Homework 7

Introduction to Black Hole Astrophysics (PHYS480)

(Due at the start of class on June 8, 2021)

Exercise 1

[Finding black hole seeds (1 pt)] In class we discussed that looking for massive black holes in dwarf galaxies could possibly distinguish the different black hole seed formation scenarios. Please read the following news about such a study and answer the following questions. The article can be found here: `https://aasnova.org/2020/08/25/investigating-black-hole-formation/`.

(1) Please summarize the current status of finding massive black holes in dwarf galaxies according to the article you read.

(2) Do their results support any of the three black hole seeding mechanisms?

Exercise 2

[Dynamical friction (1.5 pt)] In our current understanding of hierarchical galaxy formation, smaller galaxies merge into bigger and bigger galaxies over cosmic time. In the process of galaxy mergers, the two supermassive black holes (SMBHs) at their galactic centers also merge into a more massive SMBH. One of the important physical processes that brings the SMBHs closer is dynamical friction, which refers to the gravitational drag force when the SMBHs pass through the ensemble of stars in the merged galaxy. The acceleration due to dynamical friction can be expressed as

$$a_{DF} \approx -0.428 \frac{4\pi G^2 M_{BH} \rho \ln \Lambda}{v^2},$$

where the SMBH with mass M_{BH} is moving at speed v through a field of stars with density ρ, and $\Lambda = b_{max}/b_{min}$ is the ratio between the largest and smallest impact parameters in the system ($\ln \Lambda \approx 3$ for typical galaxies).

(1) Assuming that the SMBH is moving in with nearly circular orbits (so that $v \approx v_c = \sqrt{GM(r)/r}$, where $M(r)$ is the mass inside radius r), and assume that the stellar distribution is roughly spherical (i.e., $dM/dr = 4\pi r^2 \rho(r)$), show that the characteristic timescale for dynamical friction is

$$t_{DF} \approx \frac{1}{0.428} \frac{v r^2}{G M_{BH} \ln \Lambda}.$$

(2) Using the above result, estimate the timescale for a $10^8 M_\odot$ SMBH to fall to the galactic center due to dynamical friction given an initial circular velocity of 300 km/s at a radius of 100 pc.

(3) Based on your answer in (2), please comment on why SMBHs tend to lie at the center of galaxies, and whether it is easier to find SMBHs or intermediate-mass black holes (IMBHs) that are offset from their galactic centers.

Exercise 3

[Size of the black hole shadow (1.5 pt)] Recall (in HW2) that we derived the orbital equations for photons that travel around black holes in the Schwarzschild metric:

$$\left(\frac{dr}{d\tau}\right)^2 = k^2c^2 - \frac{h^2}{r^2}\left(1 - \frac{2GM}{c^2 r}\right).$$

(1) Let $b \equiv h/ck$, $\mu \equiv GM/c^2$, $V_{eff}(r) \equiv (1 - 2\mu/r)/r^2$, and recall that $r^2\dot{\phi} = r^2(d\phi/ds) \equiv h/c$ and $ds = cd\tau$, show that the above equation could be re-written as

$$\frac{1}{r^2}\frac{dr}{d\phi} = \left[\frac{1}{b^2} - \frac{1}{r^2}\left(1 - \frac{2\mu}{r}\right)\right]^{1/2}.$$

(2) Now consider a photon that just grazes the photon sphere at a radius of R. This photon would satisfy

$$\left.\frac{dr}{d\phi}\right|_{r=R} = 0.$$

Derive an expression for $b = b(R, \mu)$ for this photon.

(3) Plug in the radius for the photon sphere, $R = 3GM/c^2$, find the impact parameter b in units of the Schwarzschild radius. Your answer is the shadow radius for a non-spinning black hole.