Versatility of the Abelian Higgs Model

Ernest Ma
Physics and Astronomy Department
University of California
Riverside, CA 92521, USA

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$U(1)$ Gauge Model with Scalar Interactions

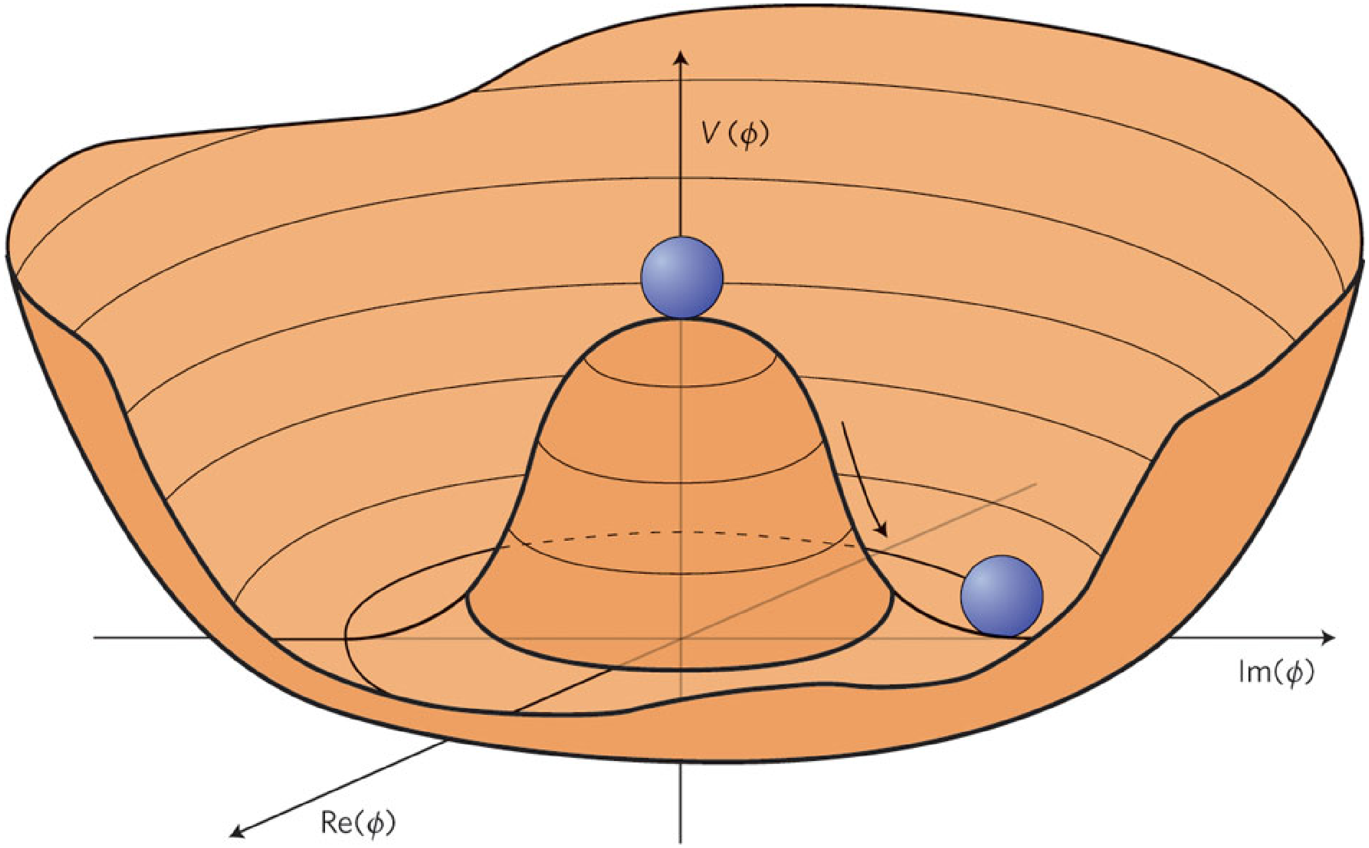
In analogy to quantum electrodynamics for photon-electron interactions, the Lagrangian of an $U(1)$ gauge model with scalar interactions is given by

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 + |(\partial_\mu - igB_\mu)\phi|^2 - V(\phi),$$

where $V = \mu^2\phi^\dagger\phi + (\lambda/2)(\phi^\dagger\phi)^2$. This Lagrangian is invariant under the $U(1)$ gauge transformation: $B_\mu \to B_\mu + \partial_\mu\Lambda$ and $\phi \to \exp(ig\Lambda)\phi$.

If $\mu^2 > 0$, this is just scalar quantum electrodynamics with $g = e$, i.e. the interaction of a charged scalar particle of mass μ with the electromagnetic field.

If $\mu^2 < 0$, then the famous phenomenon of spontaneous symmetry breaking occurs in V because $V(\phi = 0) = 0$ is no longer a minimum but a local maximum. The true minimum occurs at $|\phi| = v$, where $v^2 = -\mu^2/\lambda$, where $V(|\phi| = v) = -(1/2)\lambda v^4$. Because of the interaction of B_μ with ϕ, this makes B_μ massive with $m_B^2 = 2g^2v^2$. This is the famous Brout-Englert-Higgs (BEH) Mechanism.


V (ϕ)
Im(ϕ)
Re(ϕ)

Whereas the Goldstone (angular) degree of freedom θ in $\phi = |\phi| \exp(i\theta/v)$ is now lost to $B_\mu \to B_\mu - \partial_\mu \theta/v$, the Higgs degree of freedom, i.e. $|\phi| - v$, remains physical with $m^2 = 2\lambda v^2$ and that is what we call the Higgs boson. If this idea is generalized to the $SU(2) \times U(1)$ gauge symmetry of the Standard Model of particle interactions, the one physical Higgs boson of its realization is presumably what was discovered as a 126 GeV particle at the LHC announced on July 4, 2012.

Actually, the condensed-matter analog of the Higgs degree of freedom was discovered already in 1982.

The Condensed-Matter Higgs

In condensed-matter physics, the field $\vec{\phi}$ is an order parameter of n components. In the presence of spontaneous symmetry breaking (minimum of V at $|\vec{\phi}| = \phi_0 \neq 0$), there are two modes: the massless Goldstone mode $(\omega = ck)$ from phase fluctuations, and the Higgs mode $(\omega = \sqrt{\omega_0^2 + c^2k^2})$, where $\omega_0 = V''(\phi_0)$, from amplitude fluctuations.

However, the dynamical nature of these modes depends on their equations of motion, i.e. their Lagrangian.

For the relativistically invariant

$$\mathcal{L} = \frac{1}{2}\left(\frac{\partial \vec{\phi}}{\partial t}\right)^2 - \frac{c^2}{2}(\nabla_x \vec{\phi})^2 - V(|\vec{\phi}|),$$

the two modes are independent dynamical degrees of freedom. For the nonrelativistic

$$\mathcal{L} = i\psi^* \frac{\partial \psi}{\partial t} - \frac{c^2}{2}|\nabla_x \psi|^2 - V(|\psi|),$$

the amplitude and phase fluctuations are canonical conjugates, so there is only the Goldstone mode.

In condensed matter, $\mathcal{L}$ is generally not relativistic. However, in superconductors with the pairing energy $\Delta << E_F$, the Fermi energy, the effective action for fluctuations in the amplitude of the superconducting order, within the ordered superconductor phase only, has a relativistic form, so a Higgs mode is present, as pointed out by Littlewood/Varma(1982).

Under some circumstances, this Higgs mode couples to long-wavelength phonons, and was first observed in Raman scattering experiments in the charge-density-wave compound $NbSe_2$.

In 2012, there was also the experiment of Endres *et al.* published in Nature: *The Higgs amplitude mode at the two-dimensional superfluid/Mott insulator transition*.

This is performed with ultracold atoms in a two-dimensional optical lattice, where the quantum phase transition is described by the relativistic Higgs Lagrangian, with $\vec{\phi}$ being the 2-component superfluid order parameter.

Because it is a two-dimensional system, it is possible for the Higgs excitation to decay into multiple Goldstone modes, so this amplitude is infrared singular.

Endres *et al.* measured the response to a periodic modulation in the depth of the optical lattice across the superfluid-insulator transition.

On the insulator side, they observed the expected gapped response, vanishing upon the approach to the superfluid.

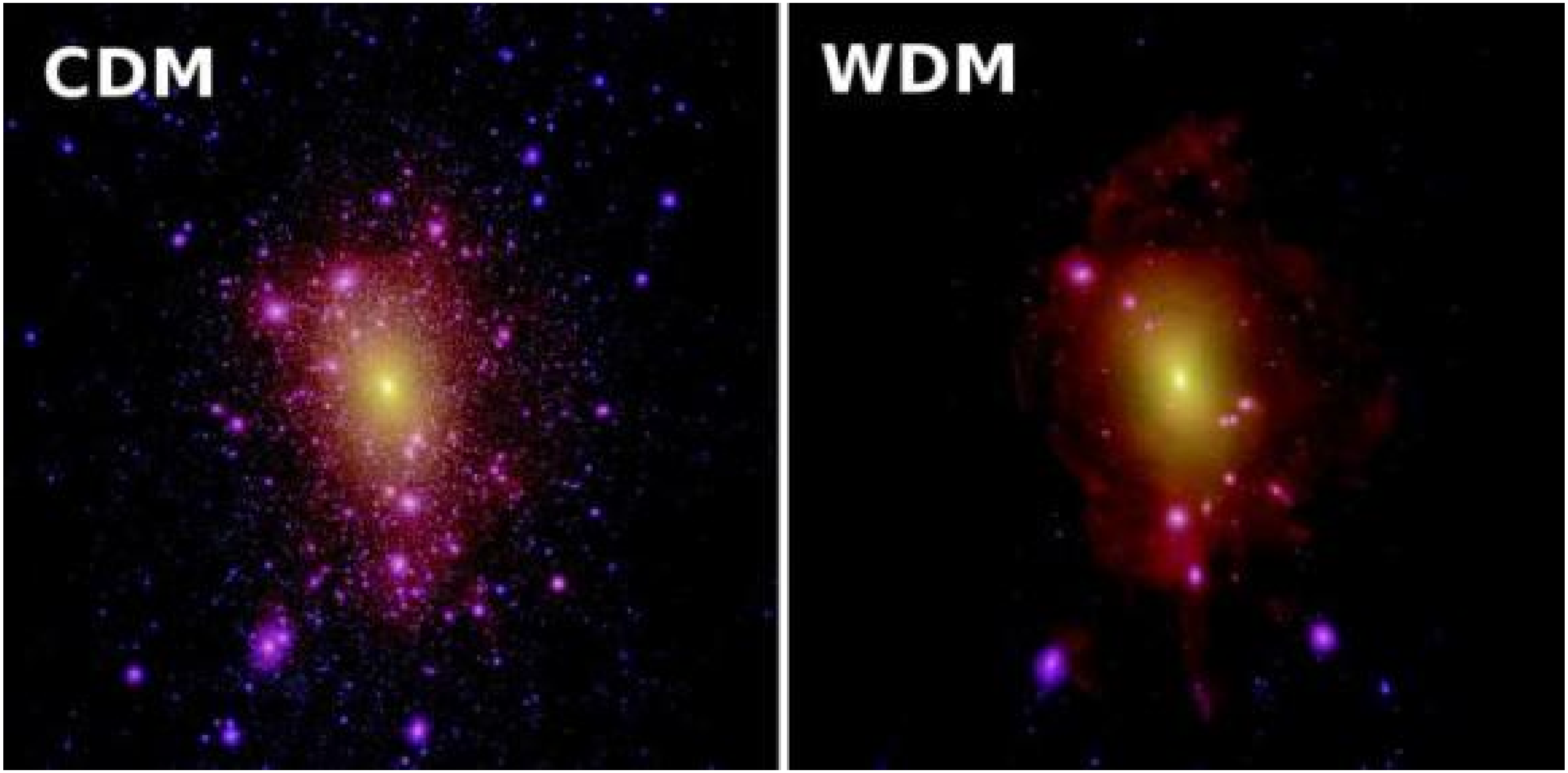
On the superfluid side, they observed the re-emergence of a finite frequency peak with a pseudo-gap at low frequencies. Their results agree with the theoretical calculations as well as Monte Carlo simulations of the existence of such a Higgs mode, including its scaling properties upon approaching the quantum critical point.

$U(1)$ Gauge Boson as Dark Mediator

There is dark matter in the Universe, but what is it? The usual assumption is that it interacts with known matter only weakly, and only at short range. If so, a stable particle with a mass of order 100 GeV having a thermal average of its annihilation cross section multiplied by its relative velocity at the time of freeze-out, i.e. $\langle\sigma v\rangle$, equal to 1 pb, would yield the correct observed dark-matter relic abundance of the Universe. This is often called the WIMP (Weakly Interacting Massive Particle) miracle, and the CDM (Cold Dark Matter) model is the default model of the Universe. If the cosmological constant is added for dark energy, then it is called the ΛCDM model. However, there is a mounting problem with this scenario, because the predicted numbers of satellite dwarf spheroidal galaxies (existing in very rich dark-matter environments) are far fewer than are observed.

Feng/Tu/Yu(2008), Feng/Kaplinghat/Tu/Yu(2009), Ackerman/Buckley/Carroll/Kamionkowski(2009):

Suppose dark matter also has long-range self-interactions in addition to gravity, then it can solve this emerging crisis in astrophysics.


CDM
WDM

The unbroken $U(1)$ gauge model of scalar interactions is a simple candidate of such a scenario. The scalar ϕ is stable dark matter and the massless B_μ is the dark mediator.

Actually B_μ may be given an explicit (Stuckelberg) mass, in which case the local $U(1)$ becomes a global symmetry, but the theory remains renormalizable since B^μ couples to the conserved current $\phi\partial_\mu\phi^* - \phi^*\partial_\mu\phi$.

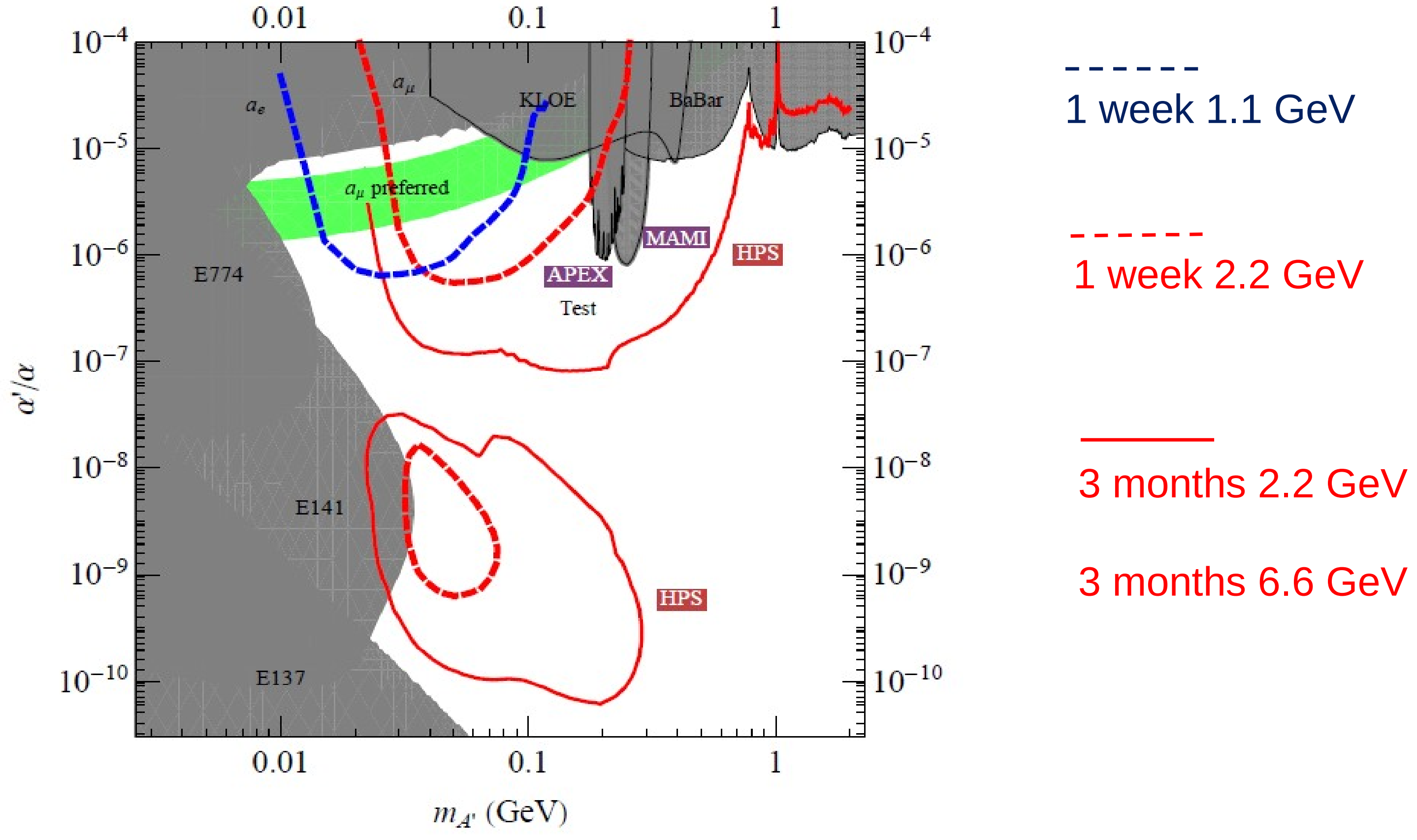
There is also an additional discrete symmetry, i.e. $B_\mu \to -B_\mu$ and $\phi \to \phi^*$, which may be used to distinguish B_μ from the electromagnetic field A_μ.

$U(1)$ Gauge Boson Mixing with Photon

Holdom(1986), Pospelov/Ritz/Voloshin(2008),
Jaekel/Redondo/Ringwald(2008),
Arkani-Hamed/Finkbeiner/Slatyer/Weiner(2009):

Without the $B_\mu \to -B_\mu$ symmetry, B_μ will have kinetic mixing with A_μ, i.e. $(\partial_\mu B_\nu - \partial_\nu B_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)$. This term is allowed by both $U(1)$ gauge transformations, but the mixing is arbitrary and may be very small.
Now B_μ will decay through its mixing with A_μ. Such a heavy or hidden or dark photon will be searched for at the HPS experiment (2014) at JLab.

HPS Reach ≈ ”Full HPS” Reach



If the mass of B_μ comes from the BEH Mechanism, then there is a corresponding dark Higgs bodon h'. The process

$$e^+e^- \to Bh', \quad \text{then } h' \to BB$$

with $B \to e^+e^-, \mu^+\mu^-$ may be searched for.

BaBar (2012): For m_B in the range 0.3 to 2.5 GeV, and $m_{h'}$ in the range 0.5 to 10 GeV, the $B - A$ mixing is less than 10^{-3} to 10^{-4}.

BELLE (2013): Similar results are expected.

$B - A$ oscillations may allow light shining through wall.

Redondo/Postma(2009):
The dark photon B_μ may have a mass in the keV range, so it may be warm dark matter. It may then be produced by its mixing with A_μ. This is analogous to the familiar notion of a sterile neutrino ν_S as warm dark matter, which is produced by its mixing with the active neutrinos. There ν_S will decay into $\nu\gamma$ through a loop of charged leptons, but its lifetime is much longer than the age of the Universe. Yet its X-ray signature is observable by CHANDRA. Here $B \to \gamma\gamma$ is forbidden, so $B \to \gamma\gamma\gamma$ is expected. Neither simple WDM scenario works well.

$U(1)$ Gauge Boson as Dark Matter

Farzan/Rezaei Akbarieh(2012):
If the $B \rightarrow -B$ and $\phi \rightarrow \phi^*$ symmetry is broken by $\langle\phi\rangle = v$, then the residual symmetry $B \rightarrow -B$ and $h' \rightarrow h'$ remains. So B may be absolutely stable dark matter. If v is around the electroweak energy scale, then B is cold dark matter. Although B only interacts through the $h'BB$ vertex, the allowed interactions of h' with the Standard-Model Higgs boson enable B to be in thermal equilibrium and freezes out with the correct relic abundance, but it will be very difficult to detect.

Josse-Michaux/Ma/Palomares Ruiz(2013):
Two more variations of the Abelian Higgs Model may also be realistic dark matter models.

(1) B is cold dark matter with mass of order 100 GeV, but h' is very light (recall $m_{h'} = 2\lambda v^2 = -2\mu^2$). The $h'BB$ interaction is then useful to cure the missing satellites problem.

(2) B is of order 10 keV, so it is warm dark matter. However, it is now thermally produced through the $h'BB$ interaction. At the time of freeze-out, it is relativistic, so its number density is simply given by

$$\frac{n_B}{n_\gamma} = \left(\frac{43/4}{g^*_{dec}}\right)\left(\frac{2}{11/2}\right)\frac{3}{2}.$$

The abundance of B fixes its density through $m_B n_B$, hence $\Omega_B h^2 \simeq 21.4(m_B/\mathrm{keV})$. For $m_B = 10$ keV to be safely above the Lyman-α forest bound, this would mean an overproduction factor of 2000. The usual remedy for this disaster is to postulate a late decaying particle which decouples after B and dilutes n_B to just the right amount through its decay to standard-model particles. This may be achieved by h' itself through its allowed mixing with the standard-model Higgs boson.

Conclusion

The original Abelian Higgs Model, i.e. a complex scalar field ϕ interacting with a vector gauge boson B, is remarkably versatile. It provides the simplest example of the BEH Mechanism. It has direct application in condensed-matter physics. It is now also studied in dark-matter physics. B may be a dark mediator, providing long-range interactions which could save the cold dark matter paradigm. It may mix with the photon and be observed. It may itself be dark matter, either cold or warm. The Higgs story is not yet over!