Homework 1

Introduction to Black Hole Astrophysics (PHYS480)

(Due at the start of class on March 9, 2021)

Exercise 1

[Escape velocities (1 pt)] Escape velocity is the minimum speed needed for an object to escape from the gravitational influence of a massive body to reach an infinite distance from it. Assuming the massive body has mass M and the object is initially at a distance r from the massive body, then the escape velocity can be expressed as $v_{esc} = \sqrt{2GM/r}$.
(1) Please derive the above equation using the definition for escape velocity.
(2) What is the escape velocity to leave the Sun (a) from the Sun's surface, (b) from 1AU (Astronomical Unit, i.e., the Earth's distance from the Sun), and (3) from 180 AU?
(3) `Voyager-1` is a space probe that was launched by NASA in 1977. It is currently at a distance of 180 AU from the Sun with a speed of 17 km/s. Is it able to escape the gravitational pull of the Sun?
(4) (No credit; this is for your own pleasure.) Research and describe how `Voyager-1` was able to reach its current velocity.

Exercise 2

[Time dilation (0.5 pt)] On the International Space Station (ISS), the astronauts are orbiting the Earth at a speed of about 7.7 km/s. Based on special relativity (ignoring any acceleration needed to send the astronauts to space and bring them back), how much younger would the astronauts be compared to people on the Earth after one year?

Exercise 3

[The pole and barn paradox (1 pt)] A pole vaulter carries a 20-m long pole. He runs so incredibly fast that it is Lorentz contracted to just 10m in Earth's reference frame. He runs into a 10m long barn. Just at the moment when the pole is entirely contained inside the barn, the doors slammed closed, trapping both runner and pole inside. Then the doors opened to let the runner out. However, from the runner's point of view, it is the barn that is contracted to half its length. How can a 20-m pole fit inside a 5-m barn?
(1) Please explain in your own words (and/or figures) how this paradox may be resolved.
(2) Express your explanation using the spacetime diagram (you are only required to give qualitative answers).

Exercise 4

[Hadronic collisions (1.5 pt)] Hadronic collisions between high-energy protons and another proton in the interstellar medium is an important process for producing high-energy radiation. There is a threshold energy above which the reaction could occur. We are going to derive this threshold energy in this exercise.

Consider the reaction

$$p + p \longrightarrow p + n + \pi^+,$$

in which an incoming proton p (rest mass ≈ 938 MeV/c^2) of (total) energy E_p hits a target proton at rest, to create a proton, a neutron (also of rest mass ≈ 938 MeV/c^2) and a positive pion (rest mass 139 MeV/c^2). What is the minimum (threshold) kinetic energy that the incident proton requires for this reaction to take place?