Homework 6

Introduction to Black Hole Astrophysics (PHYS480)

(Due at the start of class on May 25, 2021)

Exercise 1

[Superluminal motion (2 pt)] Superluminal motion is the apparently faster-than-light motion seen in some black hole jets, inferring that they are traveling at relativistic speeds. In class we discussed about a specific geometry, in this exercise you will derive a general expression for the apparent speed, and find the configuration that enables the maximal apparent speed.

(1) Assuming a radio-emitting knot is moving with speed v from point A to B, where the angle between its motion and the line of sight is θ (same configuration as in the example discussed in class but with an arbitrary angle; see p.13 of Lecture 11). Show that this knot would appear to move in the sky with a velocity

$$v' = \frac{v \sin\theta}{1 - \beta\cos\theta},$$

where $\beta \equiv v/c$.

(2) Given the above expression, find the angle θ_{max} that would maximize the apparent speed v'. Please also write down the maximal apparent speed v'_{max} when $\theta = \theta_{max}$, and comment on why it could be greater than c. [Hint: You may want to use the Lorentz factor $\gamma = 1/\sqrt{1-\beta^2}$ to express your answer.]

Exercise 2

[Telescope resolution (0.5 pt)] Theoretical models suggest that the quasar SDSS J0100+2802 has an accretion disk that, at optical wavelengths, should span about 2×10^{-6} arcseconds (1 degree = 60×60 = 3600 arcseconds) on the sky. If the wavelength of optical light is about 500 nanometers, calculate the (effective aperture) diameter of the telescope that is needed to actually resolve and hence image this accretion disk. Comment on your answer (see `https://www.eso.org/public/teles-instr/technology/interferometry/` for some information about the most advanced optical/infrared telescopes achievable by humans given current technology).

Exercise 3

[Power of black-hole jets (0.5 pt)] The radio galaxy Cygnus-A has a spectacular jet with an estimated power of $L = 4 \times 10^{39}$W. Suppose that the jet is powered by the spin of a very rapidly spinning supermassive black hole of $3 \times 10^9 \, M_\odot$. Using the fact that up to 30% of the mass-energy of a black hole can be in the form of spin energy and tapped to power the jet, estimate how long (in units of years) the black hole can drive a jet of this power.

Exercise 4

[Visualizing AGN feedback in the process of galaxy formation (1 pt)] Watch a video about galaxy formation and AGN feedback simulated using state-of-the-art cosmological simulations, the Illustris simulation, and answer the following questions. The link to the animation is `https://youtu.be/Z2dy1Lyzs94`.

(1) Describe briefly what you saw in this video, and write down one thing you have learned or found interesting about the process of AGN feedback or galaxy formation.

(2) In the end of Lecture 9, we talked about the "downsizing" of AGNs, referring to the fact that there are more luminous AGNs, such as quasars, at higher redshifts (i.e., in the early universe) than the present day. From what you saw in this video, provide explanations for possible effects that may cause this downsizing effect of AGNs.