BLACK HOLES



STELLAR-MASS BH INVENTORY TO DATE

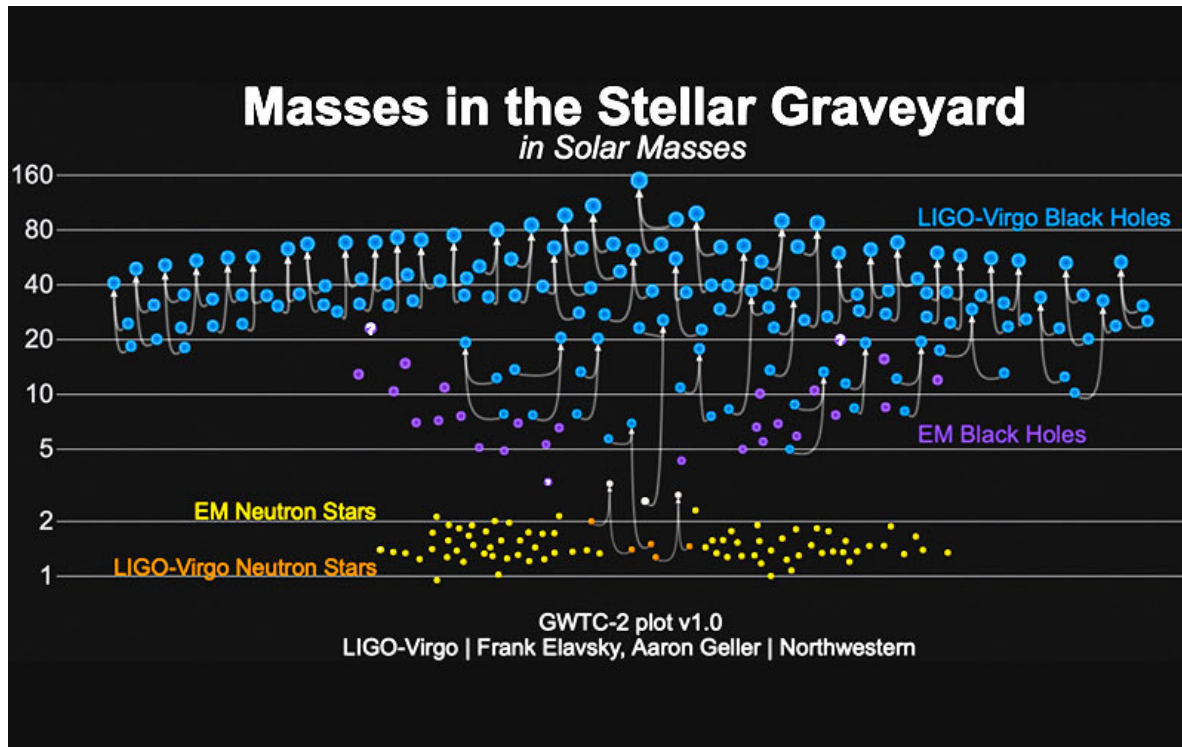


- Observed number of smbhs is determined by
 - **Intrinsic distribution** of smbhs
 - Observational **selection effects**
- Selection effects:
 - GWs tend to detect more massive smbhs because they could more easily produce spacetime ripples above the current sensitivity
 - For EM sources, fluxes decay with $1/r^2$, so can only detect nearby sources

*EM black holes: BHs that produce EM waves, i.e., light (e.g., X-ray binaries)



STELLAR-MASS BH INVENTORY TO DATE



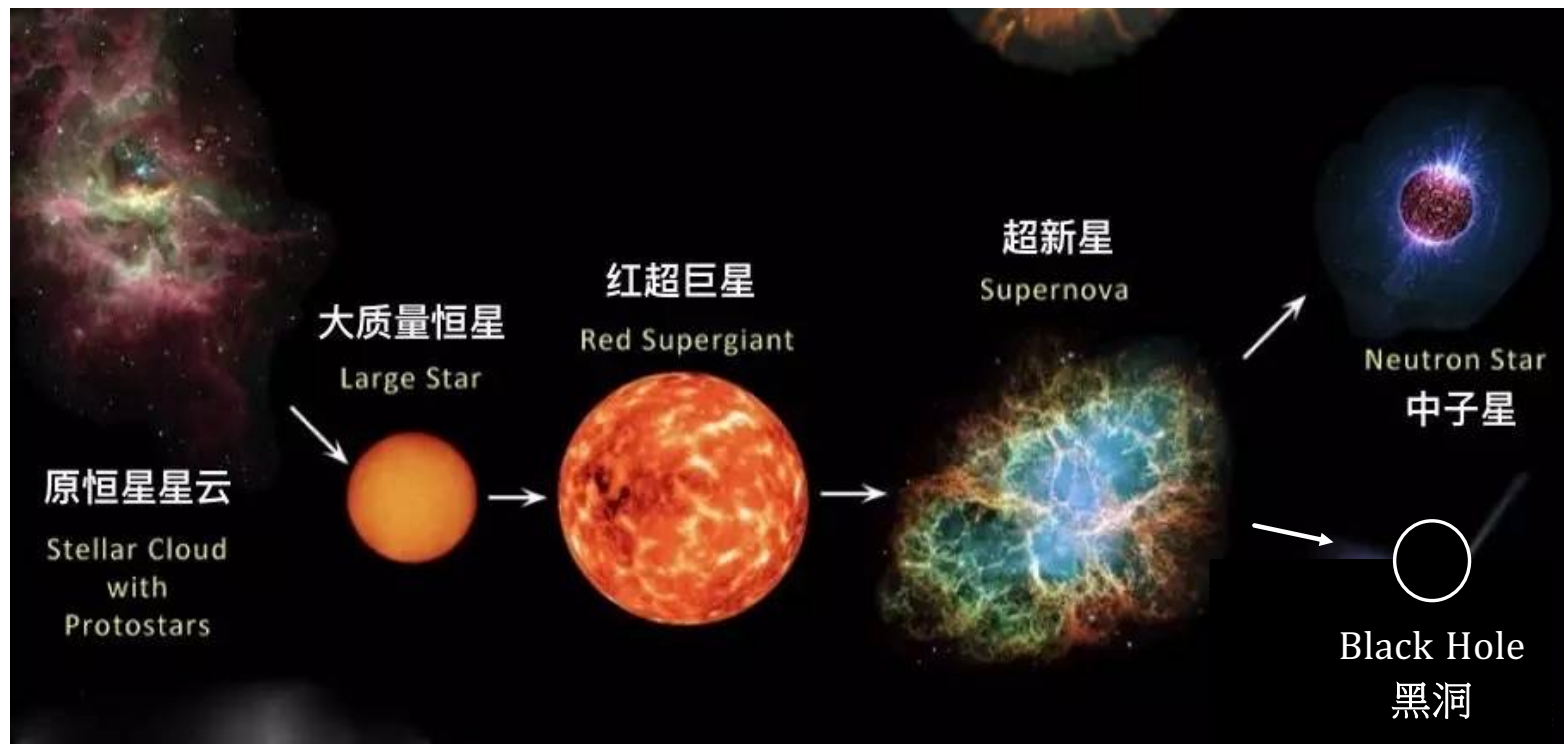
Important questions to address:

- Can stellar evolution models predict intrinsic distributions consistent with the observed distribution?
- Can models explain the **range of masses of smbhs** ($\sim 5-160 M_{\text{sun}}$) that are observed?



WHAT DO WE KNOW ABOUT THE INTRINSIC DISTRIBUTION OF BHS?

- Recall the formation process of smbh:



$$M_{\text{core}} > 3 M_{\text{sun}}$$

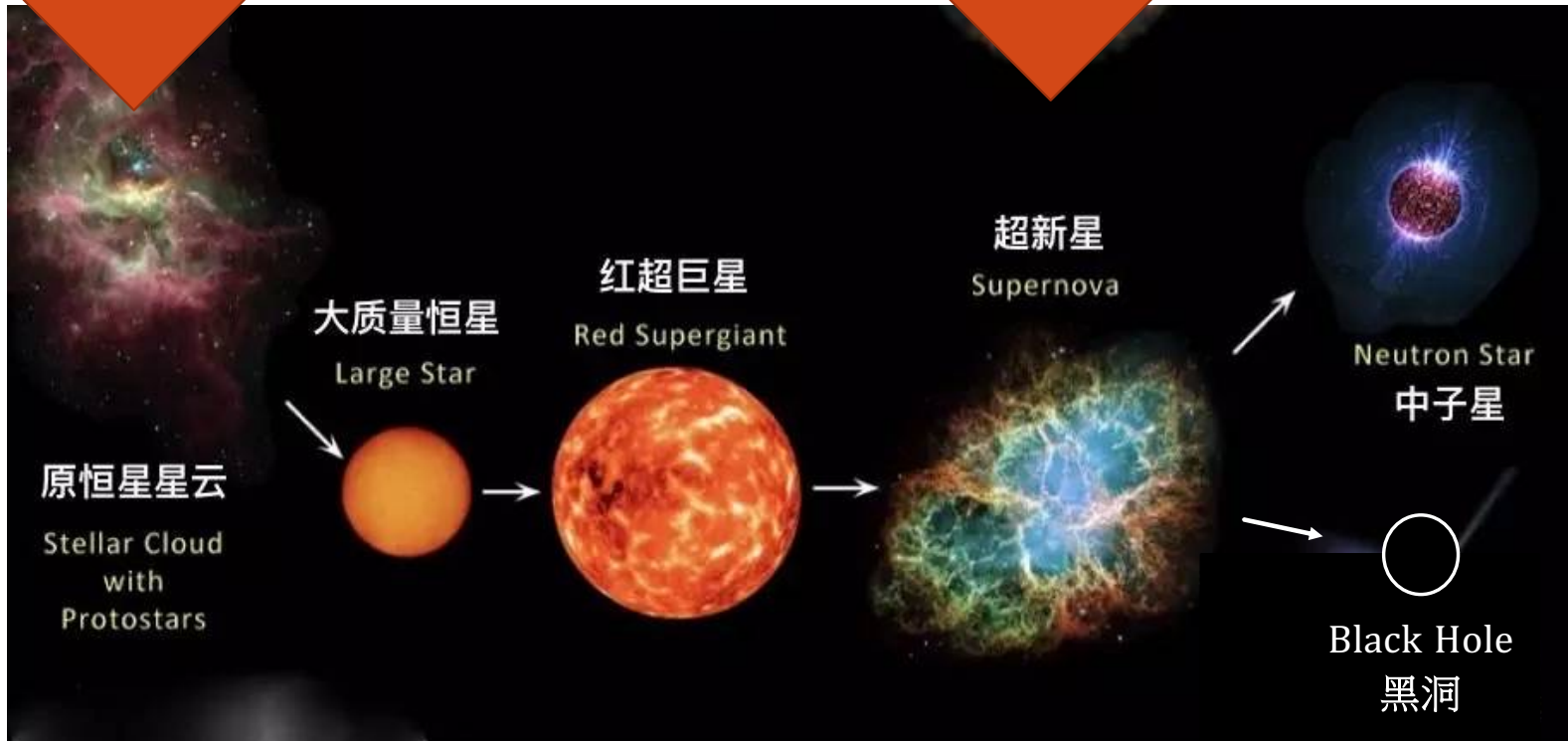


HOW DO WE KNOW THE MASS DISTRIBUTION OF STARS?

Q1: What is the mass distribution of stars when they are formed?

Q2: How much mass is left in the core during supernova explosions?

the formation process of smbh:



$$M_{\text{core}} > 3 M_{\text{sun}}$$



Q1: WHAT IS THE MASS DISTRIBUTION OF STARS WHEN THEY ARE FORMED?

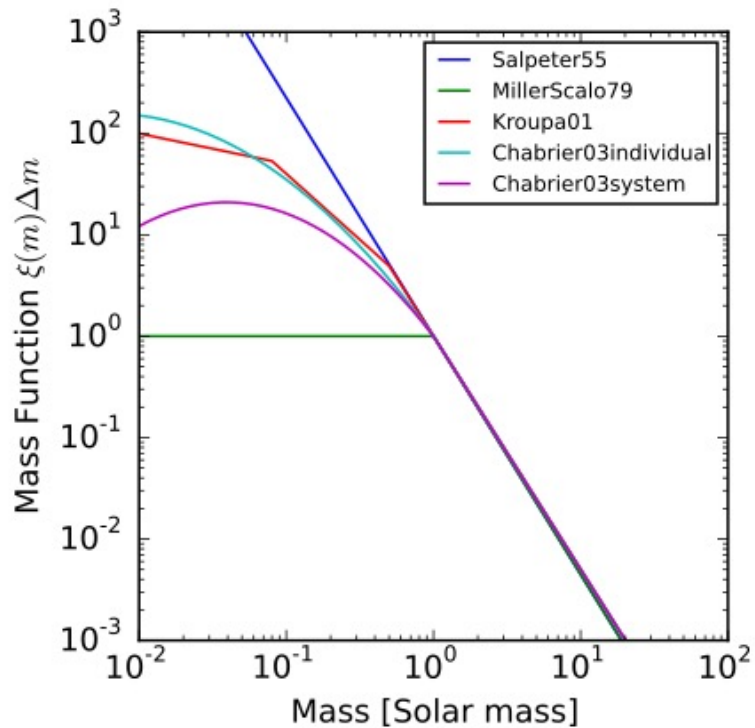
Star forming region in M17



- Stars form in dense, molecular gas
- They often form in groups
- The mass distribution of stars for a population is called the "***initial mass function (IMF)***"
- The IMF can be empirically inferred



INITIAL MASS FUNCTION

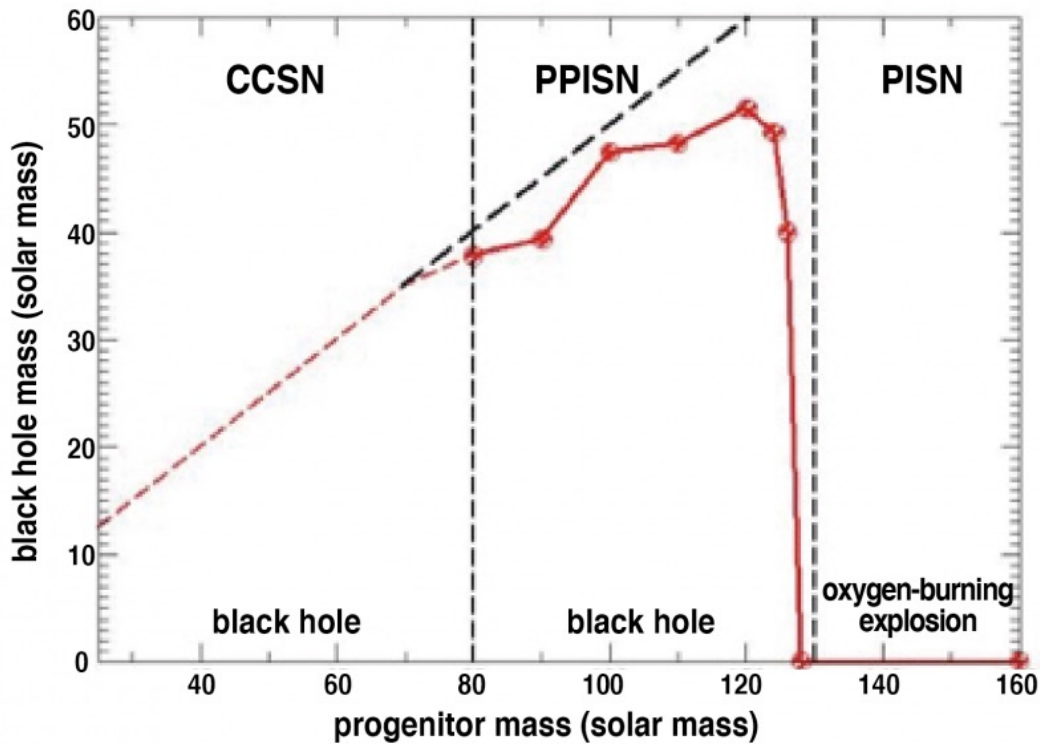


- In general, low-mass stars are abundant, and high-mass stars are rare
- Despite uncertainties for $M < M_{\text{sun}}$, for high-mass stars, the number density of stars is well described by a power-law with slope of -2.3



Q2: HOW MUCH MASS IS LEFT IN THE CORE DURING SUPERNOVA EXPLOSIONS?

Predicted relation between initial stellar mass and BH mass



- PPISN = Pulsational Pair-Instability Supernova
 - Initial mass $\sim 80\text{-}130 M_{\text{sun}}$
 - Creation of electron-positron pairs induce pulsations that cause mass loss
- PISN = Pair-Instability Supernova
 - Initial mass $\sim 130\text{-}250 M_{\text{sun}}$
 - Creation of electron-positron pairs make the star unstable
 - The explosion is so violent that no remnant is left
- For stars with initial mass $> 300 M_{\text{sun}}$, it is possible to directly collapse into a BH with $M \sim 150 M_{\text{sun}}$ without going through an explosion
- Therefore, **models predict a gap of BH masses between $\sim 50\text{-}150 M_{\text{sun}}$!!**



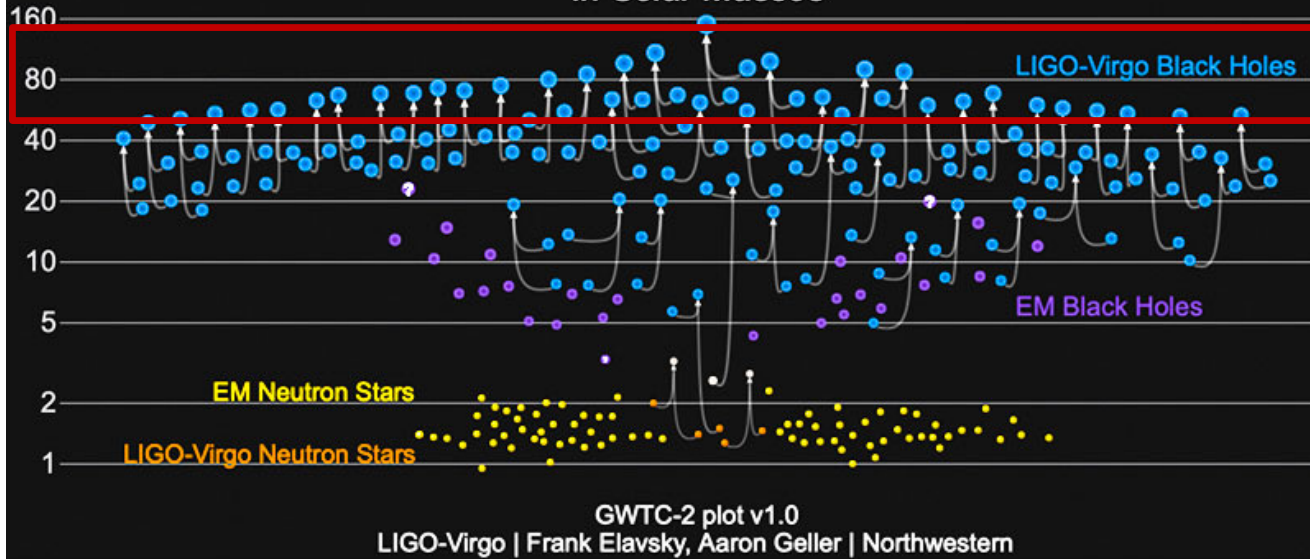
NEWS ABOUT A STELLAR-MASS BH THAT IS SO MASSIVE THAT IT SHOULDN'T EXIST?

- On 11/27/2019, a Nature paper claimed discovery of a SMBH of $M \sim 70 M_{\text{sun}}$
 - It made the news because the mass is within the “gap” of SMBH formation
- But, two weeks later, two other groups found mistakes in their measurements – the SMBH found turned out to be just a normal one...
- Great example of how science proceeds
- Read more from this news: [Impossibly Big Black Hole Was Probably Impossible After All](#)



BACK TO THE BHS FOUND BY GWS...

Masses in the Stellar Graveyard *in Solar Masses*



These guys were totally unexpected and have sparked great interests to revise the stellar-evolution models!!





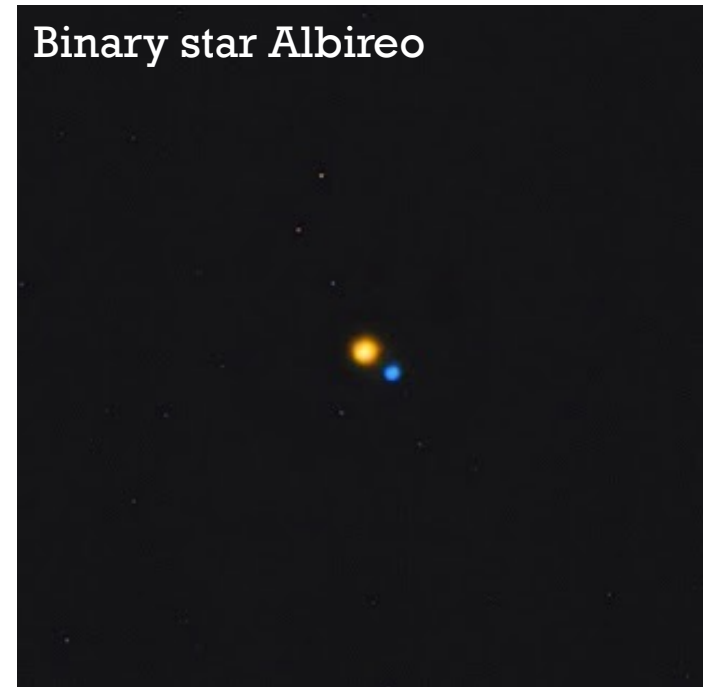
EVOLUTION OF BINARY STARS



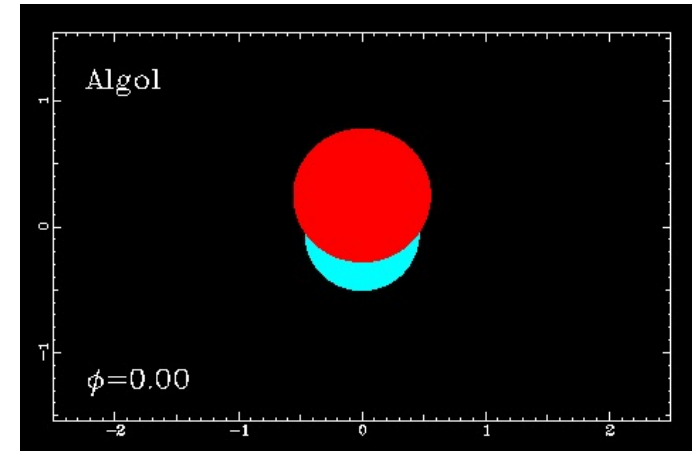
BINARY STARS

- Most stars in the Solar neighborhood are part of binaries (fraction $\sim 2/3$)
- Separations range from almost touching “*contact binaries*” to thousands of AU
- Majority are far enough apart that the stars evolve independently
- Binary paradoxes – some observed binaries have properties that are hard to understand in terms of the known evolution of single stars

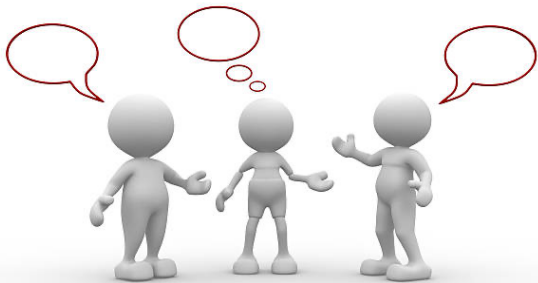
Binary star Albireo



BINARY PARADOX -- ALGOL



- Algol binary system consisting of:
 - Star A – a massive blue star on the MS
 - Star B -- a less massive red star that is on the way to becoming a red giant
- But shouldn't the more massive star evolve faster and become a giant first?

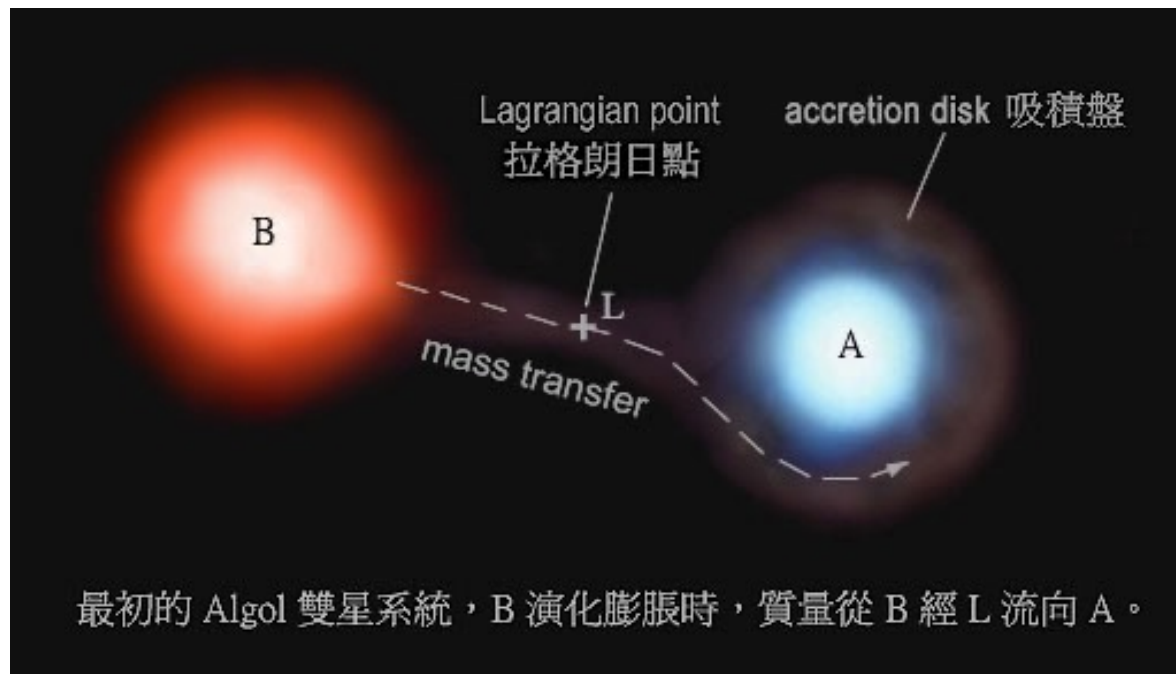


Group discussion (Q1):

Please break into groups of 3-4 people. Discuss your answers and provide an explanation. Write down your names and answers on a piece of paper and submit it to the TA after class.



THE KEY IS MASS TRANSFER!

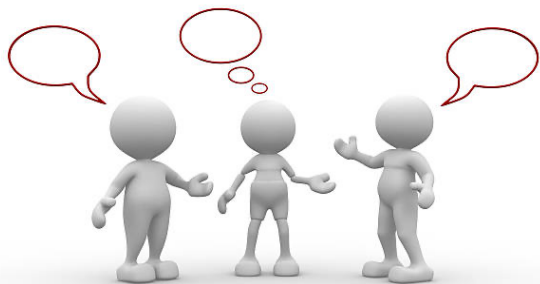
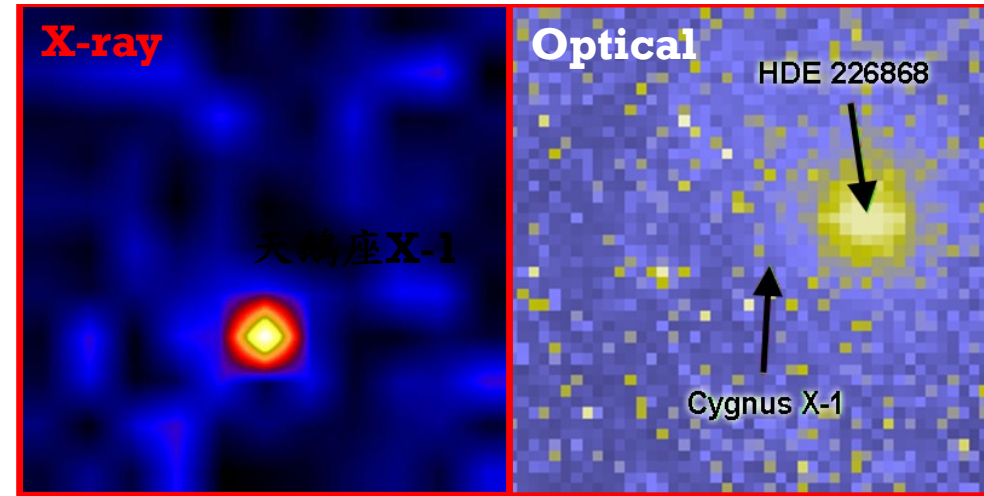


- In the beginning, B was in fact the most massive star
- As B evolves faster and became a giant star, its outer atmosphere flowed to A
- This is why A is more massive now, but still on the MS



RECALL CYGNUS X-1

- It is a BH X-ray binary
- This system has two stars:
 - Cygnus X-1: a stellar-mass BH with $M \sim 21.2 M_{\text{sun}}$
 - HDE226868: a blue supergiant star with $M \sim 40 M_{\text{sun}}$
- How exactly did the system evolve from its formation to the current stage? Please write down or draw the different stages of its evolution.

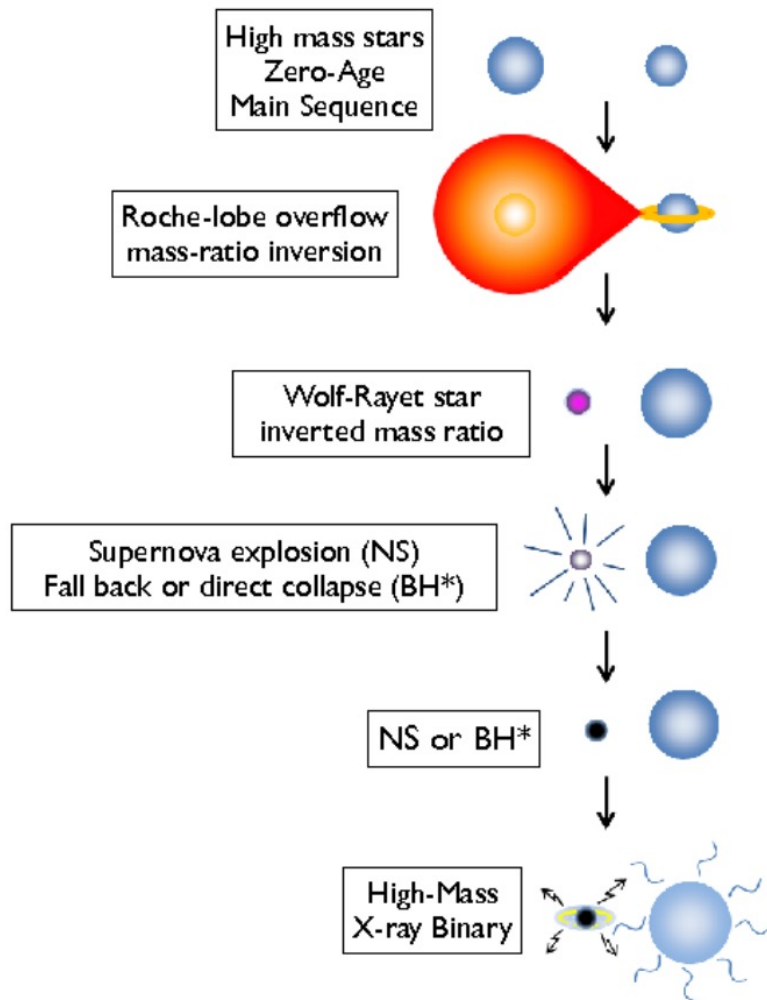


Group discussion (Q2):

Please break into groups of 3-4 people. Please discuss and write down/draw how this system evolved. Write down your names and answers on a piece of paper and submit it to the TA after class.



EVOLUTION OF CYGNUS X-1

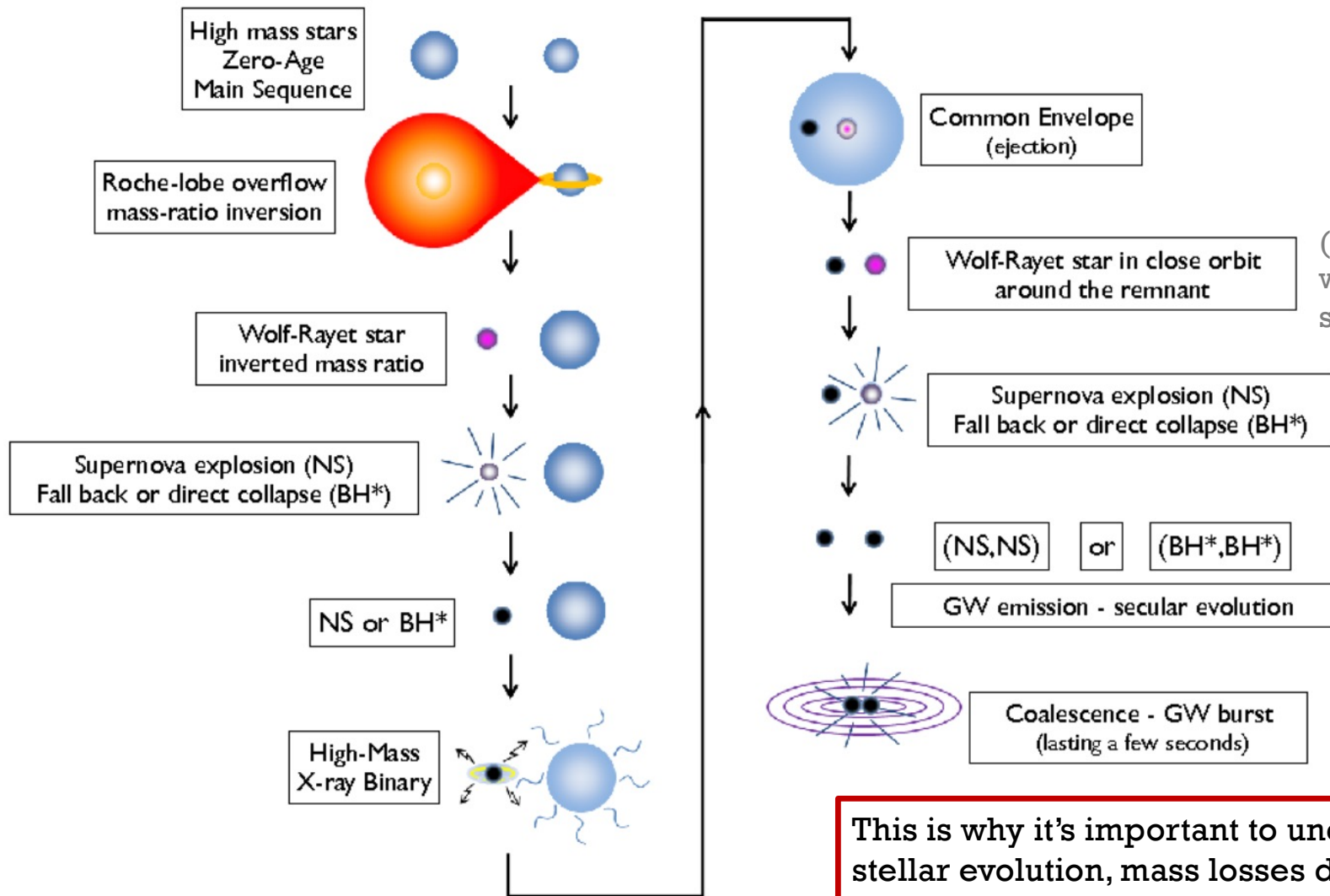


Two critical roles of *mass transfer*:

- 1) Need mass transfer to the lower-mass star so that it is not disrupted by the SN explosion of the higher-mass star
- 2) Need further mass transfer onto the NS/BH to produce observable emission via accretion



FUTURE OF CYGNUS X-1



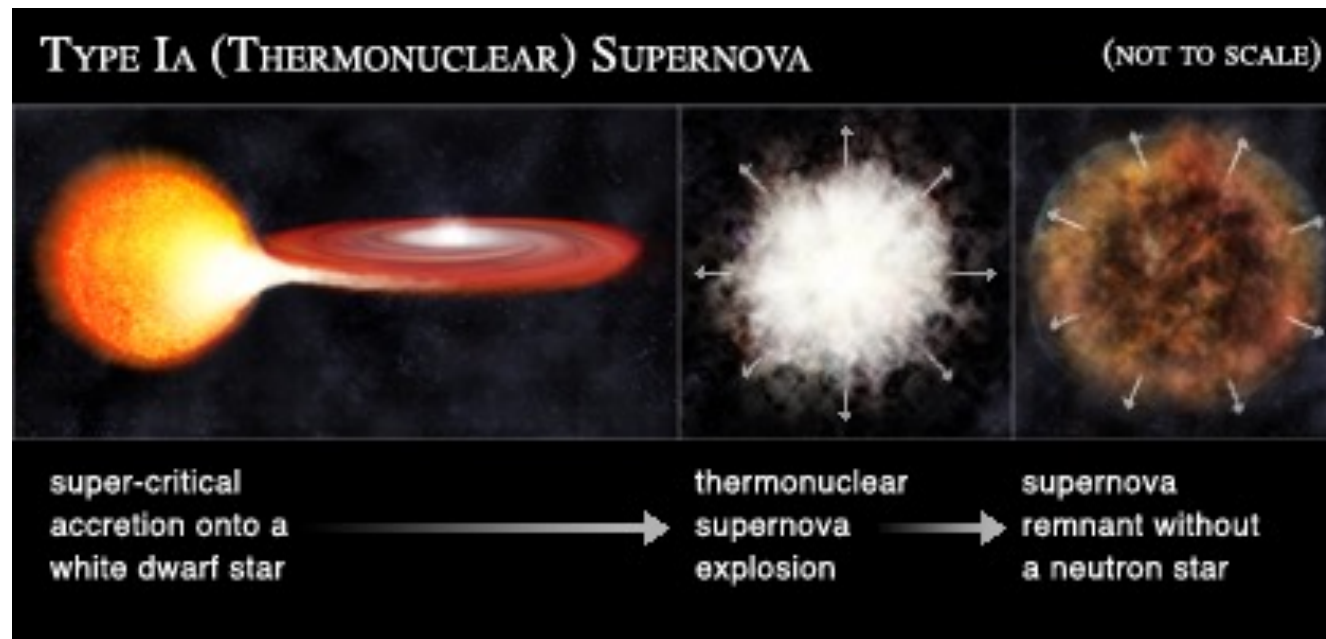
(WR stars: massive, hot stars with heavy elements & strong stellar winds)

This is why it's important to understand details of stellar evolution, mass losses due to stellar winds, supernova explosions, common envelopes, etc!



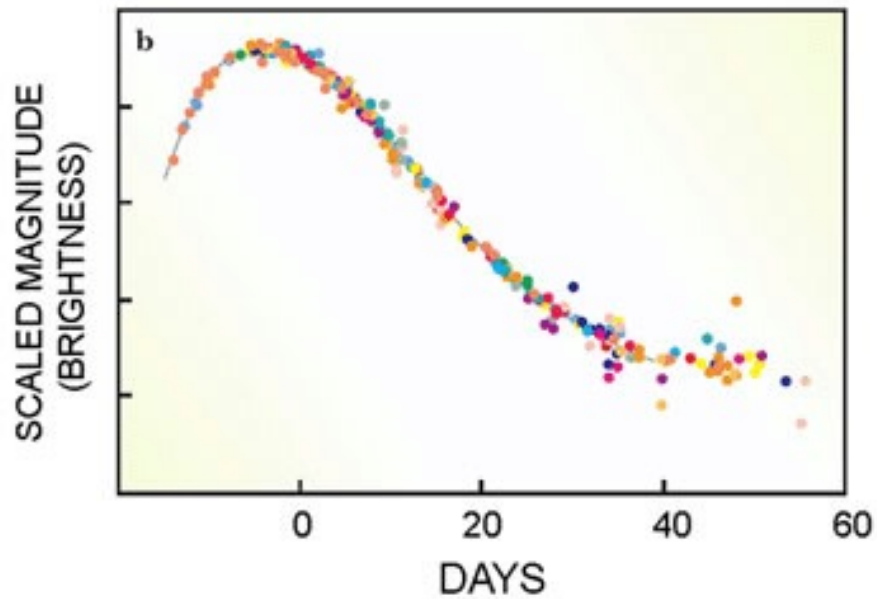
TYPE IA SUPERNOVAE

- In a binary system in which there is a WD, when it accretes from the companion star or merge with a second WD, it could reach the Chandrasakhar limit of $M \sim 1.4 M_{\text{sun}}$ and explode as a supernova



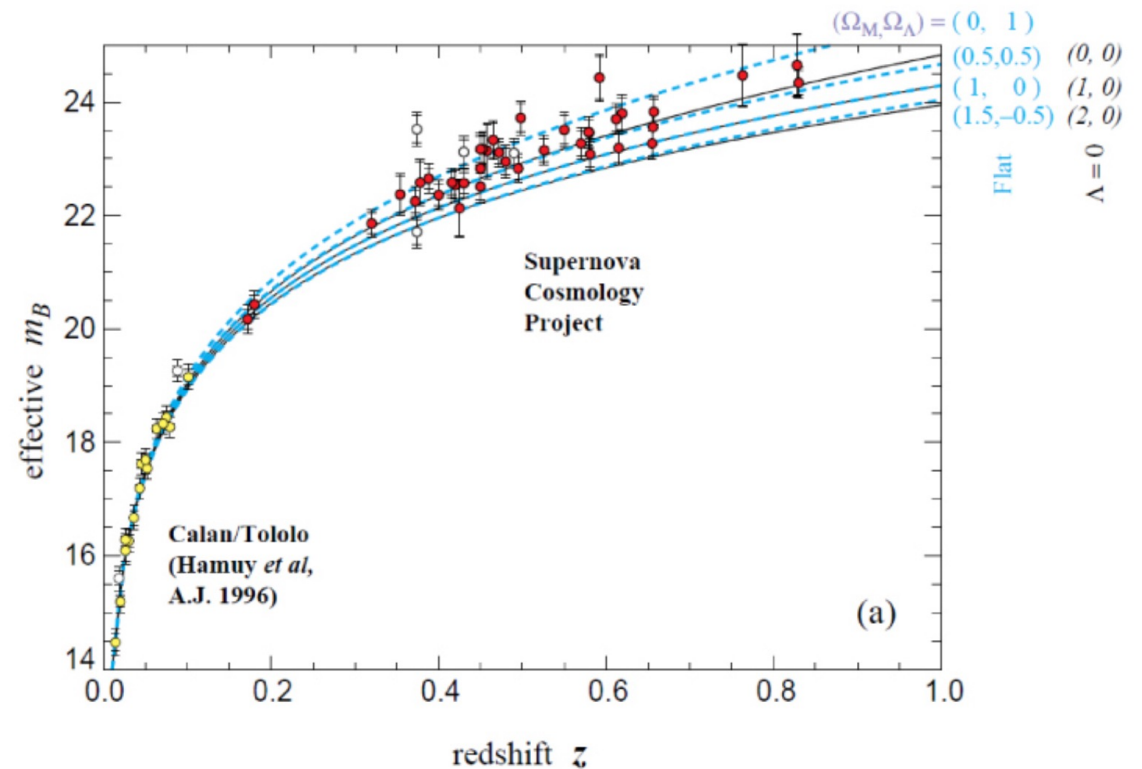
TYPE IA SUPERNOVAE ARE STANDARD CANDLES

- These Type Ia supernovae produce a consistent peak luminosity, which allows them to be used as standard candles to measure the distance in the universe



ACCELERATING EXPANSION OF UNIVERSE!

- Observations of Type Ia SNe led to the discovery of the accelerating expansion of the universe (blue lines)
- The observed relationship is inconsistent with a non-accelerating universe with $\Lambda = 0$ (black lines)
- Nobel Prize in Physics 2011



SUMMARY

- Evolution of a star is largely determined by its **mass**
 - All stars go through the **main sequence** phase of H burning
 - If $M < 8 M_{\text{sun}}$, $M_{\text{core}} < 1.4 M_{\text{sun}}$
star -> red giant -> planetary nebula -> WD
 - If $8 M_{\text{sun}} < M < 20 M_{\text{sun}}$, $1.4 M_{\text{sun}} < M_{\text{core}} < 2-3 M_{\text{sun}}$
star -> supergiant -> core-collapse (Type II) supernova -> NS
 - If $20 M_{\text{sun}} < M$, $3 M_{\text{sun}} < M_{\text{core}}$
star -> supergiant -> core-collapse (Type II) supernova -> BH
- Neutron stars
 - Properties & extreme conditions
 - Different types -- pulsars, magnetars
 - Internal structure & the unknown EOS of matter in the core – can be probed by the NICER X-ray mission by observing pulsars



SUMMARY

- Formation of stellar-mass BHs
 - Mass distributions of smbhs are determined by (1) initial mass function of star formation, and (2) core mass after supernova explosions
 - Supernova models predict a **mass gap** of smbhs in the range of **$\sim 50-150 M_{sun}$**
 - Some smbhs found by GWs are within this gap, challenging current theoretical models
- Evolution of binary stars
 - **Mass transfer** is an important process for (1) solving the binary paradox (ex: Agol), (2) explaining the emission of X-ray binaries by accretion (ex: Cygnus X-1)
 - A binary system with a WD can explode as **Type Ia SN** when the WD exceeds the Chandrasakhar limit
 - Understanding the evolution of binary systems is crucial for predicting the numbers of BH-BH, NS-NS, or BH-NS merger events that generate GWs

