Statistical Mechanics (II): Homework 3 (due on Novermber 16)

Problem 1 Goldstone theorem in magnetic system

Using the function $F(k)$ defined in proving the Mermin-Wagner theorem and replacing x component in $F(k)$ by z component, show that if $\lim_{q\to q_0} S(q) = \infty$, then the energy spectrum satisfies $\lim_{q\to q_0} E(q) = 0$. Here $S(q)$ is the same function as $S_{yy}(q)$ defined in proving the Mermin-Wagner theorem except now the index y is replaced by z.

Problem 2 Starting from the result

$$\frac{\beta F}{N} = -\log(2\cosh 2K) - \frac{1}{2\pi}\int_0^\pi d\phi \log\left[\frac{1}{2}\left(1+\sqrt{1-\kappa^2\sin^2\phi}\right)\right],$$

where $K = J/k_BT$ and $\kappa = 2/\cosh 2K \coth 2K$, show that

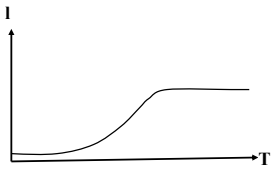
$$u = -J\coth 2K\left[1+\frac{2}{\pi}\kappa' K_1(\kappa)\right],$$

where u is the energy per site for 2D Ising model and $\kappa^2+\kappa'^2 = 1$. Show that the specific c heat near T_c is given by

$$\frac{c}{k_B} = \frac{2}{\pi}\left(\frac{2J}{k_BT_c}\right)^2\left[-\log|1-\frac{T}{T_c}| + \log\frac{k_BT_c}{2J} - (1+\frac{\pi}{4})\right].$$

Problem 3 Use the one-dimesional Ising model to describe the following observation: The length l molecules in a dilute solution of long chain-like polymer molecules is found to change with the temperature T as shown in the following picture at the end.

Problem 4 Use the cumulant expansion theorem to show the central limit theorem: $X_1, X2, ..., X_N$ are N independent random variables with $\langle X_i\rangle = 0$. If we define $Y = \frac{1}{\sqrt{N}}(X_1+X_2+...+X_N)$, in the limit of $N\to\infty$, Y is a Gaussian random variable with $\langle Y\rangle = 0$. Find the standard deviation of Y in this limit.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