

# Condensed Matter Physics (II): Homework 1

## Choose at least two problems in below, due May 18

**Ex.1 10%** Consider the 1D Anderson model. The width of the distribution for the on-site random energies is  $W$  and the nearest hopping is  $t$ . Solving the integral equation derived in the class, find the localization length for  $E = 0$  to the leading order  $W^2/t^2$

**Ex.2 20%** Consider the transfer matrix  $Q$  defined in the class for the 1D tight-binding electronic system. Show that the conductance of the system is given by

$$G = \frac{e^2}{\hbar^2} \frac{2}{QQ^\dagger + Q^\dagger Q + 2}.$$

You may consult any books or articles on mesoscopic physics.

**Ex.3 10%** Consider the scaling theory of localization. If the beta function  $\beta(g) = d - 2 - A/g$  with  $A > 0$  and  $g$  being dimensionless conductance, find the exponent of  $g \sim |\epsilon|^s$ , where  $\epsilon = 1 - g_0/g_c$ ,  $\beta(g_c) = 0$ , and  $g_0$  is some reference conductance.

**Ex.4 30%** Consider the tight-binding model with only the nearest hopping  $-t$  for an electron in 1D. Let  $G_0(x, x')$  being the Green's function for the electron in the absence of any impurity. If there are two impurities at  $x = 0$  and  $x = na$  with  $a$  being the lattice constant so that  $V(x) = U\delta_{x,0} + U\delta_{x,na}$ , find the Green's function  $G(x, x')$  in terms of  $G_0$ ,  $U$ , and  $na$ .

### Reading report

The following articles are papers that can lead you to start on searching more papers on recent development of the same subject.

**Ex.5 15%** Recent development on superconductor-insulator transition

R. W. Crane et al., Phys. Rev. B 75, 184530 (2007); Yonatan Dubi, Yigal Meir, Yshai Avishai, Nature 449, 876-880 (2007).

**Ex.6 15%** Recent development of whether electrons should be localized in the presence of Coulomb

S. V. Kravchenko et al. Phys. Rev. Lett. 77, 4938 (1996)

Y. Hanien et al. Phys. Rev. Lett., 80, 1288 (1998)