Homework 3

Introduction to Black Hole Astrophysics (PHYS480)

(Due at 13:30 on Wednesday, April 7, 2021)

Exercise 1

[Discovery of black holes (1 pt)] It can be shown that, for a spherical object with mass M and radius R, its gravitational effect is identical to a point mass with mass M for any radius $r > R$. Therefore, although all the mass of a black hole is concentrated at the singularity, it is sometimes useful to calculate an *effective* average density by spreading its mass uniformly within the Schwarzschild radius. For this exercise, ignore the spin of the black holes.

(1) Estimate the effective average density of a stellar-mass black hole (smbh) of 10 solar masses. Can you find anything on Earth with comparable density to this black hole?
(2) Estimate the effective average density of a supermassive black hole (SMBH) of 10^8 solar masses. Can you find anything on Earth with comparable density to this SMBH?
(3) Based on your answers of the above two questions, please comment on whether it is easier to propose alternative theories to a smbh or a SMBH.

Exercise 2

[Comparing the compact objects (1.5 pt)] White dwarfs (WDs), neutron stars (NSs), and stellar-mass black holes (smbhs) are *compact objects* formed at the final stages of stellar evolution. In this exercise, we will compare some of the important properties of WDs and NSs to those of smbhs (again assume no spins). Assume typical WDs have masses comparable to the Sun and radius comparable to the Earth. For NSs, assume a mass of 2 $M_\odot$ and a radius of 10 km.

(1) Estimate the escape velocity at the surface of a WD and a NS, and compare them to the Newtonian limit of escape velocity for a smbh.
(2) Estimate the gravitational acceleration at the surface of a WD and a NS, and compare them with that at the Schwarzschild radius of a smbh of 10 solar masses. Comment on whether it is possible for humans to survive landing on the surface of a WD or a NS.
(3) Recall that near a massive object, due to strongly curved spacetime, clocks would appear to tick slower and the frequency of light would decrease as viewed from a distant observer, resulting in a gravitational redshift:

$$1 + z \equiv \frac{\lambda_\infty}{\lambda} = \frac{1}{\sqrt{1 - \frac{2GM}{c^2 r}}}.$$

Compute the gravitational redshift parameter z at the surface of a WD and a NS, and compare the numbers with that of a smbh at the ISCO.

Exercise 3

[Read an article about binary evolution (1.5 pt)] The detections of gravitational waves (GWs) have brought tremendous new insights into our understanding of smbhs, especially regarding theories of binary-star evolution. Please read an article written in 2016 after the first few detections of GW events and answer the following questions based on what you read. The link to the article can be found here.

(1) What was so special about the first GW detection, GW150914? What were the questions that puzzled the astrophysicists?
(2) What are the two theoretical models that were proposed to explain this event? Briefly summarize each theory using your own words.
(3) What are the pros and cons for each model? Which is the more likely scenario, in your opinion?