2025 Homework #1

Introduction to Quantum Theory of Black Holes and Holography (I)
(National Tsing Hua University Course)

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Due date: Oct 7, 2025
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Background

In class, we first derived the spherically symmetric black hole solutions without a cosmological constant and then with a negative cosmological constant. We analyzed the physical features of these solutions.

In particular, when the cosmological constant is negative, the spacetime is not asymptotically flat but asymptotically Anti-de Sitter (AdS). We saw that this plays the role of a "box" for particles, effectively confining them within the spacetime.

Keep this discussion in mind as you work on the following problems.

Main Task (Required)

Building on the solutions of the Einstein equation derived in the lecture, now consider the spherically symmetric black hole solution with a positive cosmological constant $\Lambda > 0$ (Schwarzschild-de Sitter).

Explain how the spacetime structure differs from the $\Lambda < 0$ (AdS) case:

- Identify the horizons.
- Describe the observable region for a static observer.
- Contrast with the "box-like" behavior in AdS.

Optional Problems (try if you are interested)

1. Explain physically or intuitively (without detailed math!) **why** a new cosmological horizon appears in de Sitter (dS) ($\Lambda > 0$).
2. In AdS, global equilibrium is feasible, while in dS the black hole and cosmological horizons generally have different temperatures. Discuss the physical implications of this thermodynamical contrast.
3. For a static observer, what is the "edge" or the "boundary" of the observable world in (i) AdS and (ii) dS? Explain how boundary conditions and the behavior of perturbations differ between these two cases.

4. Consider a toy model where the cosmological constant depends on position — for example, a local de Sitter bubble embedded in AdS space.
5. Describe what happens as the black hole and cosmological horizons approach each other in Schwarzschild-de Sitter (Nariai limit). How does the metric behave?