Homework 4

Introduction to Black Hole Astrophysics (PHYS480)

(Due at the start of class on April 27, 2021)

Exercise 1

[Bondi spherical accretion (1.5 pt)] Recall that Bondi accretion describes the simplest case of spherical black hole accretion within a fluid without angular momentum and magnetic fields.

(1) Please calculate the Bondi radius and Bondi accretion rate for a supermassive black hole (SMBH) with $M = 10^6\ M_\odot$ located within a galactic medium with a characteristic density of 1.67×10^{-24} g/cm^3 and sound speed of 200 km/s. How is the Bondi radius compared to the Schwarzschild radius of this SMBH?

(2) How is the Bondi accretion rate calculated in (1) compared to the Eddington accretion rate of this SMBH (assuming a radiative efficiency of $\varepsilon = 0.1$)? From your answer, comment on whether Bondi accretion is an efficient accretion mechanism for growing a SMBH.

(3) Given the same properties of the galactic medium, how massive does the SMBH need to be in order for it to reach the Eddington accretion rate by Bondi accretion? From your answer, please comment on whether Bondi accretion is a likely scenario to explain quasars that are accreting near the Eddington rate observed in the early universe.

Exercise 2

[Radiative efficiency of thin disks (1.5 pt)] In class we discussed the standard thin-disk model proposed to explain the large observed luminosity of highly accreting systems like quasars. One of the biggest successes of the thin-disk model is that, given the model assumptions, it is possible to reach a radiative efficiency on the order of ~ 10%, consistent with the expected value derived from General Relativity. We will derive part of this important result in this exercise.

Given the assumptions of the thin-disk model (e.g., $H/R \ll 1$, parametrized viscosity, being radiatively efficient and opaque), one can derive an expression for the energy dissipation rate due to viscosity per unit area on each face of the disk as a function of radius R,

$$Q(R) = \frac{3GM\dot{M}}{8\pi R^3}\left(1 - \sqrt{\frac{R_{in}}{R}}\right).$$

(1) Assuming all this energy is radiated away, please integrate $Q(R)$ over the two faces of the disk from the inner radius R_{in} to the outer radius R_{out} and derive the total luminosity of the

disk,

$$L_d = \frac{3GM\dot{M}}{2R_{in}}\left[\frac{1}{3} - \frac{R_{in}}{R_{out}}\left(1 - \frac{2}{3}\sqrt{\frac{R_{in}}{R_{out}}}\right)\right].$$

(2) In the limit $R_{out} \gg R_{in}$,

$$L_d \approx \frac{GM\dot{M}}{2R_{in}}.$$

Using this approximation, and assume $R_{in} = R_{ISCO}$, compute the radiative efficiency $\eta \equiv L_d/\dot{M}c^2$ for (a) a Schwarzschild black hole, and (b) a maximally spinning black hole with spin parameter $a = 1$.

Exercise 3

[Ultra-luminous X-ray sources (0.5 pt)] The observed luminosity of ultra-luminous X-ray sources (ULXs) is $L \sim 10^{32} - 10^{35}$ W. Assuming they are accreting black holes at the Eddington rate, what are their inferred masses? Would you classify them as stellar-mass black holes ($M < 100\ M_\odot$), intermediate-mass black holes ($M \sim 10^2 - 10^5 M_\odot$), or supermassive black holes ($M > 10^5 M_\odot$)?

Exercise 4

[Measuring black hole spins (0.5 pt)] So far there have been no robust spin measurement for the Sgr A*, the supermassive black hole at the center of our Milky Way Galaxy. Could you please briefly explain the reasons why we have not been able to measure its spin using the three methods discussed in class?