

# Quantum electronics

中央研究院物理研究所 陳啟東

# **Outline:**

**Device fabrication**

**e-beam lithography, examples**

**Measurement electronics**

**Electron transport in**

**Ballistic systems**

**Highly disordered systems**

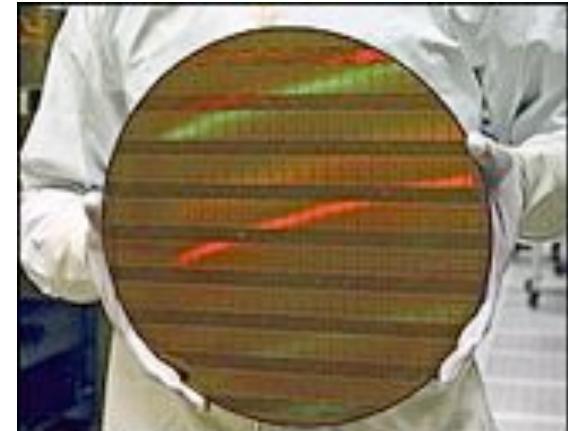
**Hopping conduction in diluted trap systems**

**Tunneling through quantum dots**

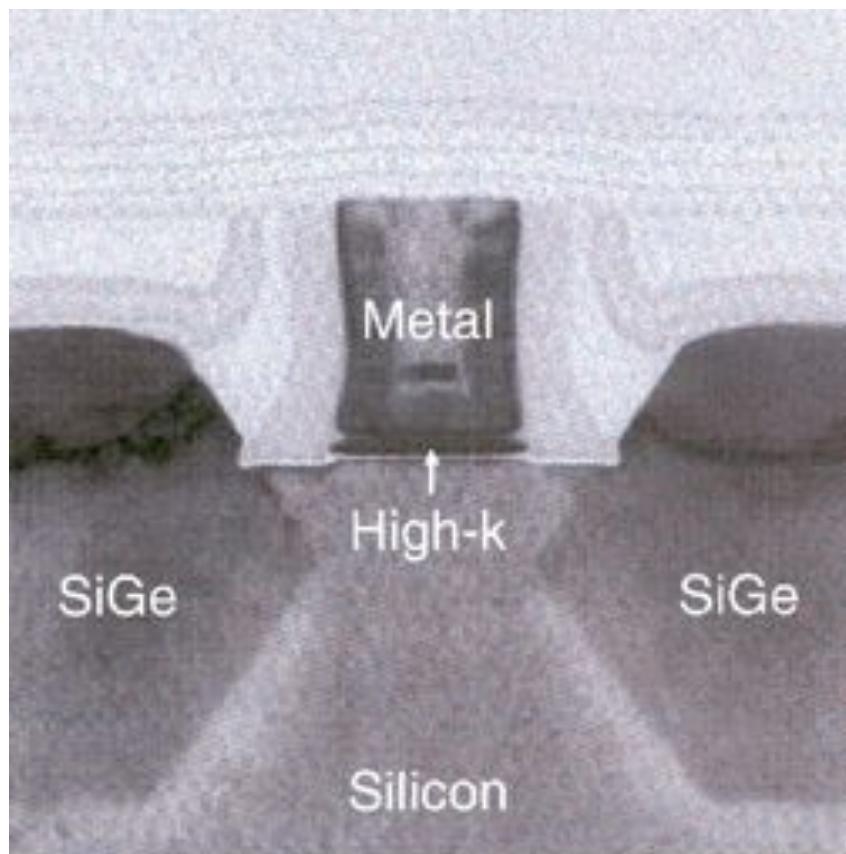
**Quantum bits:**

**Resonant tunneling between coupled quantum-dots  
from small Josephson junctions to charge qubits**

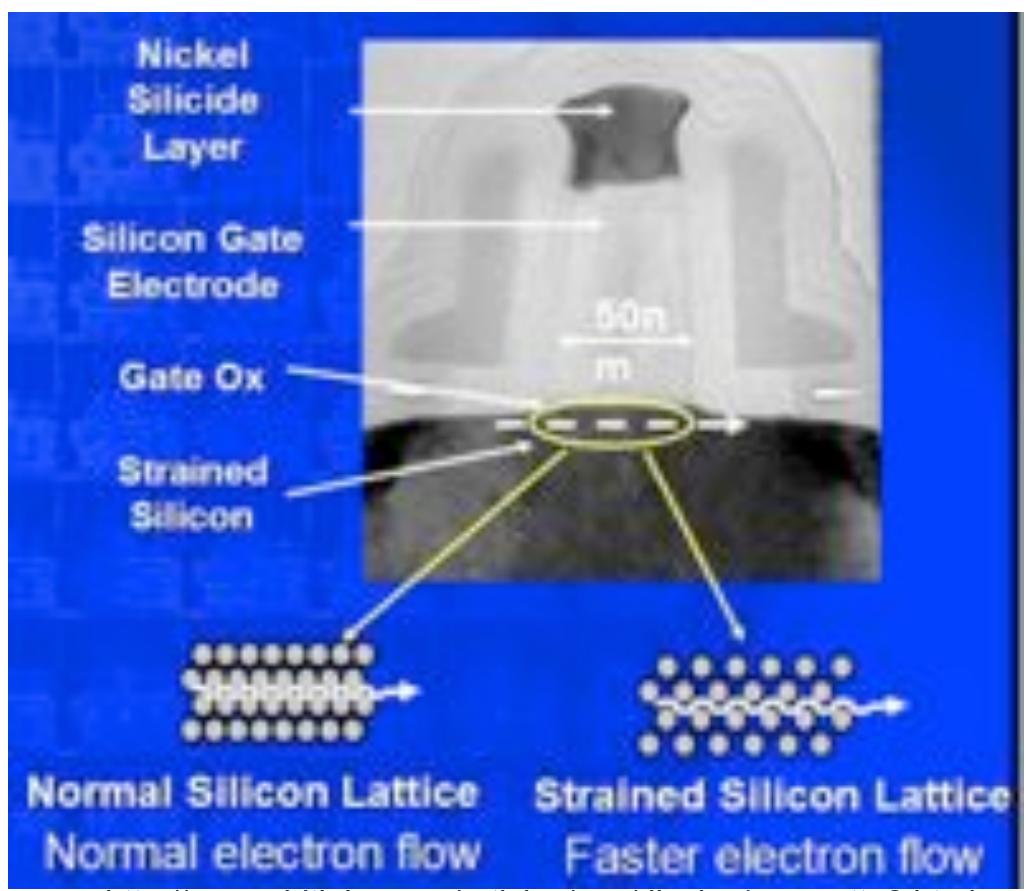
# Present-day semiconductor industry



Scientific American, January 30, 2007



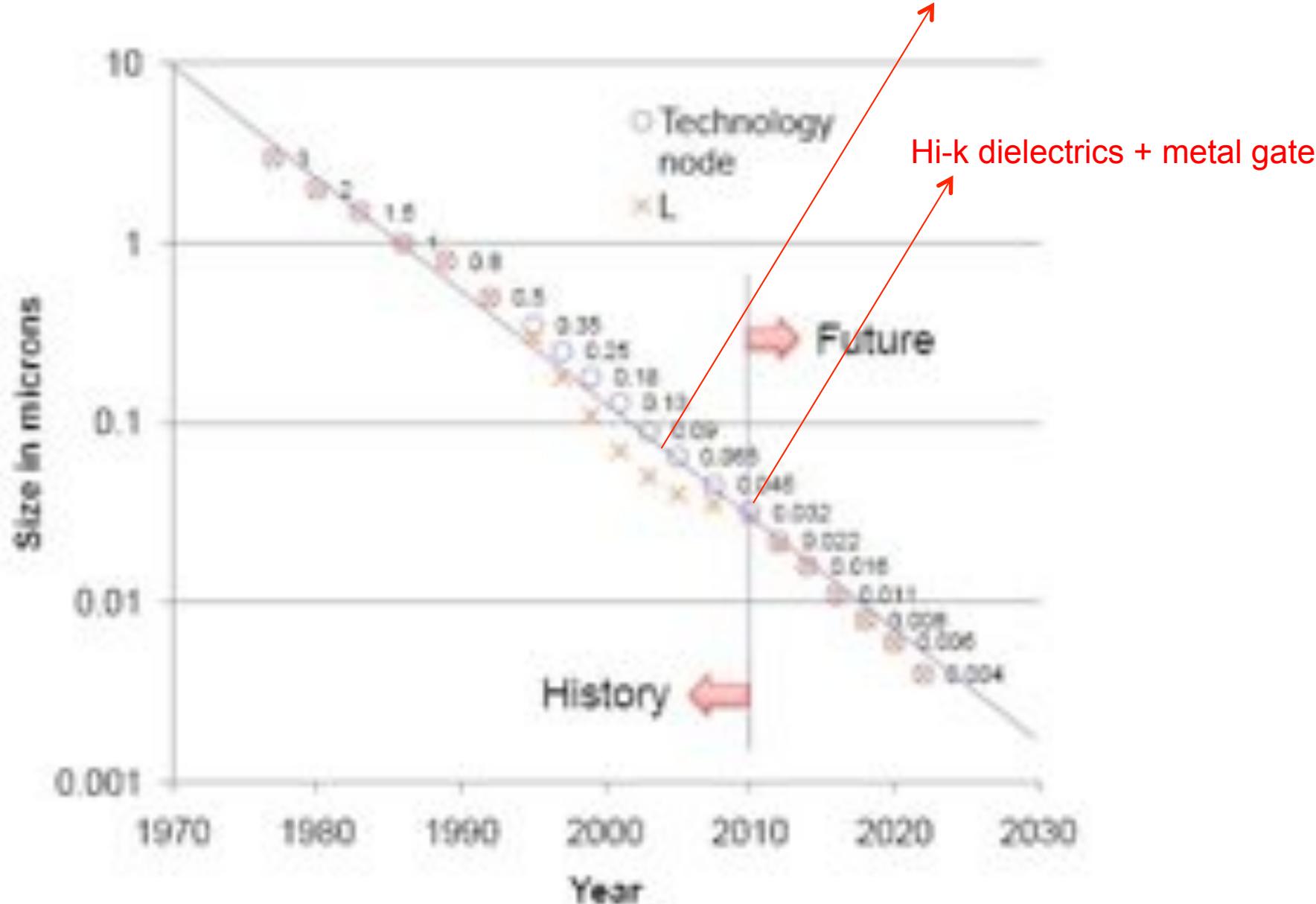
<http://channel.hexus.net/content/item.php?item=12389>



[http://www.xbitlabs.com/articles/cpu/display/prescott\\_2.html](http://www.xbitlabs.com/articles/cpu/display/prescott_2.html)

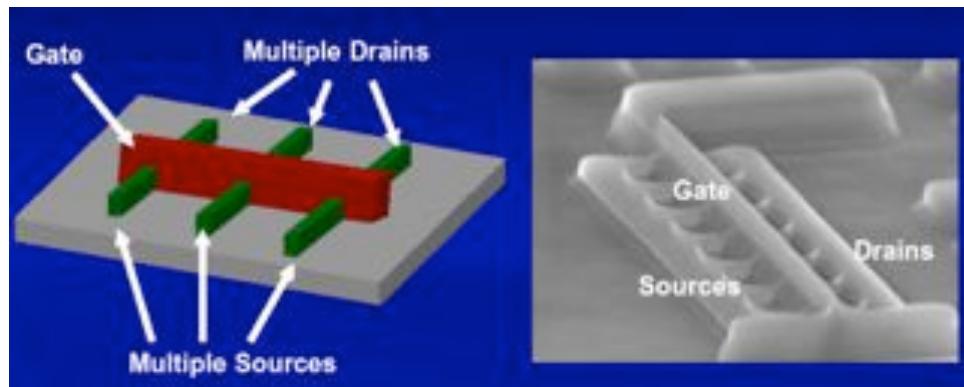
## Gate length map

### Strained Silicon

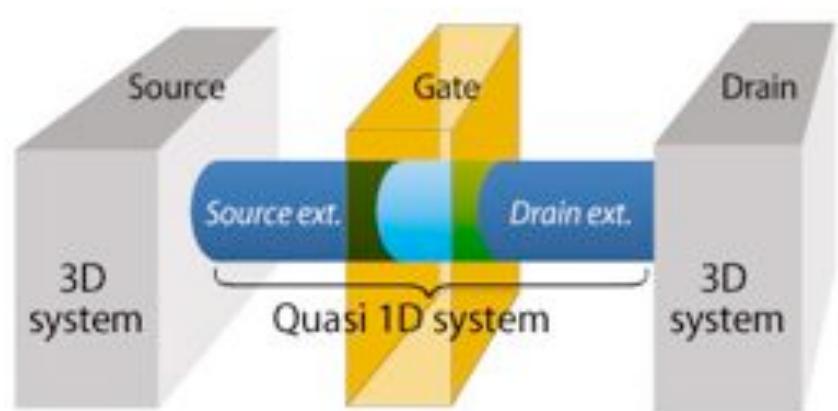


# Latest development and future technologies

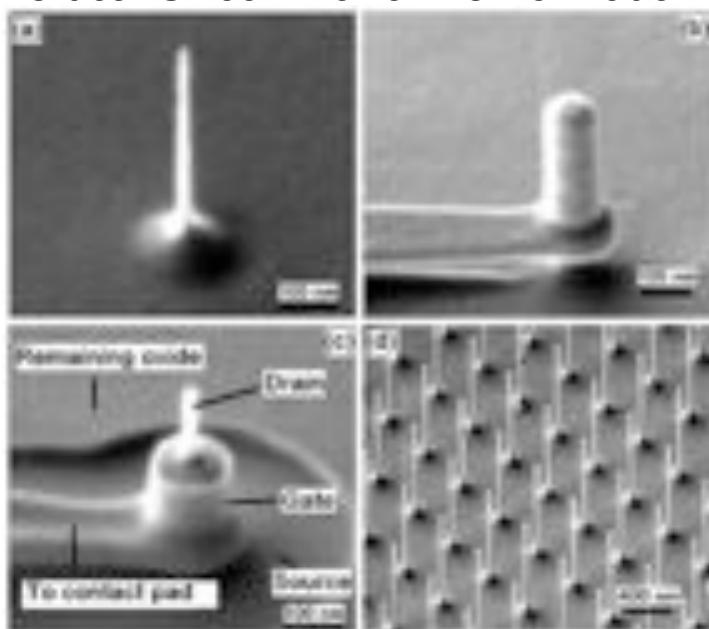
FinFET or Multi-gate FET (MuGFET)



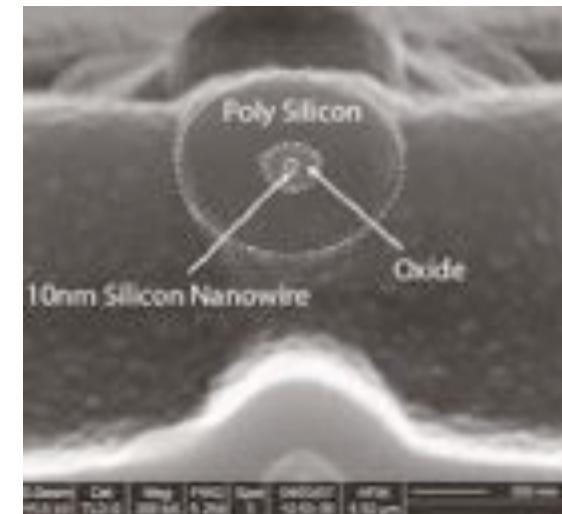
Gate-All-Around Field-Effect-Transistors



Vertical Silicon-Nanowire Formation



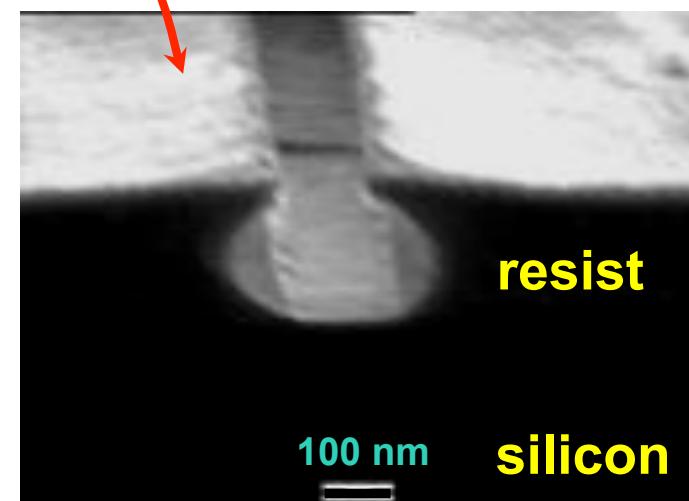
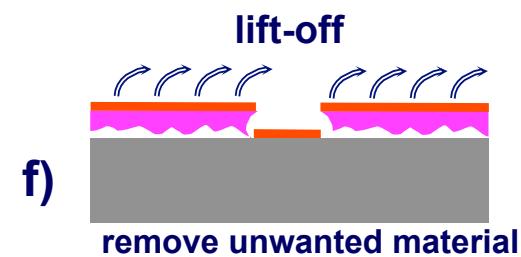
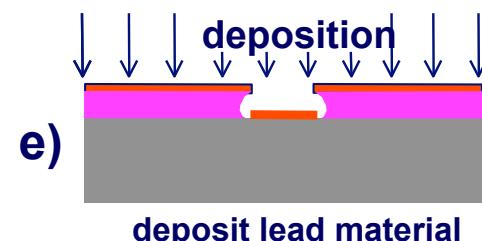
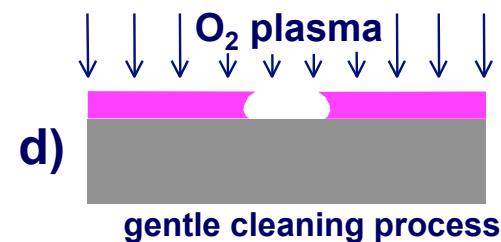
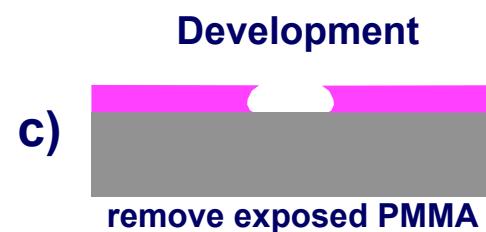
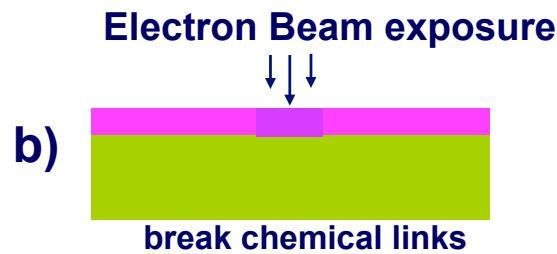
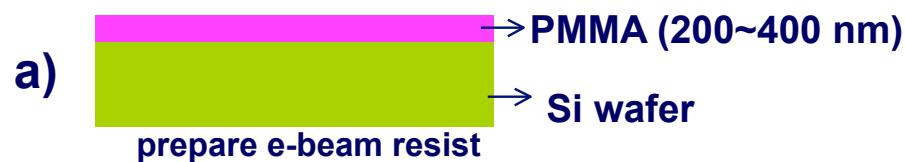
<http://www.ime.a-star.edu.sg/html/newsrelease/2008/Shortcuts-May08Jun08.htm>



[http://www.advancedsubstratenews.com/2010/12/  
Advanced Substrate Corners](http://www.advancedsubstratenews.com/2010/12/Advanced_Substrate_Corners)

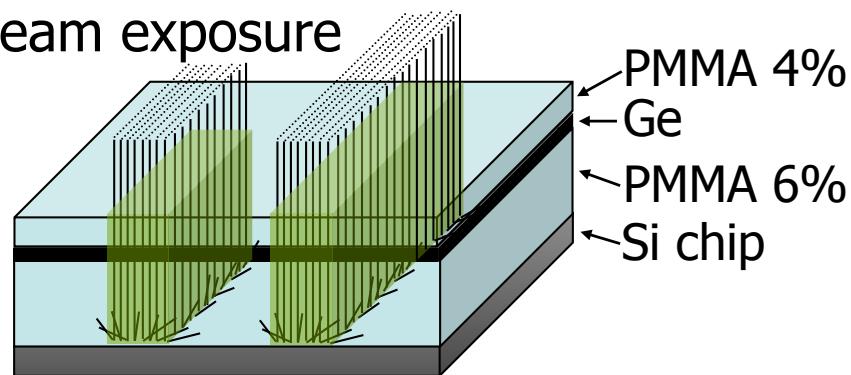
Posted by [Professor Ru HUANG](#) on December 8, 2010

# Electron beam lithography and lift-off technique

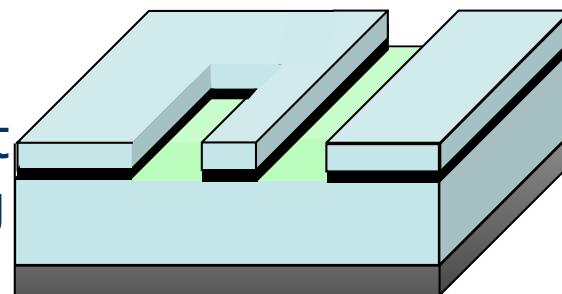


# Resist profile engineering: Fabrication of Aluminum tunnel junctions

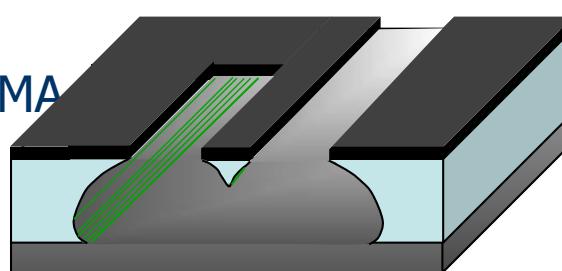
1. e-beam exposure



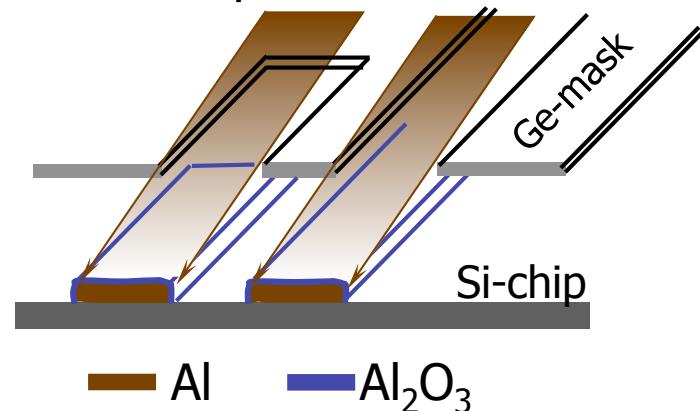
2. development  
and Ge-etching



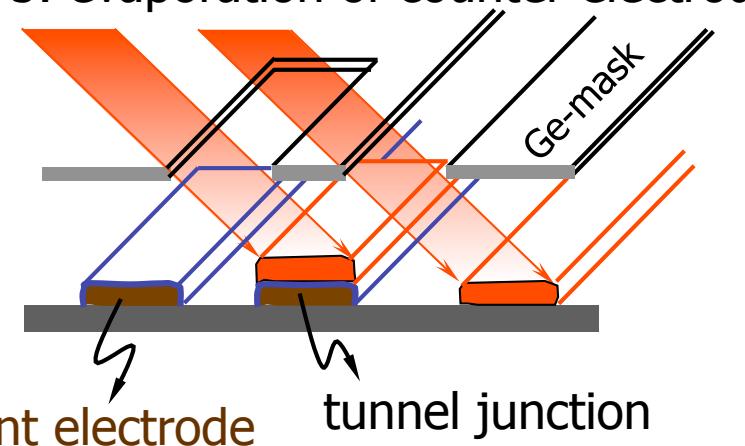
3. O<sub>2</sub> dry etching  
of the bottom PMMA



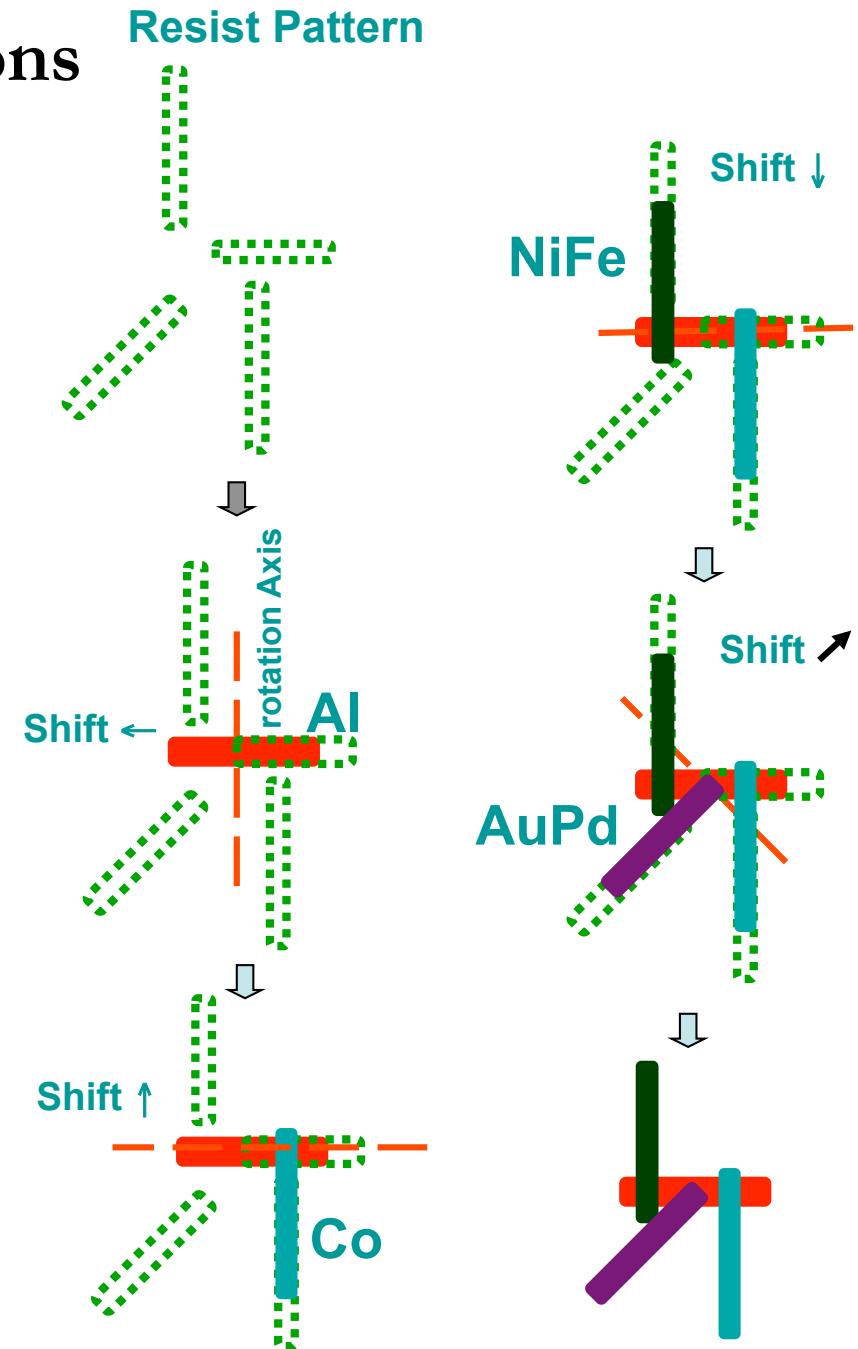
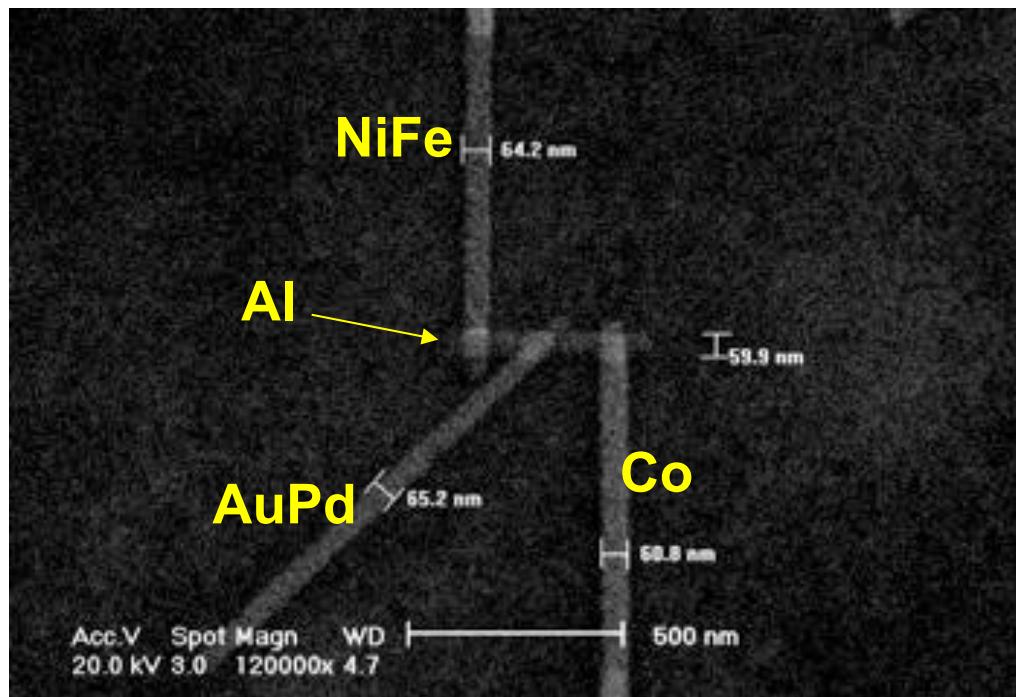
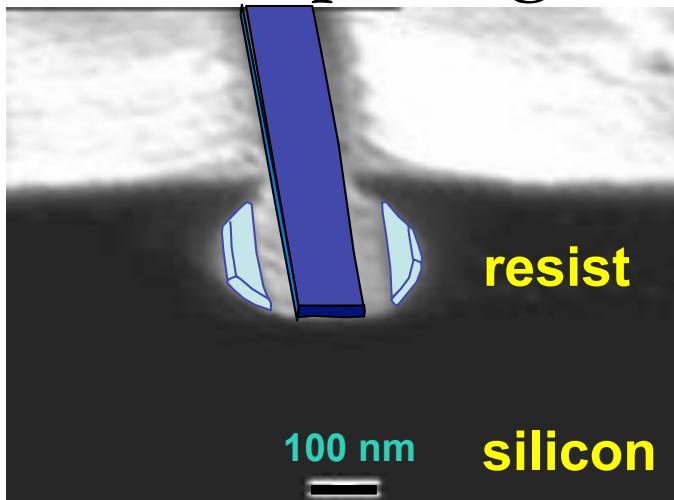
4. Al- evaporation + oxidation



5. evaporation of counter electrode

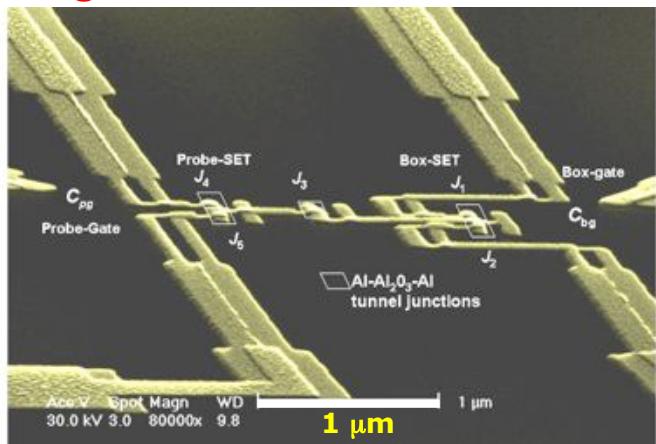


# Multiple angle evaporation

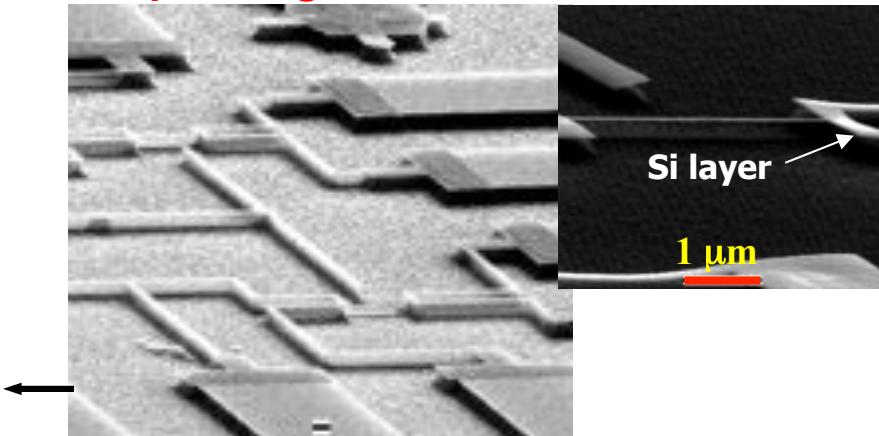


## Metallic electronics

### Single electron devices

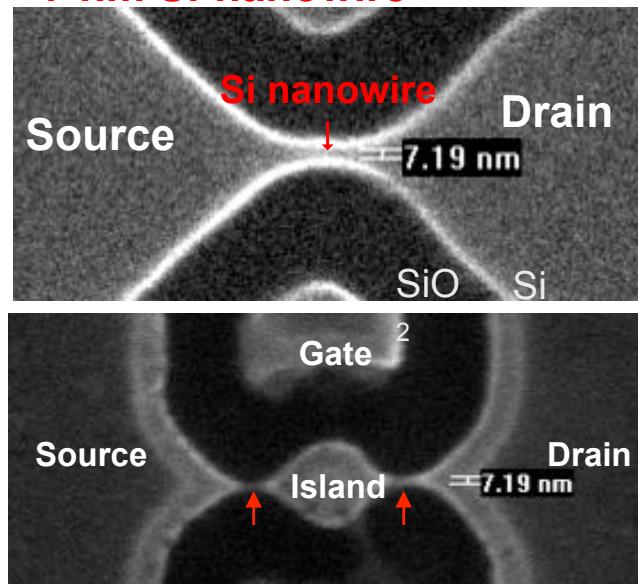


### Suspending wire devices

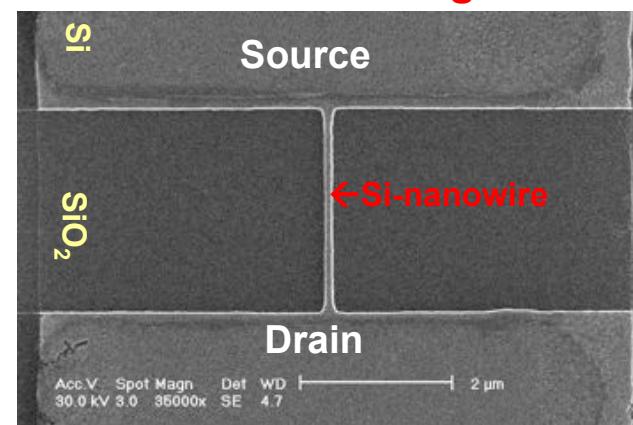


## silicon nanoelectronics

### 7 nm Si-nanowire

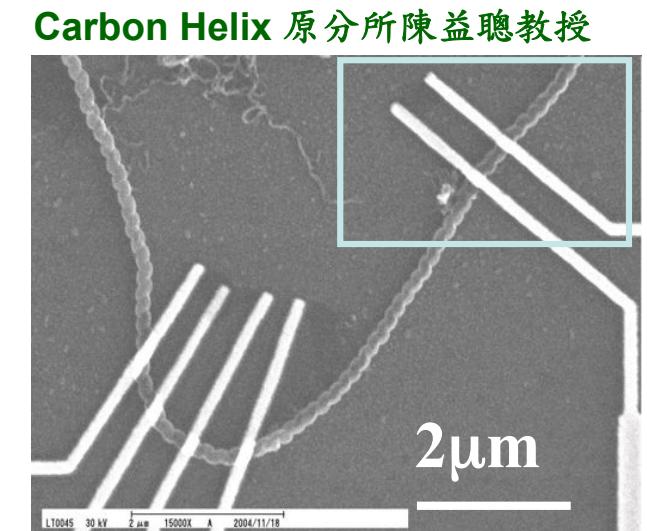
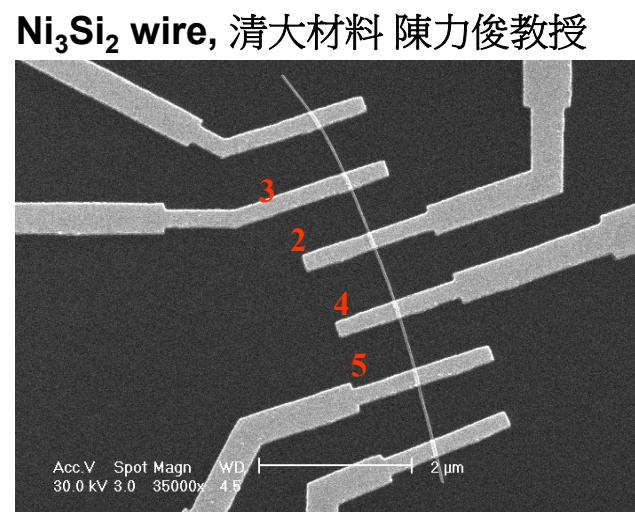
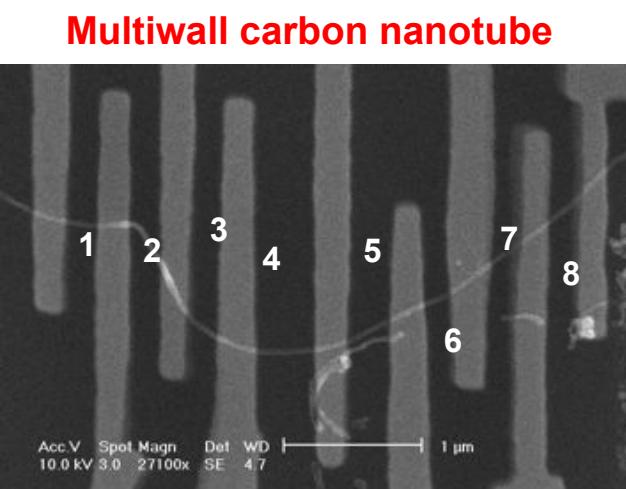
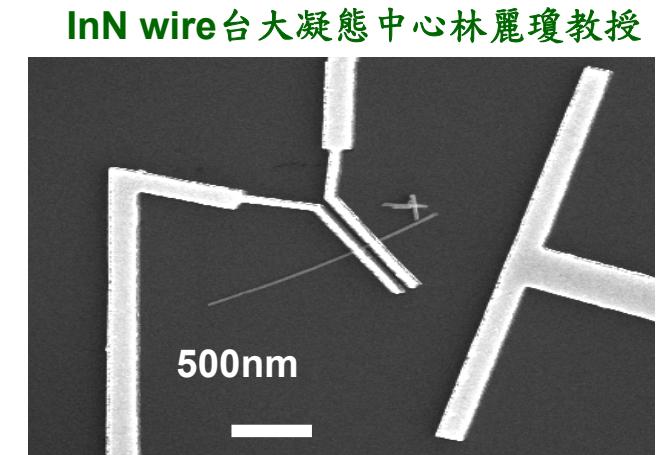
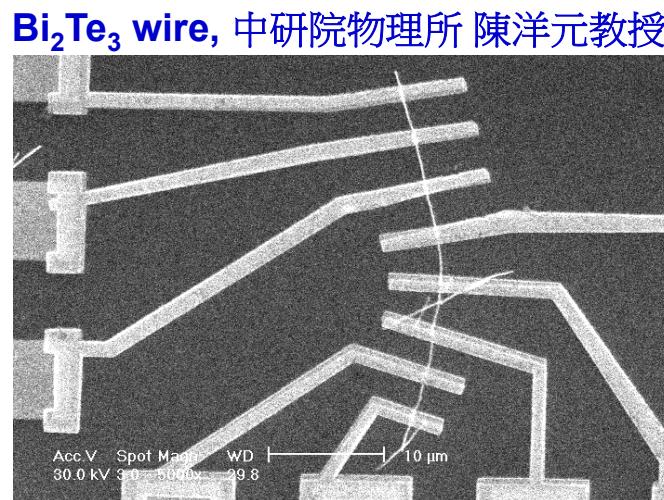
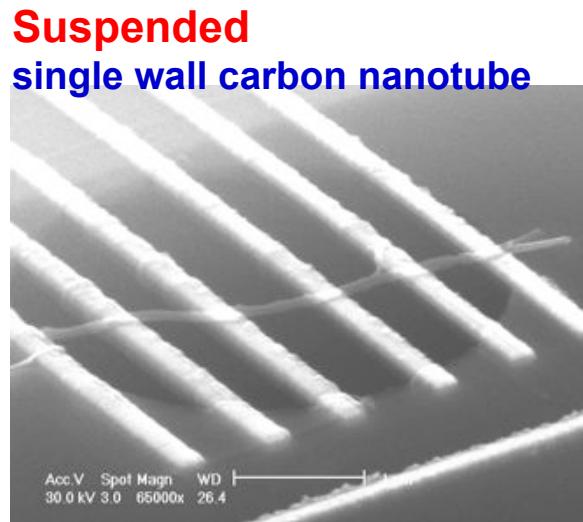


### 100nm Si-nanowire charge sensor



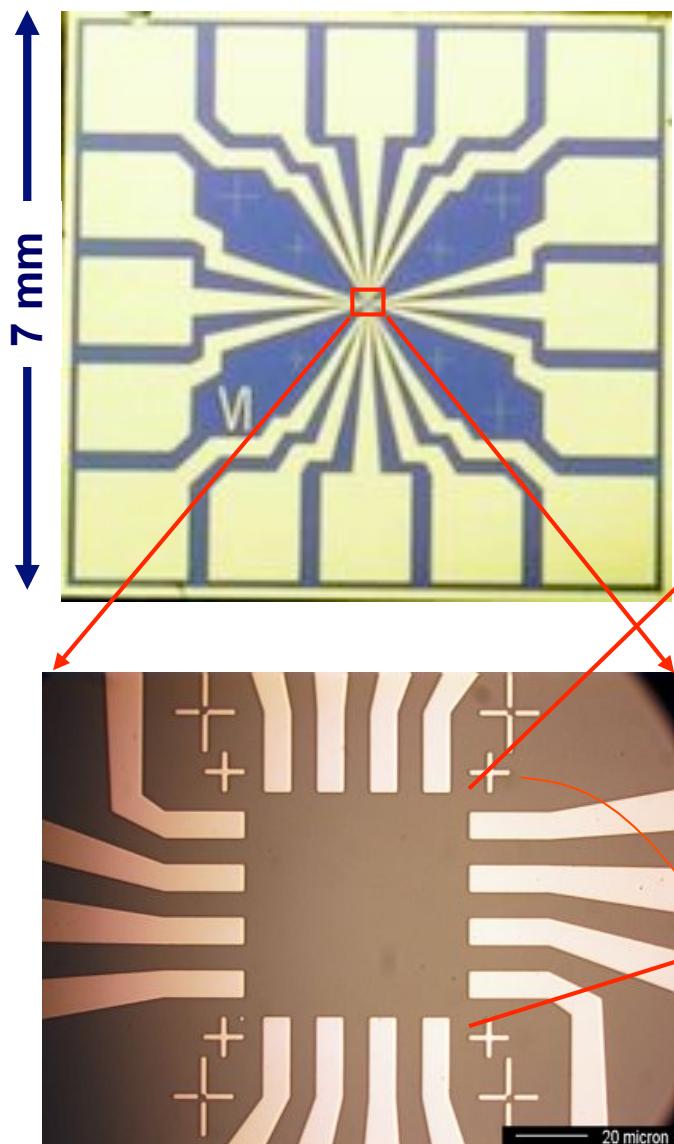
(Silicon On Insulator)

# Nanowire electronic devices

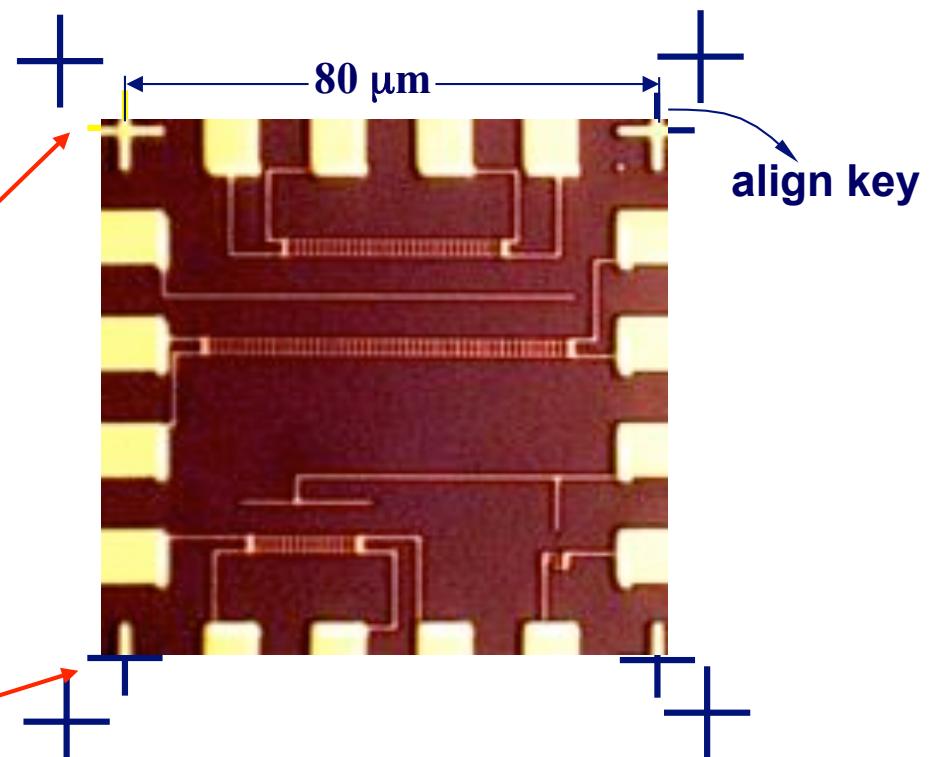


# Mix and Match technology

## Photolithography



## E-beam lithography

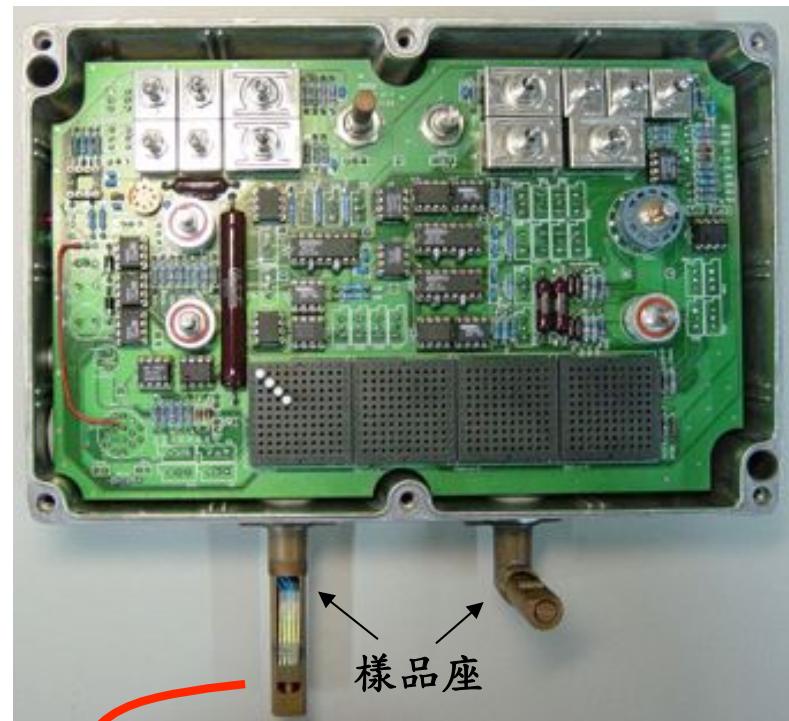


align key

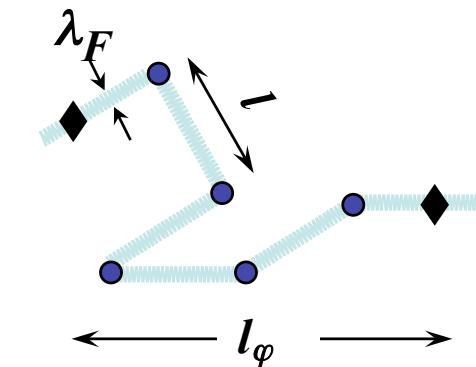
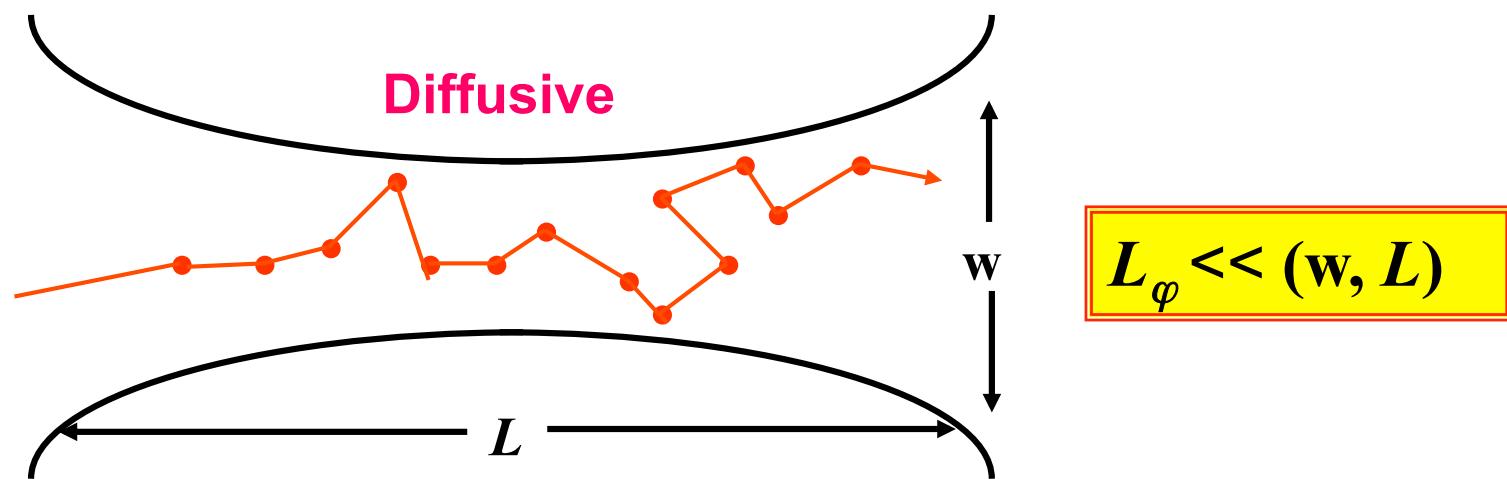
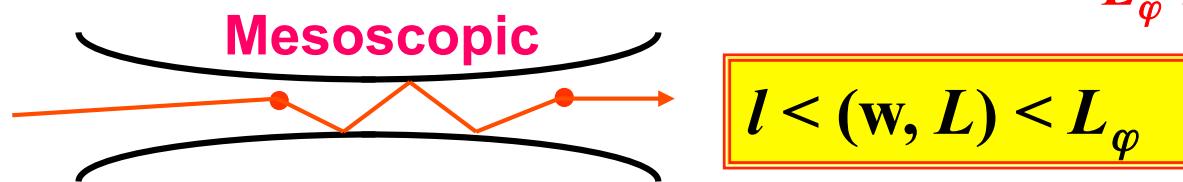
# measurement setup



Available: 40mK, 5T



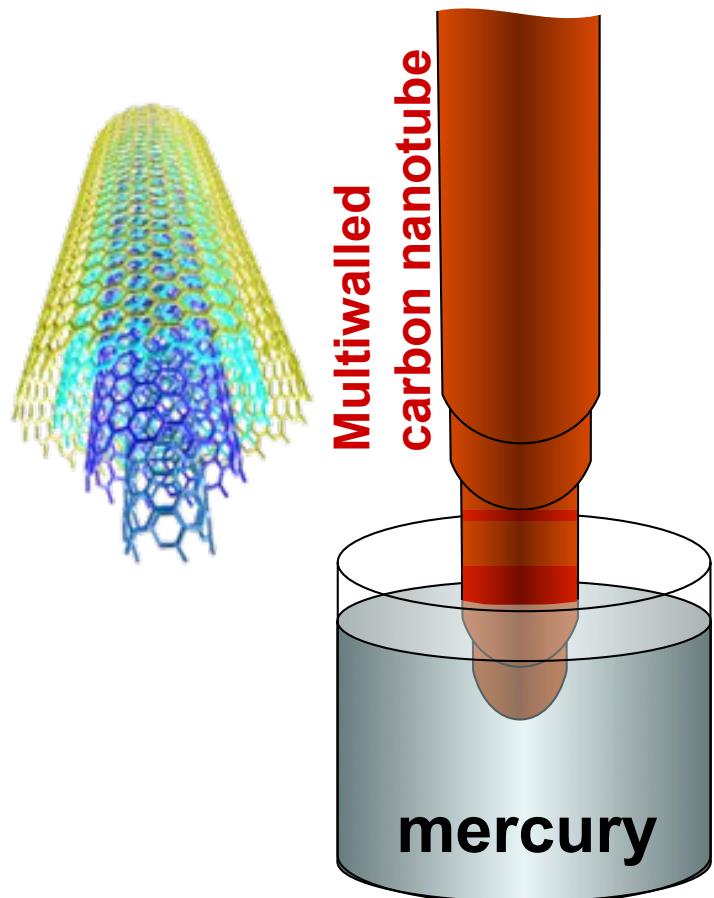
# Characteristic length scales



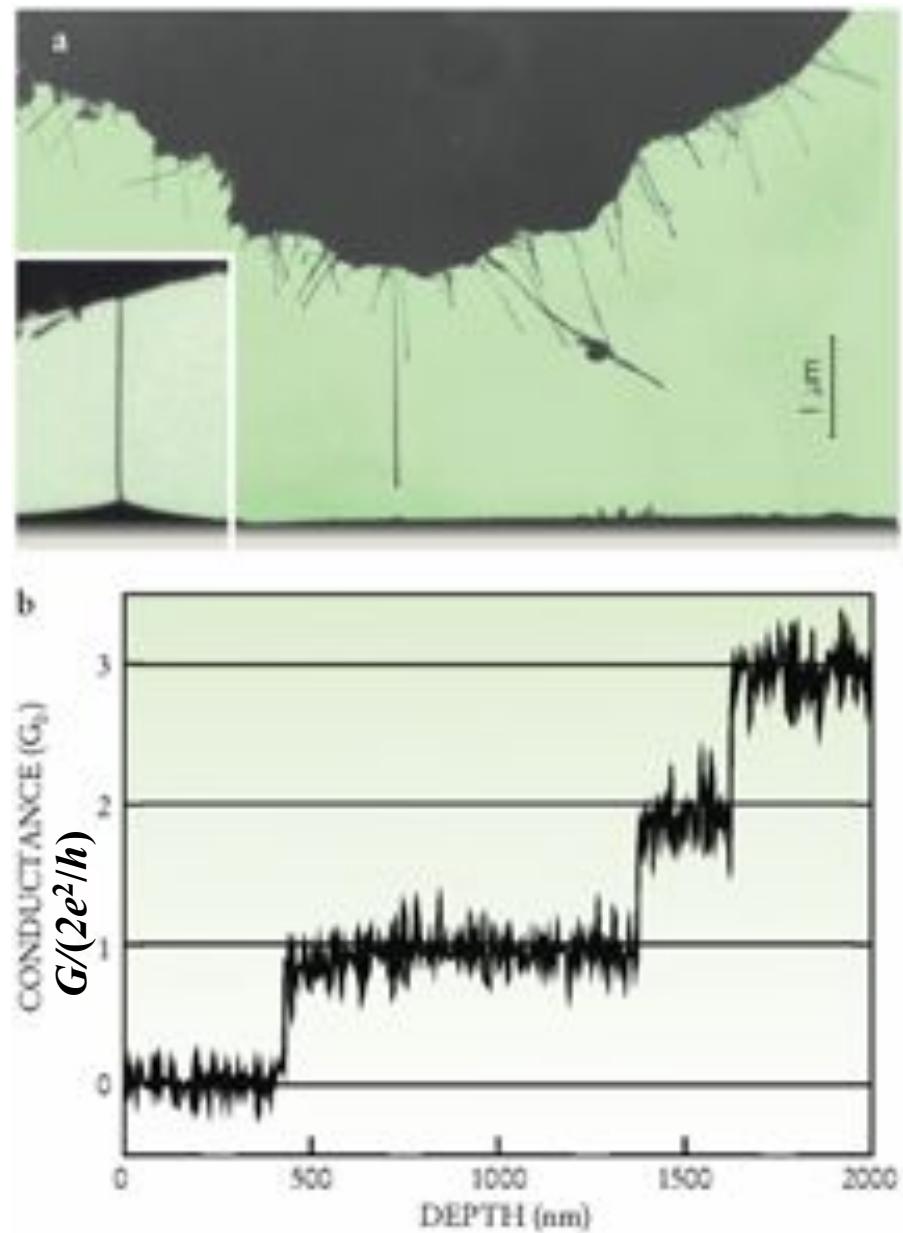
$l$  : elastic mean free path

$L_\varphi$  : phase-breaking length

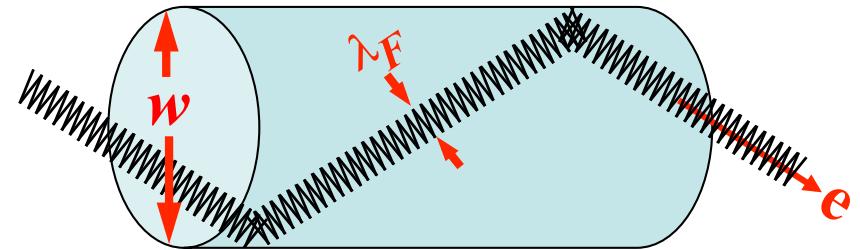
# Ballistic transport: Length independent conductance



Walt de Heer,  
Georgia Institute of Technology



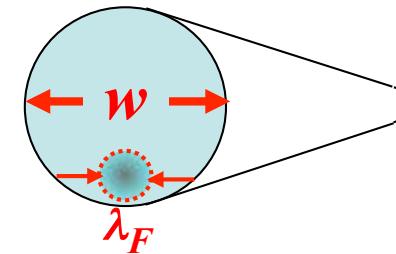
# Ballistic system:



Devices with

1. Small diameter → only a few conduction channels

$$N \approx \pi w^2 / \pi \lambda_F^2$$



2. Small electron density → no  $e$ - $e$  scattering
3. No impurity, no defect → no impurity scattering

$$G_0 = N 2e^2/h$$

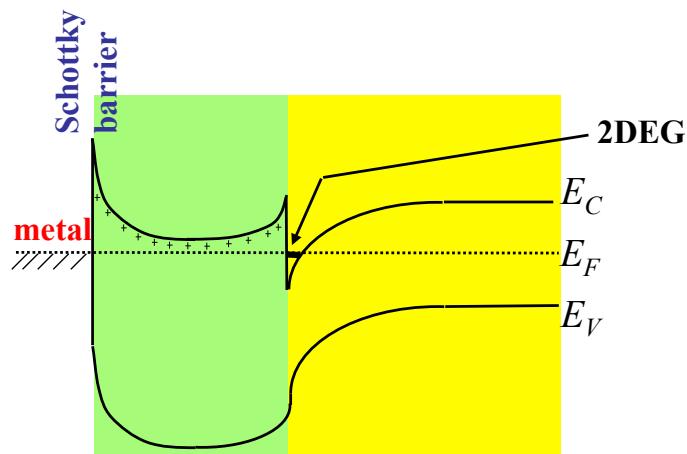
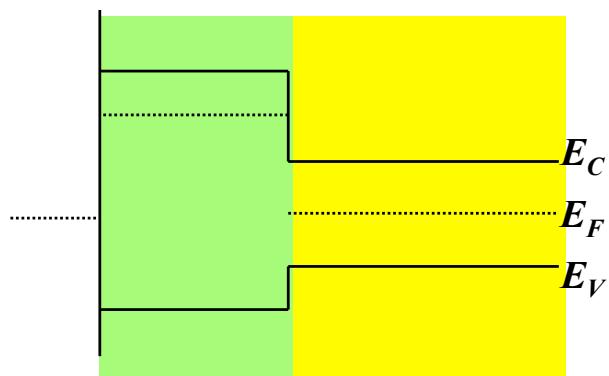
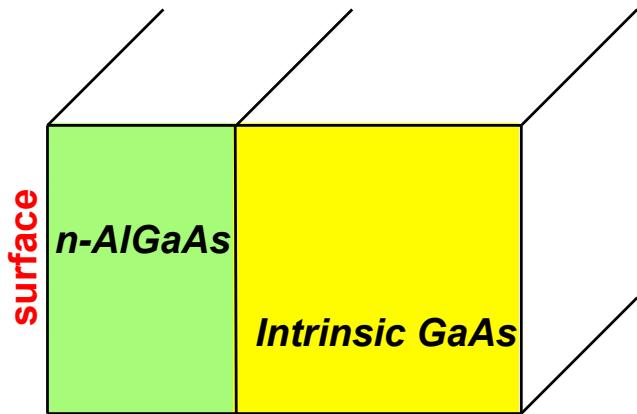
$$R_0 = \frac{1}{N} \frac{h}{2e^2} = \frac{R_Q}{N} \approx \frac{6.5k\Omega}{N}$$

$R_Q$ : quantum resistance

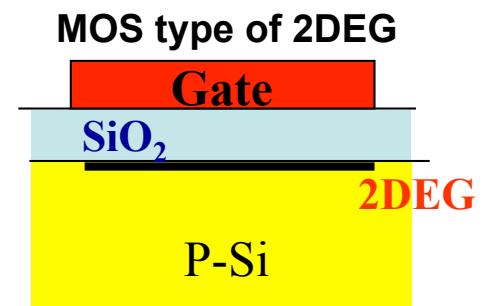
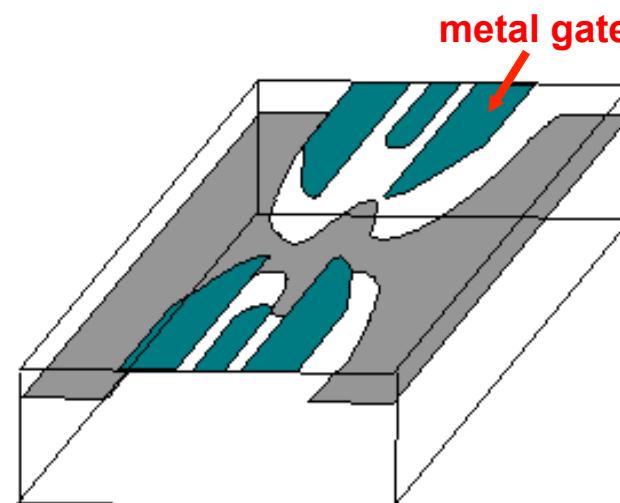
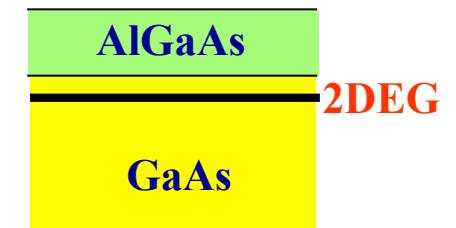
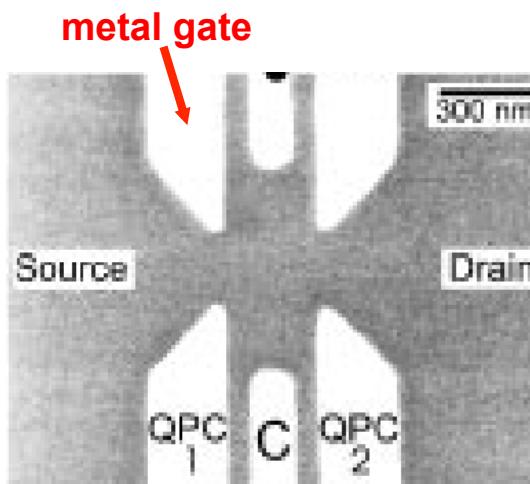
Resistance independent of the length,  
only determined by number of channels  $N$

Resistance takes place at the macroscopic contacts

# Two Dimensional Electron Gas (2DEG)



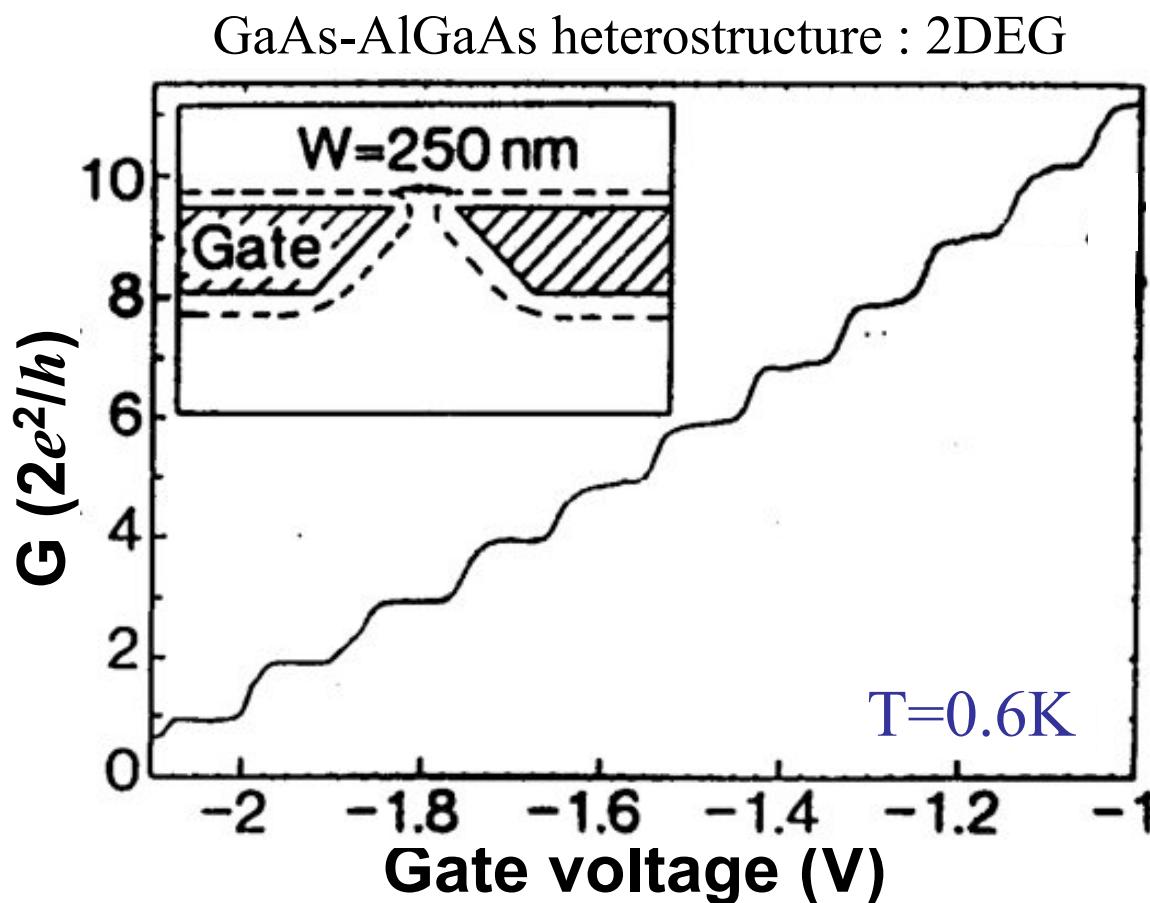
**Formation:**



# Conductance Quantization

## Experiments on Quantum Point Contacts

B.J.van Wees et al. PRL 60, 848 (1988)



$$G = \frac{2e^2}{h} \sum_n^N T_n(E_F)$$

$$T_n(E_F) = \sum_{n=1}^N |t_{m \rightarrow n}|^2$$

For adiabatic constriction

$$t_{m \rightarrow n} = \delta_{nm}$$

For abrupt constriction

$$t_{m \rightarrow n} \neq 1$$

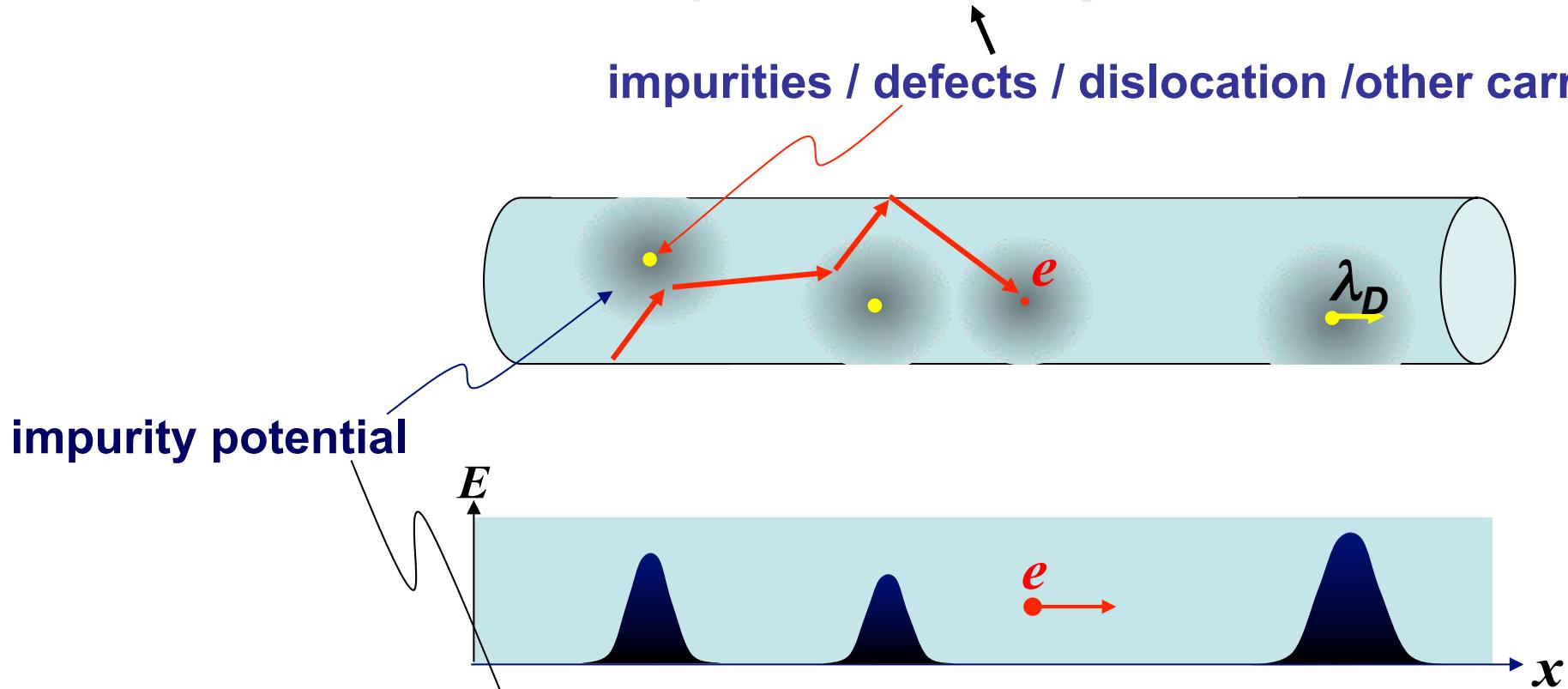
Optimized gate length

$$L_{opt} \approx 0.4\sqrt{\omega \lambda_F}$$

# Origin of resistance

electron transport in disorder systems

impurities / defects / dislocation /other carriers

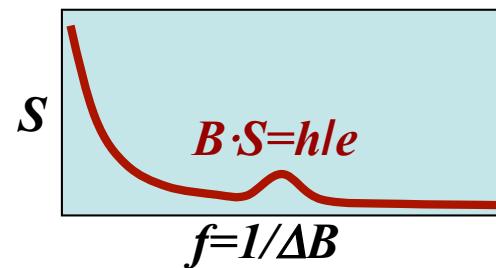
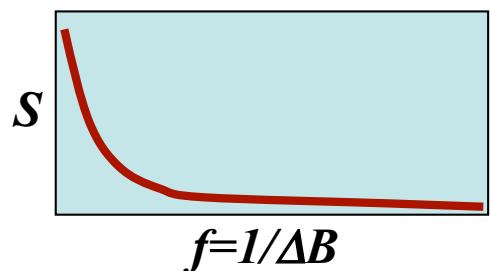
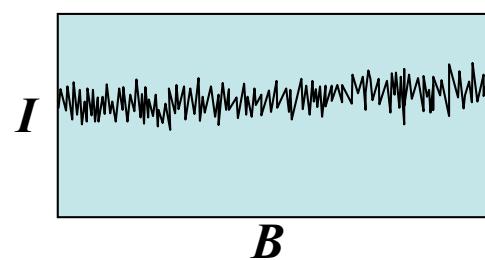
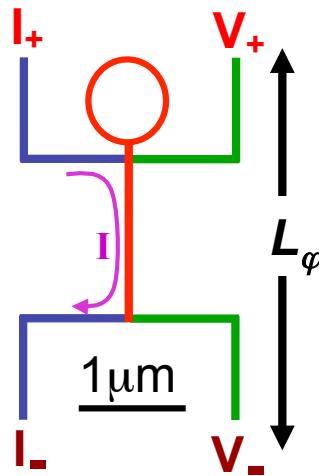
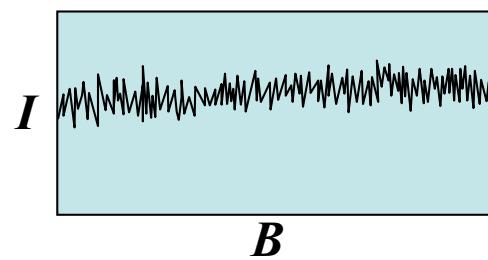
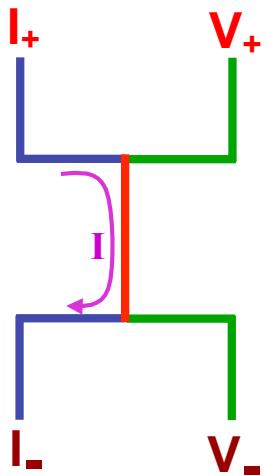


$$i\hbar \frac{\partial \Psi}{\partial t} = \left( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V \right) \Psi$$

**Debye length**  $\lambda_D = \sqrt{\epsilon k_B T / e^2 N_i}$

$\epsilon$  = dielectric permittivity,  
 $k_B$  = Boltzmann constant,  
 $T$  = absolute temperature,  
 $e$  = electron charge

# Quantum correction to the conduction



The quantum correction to the conductance can be strongly influenced by phase coherent regions extending beyond the probes and outside the classical current paths

## Observation of $h/e$ Aharonov-Bohm Oscillations in Normal-Metal Rings

R. A. Webb, S. Washburn, C. P. Umbach, and R. B. Laibowitz  
*IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598*  
 (Received 27 March 1985)

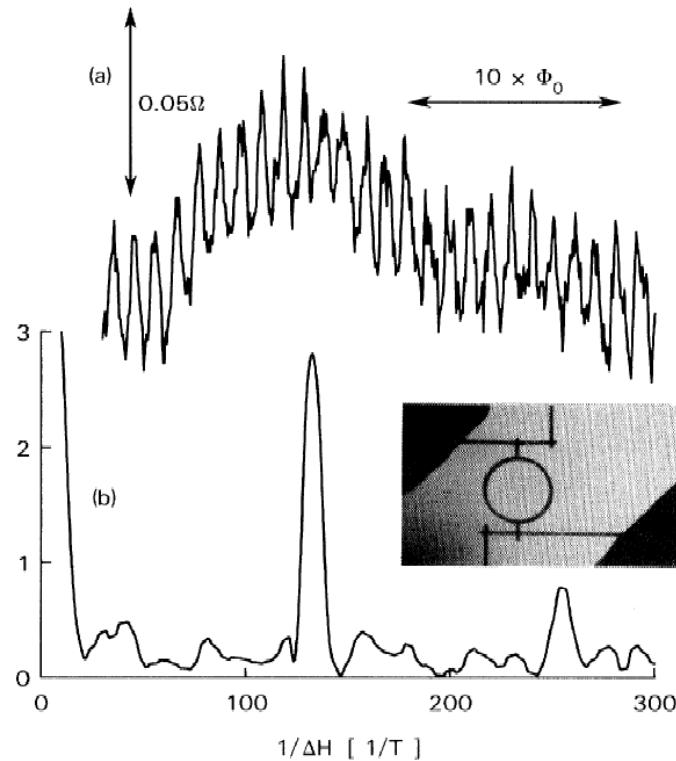


FIG. 1. (a) Magnetoresistance of the ring measured at  $T = 0.01$  K. (b) Fourier power spectrum in arbitrary units containing peaks at  $h/e$  and  $h/2e$ . The inset is a photograph of the larger ring. The inside diameter of the loop is 784 nm, and the width of the wires is 41 nm.

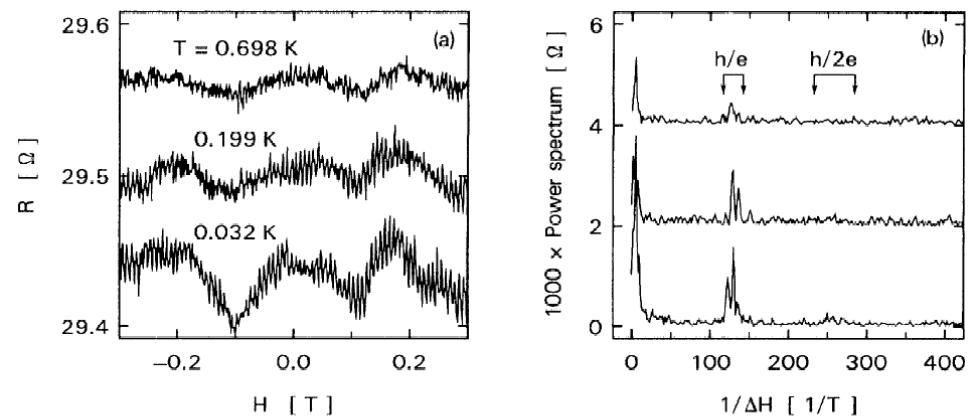
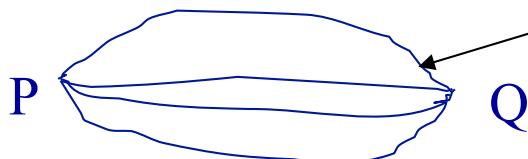


FIG. 2. (a) Magnetoresistance data from the ring in Fig. 1 at several temperatures. (b) The Fourier transform of the data in (a). The data at  $0.199$  and  $0.698$  K have been offset for clarity of display. The markers at the top of the figure indicate the bounds for the flux periods  $h/e$  and  $h/2e$  based on the measured inside and outside diameters of the loop.

# Aharanov-Bohm effect



Probability for  $P \rightarrow Q$

electron trajectory

$$W_{P \rightarrow Q} = \left| \sum_i A_i \right|^2 = \underbrace{\sum_i |A_i|^2}_{\text{Clasical}} + \underbrace{\sum_{i \neq j} A_i A_j^*}_{\text{Quantum diffusion interference}}$$

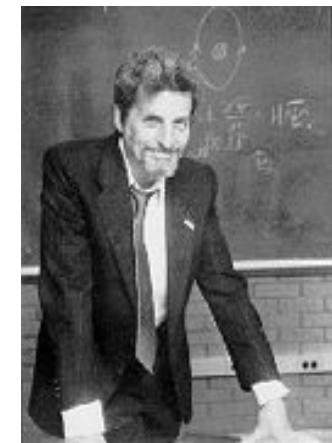
electrons pick a phase  $\varphi$   
when traveling along a path  $P$

$$\varphi = \frac{e}{\hbar} \int_P A \cdot dx$$

phase difference between two paths with the same ends

$$\phi = \frac{1}{\hbar} \int_P^Q A \cdot dl_1 - \frac{1}{\hbar} \int_P^Q A \cdot dl_2 = \frac{1}{\hbar} \int_P^Q A \cdot dl_1 + \frac{1}{\hbar} \int_Q^P A \cdot dl_2 = \frac{1}{\hbar} \oint A \cdot dl$$

$$= \frac{2e}{\hbar} \int (\nabla \times A) \cdot dS = \frac{2e}{\hbar} \int B dS = 2\pi \frac{B \cdot S}{h/2e}$$



Yakir Aharonov

Probability for back-scattering

$$W_{P \leftrightarrow P} = 2|A|^2 + 2|A|^2 \cos\left(2\pi \frac{B \cdot A}{h/2e}\right) \rightarrow \text{period} = h/2e$$

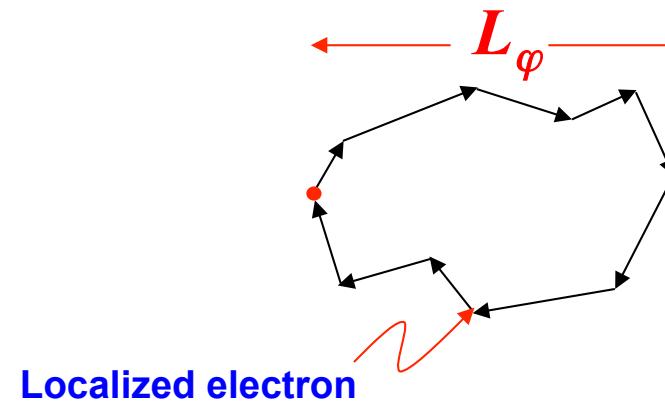
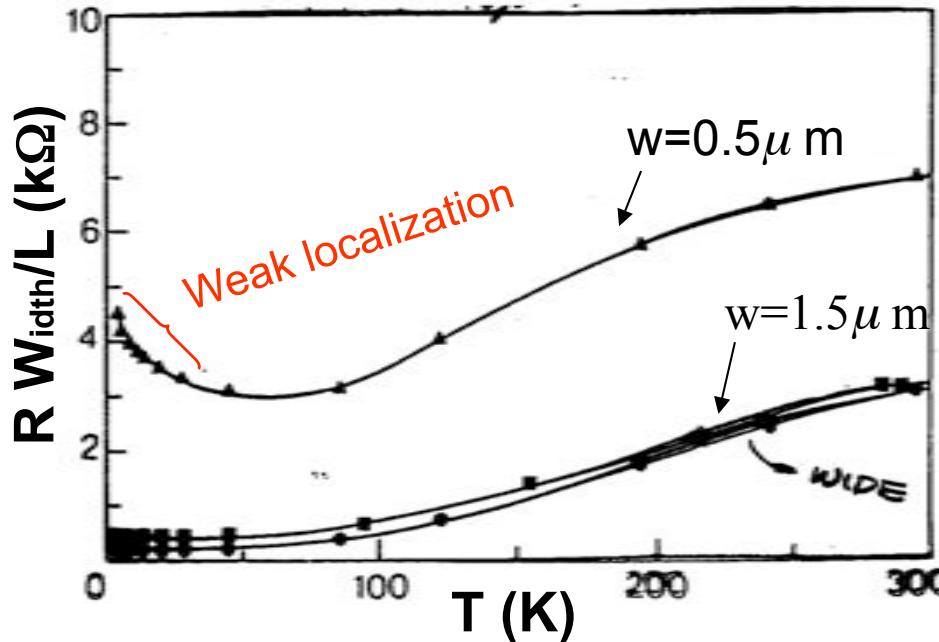


Aharonov-Casher effect

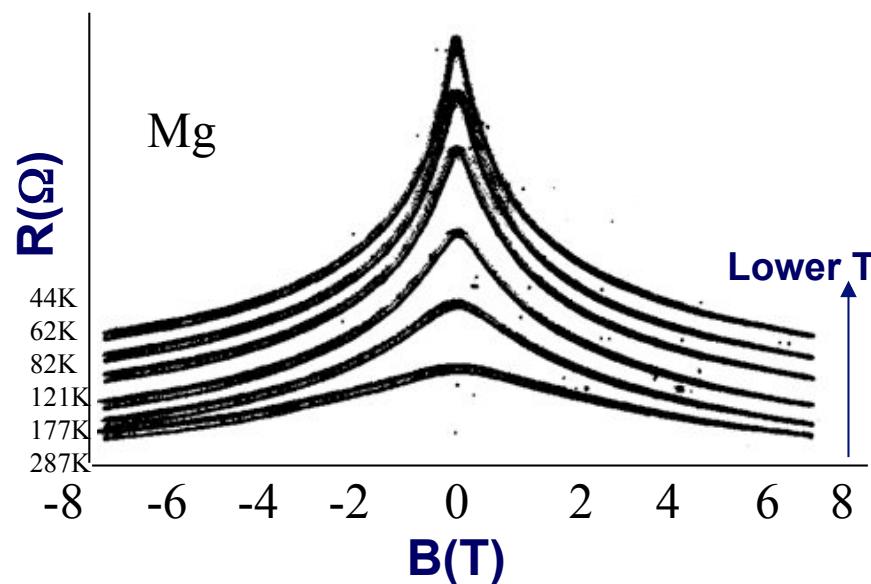


Aharonov-Bohm effect

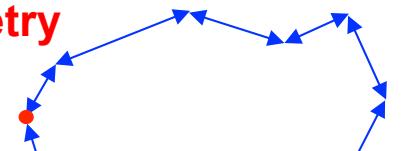
**Weak localization** takes place for length scale  $< L_\phi$



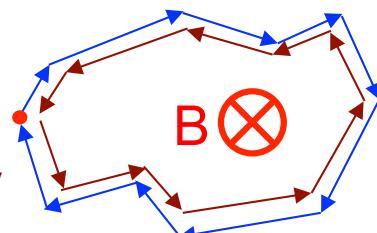
Other possible explanations:  
electron-electron interaction  
Kondo-effect



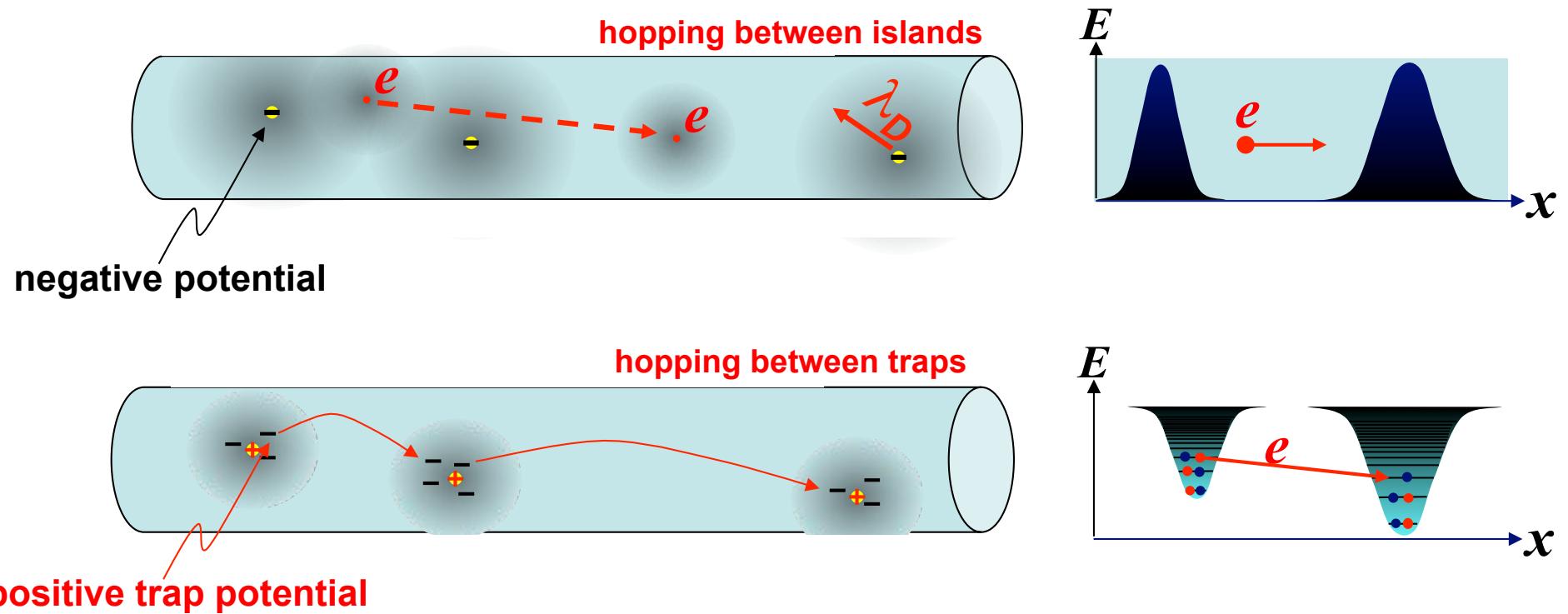
time reversal symmetry



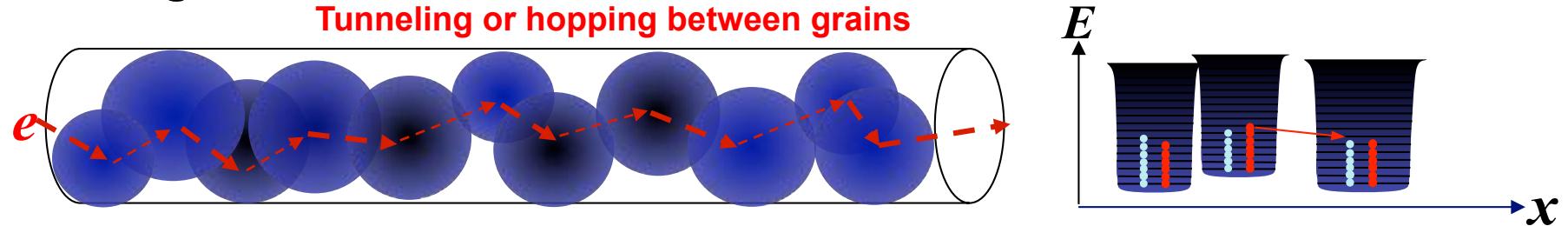
Magnetic field:  
break the  
time reversal symmetry



# Hopping conduction in disordered systems

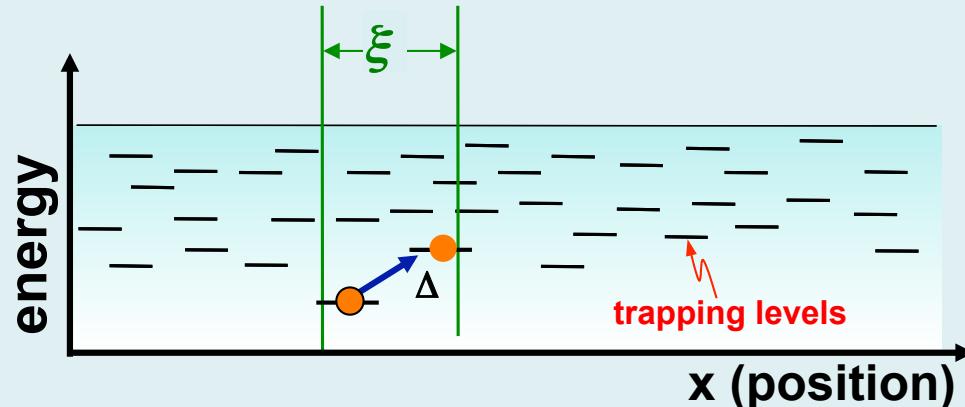


## Metallic grains



# Hopping transport $R_0$ increases with decreasing temperature

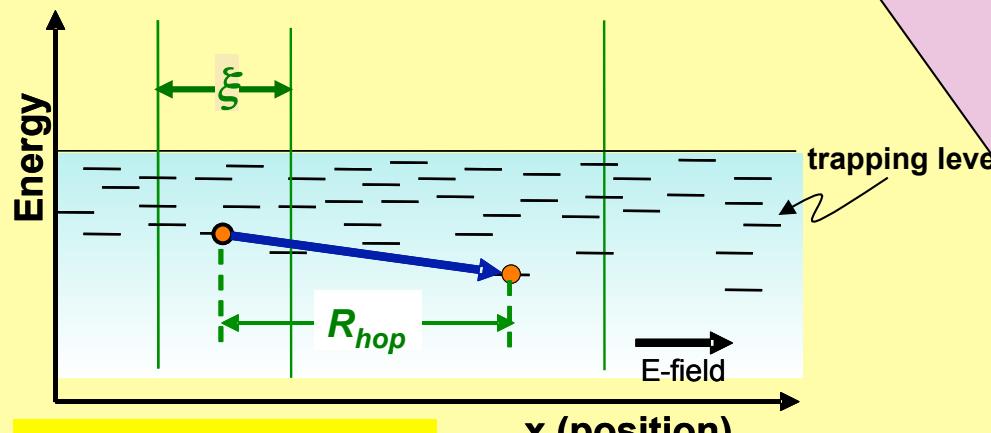
## Nearest neighbor hopping (Arrhenius form)



$$R_0 = R_A \exp\left(-\frac{\Delta}{k_B T}\right)$$

Presence of a **hopping barrier**; thermally activated hopping

## Mott Variable range hopping



$$R_0 = R_A T^S \exp\left[-\left(\frac{T_{Mott}}{T}\right)^p\right]$$

$T_{Mott}$  = Mott characteristic temperature

$$p = \frac{1}{1+d} \quad d = \text{system dimensionality}$$

## Efros-Shklovskii Variable range hopping

when Coulomb interaction is considered

$$R_0 = R_A T^S \exp\left[\left(\frac{T_{ES}}{T}\right)^{1/2}\right]$$

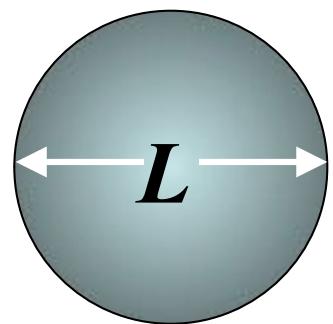
power-law dependence in DOS near  $E_F$

$$N(E) = N_0 |E - E_F|^\gamma$$

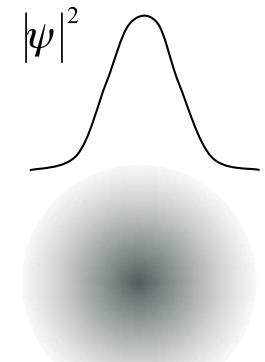
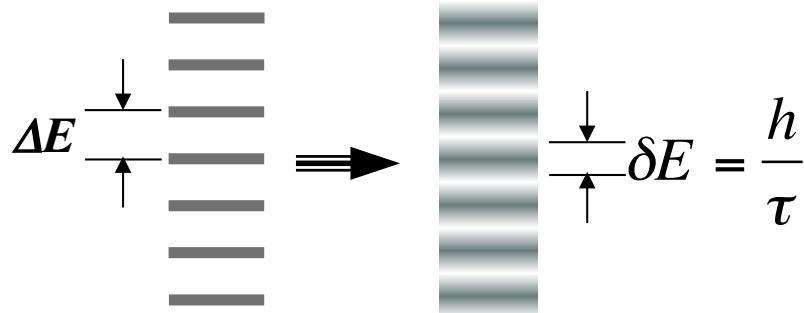
For 3D,  $\gamma=2$

Nonlinear  $IV_b$  characteristics

# Quantum origin of energy level broadening



broadening of quantized levels



$\tau$  = time for electron to travel through the grain

in ballistic regime:  $\tau = L/v_F$

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in diffusive regime:  $\tau = L^2/D$  for  $D$ : Einstein relation:  $\sigma = e^2 D N_0(E_F)$

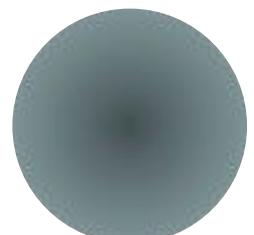
$$\Rightarrow \delta E = \frac{h}{\tau} = \frac{hD}{L^2} = \frac{h}{e^2} \frac{\sigma}{L^2 N_0(E_F)}$$

Level spacing :  $\Delta E = [L^d N_0(E_F)]^{-1}$

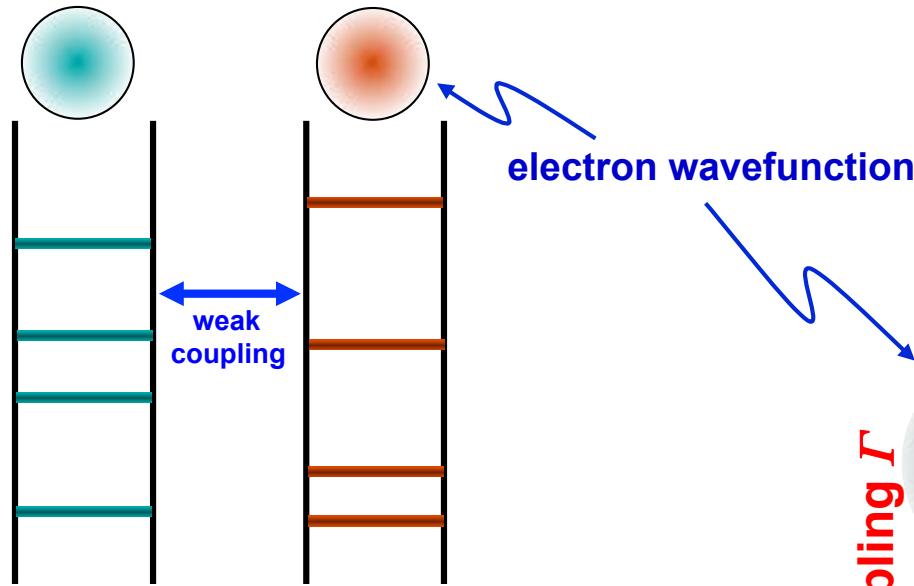
$$\text{Thouless ratio } g = \frac{\delta E}{\Delta E} \approx \frac{h}{e^2} \sigma L^{d-2} = \frac{G}{e^2/h}$$

$g < 1$  ( $R > 26k\Omega$ )  $\Rightarrow$  localized state

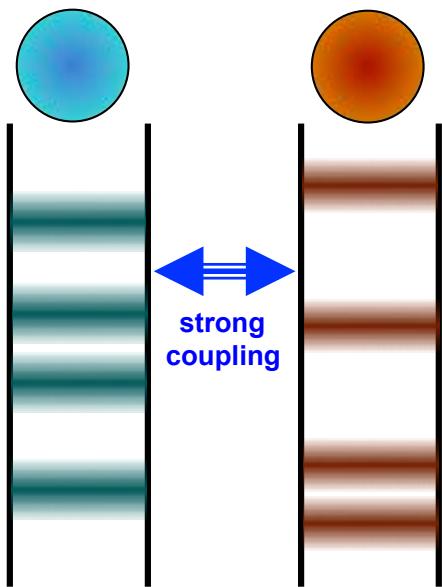
$g > 1$  ( $R < 26k\Omega$ )  $\Rightarrow$  extended state



# Inter-grain coupling induced level broadening

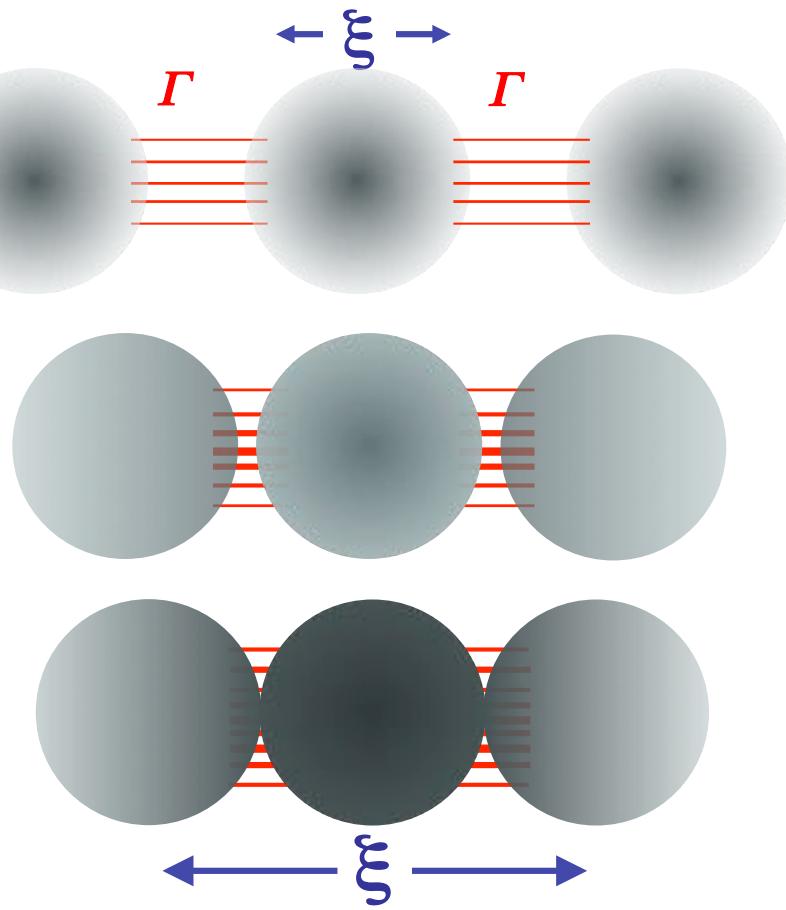


Level broadening  $\delta E = \Gamma$



Increasing inter-grain coupling  $\Gamma$

$\xi = \text{localization length}$



# Resonate tunneling between coupled quantum dots

For very weak inter-coupling, two possible states exist :  $|R\rangle$  and  $|L\rangle$   
With finite coupling  $\Gamma$ ,  $|R\rangle$  and  $|L\rangle$  mix and form two eigenstates  $|A\rangle$  and  $|S\rangle$

Coupling strength =  $\Gamma$   
tunneling rate  $\gamma = \Gamma/\hbar$   
level broadening  $\delta E = \Gamma$

