Quantum Optics with Propagating Microwaves in Superconducting Circuits I

Io-Chun, Hoi
Outline

1. Introduction to quantum electrical circuits
2. Introduction to superconducting artificial atom
3. Quantum optics with superconducting circuits
4. Single atom scattering

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Introduction to quantum electrical circuits
Coherent superposition states:

- Charge $Q$
- Flux $\Phi$

Properties:

- The superposition states collapse when measure.
- Probabilistic character.

Charge on a capacitor:

\[ \frac{1}{\sqrt{2}} \left( \begin{array}{c} \text{charge} \\ \text{capacitor} \end{array} \right) + \begin{array}{c} \text{charge} \\ \text{capacitor} \end{array} \]

Current or magnetic flux in an inductor:

\[ \frac{1}{\sqrt{2}} \left( \begin{array}{c} \text{magnetic flux} \\ \text{inductor} \end{array} \right) + \begin{array}{c} \text{magnetic flux} \\ \text{inductor} \end{array} \]

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Conventional electrical circuits

Basic elements:

- Figure from Intel

Properties:
* Deterministic
* No quantum mechanics
* No superposition principle
* No quantization of fields

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Introduction to superconducting artificial atom

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Superconducting circuits are like LEGOS!

What's good about circuits?

• Circuits are like LEGOs!

a few elementary building blocks, gazillions of possibilities!
Basic Elements of Superconducting Circuits

Dissipationless!

Capacitance

Inductance

Josephson Junction: Non-dissipative nonlinear inductance

Tunnel barrier between two superconductors

\[ I = I_c \sin \phi \]

\[ \frac{d\phi}{dt} = \frac{2e}{h} V \]

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Fabrication of Josephson Junction

1. electron beam writing
2. development
3. first aluminum evaporation
4. oxidation
5. second aluminum evaporation
6. lift-off

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Constructing linear quantum electrical circuits

Classical physics:

\[ H = \frac{Q^2}{2C} + \frac{\Phi^2}{2L} \]
\[ H = \frac{p^2}{2m} + \frac{1}{2} kx^2 \]

Analogy with a moving particle in a harmonic potential

Quantum mechanics:

\[ H = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L} \]
\[ H = \hbar \omega (a^\dagger a + \frac{1}{2}) \]
\[ [\hat{\Phi}, \hat{Q}] = i\hbar \]

\[ \omega = \frac{1}{\sqrt{LC}} \sim \text{GHz} \]


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Constructing nonlinear Quantum circuit:

**Artificial atom**

Replace linear inductance by Josephson junction (Nonlinear inductance)

\[ U = -E_J \cos \phi \]

\[ L_J = \frac{\hbar}{4eI_c \cos \left( \pi \frac{\Phi_{\text{ext}}}{\Phi_0} \right)} \]

\[ C \quad \square \quad L_J \]

Transition become addressable!

Emission spectrum

Anharmonicity:

\[ \alpha = \omega_{01} - \omega_{12} \]
How to operate electrical circuits quantum mechanically?

Avoid dissipation
Avoid broaden energy levels

Work at low temperatures
Provide reset of the circuit (Ground state)

\[ k_B T \ll \hbar \omega \ll \Delta \]
Superconducting gap energy

\[ \omega / 2\pi \sim 4 - 8 \text{GHz} \quad T @ mK \]

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Family of superconducting artificial atom

Focus on Cooper Pair Box and Transmon!

Fig. from Michel Devoret. Linneaus summer school in quantum engineering. 2010.


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Artificial atom I: The Single-Cooper Pair Box

$$H = -\frac{1}{2} E_{ch} \sigma_z - \frac{1}{2} E_J \sigma_x$$

Map to a spin 1/2 particle in magnetic field.

$V_g$

$C_g$

$C_j$

$E_J = \frac{\Phi_0 I_c}{2\pi} \left| \cos \frac{\pi \Phi}{\Phi_0} \right|$  
$C_\Sigma = C_g + C_j$

$E_{ch} = E_Q (1 - 2n_g)$  
$E_Q = 4E_c = \frac{(2e)^2}{2C_\Sigma}$  
$n_g = C_g V_g / (2e)$

$E_J / E_c < 1$

$1/\sqrt{2}(|0>-|1>)$

$1/\sqrt{2}(|0>+|1>)$


But the coherence time is short (few ns) due to charge noise!
Decoherence of artificial atom
(Effect from the environment)

Relaxation rate $\Gamma_{01}$

Pure dephasing rate $\Gamma_{\varphi}$

Random switching $|1\rangle \rightarrow |0\rangle$

Phase randomization $\omega_{01} \rightarrow \omega_{01} + \delta\omega_{01}(t)$

$e^{-i\omega_{01}t}$

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Artificial atom II: The transmon

\[20 < \frac{E_J}{E_c} < 100\]

Insensitive to the charge noise
Long coherence time.

Jens Koch et al.

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Studying/Engineering the matter-light interaction

Natural atom
Optical photons

Superconducting artificial atom
Microwave photons

Compare with optical photon, the frequency of microwave photon is $10^6$ less.

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Comparison of the toolboxes

Quantum optics          Superconducting circuits

Optical photons          Microwave photons

Detect I, Q

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Advantages of quantum circuit

1. Photons and “atom” interaction can be engineered
2. The photons can be guided by waveguides; beam alignment is not needed.
3. Large vacuum field $E_{0,rms} \approx 0.2V/m$ due to small mode volume
4. Standard on-chip fabrication technique
5. Tunable transition energy of the “atom”
6. Mechanical stable

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Quantum optics with superconducting circuits

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Atoms ⇒ Qubits
3D Cavity ⇒ 1D on-chip resonator

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Resonant scattering

Resonant scattering in 3D space

Incoming light

Atom/dipole emits light

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Resonant scattering in 3D space

The extinction signal is due to interference

Incoming light

Atom/dipole emits light

Resonant scattering in 3D space

Sum

Spatial mode mismatch


Fig. from U. Håkanson

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Resonant scattering in 1D waveguide

Fully coherent: no transmission, perfect reflection.

Resonant scattering in 1D waveguide

Fully coherent: no transmission, perfect reflection.

Point like atom/dipole! \( \lambda >> d \)
\[ \lambda \sim \text{cm} \quad \text{Wavelength of EM field} \]
\[ d \sim \mu \text{m} \quad \text{Size of “atom”} \]

Relaxation dominated by transmission line.

IoChun, Hoi et al. PRL 107, 073601 (2011)
Resonant scattering in 1D waveguide

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Relaxation dominated by transmission line.


IoChun, Hoi et al. PRL 107, 073601 (2011)

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Quantum circuit model

Relaxation rate into 1D transmission line, indicates the strength of coupling!

\[ \Gamma_{10} \approx \frac{\omega_{01}^2 C_c^2 Z}{4 C_\Sigma} \quad \quad C_\Sigma = C_c + C_{JS} \quad \quad Z = \sqrt{\frac{L_0}{C_0}} \]

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Transmission and reflection

\[ r = \frac{\langle V_R \rangle}{\langle V_{in} \rangle} \quad t = \frac{\langle V_T \rangle}{\langle V_{in} \rangle} \]

Reflection coefficient

\[ r = -\frac{\Gamma_{10}}{2\gamma_{10}} \left[ \frac{1 - i\delta\omega_p / \gamma_{10}}{1 + \left(\frac{\delta\omega_p / \gamma_{10}}{1 + \Omega_p^2 / \Gamma_{10}\gamma_{10}} \right)^2} \right] \]

n resonance, low power

\[ \left| r\left(\delta\omega_p = 0, \Omega_p \ll \gamma_{10}\right) \right| = \frac{\Gamma_{10}}{2\gamma_{10}} = \frac{1}{1 + 2\Gamma_{\phi} / \Gamma_{10}} \]

Strong interaction limit:

\[ \Gamma_{10} \gg \Gamma_{\phi} \quad \left| r\left(\delta\omega_p = 0, \Omega_p \ll \gamma_{10}\right) \right| \approx 1 \quad \text{Fully coherent.} \]

\[ \delta\omega_p \text{ : Detuning} \]
\[ \Gamma_{10} \text{ : Relaxation} \]
\[ \Gamma_{\phi} \text{ : Pure dephasing} \]
\[ \gamma_{10} = \Gamma_{10} / 2 + \Gamma_{\phi} \]

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Saturation of transmission

\[ T = |t|^2 \]

Almost full reflection at low power

Almost full transmission at high power

\[ \langle N_P \rangle = \frac{P_p}{\hbar \omega_p (\Gamma_{10} / 2\pi)} \]

Nonlinear nature of the atom!

<table>
<thead>
<tr>
<th>Sample</th>
<th>( E_J/\hbar )</th>
<th>( E_C/\hbar )</th>
<th>( E_J/E_C )</th>
<th>( \omega_{10}/2\pi )</th>
<th>( \omega_{21}/2\pi )</th>
<th>( \Gamma_{10}/2\pi )</th>
<th>( \Gamma_{\phi}/2\pi )</th>
<th>Ext.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.7</td>
<td>0.59</td>
<td>21.6</td>
<td>7.1</td>
<td>6.38</td>
<td>0.073</td>
<td>0.018</td>
<td>90%</td>
</tr>
<tr>
<td>2</td>
<td>10.7</td>
<td>0.35</td>
<td>31</td>
<td>5.13</td>
<td>4.74</td>
<td>0.041</td>
<td>0.001</td>
<td>99%</td>
</tr>
</tbody>
</table>

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Transmission comparing to theory

Sample 1

Experiment
\[ \omega_p / 2\pi \ [\text{GHz}] \]

Theory
\[ \omega_p / 2\pi \ [\text{GHz}] \]

\[ P_p \ [\text{dBm}] \]

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Coherent vs Incoherent scattering

Sample 2

\[ \Omega_p/2\pi [\text{MHz}] \]

\[ P_p [\text{dBm}] \]

Total scattered
\[ \langle V_x^2 \rangle \]
- Blue: BW=10MHz
- Green: BW=100MHz

\[ \langle V_x \rangle \Delta \text{Elastic scattered} \]

\[ \langle V_x \rangle \]

- Red: Elastic scattered
- Dotted line: Input field

\[ \omega_{10} \rightarrow \Omega_p \rightarrow 0 \]

\[ \delta \omega_p/2\pi \]

- 30 MHz
- 83 MHz
- 250 MHz

\[ \Omega_p \ll \gamma_{10} \]

\[ \langle V_{in} \rangle^2 \approx \langle V_R \rangle^2 \approx \langle V_R^2 \rangle \quad |r_{p,1}| \sim 1 \]


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Tunable artificial atom

Low power

Only 0-1 transition occurs!

High power

Only two-photon transition occurs!

Extract:

- $E_{J,\text{Max}} = 13\,\text{GHz}$
- $E_c = 590\,\text{MHz}$
- $E_J / E_c = 23$

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Fully coherent: perfect reflected by the atom.

\[ r_{p,2} = \frac{\langle V_R \rangle}{\langle V_{in} \rangle} \]

Device 2

Emitted fields can propagate in one direction

\( r_p \) measure the phase coherent signal.
\[ \Omega_p \ll \gamma_1 \]

Transmon at the end of transmission line

The missing power are incoherent scattered.

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Two-Tone Spectroscopy
Higher level effect

\[ \omega_p = \omega_{12} \]

Anharmonicity:
\[ \alpha = \omega_{01} - \omega_{12} \approx 720 \text{MHz} \]

\[ \omega_{12} / 2\pi = 6.38 \text{GHz} \]
\[ \omega_{10} / 2\pi = 7.1 \text{GHz} \]

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Mollow triplet

\[ P_{01} \text{ [dBm]} \]

\[ \omega \]

\[ \omega_p / 2\pi \text{ [GHz]} \]

\[ T_{p,l} \text{ [2\pi]} \]


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Autler-Townes Splitting

A. A. Abdumalikov, Jr et al. PRL 104, 193601 (2010)

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To be continued…