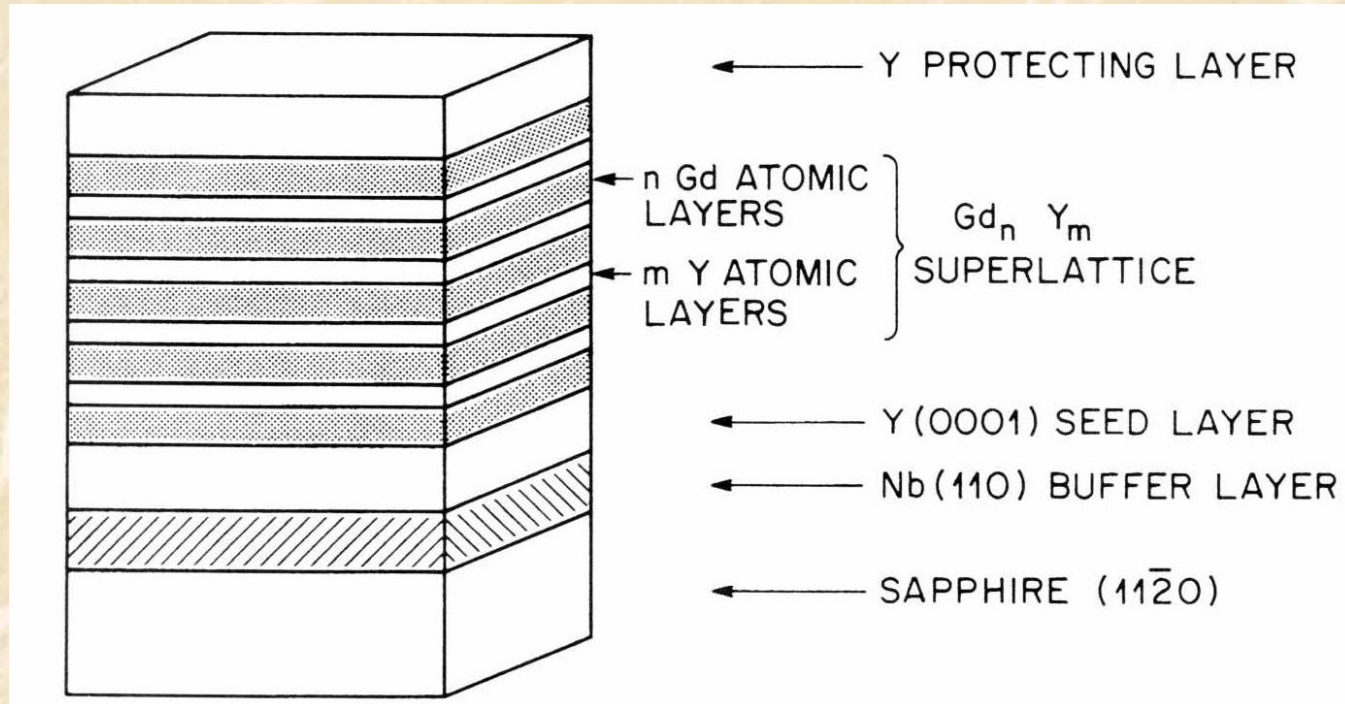


Artificial Superlattice

- Matching the structural periodicity with physical length scale of **superconductivity and magnetism**
- Modulation of physical properties

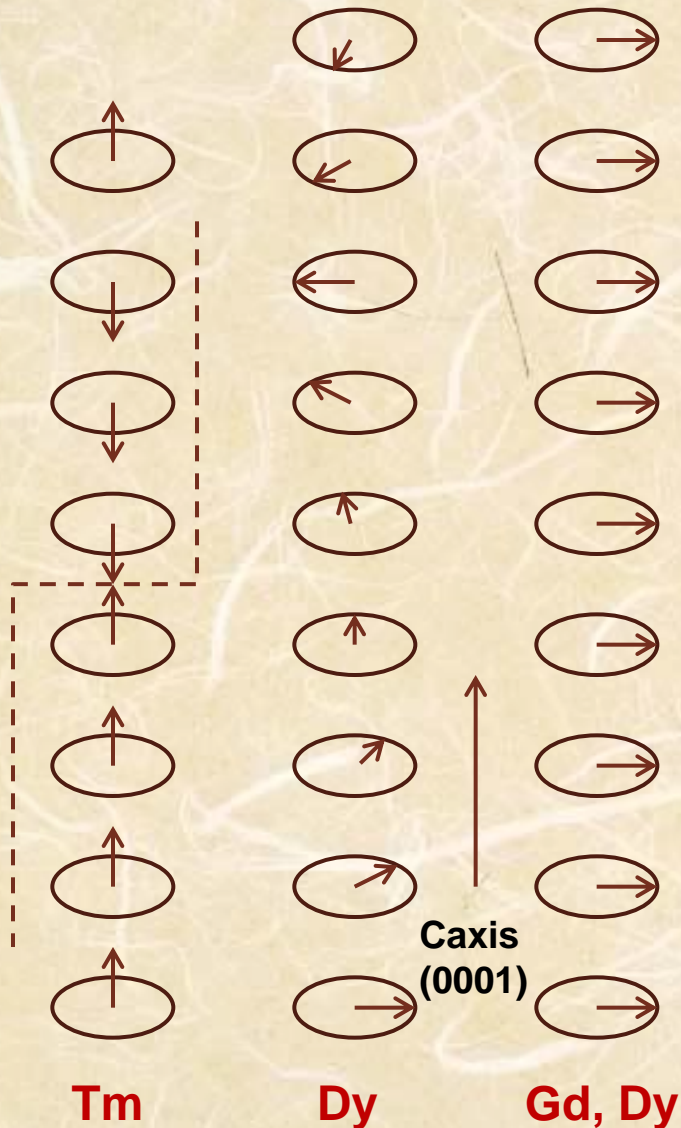


Invention of metal molecular beam epitaxy in 1981

- Single crystal epitaxial superlattices with Atomically abrupt interfaces

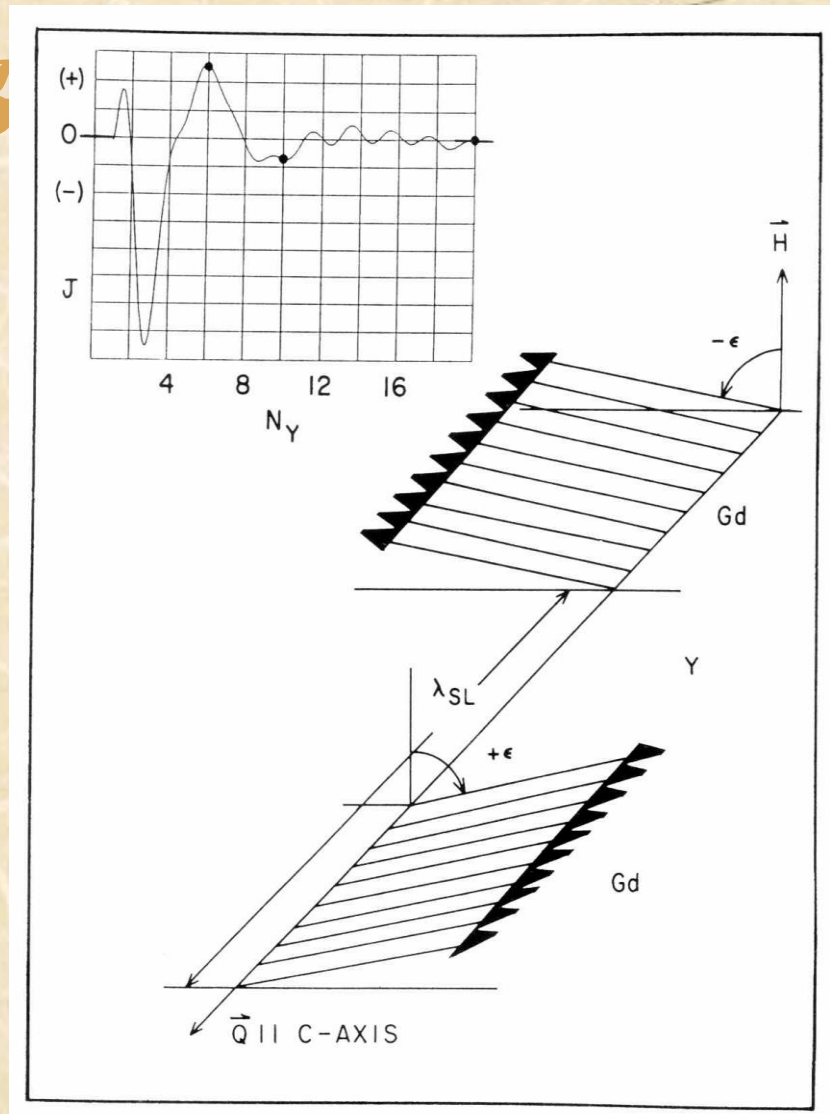
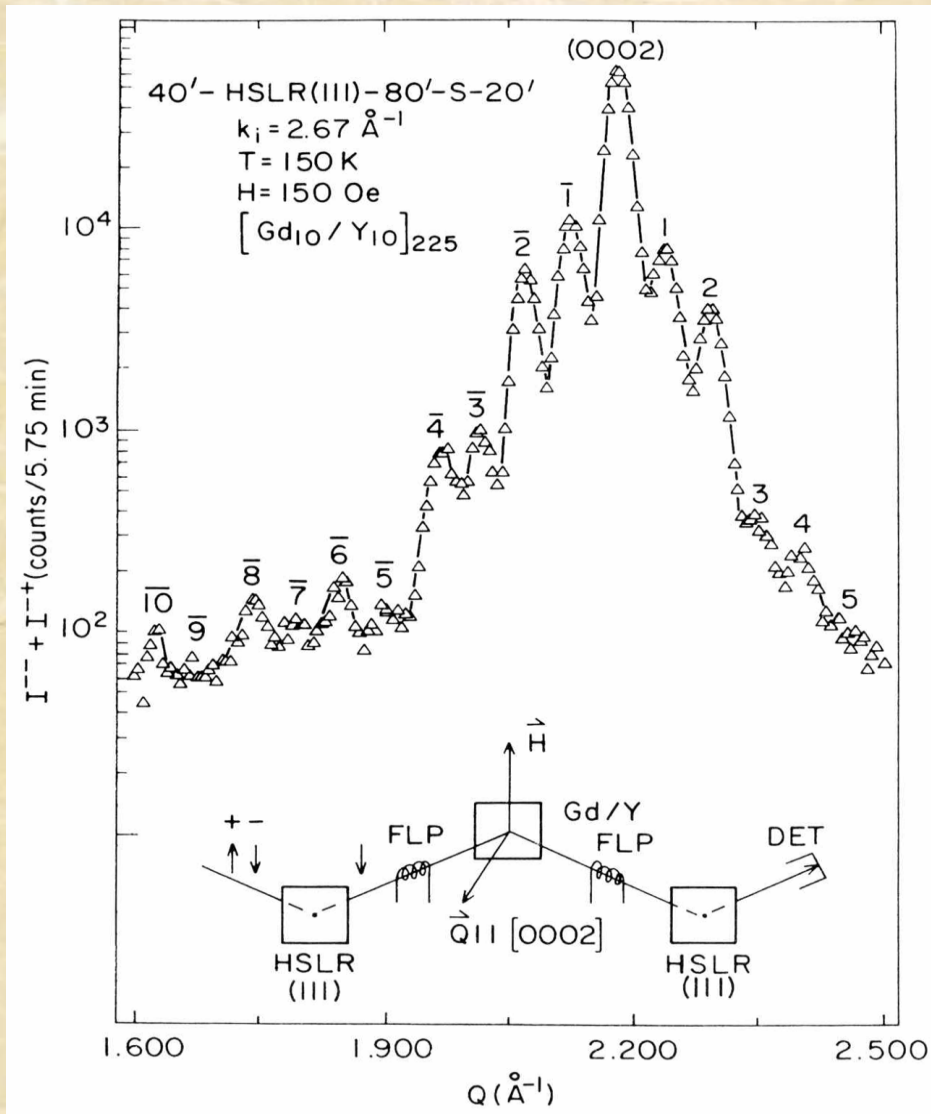
HCP crystal structure

- Similar crystal- chemical nature of rare earth forms coherent superlattices
- Metallic superlattice effect
 - Long range nature of the indirect exchange interaction
 - Magnetic coupling of magnetic rare earth through non – magnetic Y, Lu
 - Modulation of magnetic properties of Gd - Y Superlattices
 - Spin structure modification of Tm - Y, Dy - Y Superlattices
- 2-dimensional magnetism
- Interfacial magnetism



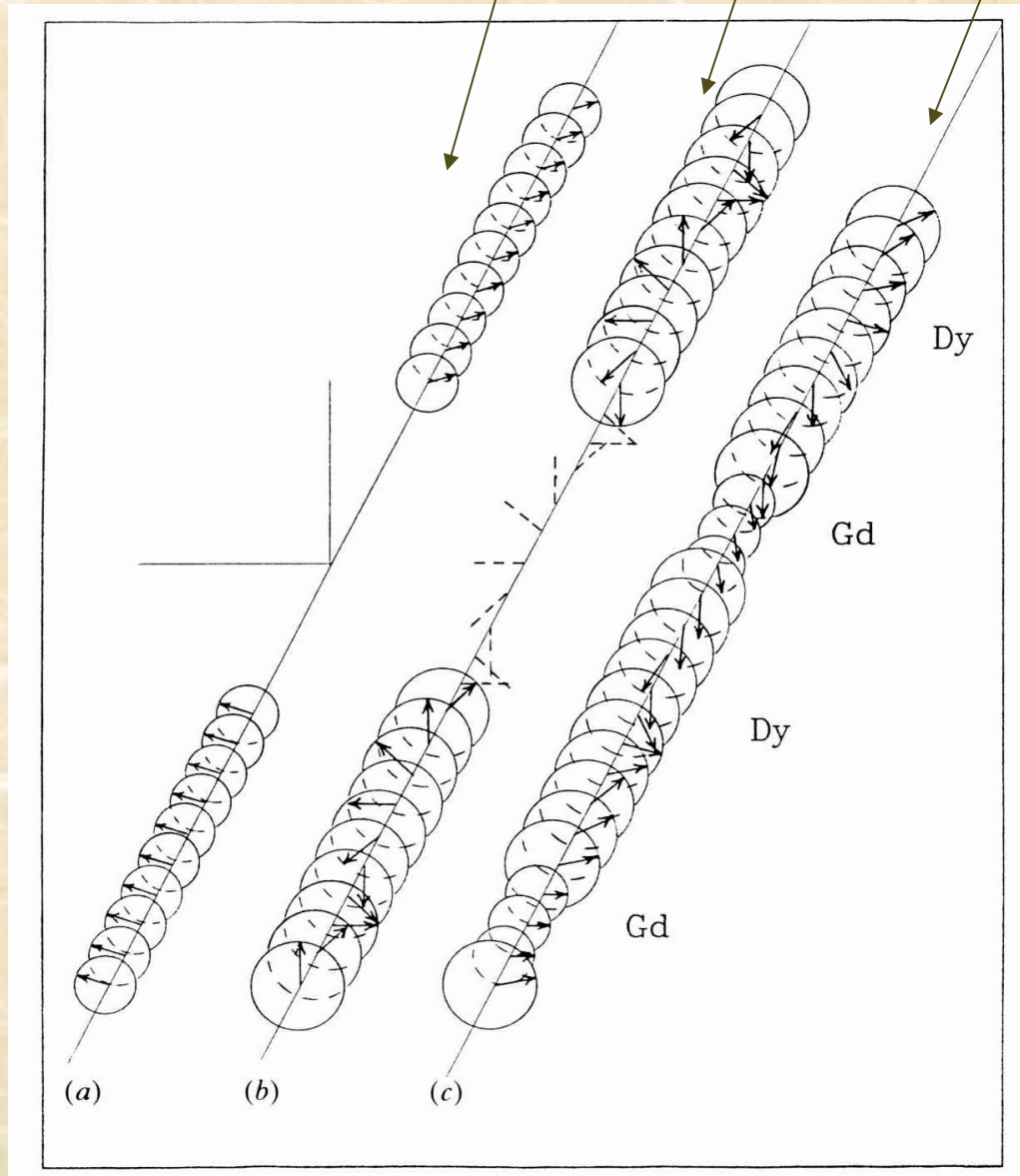
Spin structures of heavy rare - earths

Neutron Diffraction Studies of the $\text{Gd}_5\text{-Y}_{10}$ Magnetic Superlattice – Antiferromagnetically coupled



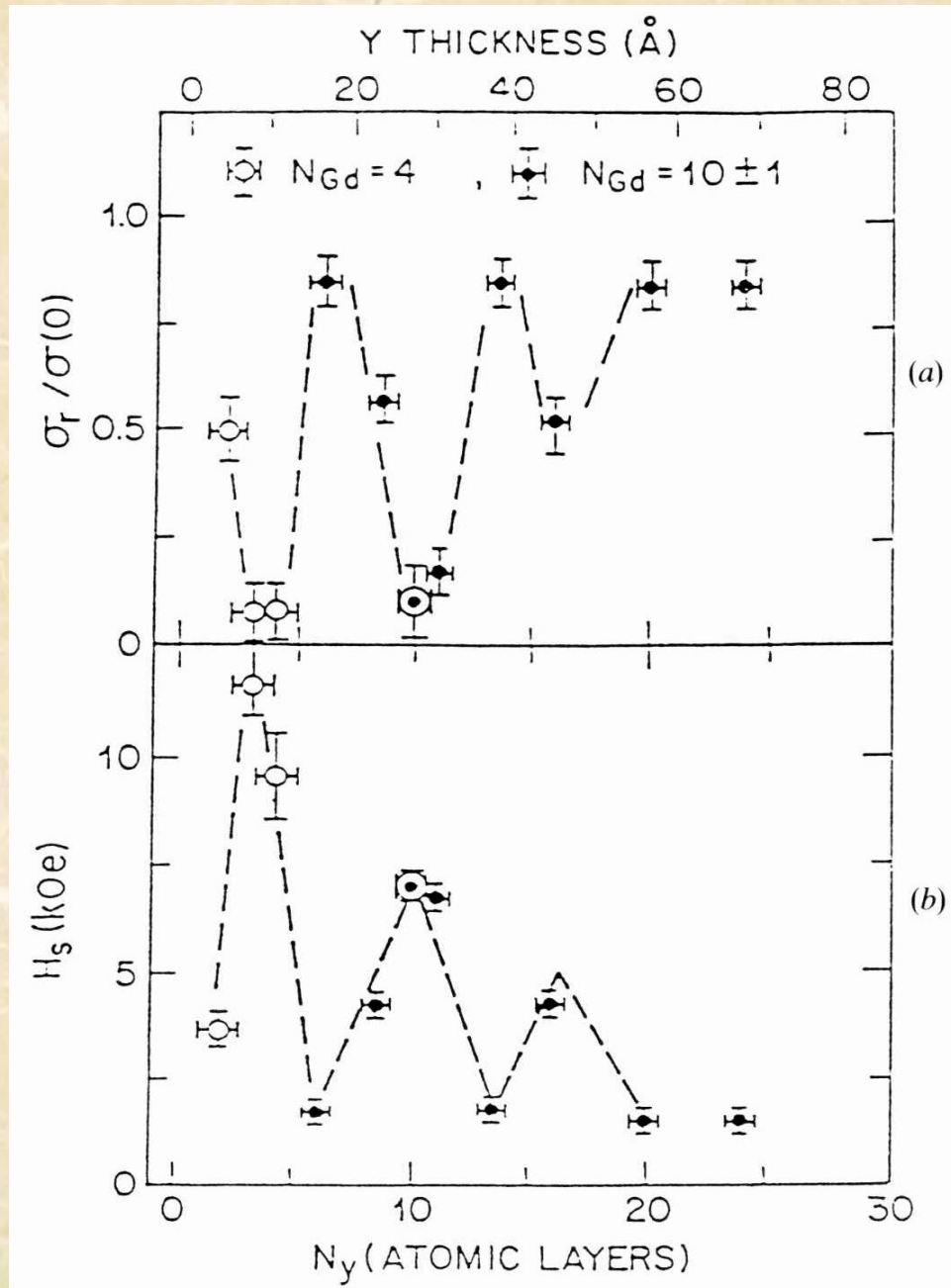
Spin Structure Tailoring in artificial Superlattices

Gd-Y Dy-Y Gd-Dy



1984-1989

Modulation of Magnetic Properties



Chapter 12: Superconductivity

References:

1. C. Kittel ; Introduction to solid state physics
2. M. Tinkham: Introduction to superconductivity
3. J. R. Schrieffer: Superconductivity

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SUMMARY

Table 1 Superconductivity parameters of the elements

An asterisk denotes an element superconducting only in thin films or under high pressure in a crystal modification not normally stable. Data courtesy of B. T. Matthias, revised by T. Geballe.

[illegible]

***Experimental Survey of
Superconductivity Phenomenon***

WHAT IS A SUPERCONDUCTOR?

- 1. Zero resistance**
- 2. Complete expulsion of magnetic flux**

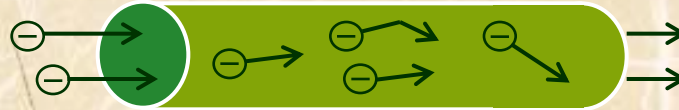
SUPERCONDUCTIVITY

Type of material

What happens in a wire?

Result

Conductor



Electrons flow easily
(like water through a
garden hose)

Collisions cause
dissipation (heat)

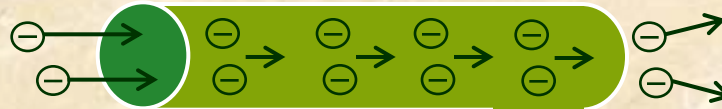
Insulator



Electrons are tightly bound no flow
(like a hose plugged with cement)

No current flow
at all

Superconductor



Electrons bind into pairs and
cannot collide
(a frictionless hose)

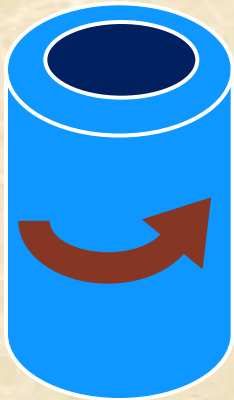
No collisions
No dissipation
No heat
No resistance

HOW SMALL IS THE RESISTANCE?



Copper Cylinder

- 1) Induce current
- 2) Current decays in about 1/1000 second



Superconducting Cylinder

- 1) Induce current
- 2) Current does not decay
(less than 0.1% in a year)
so, resistance is smaller than copper
by $\frac{1000 \text{ years}}{1/1000 \text{ second}}$
i.e., at least 1 trillion times!

Why Superconductivity is so fascinating ?

- ❖ Fundamental SC mechanism
- ❖ Novel collective phenomenon at low temp
- ❖ Applications

Bulk: - Persistent current, power storage
- Magnetic levitation
- High field magnet, MRI

Electronics:
- SQUID magnetometer
- Josephson junction electronics

POSSIBLE IMPACT OF SUPERCONDUCTIVITY

● Energy

- Superconductivity generators & motors
- Power transmission & distribution
- Energy storage systems
- Magnets for fusion power
- Magnets for magneto-hydrodynamic power

● Transportation

- Magnets for levitated trains
- Electro-magnetic powered ships
- Magnets for automobiles

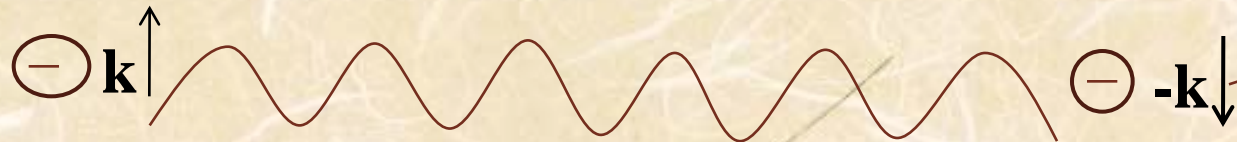
● Health care

- Magnetic resonance imaging

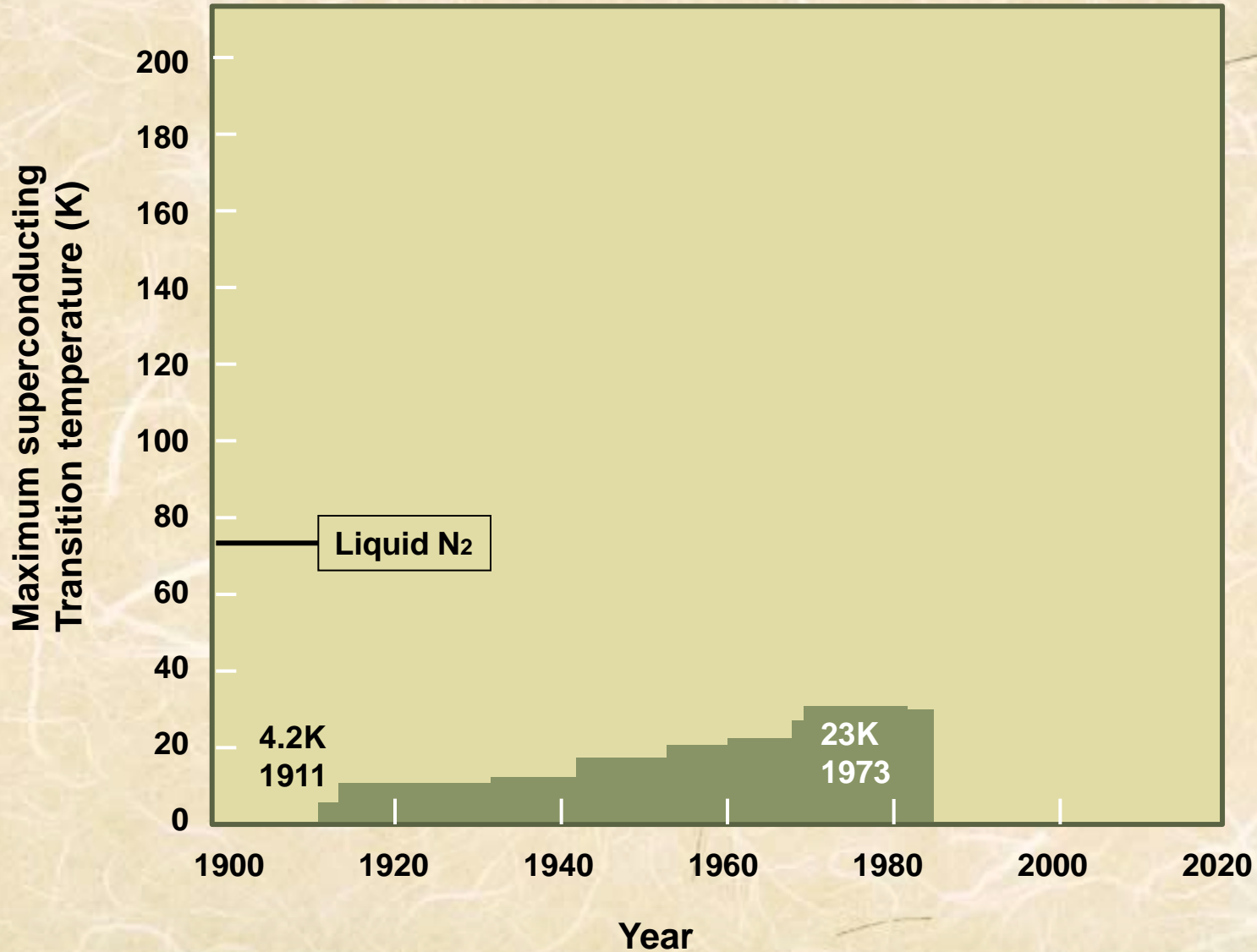
Low- T_c Superconductivity Mechanism



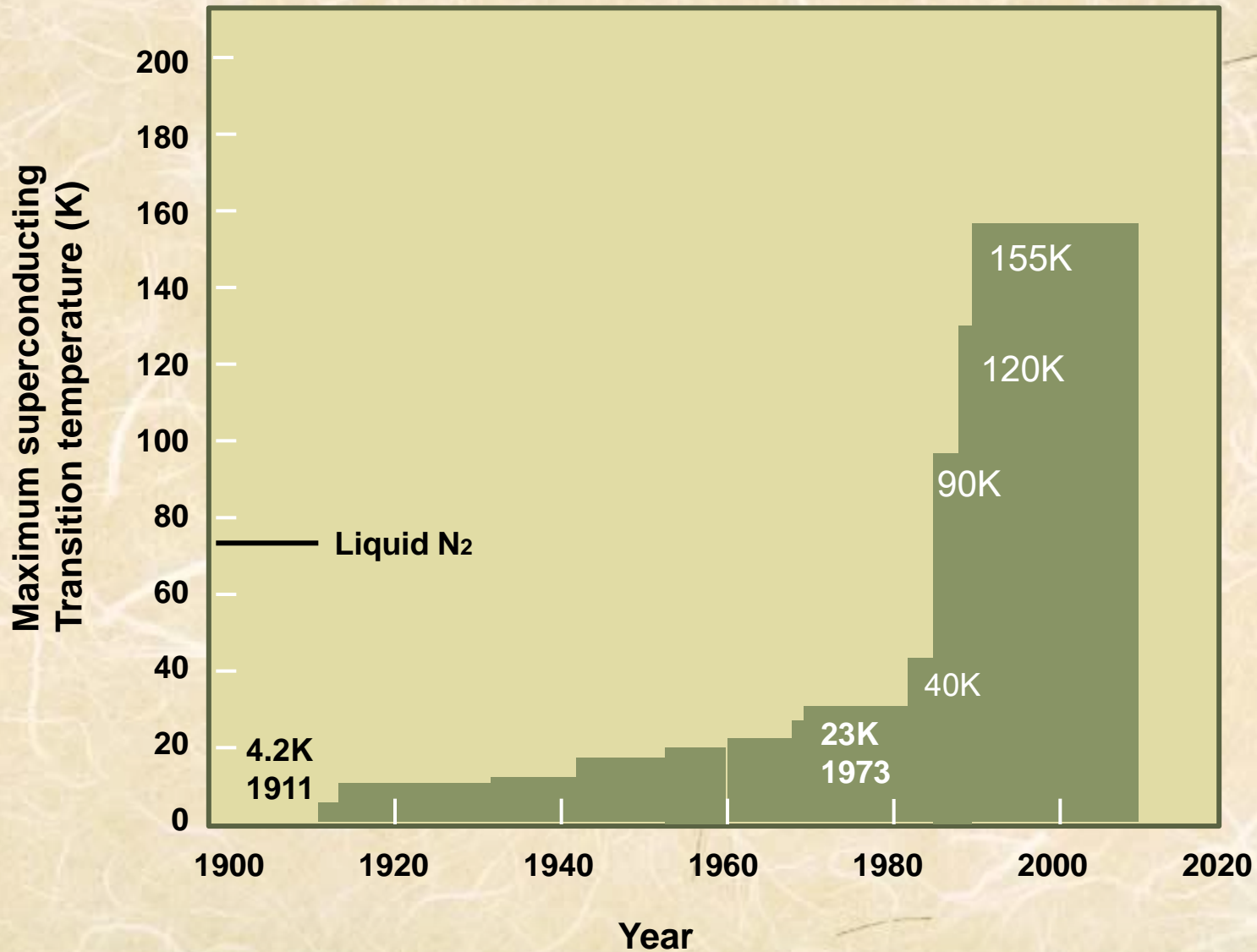
Electron phonon coupling



PROGRESS IN SUPERCONDUCTIVITY



PROGRESS IN SUPERCONDUCTIVITY



Low temperature Superconductors

-- Mediated by electron phonon coupling

-- the critical temperature T_c in the strong electron-phonon coupling limit

$$T_c = \frac{\Theta_D}{1.45} \exp \left\{ - \left[\frac{(1 + \lambda_{ep})}{\lambda_{ep} - \mu^*(1 + 0.62\lambda_{ep})} \right] \right\}$$

λ : electron phonon coupling constant

μ^* : Coulomb repulsion of electrons

$\lambda \propto N(0) \langle I^2 \rangle / M \langle \omega^2 \rangle$

Experimental Electrical Resistivity of Metals

The electrical resistivity of most metals is dominated at room temperature (300 K) by collisions of the conduction electrons with lattice phonons and at liquid helium temperature (4 K) by collisions with impurity atoms and mechanical imperfections in the lattice (Fig. 11).

$$\frac{1}{\tau} = \frac{1}{\tau_L} + \frac{1}{\tau_i},$$

Lattice phonons Imperfections

To a good approximation the rates are often independent.
And can be summed together

where τ_L and τ_i are the collision times for scattering by phonons and by imperfections, respectively.

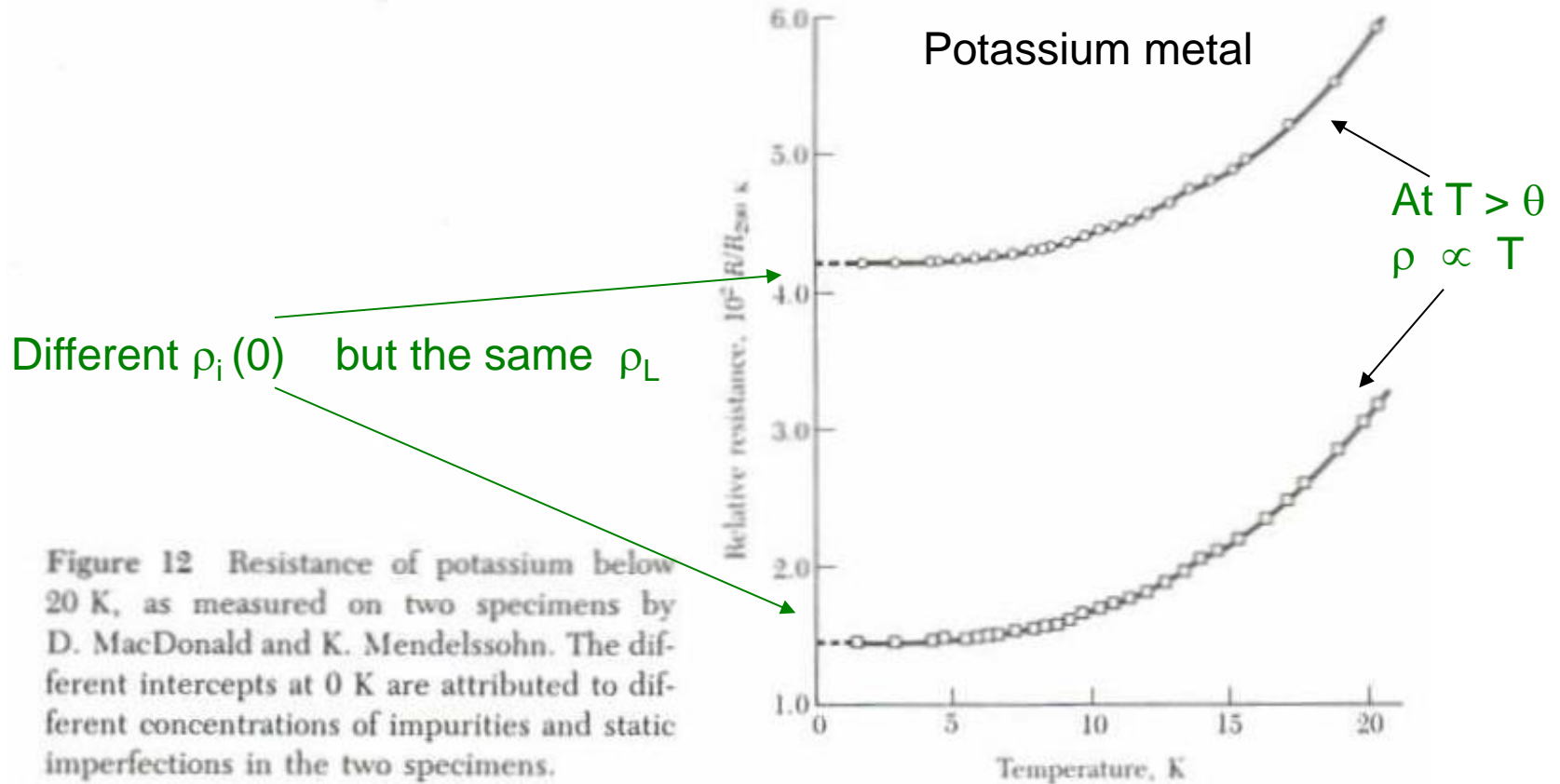
The net resistivity is given by

$$\rho = \rho_L + \rho_i,$$

$$\text{Since } \rho \sim 1/\tau \quad (47)$$

Often ρ_L is independent of the number of defects when their concentration is small, and often ρ_i is independent of temperature.

This empirical observation expresses **Matthiessen's Rule.**



The temperature-dependent part of the electrical resistivity is proportional to the rate at which an electron collides with thermal phonons

One simple limit is at temperatures over the Debye temperature θ :

here the phonon concentration is proportional to the temperature T , so that $\rho \propto T$ for $T > \theta$.

$$N_{ph} \propto T \quad \text{hence} \quad \rho \propto 1/\tau \propto N_{ph} \propto T$$

The electrical resistivity of many metals and alloys drops suddenly to zero when the specimen is cooled to a sufficiently low temperature, often a temperature in the liquid helium range. This phenomenon, called superconductivity, was observed first by Kamerlingh Onnes¹ in Leiden in 1911, three years after he first liquified helium.

First SC found in Hg by 1911 !

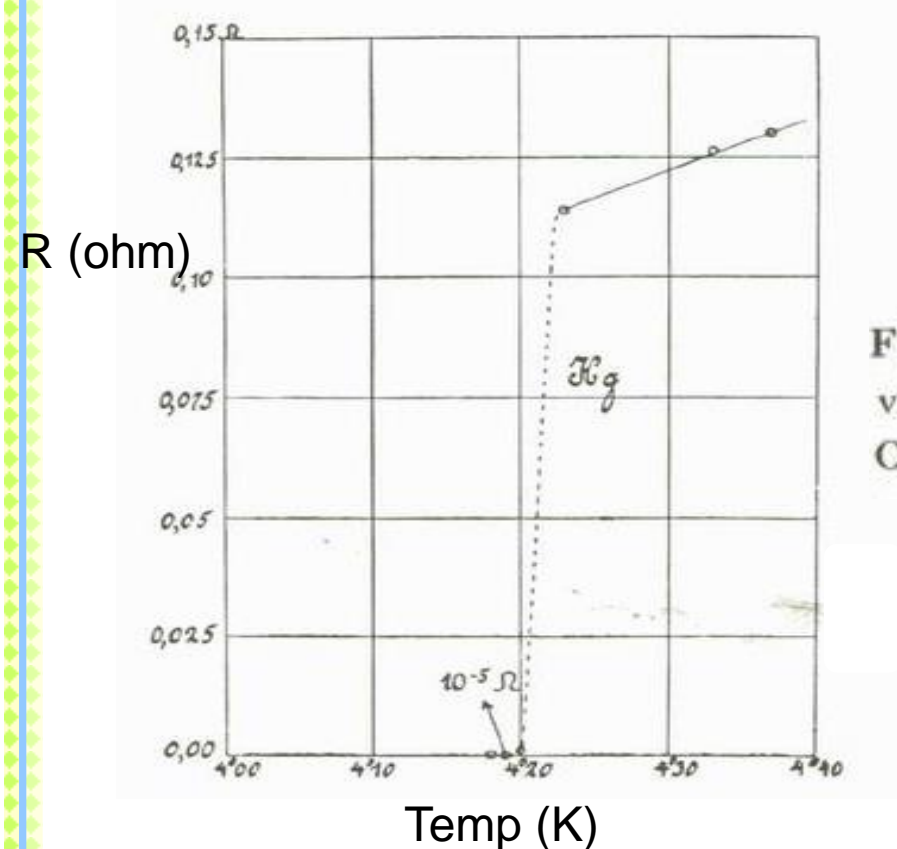


Figure 1 Resistance in ohms of a specimen of mercury versus absolute temperature. This plot by Kamerlingh Onnes marked the discovery of superconductivity.

Decay of persistent current from 1 year up to 10^5 year

Meissner Effect

It is an experimental fact that a bulk superconductor in a weak magnetic field will act as a perfect diamagnet, with zero magnetic induction in the interior. When a specimen is placed in a magnetic field and is then cooled through the transition temperature for superconductivity, the magnetic flux originally present is ejected from the specimen. This is called the Meissner effect.

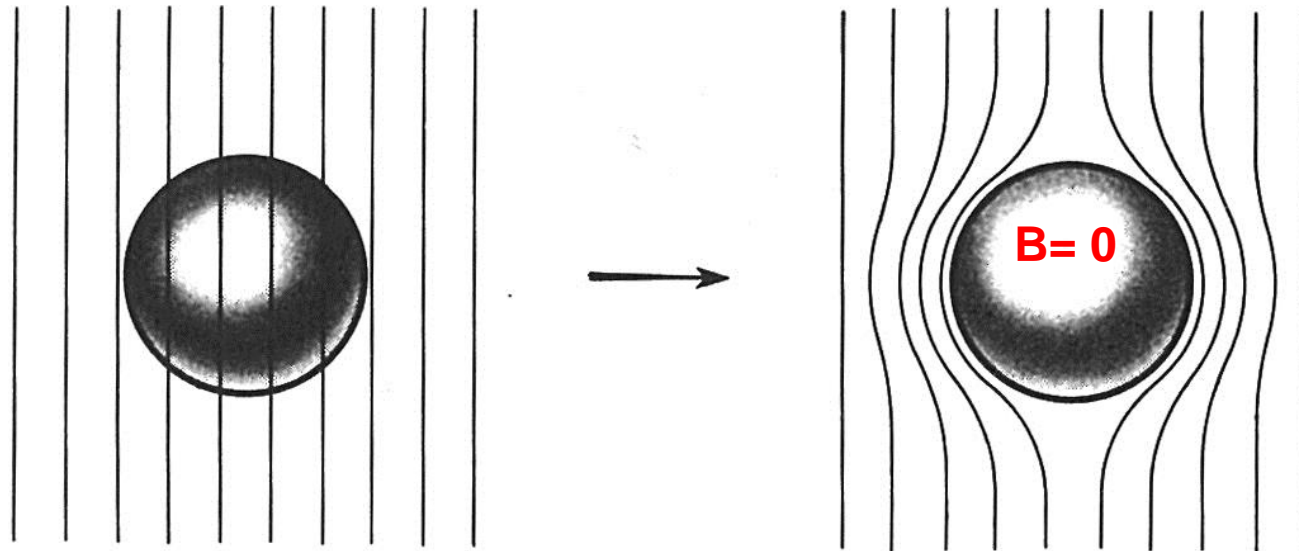


Figure 2 Meissner effect in a superconducting sphere cooled in a constant applied magnetic field; on passing below the transition temperature the lines of induction \mathbf{B} are ejected from the sphere.

Fundamental Mechanism

The superconducting state is an ordered state of the conduction electrons of the metal.

Electron-Phonon Coupling

Cooper Pair formed by two electrons k and $-k$ with opposite spins, as coupled through **phonons** of the lattice

The nature and origin of the ordering was explained by Bardeen, Cooper, and Schrieffer.³

BCS Theory, 1957

³J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. 106, 162 (1957); 108, 1175 (1957).

The Discovery of Superconductivity

- Early 90's -- elemental SP metals like Hg, Pb, Al, Sn, Ga, etc.
- Middle 90's -- transitional metals, alloys, and compounds like Nb, NbN, Nb₃Sn, etc.
- Late 90's -- in perovskite oxides

Table 2 Superconductivity of selected compounds

Compound	T_c , in K	Compound	T_c , in K
Nb ₃ Sn	18.05	V ₃ Ga	16.5
Nb ₃ Ge	23.2	V ₃ Si	17.1
Nb ₃ Al	17.5	YBa ₂ Cu ₃ O _{6.9}	90.0
NbN	16.0	Rb ₂ CsC ₆₀	31.3
K ₃ C ₆ O	19.2	La ₃ In	10.4

A-15

B1

HTSC

A-15 compound A_3B , with $T_c = 15-23$ K

With three perpendicular linear chains of **A** atoms on the cubic face, and B atoms are at body centered cubic site

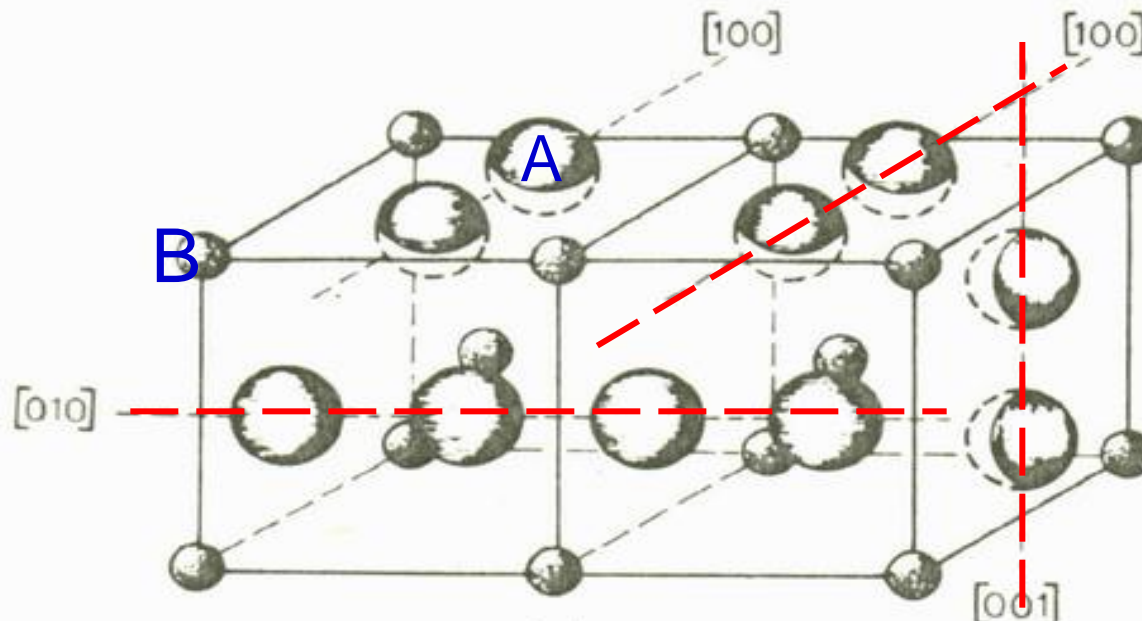


FIG. 34. (a) The position of A and B atoms in the unit cell of an A_3B compound possessing the β -W structure. (b) The fermi surface of an A_3B compound in the tight binding, nearest neighbors approximation. There are three degenerate bands corresponding to electrons localized on the three families of chains. (c) The first Brillouin zone (BZ) of the β -W lattice. The high symmetry points (Γ , X, M, R) and the high symmetry lines (Δ , Σ , A , Z , S , T) are indicated.

1973 Nb_3Ge , 23K !

Low temperature Superconductors

- Mediated by Electron phonon coupling
- McMillian formula for T_c

$$T_c = \frac{\Theta_D}{1.45} \exp \left\{ - \left[\frac{(1 + \lambda_{ep})}{\lambda_{ep} - \mu^*(1 + 0.62\lambda_{ep})} \right] \right\}$$

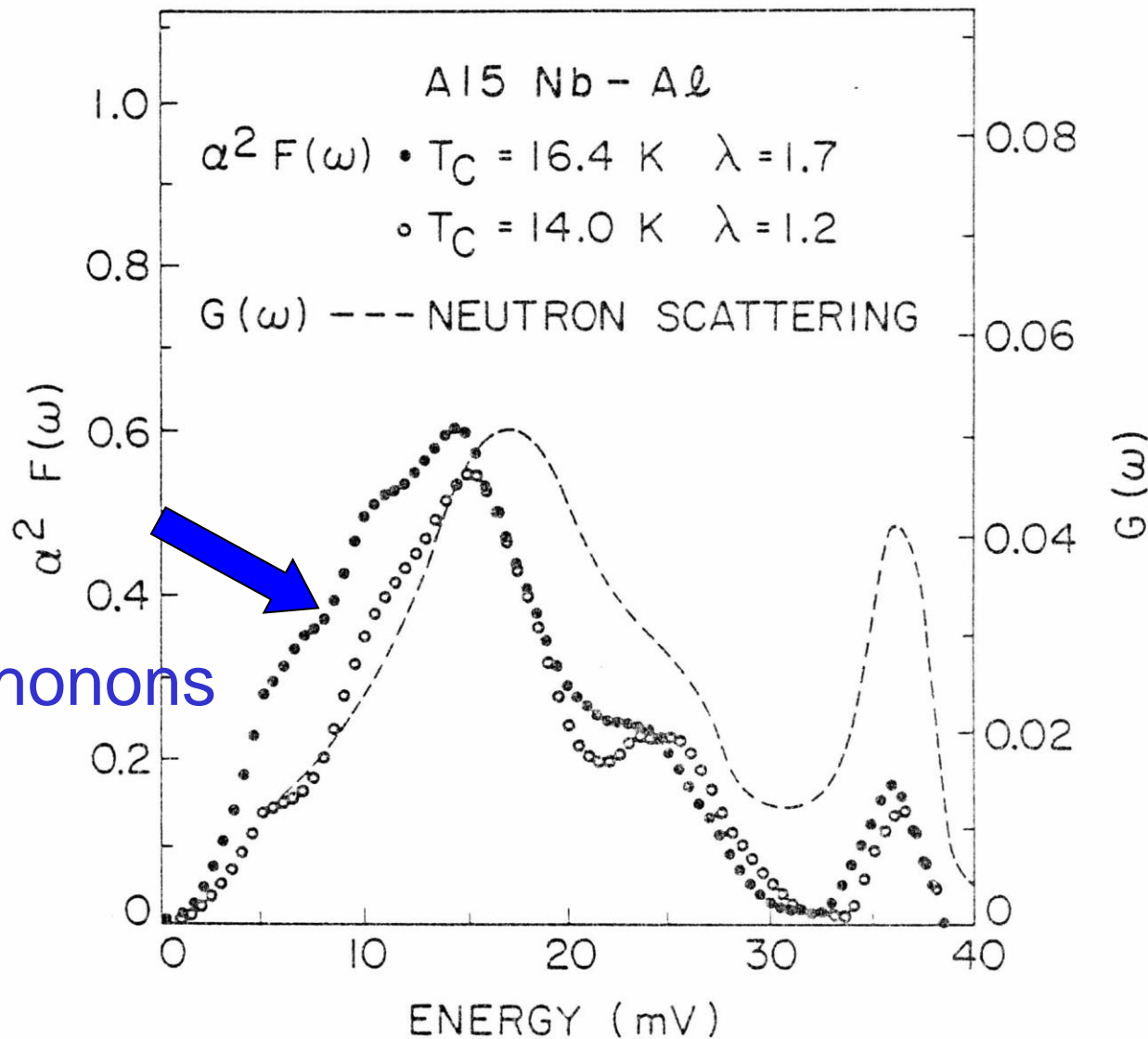
λ : electron phonon coupling constant

μ^* : Coulomb repulsion of electrons

$$\lambda \propto N(0) \langle I^2 \rangle / \omega^2$$

Are electrons or phonons more important?

The Phonon Spectrum of the low T_c A-15 compound Nb_3Al



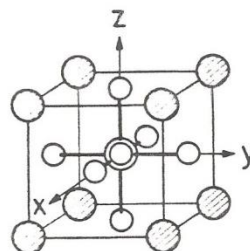
*Can we raise the T_c higher
than 30K?*

*Are we reaching the limitation
of the BCS Theory ?*

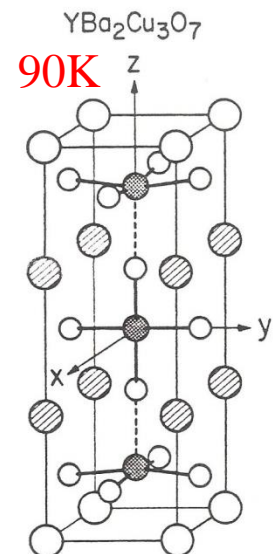
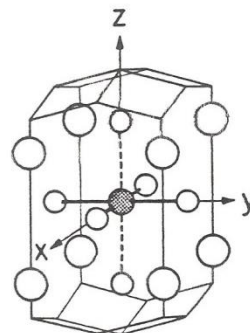
*Breakthrough in late 1986
By Bednorz and Muller*

Start the HTSC Era !

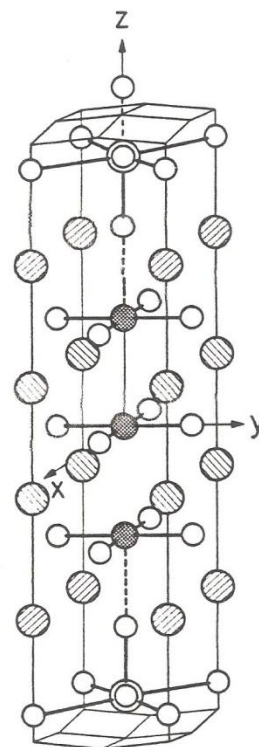
BaBiO_3
30K



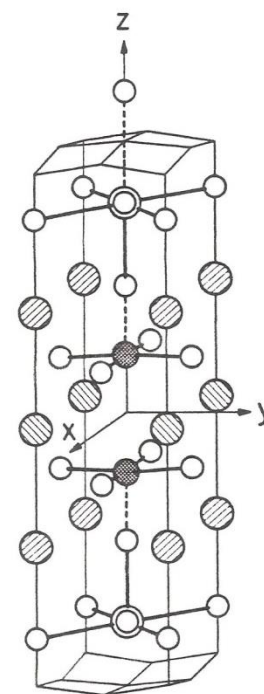
La_2CuO_4
40K



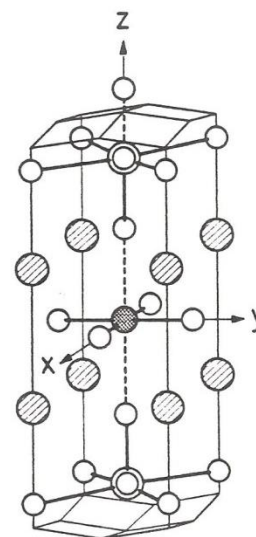
- La, Y
- Ba, Sr
- ▨ Ca
- ⊙ Bi, Tl
- Cu
- O



$\text{Ca}_2\text{Ba}_2\text{Tl}_2\text{Cu}_3\text{O}_{10}$
120K

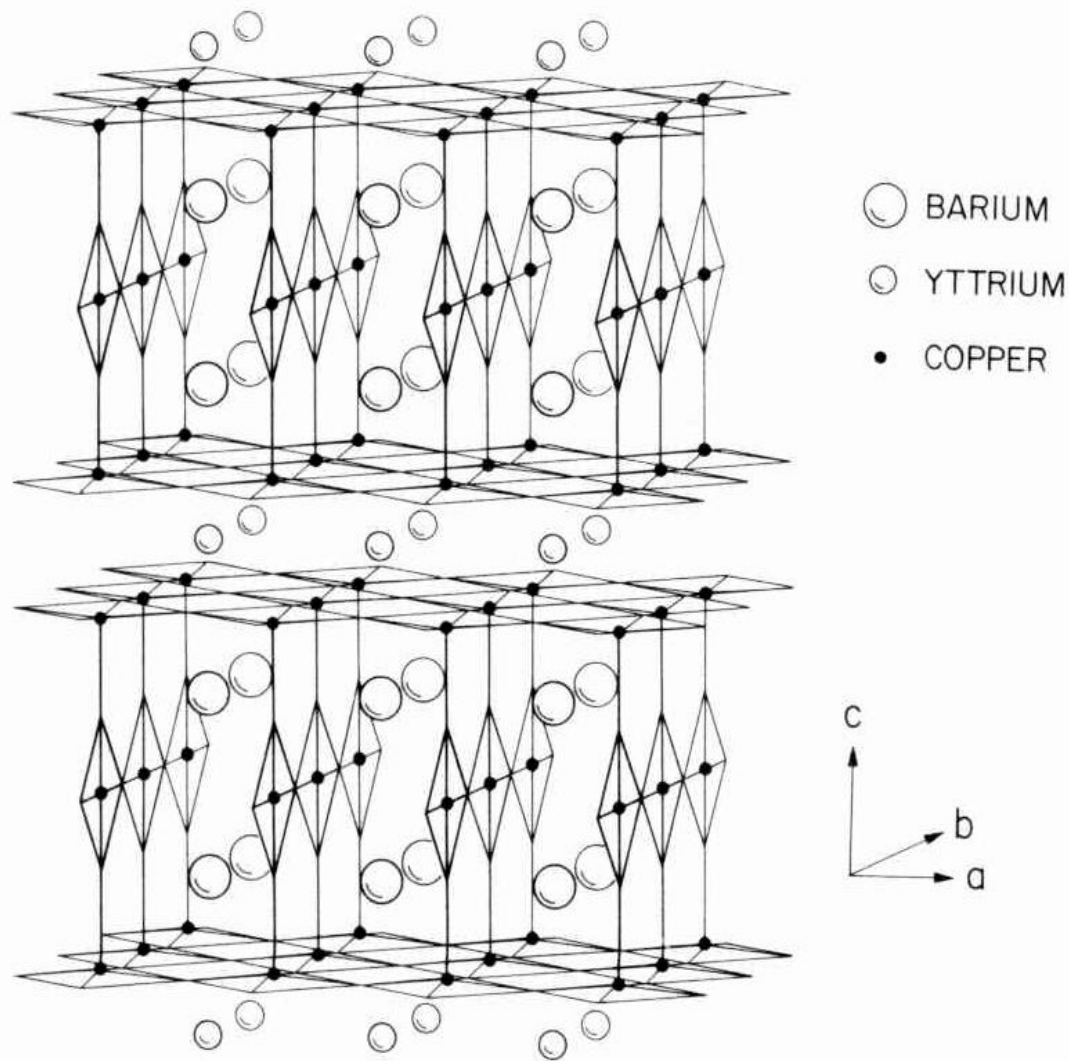


$\text{CaSr}_2\text{Bi}_2\text{Cu}_2\text{O}_8$
 $\text{CaBa}_2\text{Tl}_2\text{Cu}_2\text{O}_8$
80K



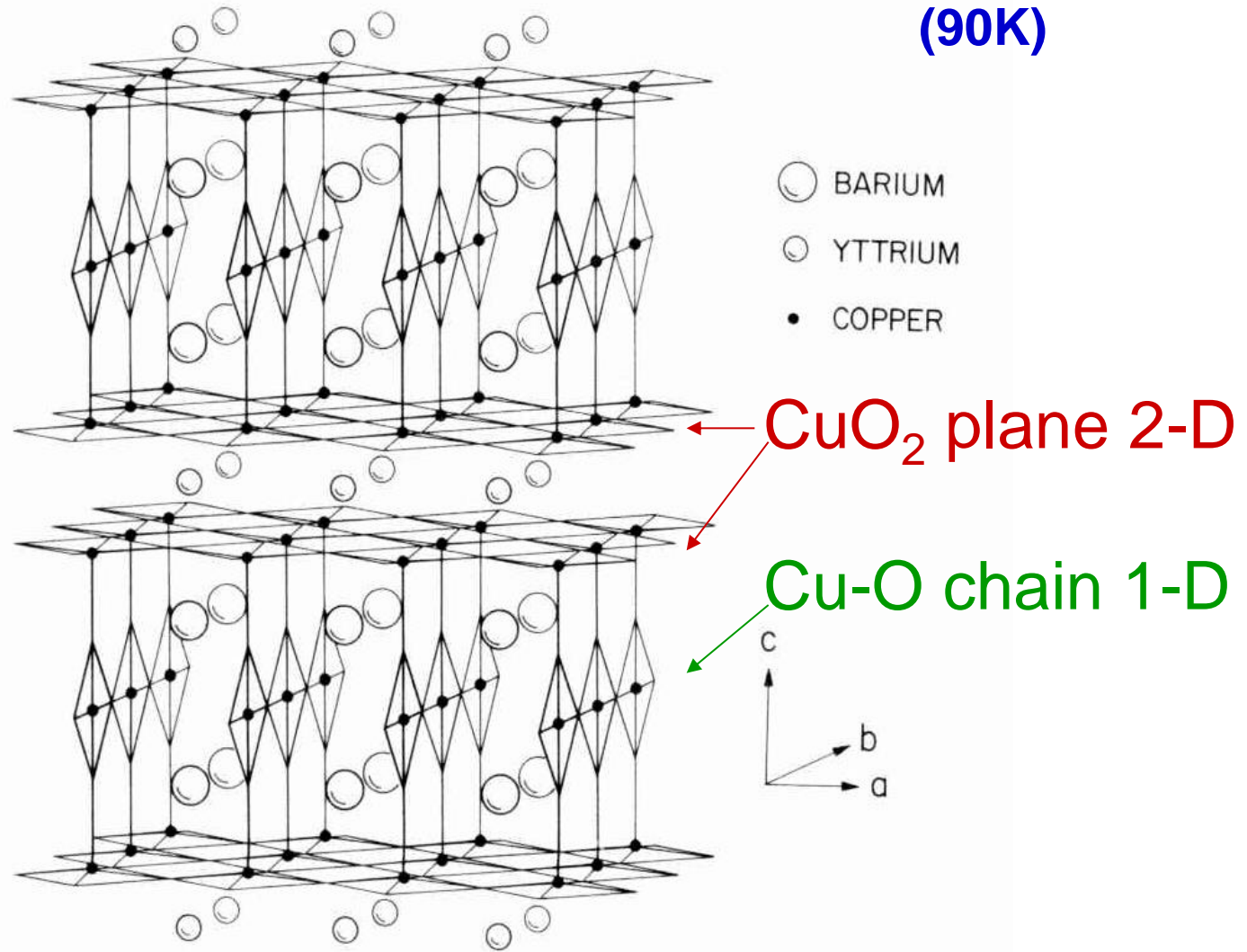
$\text{Sr}_2\text{Bi}_2\text{CuO}_6$
 $\text{Ba}_2\text{Tl}_2\text{CuO}_6$
40K

High Temperature Superconductor $\text{YBa}_2\text{Cu}_3\text{O}_7$



**Invention of Oxide Molecular Beam Epitaxy in 1988
For HTSC Single Crystal Films.**

High Temperature Superconductor $\text{YBa}_2\text{Cu}_3\text{O}_7$ (90K)



**Invention of Oxide Molecular Beam Epitaxy
For HTSC Single Crystal Films.**

Will all non magnetic metal become SC at low T?

(I) Destruction of Superconductivity by Magnetic Impurities

It is important to eliminate from the specimen even trace quantities of foreign paramagnetic elements

(II) Destruction of Superconductivity by Magnetic fields

At the critical temperature the critical field is zero: $H_c(T_c)=0$

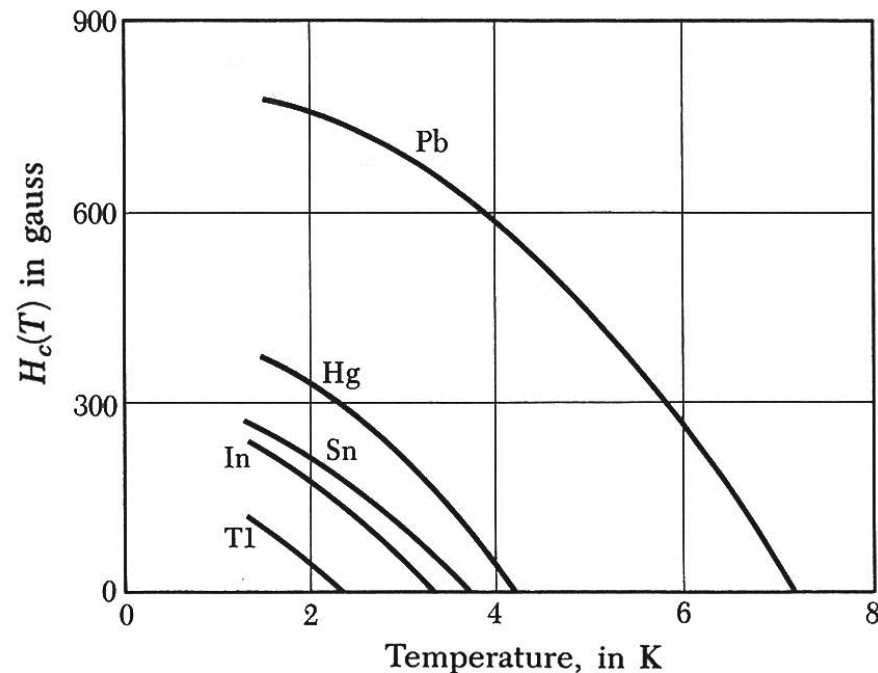


Figure 3 Experimental threshold curves of the critical field $H_c(T)$ versus temperature for several superconductors. A specimen is superconducting below the curve and normal above the curve.

Meissner Effect

It is an experimental fact that a bulk superconductor in a weak magnetic field will act as a perfect diamagnet, with zero magnetic induction in the interior. When a specimen is placed in a magnetic field and is then cooled through the transition temperature for superconductivity, the magnetic flux originally present is ejected from the specimen. This is called the Meissner effect.

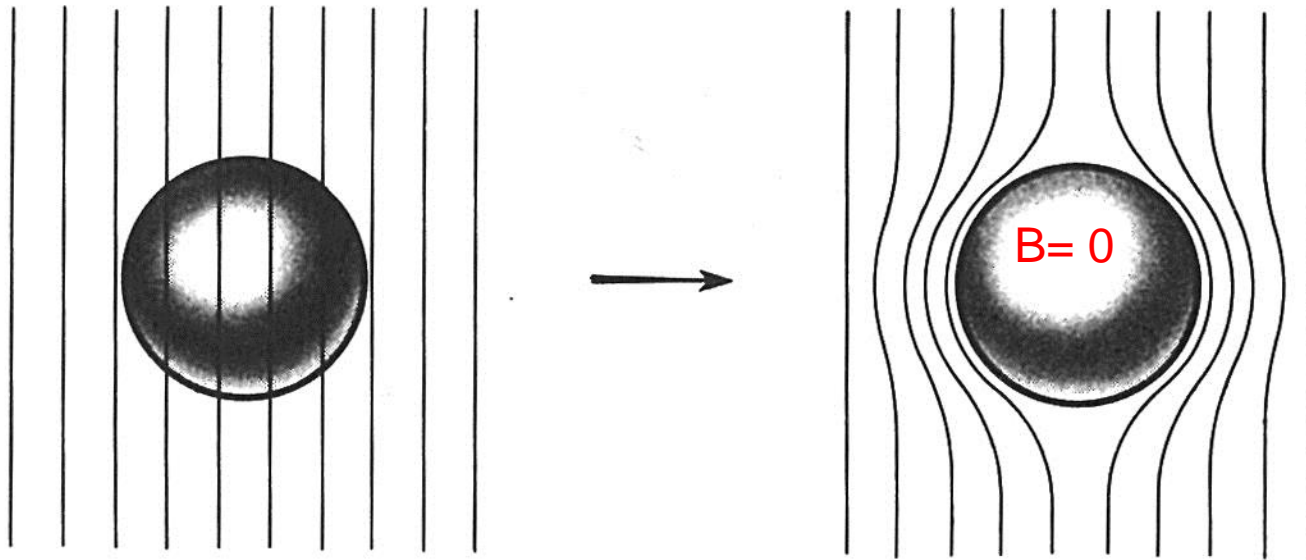


Figure 2 Meissner effect in a superconducting sphere cooled in a constant applied magnetic field; on passing below the transition temperature the lines of induction \mathbf{B} are ejected from the sphere.

Meissner Effect

$$B = B_a + 4\pi M = 0 ;$$

$$\frac{M}{B_a} = -\frac{1}{4\pi}$$

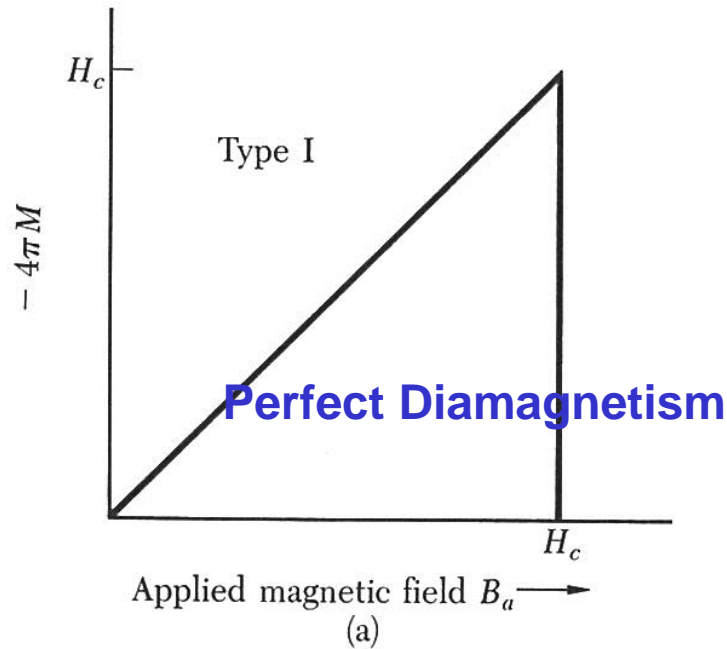
Eq.(1)

Perfect Diamagnetism

The magnetic properties cannot be accounted for by the assumption that a superconductor is a normal conductor with zero electrical resistivity.

The result $B = 0$ cannot be derived from the characterization of a superconductor as a medium of zero resistivity. From Ohm's law, $\mathbf{E} = \rho \mathbf{j}$, we see that if the resistivity ρ goes to zero while \mathbf{j} is held finite, then \mathbf{E} must be zero. By a Maxwell equation $d\mathbf{B}/dt$ is proportional to curl \mathbf{E} , so that zero resistivity implies $d\mathbf{B}/dt = 0$. This argument is not entirely transparent, but the result predicts that the flux through the metal cannot change on cooling through the transition. The Meissner effect contradicts this result and suggests that perfect diamagnetism is an essential property of the superconducting state.

Type I superconductor



Type II superconductor

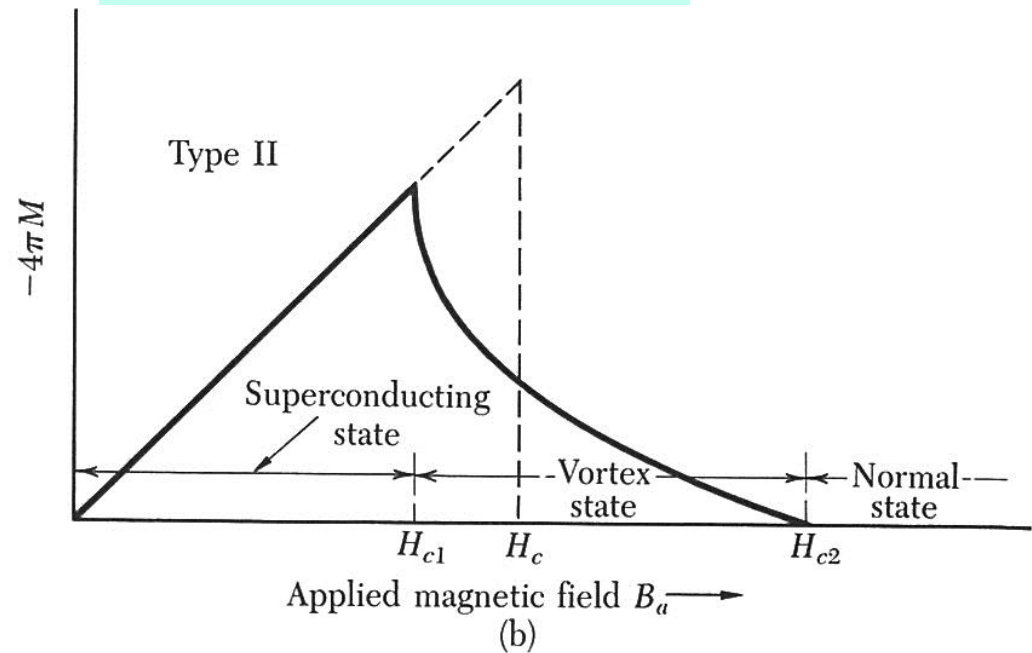


Figure 4 (a) Magnetization versus applied magnetic field for a bulk superconductor exhibiting a complete Meissner effect (perfect diamagnetism). A superconductor with this behavior is called a type I superconductor. Above the critical field H_c the specimen is a normal conductor and the magnetization is too small to be seen on this scale. Note that minus $4\pi M$ is plotted on the vertical scale: the negative value of M corresponds to diamagnetism. (b) Superconducting magnetization curve of a type II superconductor. The flux starts to penetrate the specimen at a field H_{c1} lower than the thermodynamic critical field H_c . The specimen is in a vortex state between H_{c1} and H_{c2} , and it has superconducting electrical properties up to H_{c2} . Above H_{c2} the specimen is a normal conductor in every respect, except for possible surface effects. For given H_c the area under the magnetization curve is the same for a type II superconductor as for a type I. (CGS units in all parts of this figure.)

Type II superconductors

- (1) They tend to be alloys (as in Fig. 5a) or transition metals with high values of the electrical resistivity in the normal state: that is, the electronic mean free path in the normal state is short.
- (2) Type II superconductors have superconducting electrical properties up to a field denoted by H_{c2} .
- (3) Between the lower critical field H_{c1} and the upper critical field H_{c2} the flux density $B \neq 0$ and the Meissner effect is said to be incomplete.
- (4) In the region between H_{c1} and H_{c2} the superconductor is threaded by flux lines and is said to be in the vortex state.

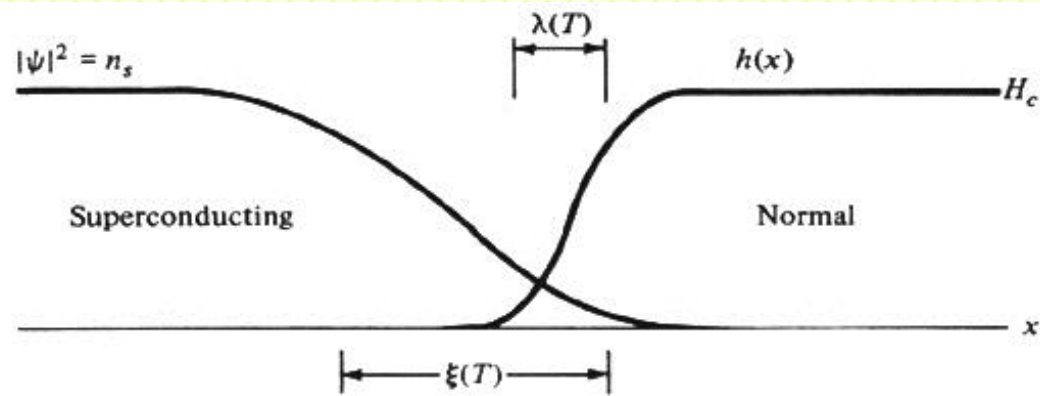


FIGURE 1-4
Interface between superconducting and normal domains in the intermediate state.

$$\kappa = \frac{\lambda_{\text{eff}}(T)}{\xi(T)} = \frac{2\sqrt{2}\pi H_c(T)\lambda_{\text{eff}}^2(T)}{\Phi_0}$$

Ginsburg Landau
Parameter

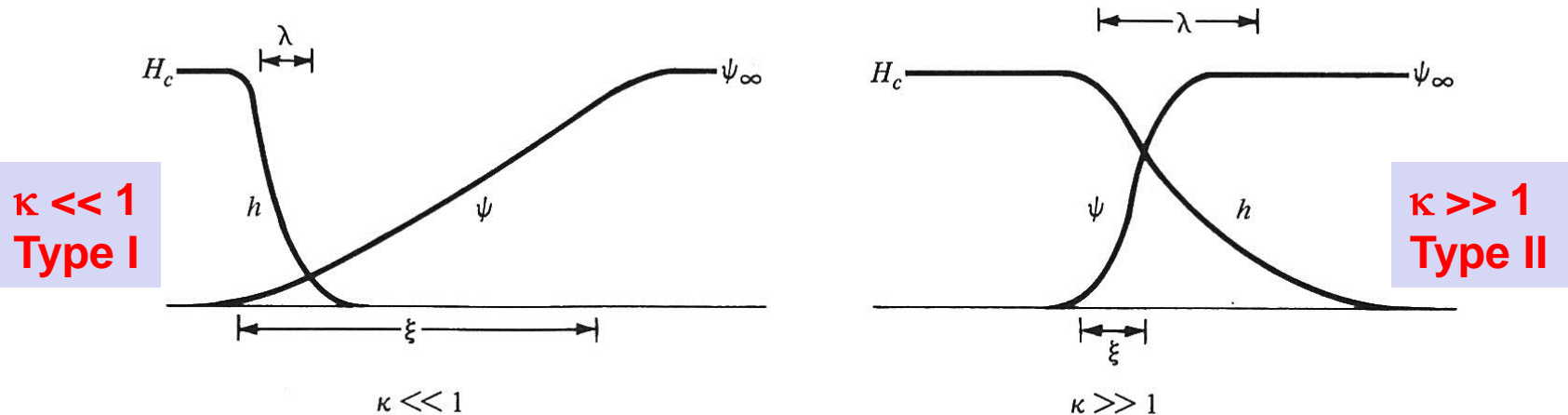


FIGURE 4-2
Schematic diagram of variation of h and ψ in a domain wall. The case $\kappa \ll 1$ refers to a type I superconductor (positive wall energy); the case $\kappa \gg 1$ refers to a type II superconductor (negative wall energy).

Doping in Pb with In

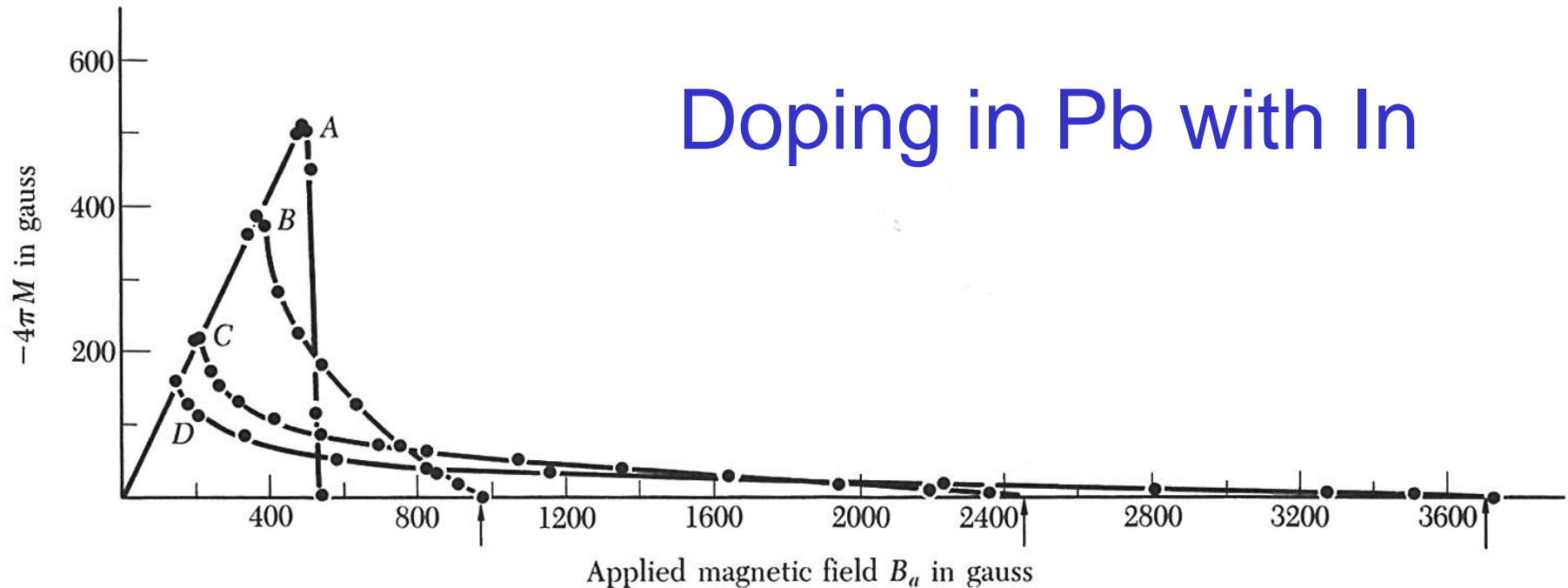


Figure 5a Superconducting magnetization curves of annealed polycrystalline lead and lead-indium alloys at 4.2 K. (A) lead; (B) lead-2.08 wt. percent indium; (C) lead-8.23 wt. percent indium; (D) lead-20.4 wt. percent indium. (After Livingston.)

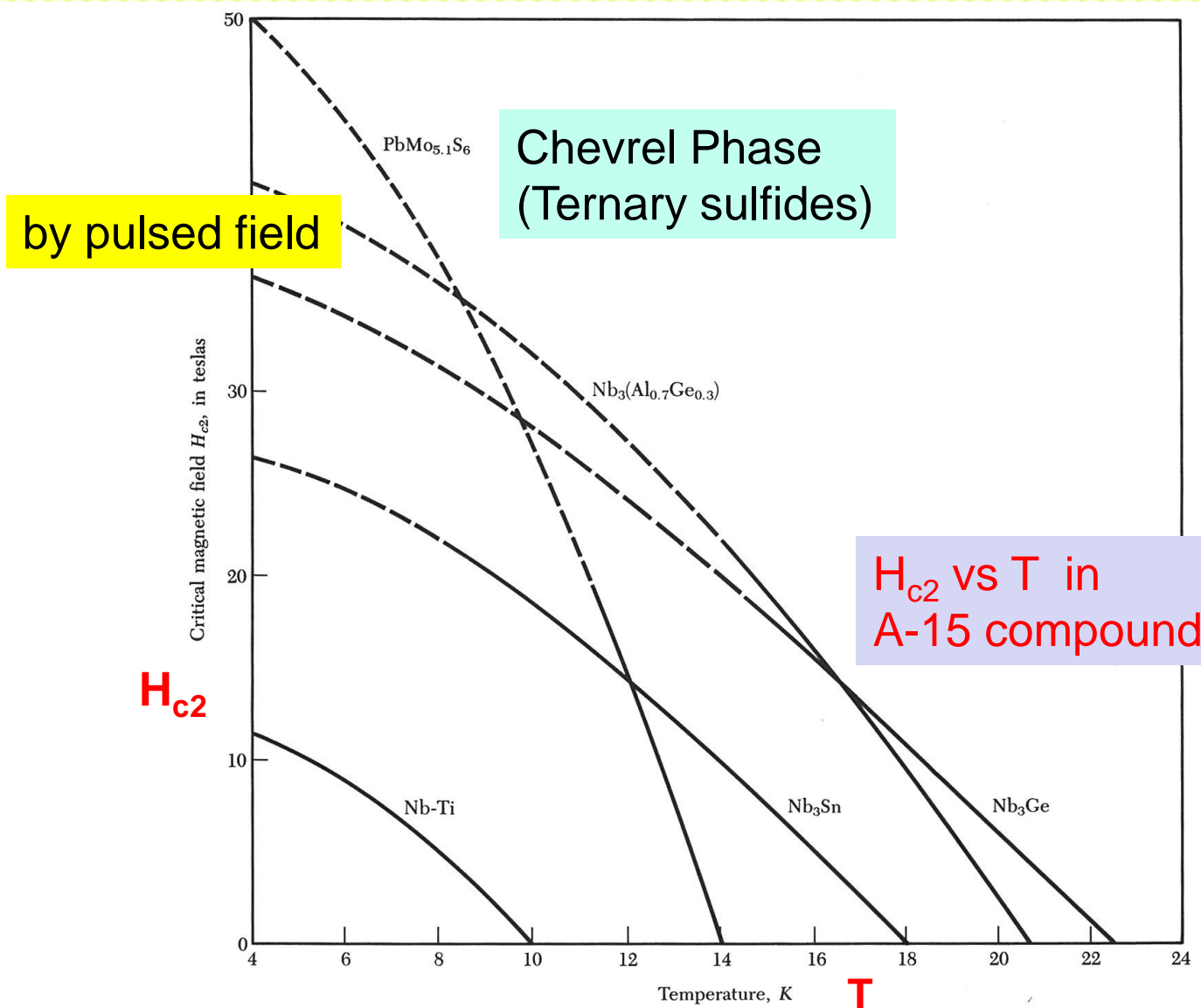


Figure 5b Stronger magnetic fields than any now contemplated in practical superconducting devices are within the capability of certain Type II materials. These materials cannot be exploited, however, until their critical current density can be raised and until they can be fabricated as finely divided conductors. (Magnetic fields of more than about 20 teslas can be generated only in pulses, and so portions of the curves shown as broken lines were measured in that way.)

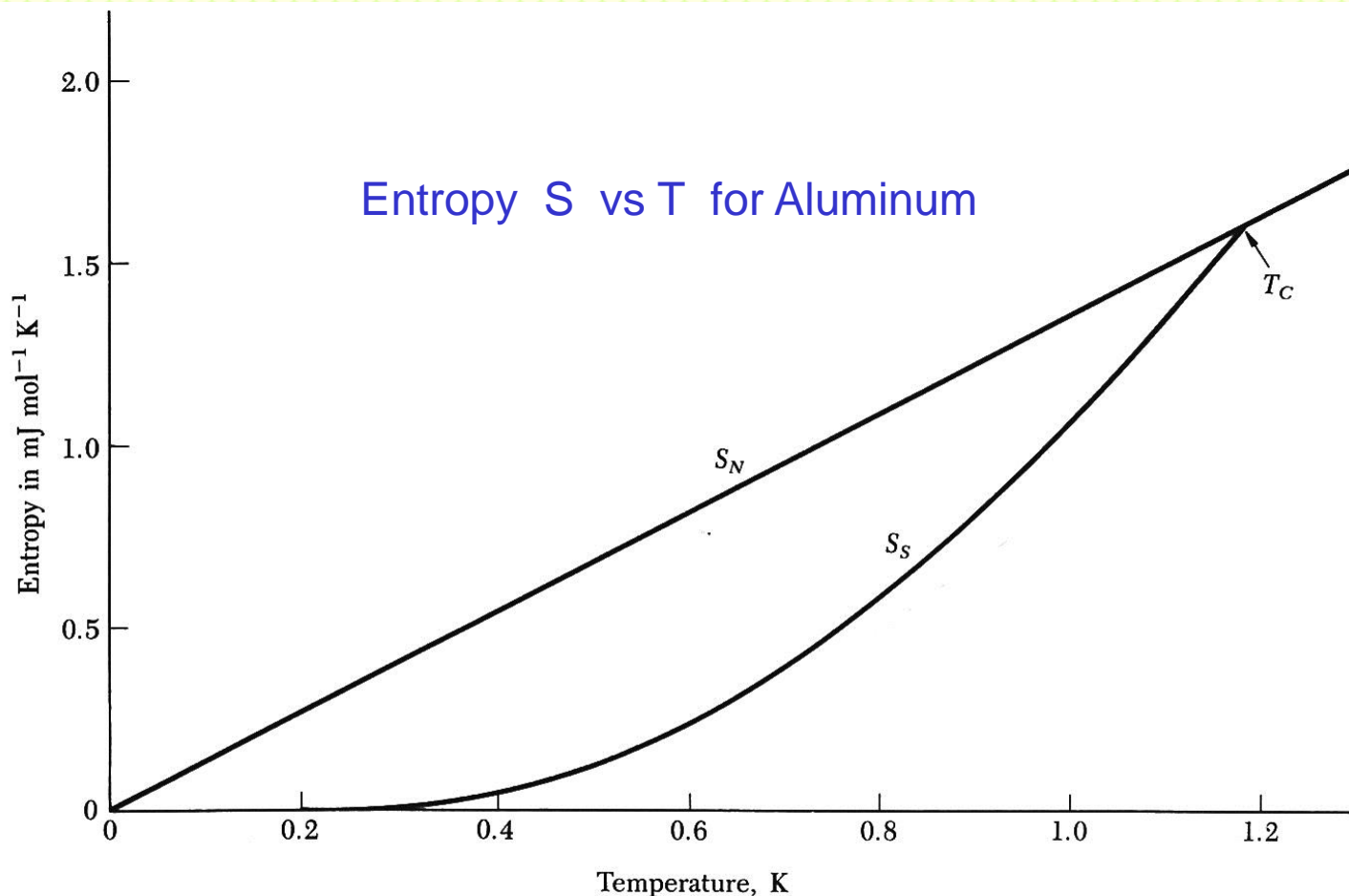
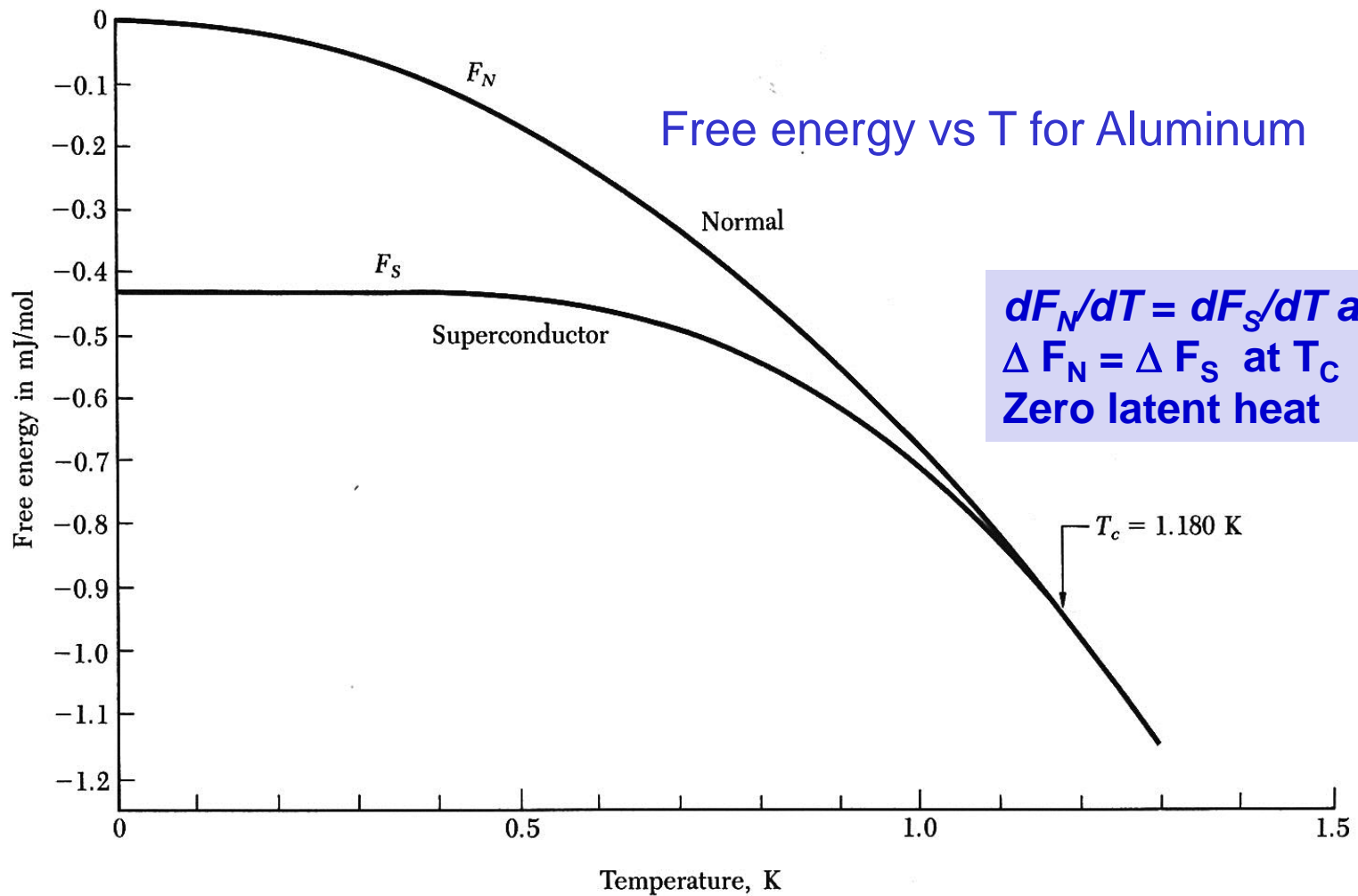


Figure 6 Entropy S of aluminum in the normal and superconducting states as a function of the temperature. The entropy is lower in the superconducting state because the electrons are more ordered here than in the normal state. At any temperature below the critical temperature T_c the specimen can be put in the normal state by application of a magnetic field stronger than the critical field.

The small entropy change must mean that only a small fraction (of the order of 10^{-4}) of the conduction electrons participate in the transition to the ordered superconducting state.



So that the phase transition is second order (there is no latent heat of transition at T_c).

heat capacity of an electron gas is

$$C_{el} = \frac{1}{3} \pi^2 D(\epsilon_F) k_B^2 T \quad (34)$$

$$D(\epsilon_F) = 3N/2\epsilon_F = 3N/2k_B T_F \quad (35)$$

$$C_{el} = \frac{1}{2} \pi^2 N k_B T / T_F$$

$$(36) \quad \text{Compare with } C_V = 2Nk_B T / T_F$$

$$\text{where } \mathcal{E}_F = k_B T_F$$

T_F is called the Fermi temperature,

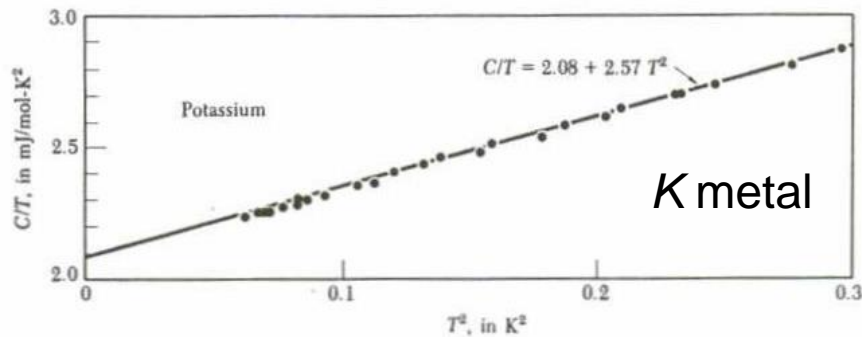


Figure 9 Experimental heat capacity values for potassium, plotted as C/T versus T^2 . (After W. H. Lien and N. E. Phillips.)

$$\gamma = \frac{1}{2} \pi^2 N k_B T / T_F \quad \text{Since } \epsilon_F \propto T_F \propto 1/m \quad \therefore \gamma \propto m \quad (\text{See Eq. 17})$$

At temperatures much below both the Debye temperature and the Fermi temperature, the heat capacity of metals may be written as the sum of electron and phonon contributions: $C = \gamma T + AT^3$

$$C/T = \gamma + AT^2 \quad (37)$$

γ , called the Sommerfeld parameter

At low T , the electronic term dominates,

Heat Capacity of Ga at low T

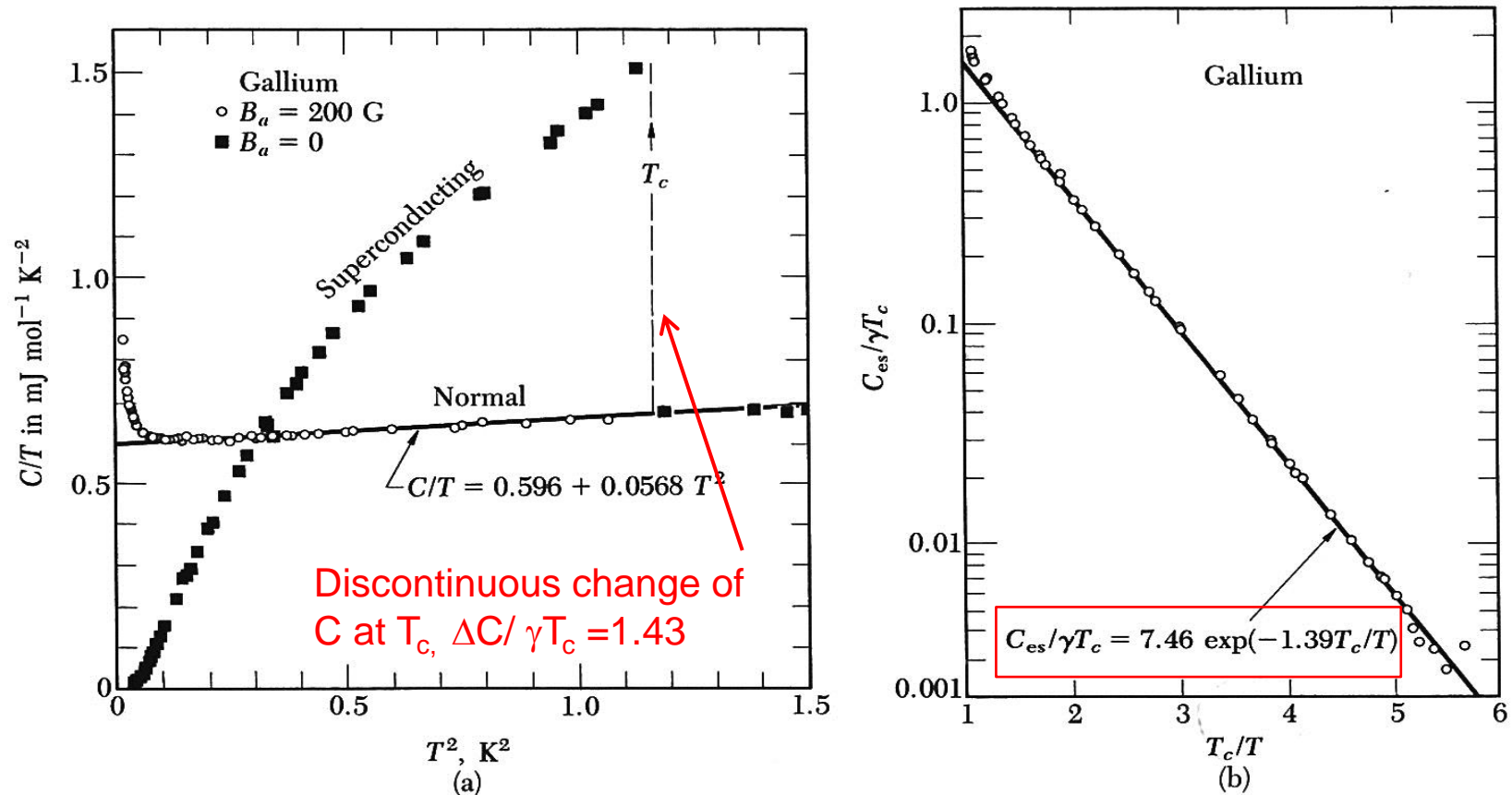


Figure 8 (a) The heat capacity of gallium in the normal and superconducting states. The normal state (which is restored by a 200 G field) has electronic, lattice, and (at low temperatures) nuclear quadrupole contributions. In (b) the electronic part C_{es} of the heat capacity in the superconducting state is plotted on a log scale versus T_c/T : the exponential dependence on $1/T$ is evident. Here $\gamma = 0.60 \text{ mJ mol}^{-1} \text{ deg}^{-2}$. (After N. E. Phillips.)

Electronic part of heat capacity in SC state: $C_{es} / \gamma T_c \propto \exp(-b T_c / T)$

proportional to $-1/T$, suggestive of excitation of electrons across an energy gap

Energy Gap

In a superconductor the important interaction is the electron-electron interaction which orders the electrons in k space with respect to the Fermi gas of electrons.

the exponential factor in the electronic heat capacity of a superconductor is found to be $-E_g/2k_B T$

The transition in zero magnetic field from the superconducting state to the normal state is observed to be a second-order phase transition.

Energy Gap of superconductors in Table 3

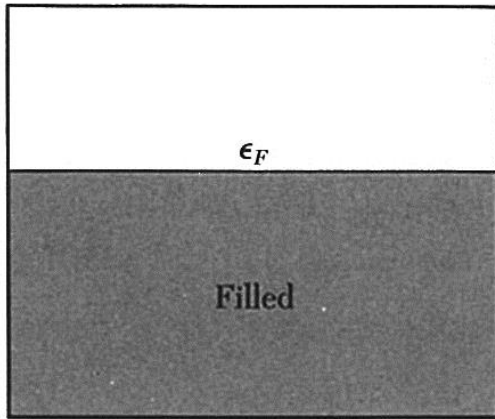
$E_g(0)/k_B T_c = 3.52$ Weak electron-phonon coupling

$E_g(0)/k_B T_c > 3.52$ Strong electron-phonon coupling

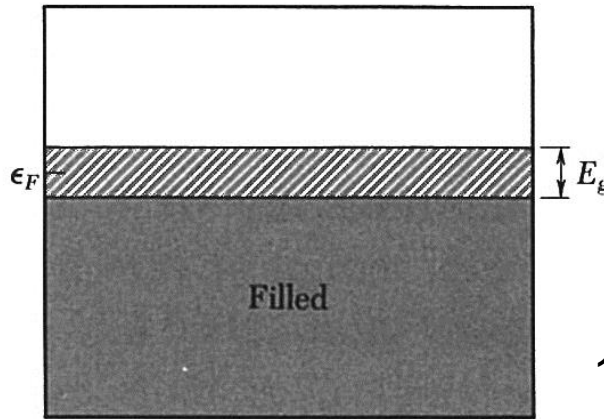
Table 3 Energy gaps in superconductors, at $T = 0$

										Al	Si
										3.4	
										3.3	
Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge
		16.							2.4	3.3	
		3.4							3.2	3.5	
Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn ^(w)
		30.5	2.7						1.5	10.5	11.5
		3.80	3.4						3.2	3.6	3.5
La _{fcc}	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg _(n)	Tl	Pb
19.		14.							16.5	7.35	27.3
3.7		3.60							4.6	3.57	4.38

$E_g(0)$ in 10^{-4}eV .
 $E_g(0)/k_B T_c$.



Normal
(a)

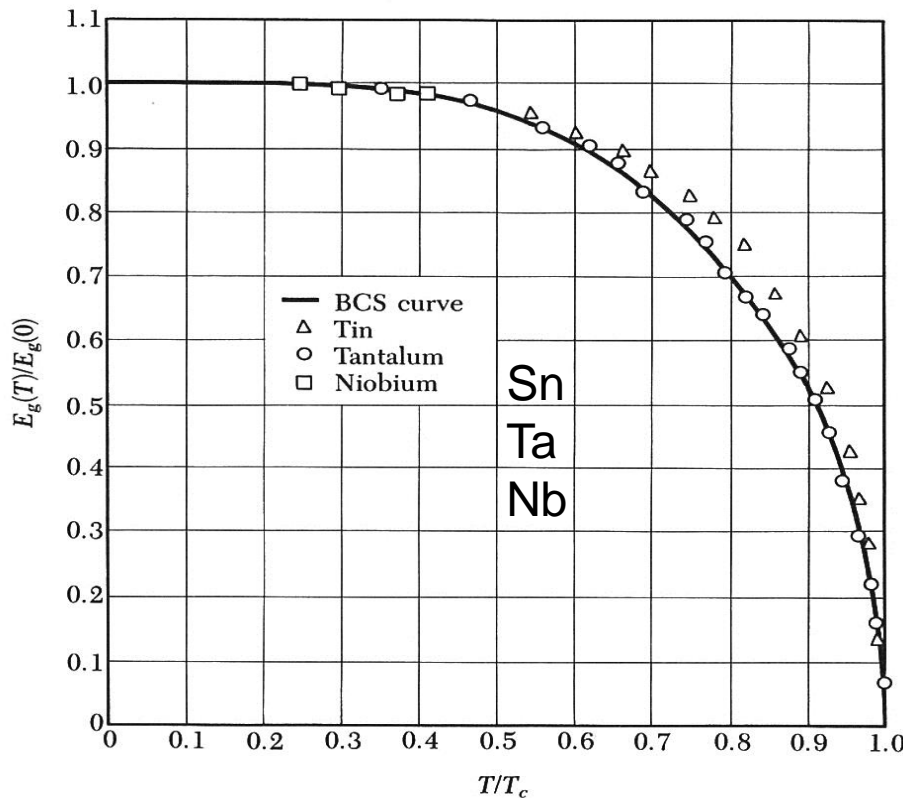


Superconductor
(b)

$$E_g \sim 10^{-4} \epsilon_F$$

1-5 meV,

3-10 eV



$$\Delta(T) / \Delta(0) = (1 - T/T_c)^{1/2}$$

Mean field theory

Figure 10 Reduced values of the observed energy gap $E_g(T)/E_g(0)$ as a function of the reduced temperature T/T_c , after Townsend and Sutton. The solid curve is drawn for the BCS theory.

$E_g(T)$ as the **order parameter**, goes smoothly to zero at T_c

-- second order phase transition

Isotope Effect

It has been observed that the critical temperature of superconductors varies with isotopic mass.

The experimental results within each series of isotopes may be fitted by a relation of the form

$$M^{\alpha}T_c = \text{constant} \quad \alpha \sim 0.5$$

Table 4 Isotope effect in superconductors

Experimental values of α in $M^{\alpha}T_c = \text{constant}$, where M is the isotopic mass.

Substance	α	Substance	α
Zn	0.45 ± 0.05	Ru	0.00 ± 0.05
Cd	0.32 ± 0.07	Os	0.15 ± 0.05
Sn	0.47 ± 0.02	Mo	0.33
Hg	0.50 ± 0.03	Nb ₃ Sn	0.08 ± 0.02
Pb	0.49 ± 0.02	Zr	0.00 ± 0.05

From the dependence of T_c on the isotopic mass we learn that lattice vibrations and hence electron-lattice interactions are deeply involved in superconductivity.

$$\theta \propto \nu \propto M^{-1/2} \quad T_c \propto \theta_{\text{Debye}} \propto M^{-1/2}, \text{ so that } \alpha = \frac{1}{2}$$