

## Spin and Orbital Magnetic Moments of $\text{Fe}_3\text{O}_4$

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We present measurements of the spin and orbital magnetic moments of  $\text{Fe}_3\text{O}_4$  by using SQUID and magnetic circular dichroism in soft x-ray absorption. The measurements show that  $\text{Fe}_3\text{O}_4$  has a noninteger spin moment, in contrast to its predicted half-metallic feature.  $\text{Fe}_3\text{O}_4$  also exhibits a large unquenched orbital moment. Calculations using the local density approximation including the Hubbard  $U$  method and the configuration interaction cluster-model suggest that strong correlations and spin-orbit interaction of the  $3d$  electrons result in the noninteger spin and large orbital moments of  $\text{Fe}_3\text{O}_4$ .

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Magnetite ( $\text{Fe}_3\text{O}_4$ ) exhibits many interesting properties such as charge ordering, mixed valence, and metal-insulator transition known as the Verwey transition [1], in which the conductivity decreases by 2 orders of magnitude upon cooling through the transition temperature  $T_V \sim 120$  K. In spite of intensive studies on its electronic structure, surprisingly, no consensus has been reached concerning the electronic nature of  $\text{Fe}_3\text{O}_4$ . Experimental studies, including neutron diffuse scattering [2], NMR [3], and x-ray scattering [4,5], indicate that  $\text{Fe}_3\text{O}_4$  should be considered as an itinerant magnet rather than a fluctuating mixed-valence material. According to band theory,  $\text{Fe}_3\text{O}_4$  is a half-metal above  $T_V$ ; its minority-spin electrons are conducting, whereas the majority-spin ones are insulating [6]. In addition,  $\text{Fe}_3\text{O}_4$  would have an integral spin moment per formula unit (f.u.), i.e.,  $4.0\mu_B$ ; the orbital moment of metallic  $\text{Fe}_3\text{O}_4$  would be quenched.

On the other hand, charge ordering of the octahedral ( $B$ -site) Fe in  $\text{Fe}_3\text{O}_4$  has been suggested by the refinements of x-ray and neutron diffraction data [7], implying that the  $3d$  electrons of  $\text{Fe}_3\text{O}_4$  have a strong localized character.  $\text{Fe}^{2+}$  in  $\text{Fe}_3\text{O}_4$  is thus expected to exhibit a large unquenched orbital moment, like  $\text{Fe}^{2+}$  in FeO [8]. Theoretical and experimental works show that localization of the  $3d$  electrons of transition-metal compounds leads to giant orbital moments. For example, FeO [8], CoO [9], Fe impurities in alkali metals [10], and Fe nitridometalates [11] are shown to have giant or unquenched orbital moments. In addition, calculations based on atomic multiplet theory show that the localized nature of the open  $3d$  shell of  $\text{Fe}_3\text{O}_4$  sets a limit of  $-66.7\%$  on the spin polarization of conduction electrons [12], rather than  $-100\%$  predicted by band theory. Results of spin-resolved photoemission from epitaxial

thin films and single crystals of  $\text{Fe}_3\text{O}_4$  support the conclusion of multiplet calculations [13–16], in contrast to the conclusion from spin-resolved photoemission of  $\text{Fe}_3\text{O}_4(111)$  thin films grown on W(110) [17].

Measurements of orbital and spin moments therefore provide an opportunity to explore the character of  $3d$  electrons in  $\text{Fe}_3\text{O}_4$  [18,19]. Examining whether  $\text{Fe}_3\text{O}_4$  has a quenched orbital moment and an integral spin moment is important in revealing its electronic nature.

In this Letter, we present studies of the spin and orbital moments of  $\text{Fe}_3\text{O}_4$  single crystals by combining magnetic circular dichroism (MCD) in soft x-ray absorption spectroscopy (XAS) and measurements using a superconducting quantum interference device (SQUID) magnetometer. In addition, we performed cluster-model calculations in the configuration interaction (CI) approach and band-structure calculations in the local spin density approximation including the on-site Coulomb interaction  $U$  (LDA +  $U$ ) [20,21] to unravel the underlying physics of the magnetic moments of  $\text{Fe}_3\text{O}_4$ .

Single crystals of  $\text{Fe}_3\text{O}_4$  were grown by the floating zone method and fully characterized by x-ray diffraction. Temperature-dependent measurements of the resistance of the crystal show an abrupt change at 120 K, as plotted in Fig. 3(a), revealing the Verwey transition of  $\text{Fe}_3\text{O}_4$ . We measured the total magnetic moment of a 21.59-mg  $\text{Fe}_3\text{O}_4$  single crystal with an applied field of 1 T along the [111] direction using a SQUID magnetometer.

We measured MCD in XAS on  $\text{Fe}_3\text{O}_4$  at various temperatures under an applied magnetic field of 1 T along the [111] direction using the Dragon beam line at the National Synchrotron Radiation Research Center in Taiwan. XAS spectra of  $\text{Fe}_3\text{O}_4$  were taken in the total electron yield (TEY) mode with a photon-energy resolution of 0.2 eV

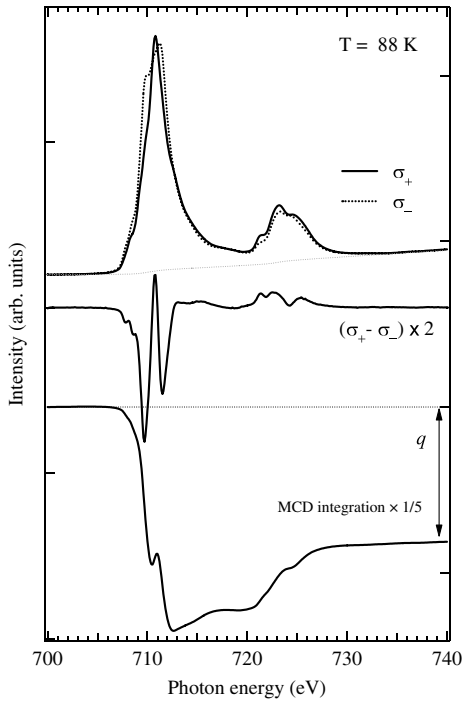


FIG. 1. Fe  $L_{2,3}$ -edge XAS and MCD spectra of  $\text{Fe}_3\text{O}_4$  with correction for the saturation effect. Top: XAS spectra with spin of photons parallel and antiparallel to that of Fe  $3d$  majority electrons. The XAS background is depicted in a thin dotted line. Middle: MCD spectrum, i.e.,  $(\sigma_+ - \sigma_-) \times 2$ . Bottom: integration of MCD. Spectra of MCD and MCD integration are plotted with different vertical offsets for clarity.

and an incident angle of  $60^\circ$ . The degree of circular polarization of the incident light was 80%. The crystal was freshly cleaved in an ultrahigh vacuum at 90 K; the fracture plane of the sample is normal to the  $[110]$  direction. We take our MCD measurements to be representative of bulk  $\text{Fe}_3\text{O}_4$ , because the probing depth of the TEY method is around  $50 \text{ \AA}$  or deeper.

The sum rules of MCD in x-ray absorption permit an element-selective separation of the spin and orbital contributions to the total magnetic moment of materials [22–26]. The total orbital moment  $m_{\text{orb}}$  per formula of  $\text{Fe}_3\text{O}_4$  can be expressed as [27]

$$m_{\text{orb}} = -\frac{4}{3} \frac{\int_{L_{2,3}} (\sigma_+ - \sigma_-) d\omega}{\int_{L_{2,3}} (\sigma_+ + \sigma_-) d\omega} N^h, \quad (1)$$

in which  $\sigma_+$  and  $\sigma_-$  are the absorption cross sections taken with the projection of spin of incident photons parallel and antiparallel to those of the majority of  $3d$  electrons, respectively. In addition,  $\omega$  is the photon energy;  $N^h$  is the total number of Fe  $3d$  holes per formula unit.

XAS recorded with the TEY method suffers typically from the saturation effects, leading to an inaccurate measure of orbital moments [28]. The measured absorption  $I_{\text{TEY}}$  in a TEY measurement is reduced by a factor of

$f = 1/(1 + \lambda_e/\lambda_x \cos\theta)$ , where  $\lambda_x$  and  $\lambda_e$  are the photon penetration depth and the electron sampling depth, respectively, and  $\theta$  is the incidence angle of x ray with respect to the surface normal [28].  $\lambda_e$  was estimated to be  $50 \text{ \AA}$  for  $\text{Fe}_3\text{O}_4$  [29]. By using quasitransmission measurements of XAS, we have determined the photon-energy-dependent  $\lambda_x$  of  $\text{Fe}_3\text{O}_4$  to correct our XAS and MCD measurements for the saturation effects [30].  $\lambda_x$  at the  $L_3$  and  $L_2$  edges are, respectively, 254 and  $653 \text{ \AA}$ .

Figure 1 displays Fe  $L_{2,3}$ -edge XAS and MCD spectra of  $\text{Fe}_3\text{O}_4$  single crystals measured at 88 K using the TEY method. Our XAS and MCD spectra are similar to those of epitaxial  $\text{Fe}_3\text{O}_4$  thin films [31,32]. The XAS background shown in Fig. 1 is composed of an arctangent-like edge-jump function and a linear function. With  $N^h = 13.5$  [33] and taking the geometric effect in absorption and the degree of circular polarization of incident photons into account, we obtained  $m_{\text{orb}}$  of  $\text{Fe}_3\text{O}_4$  at various temperatures, as summarized in Table I. The uncertainty in determining  $m_{\text{orb}}$  originates mainly from the background function of XAS. Our measurements unravel that  $\text{Fe}_3\text{O}_4$  exhibits an unquenched orbital moment. For example, the measured orbital moment  $m_{\text{orb}}$  is  $0.65 \pm 0.07$  at  $T = 145 \text{ K}$ . In other words, the average orbital moment per  $B$ -site Fe is  $0.33 \pm 0.04 \mu_B$ , because the orbital moment of  $A$ -site  $\text{Fe}^{3+}$  is insignificant according to Hund's rule and the local density approximation (LDA) and the local density approximation with Hubbard  $U$  (LDA +  $U$ ) calculations described later; such an unquenched orbital moment is much larger than that of Fe metal,  $0.09 \mu_B$  [25].

To comprehend the underlying physics of an unquenched orbital moment of  $\text{Fe}_3\text{O}_4$ , we performed band-structure calculations on its cubic phase using the all-electron full potential linear muffin-tin orbital method including the spin-orbit interaction [35] within the LDA and LDA +  $U$  schemes. Both LDA [6] and LDA +  $U$  calculations [36] conclude that cubic  $\text{Fe}_3\text{O}_4$  is half-metallic and has a spin moment of  $4.0 \mu_B$  per f.u. as summarized in Table II. Also the orbital moment of  $A$ -site Fe ions is insignificant ( $\sim -0.02 \mu_B$ ), as expected from Hund's coupling of a half-filled  $\text{Fe}^{3+}$ . LDA calculations give rise to a nearly quenched orbital moment of  $\text{Fe}_3\text{O}_4$ . On the other hand, an unquenched orbital moment of  $0.21 \mu_B$  per  $B$ -site Fe atom was obtained by the LDA +  $U$  calculations [37], indicating that the Coulomb interactions of  $3d$  electrons lead to the unquenched orbital moment. To demonstrate such an effect, we calculated the occupation numbers and charge densities of the  $B$ -site  $3d$

TABLE I. Measured  $m_{\text{orb}}$  of  $\text{Fe}_3\text{O}_4$  from MCD at various temperatures.

T (K)	88	100	145	200
$m_{\text{orb}}$	$0.76 \pm 0.09$	$0.66 \pm 0.07$	$0.65 \pm 0.07$	$0.67 \pm 0.08$

TABLE II. Calculated and measured (at  $T = 145$  K) magnetic moments of  $\text{Fe}_3\text{O}_4$ . Total spin ( $m_{\text{spin}}$ ) and orbital ( $m_{\text{orb}}$ ) moments per f.u. of  $\text{Fe}_3\text{O}_4$ , and average orbital moment ( $m_{\text{orb}}^B$ ) per  $B$ -site Fe atom are displayed in units of  $\mu_B$ .

	$m_{\text{spin}}$	$m_{\text{orb}}$	$m_{\text{orb}}^B$	$m_{\text{orb}}/m_{\text{spin}}$
LDA	4.0	0.06	0.04	0.015
LDA + U	4.0	0.43	0.21	0.108
Expt.	$3.68 \pm 0.09$	$0.65 \pm 0.07$	$0.33 \pm 0.04$	0.18

down-spin states projected to orbitals with different magnetic quantum numbers  $m$ . Without inclusion of the Coulomb interaction  $U$ , the projected occupation number of  $B$ -site  $3d$  electrons distributes almost evenly onto orbitals of different magnetic quantum numbers; the orbital moment is thus quenched. With inclusion of  $U$ , on the other hand, the occupation number of the  $m = 1$  state is drastically enhanced, and that of the  $m = -1$  state is suppressed, resulting in a strong orbital polarization and a large orbital moment of  $B$ -site Fe, as presented in Fig. 2.

We also calculated the magnetic moments of octahedral Fe using a configuration interaction (CI) cluster model including spin-orbit interaction [32,39]. The results suggest that the octahedral  $\text{Fe}^{2+}$  in a  $\text{FeO}_6^{10-}$  cluster exhibits a spin moment of  $3.74\mu_B$  and an orbital moment of  $0.97\mu_B$ , implying an average orbital moment of  $0.48\mu_B$  per  $B$ -site Fe atom in  $\text{Fe}_3\text{O}_4$  [32]. The measured average orbital moment of  $0.33 \pm 0.04\mu_B$  per  $B$ -site Fe thus indicates that the  $3d$  electrons of  $\text{Fe}_3\text{O}_4$  have a strongly correlated electronic nature even at temperatures above  $T_V$ .

To quantitatively determine the total spin moment  $m_{\text{spin}}$  per formula unit of  $\text{Fe}_3\text{O}_4$ , we measured also the total magnetic moment of  $\text{Fe}_3\text{O}_4$  at the temperatures between 80 and 200 K by using a SQUID magnetometer, as plotted in Fig. 3(b). Combining the MCD and SQUID measure-

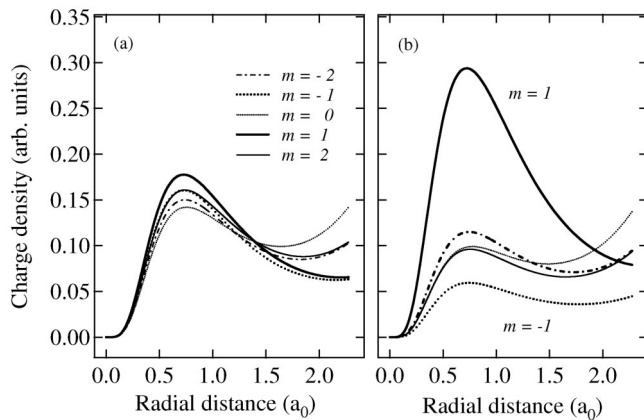


FIG. 2. Charge densities of cubic  $\text{Fe}_3\text{O}_4$  versus radial distance in units of atomic radius  $a_0$ . The charge densities projected to different orbitals with magnetic quantum number  $m$  were obtained from (a) LDA and (b) LDA + U calculations.

ments, we then obtained  $m_{\text{spin}}$  at various temperatures, because both LDA and LDA + U calculations conclude that the orbital moment of oxygen is negligible. Our measurements indicate that the spin and orbital moments of  $\text{Fe}_3\text{O}_4$  do not change significantly around the temperature  $T_V$ . In addition,  $\text{Fe}_3\text{O}_4$  exhibits a noninteger spin moment. For example, the total magnetic moment of  $\text{Fe}_3\text{O}_4$  at  $T = 145$  K is  $4.33\mu_B$ ; with the measured  $m_{\text{orb}}$  of  $0.65 \pm 0.07$ , remarkably  $m_{\text{spin}}$  per f.u. of  $\text{Fe}_3\text{O}_4$  is  $3.68 \pm 0.09\mu_B$ , as displayed in Table II, in contrast to the integral spin moment of  $4.0\mu_B$  as a result of half-metallic behavior predicted by band theory. With CI calculations, we found that the spin moment of octahedral  $\text{Fe}^{2+}$  in a  $\text{FeO}_6^{10-}$  cluster is suppressed by  $\sim 5\%$  if the strength of the spin-orbit interaction of  $3d$  electrons is doubled, whereas the integral spin moment of  $\text{Fe}_3\text{O}_4$  obtained from LDA + U calculations is rather insensitive to the strength of spin-orbit coupling. This observation suggests that the observed noninteger spin moment is beyond the Bloch electron picture and might result from a combined effect of the spin-orbit interaction and strong correlations of the  $3d$  electrons in  $\text{Fe}_3\text{O}_4$ .

With measurements of SQUID and MCD in soft x-ray absorption, we can also study the spin moment of oxygens in  $\text{Fe}_3\text{O}_4$ . The spin sum rule of MCD [23] correlates the total spin moment  $m_{\text{spin}}^{\text{Fe}}$  of Fe in  $\text{Fe}_3\text{O}_4$  to the MCD data as  $m_{\text{spin}}^{\text{Fe}} + 7\langle T_z \rangle = -\frac{6p-4q}{I} N^h$ , in which  $p$  and  $q$  are defined as MCD integrations  $\int_{L_3} (\sigma_+ - \sigma_-) d\omega$  and  $\int_{L_3+L_2} (\sigma_+ - \sigma_-) d\omega$ , respectively, and  $I$  as the XAS integration. In addition,  $\langle T_z \rangle$  is the expectation value of magnetic dipole

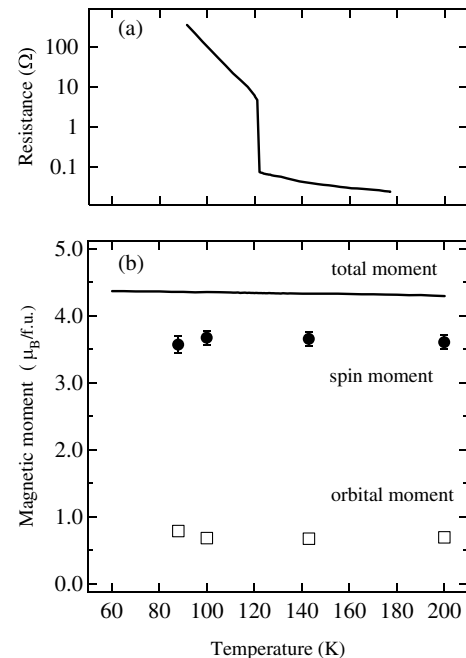


FIG. 3. (a) Resistance of a  $\text{Fe}_3\text{O}_4$  single crystal in the vicinity of the Verwey transition. (b) Total magnetic moment, spin, and orbital moments of  $\text{Fe}_3\text{O}_4$  at various temperatures.

operator. By using the spin sum rule with correction for its deviation resulting from the intermixing between the  $L_2$  and  $L_3$  edges [40], we obtained a value of  $3.55\mu_B$  for  $m_{\text{spin}}^{\text{Fe}} + 7\langle T_z \rangle$  at  $T = 145$  K. Our LDA + U calculations disclose that  $7\langle T_z \rangle$  per  $B$ -site Fe atom is  $0.155\mu_B$ , whereas  $7\langle T_z \rangle$  of the  $A$ -site Fe atoms is  $-0.0001\mu_B$ . The significant  $7\langle T_z \rangle$  value of the  $B$ -site Fe atoms is caused by their strong orbital polarization as shown in Fig. 2. The total spin moment  $m_{\text{spin}}^{\text{Fe}}$  of Fe in  $\text{Fe}_3\text{O}_4$  is therefore  $3.24\mu_B$ , leading to a spin moment of  $0.44\mu_B$  originating from oxygen atoms per  $\text{Fe}_3\text{O}_4$ , i.e., a spin moment of  $0.11\mu_B$  per O atom in  $\text{Fe}_3\text{O}_4$ . This deduced spin moment of oxygen is close to the calculated spin moment of  $0.07\mu_B$  per O atom from our LDA + U calculations and consistent with LDA calculations [6,42].

In conclusion, we have measured the orbital and spin magnetic moments of  $\text{Fe}_3\text{O}_4$  by combining SQUID and MCD. We found that  $\text{Fe}_3\text{O}_4$  has a noninteger spin moment, in contrast to its predicted half-metallic feature, and that the average orbital moment of  $B$ -site Fe in  $\text{Fe}_3\text{O}_4$  is significantly larger than that of Fe metal. As evidenced by LDA + U calculations, the on-site Coulomb interactions of  $3d$  electrons result in the unquenched orbital moment and magnetic dipole moment of  $\text{Fe}_3\text{O}_4$ . Our results suggest that spin-orbit interaction and electron correlations of  $3d$  electrons play an important role in the spin and orbital moments of  $\text{Fe}_3\text{O}_4$ . We call for further theoretical work on the magnetic moments of  $\text{Fe}_3\text{O}_4$ .

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