Chapter 12: Superconductivity

References:

1. C. Kittel, Introduction to solid state physics
2. M. Tinkham, Introduction to superconductivity
3. Paul Hansma, Tunneling Spectroscopy

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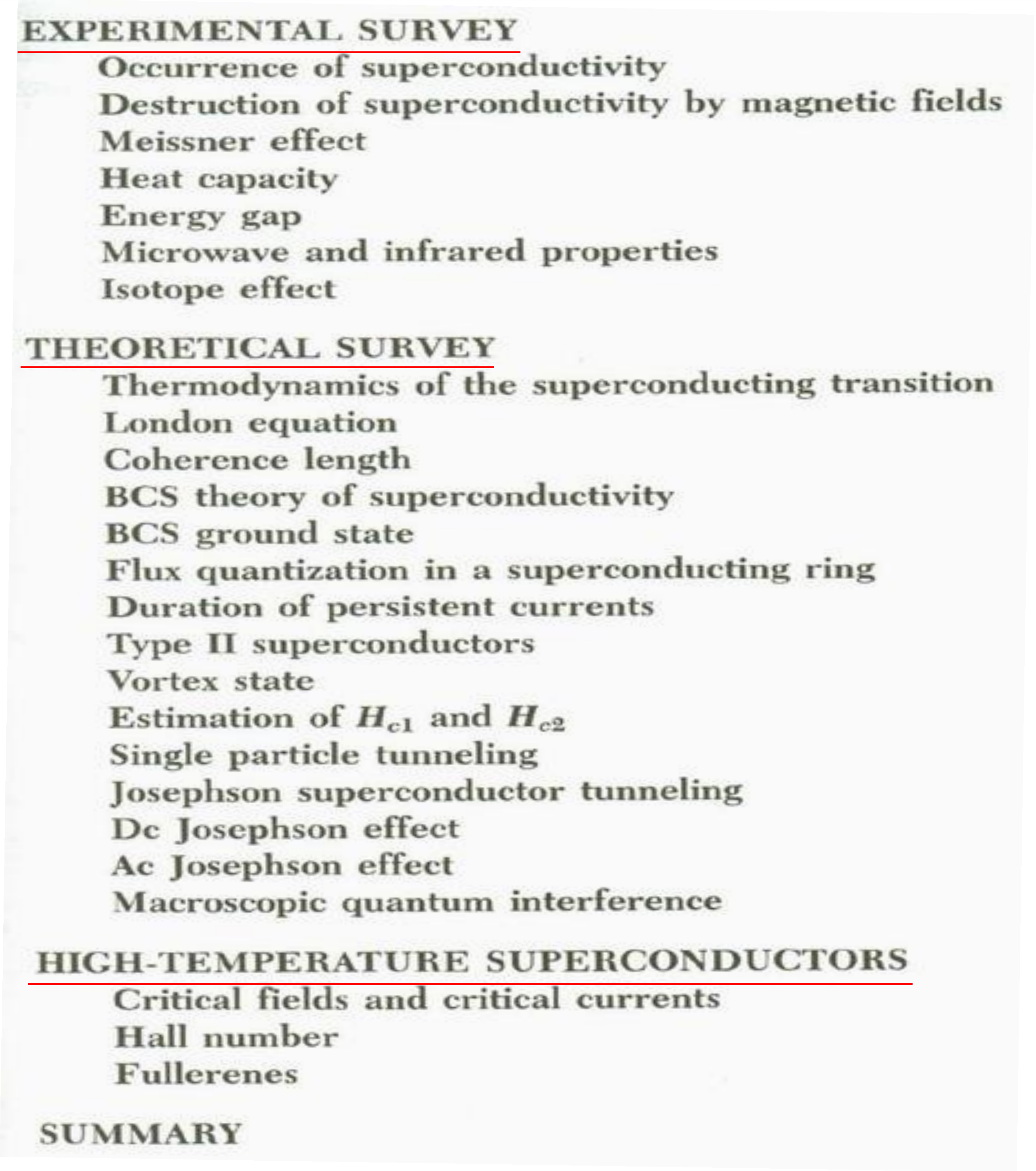


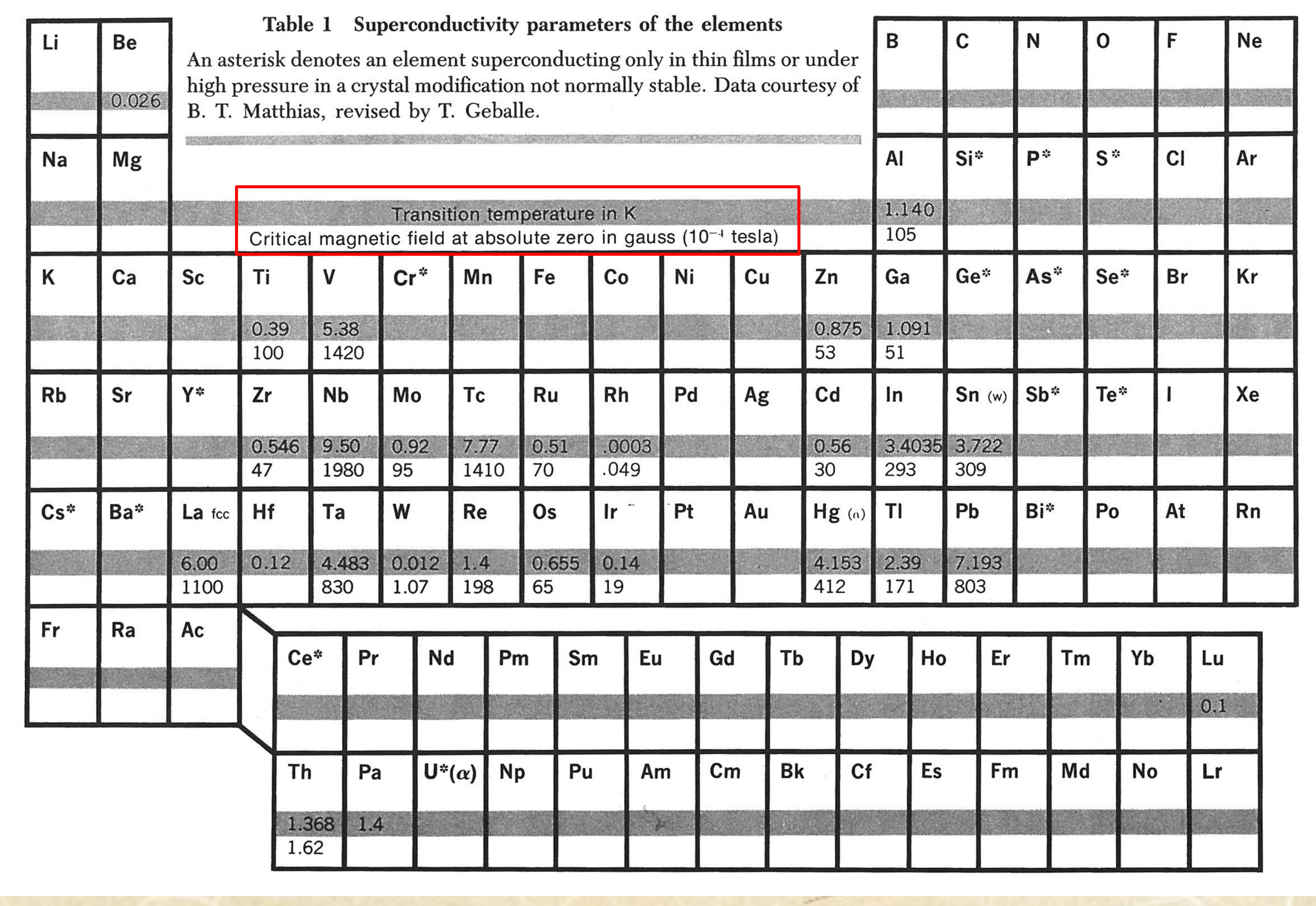


Table 1 Superconductivity parameters of the elements

An asterisk denotes an element superconducting only in thin films or under high pressure in a crystal modification not normally stable. Data courtesy of B. T. Matthias, revised by T. Geballe.

Transition temperature in K

Critical magnetic field at absolute zero in gauss (10^{-4} tesla)

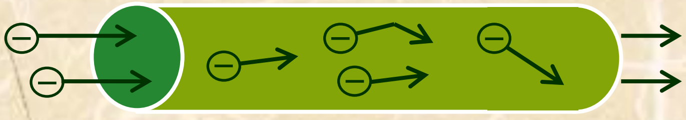
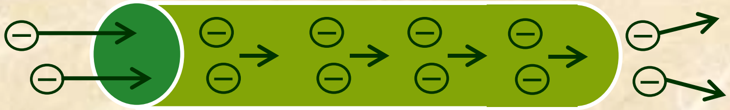
Li	Be 0.026											B	C	N	O	F	Ne
Na	Mg											Al 1.140 105	Si*	P*	S*	Cl	Ar
K	Ca	Sc	Ti 0.39 100	V 5.38 1420	Cr*	Mn	Fe	Co	Ni	Cu	Zn 0.875 53	Ga 1.091 51	Ge*	As*	Se*	Br	Kr
Rb	Sr	Y*	Zr 0.546 47	Nb 9.50 1980	Mo 0.92 95	Tc 7.77 1410	Ru 0.51 70	Rh .0003 .049	Pd	Ag	Cd 0.56 30	In 3.4035 293	Sn (w) 3.722 309	Sb*	Te*	I	Xe
Cs*	Ba*	La fcc 6.00 1100	Hf 0.12	Ta 4.483 830	W 0.012 1.07	Re 1.4 198	Os 0.655 65	Ir 0.14 19	Pt	Au	Hg (α) 4.153 412	Tl 2.39 171	Pb 7.193 803	Bi*	Po	At	Rn
Fr	Ra	Ac															
			Ce*	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu 0.1	
			Th 1.368 1.62	Pa 1.4	U*(α)	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr	

Experimental Survey of Superconductivity Phenomenon

WHAT IS A SUPERCONDUCTOR?

1. **Zero resistance**
2. **Complete expulsion of magnetic flux**

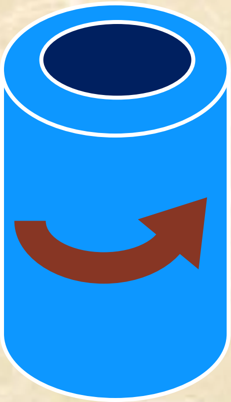
SUPERCONDUCTIVITY

Type of material	What happens in a wire?	Result
Conductor	 Electrons flow easily (like water through a garden hose)	Collisions cause dissipation (heat)
Insulator	 Electrons are tightly bound no flow (like a hose plugged with cement)	No current flow at all
Superconductor	 Electrons bind into pairs and cannot collide (a frictionless hose)	No collisions No dissipation No heat No resistance

HOW SMALL IS THE RESISTANCE?

Copper Cylinder

1) Induce current
2) Current decays in about 1/1000 second



Superconducting Cylinder

1) Induce current
2) Current does not decay
   (less than 0.1% in a year)
   so, resistance is smaller than copper
   by $\dfrac{\text{1000 years}}{\text{1/1000 second}}$
   i.e., at least 1 trillion times!

Why Superconductivity is so fascinating ?

- Fundamental SC mechanism

- Novel collective phenomenon at low temp

- Applications

  Bulk: - Persistent current, power storage
  - Magnetic levitation
  - High field magnet, MRI

  Electronics:
  - SQUID magnetometer
  - Josephson junction electronics

POSSIBLE IMPACT OF SUPERCONDUCTIVITY

● **Energy**

- Superconductivity generators & motors
- Power transmission & distribution
- Energy storage systems
- Magnets for fusion power
- Magnets for magneto-hydrodynamic power

● **Transportation**

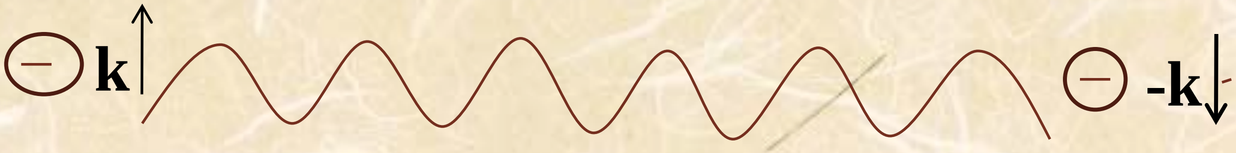
- Magnets for levitated trains
- Electro-magnetic powered ships
- Magnets for automobiles

● **Health care**

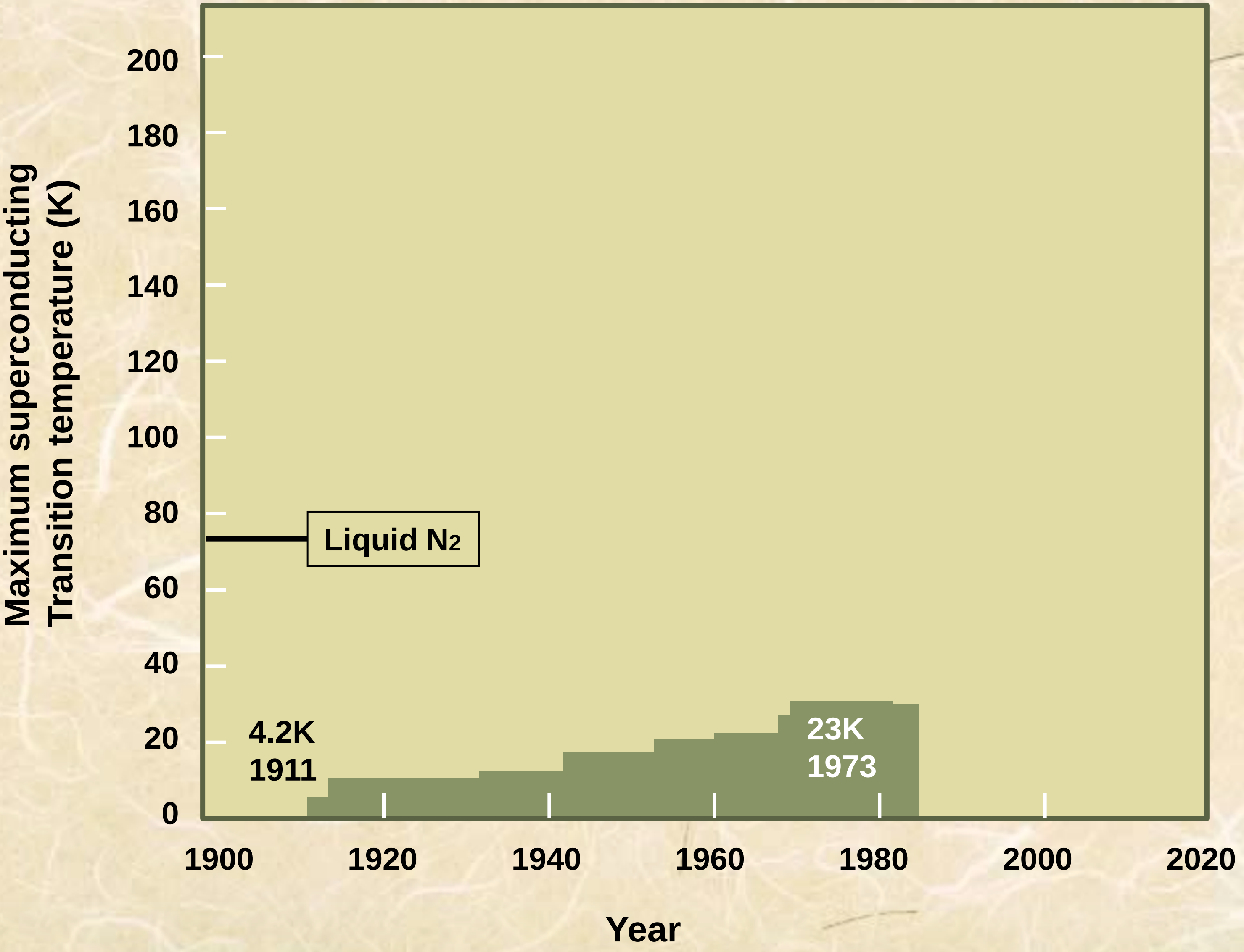
- Magnetic resonance imaging

Low-T_c Superconductivity Mechanism

Electron phonon coupling



PROGRESS IN SUPERCONDUCTIVITY

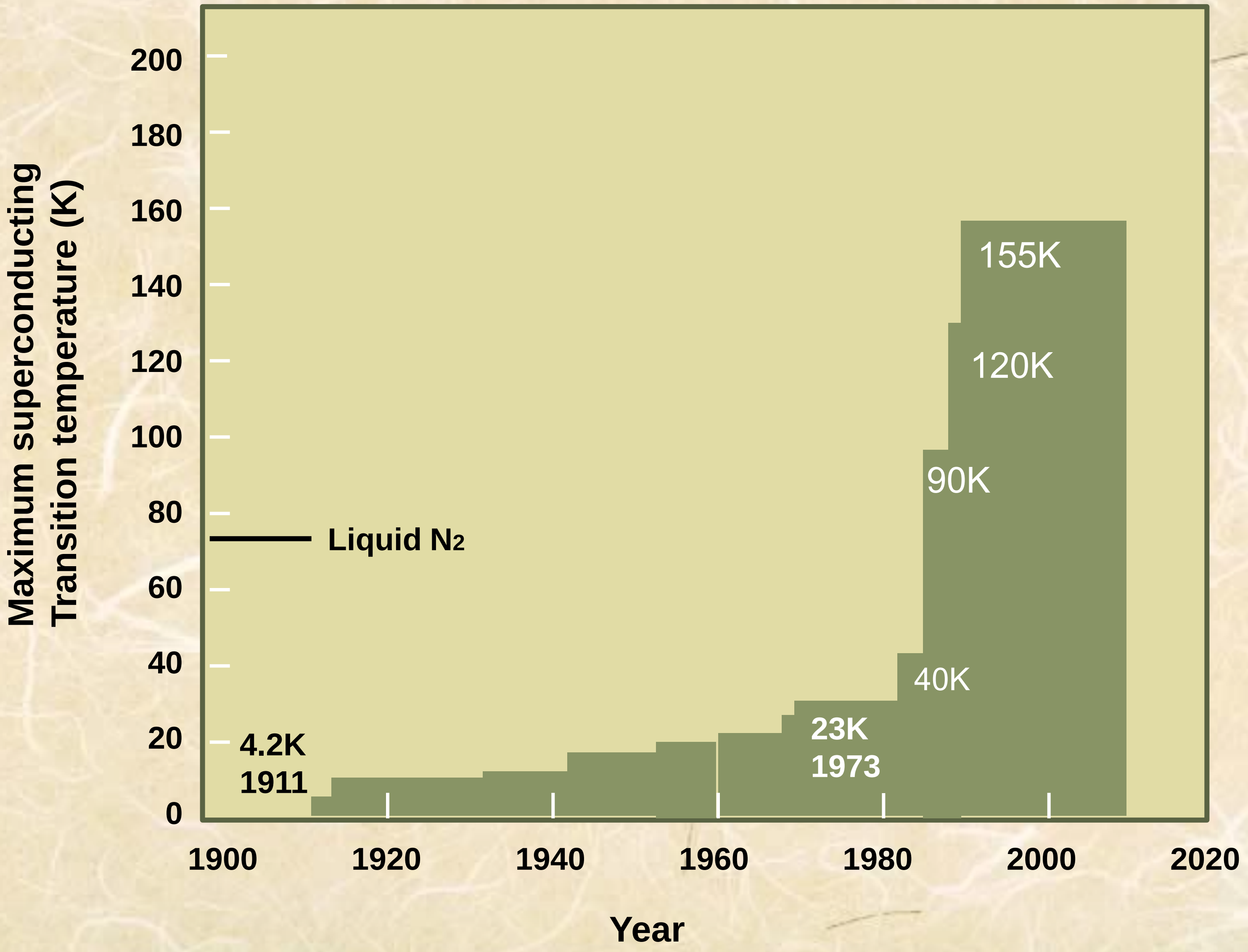


A legacy of Superconductivity

Ted H. Geballe

A legacy of Superconductivity

PROGRESS IN SUPERCONDUCTIVITY



Low temperature Superconductors

-- Mediated by electron phonon coupling

-- the critical temperature T_c in
the strong electron-phonon coupling limit

$$T_c = \frac{\Theta_D}{1.45} \exp\left\{ - \left[\frac{(1+\lambda_{\mathrm{ep}})}{\lambda_{\mathrm{ep}} - \mu^*(1+0.62\lambda_{\mathrm{ep}})} \right] \right\}$$

λ : electron phonon coupling constant

μ^* : Coulomb repulsion of electrons

$\lambda \propto N(0) < I^2 > / M<\omega^2>$

Experimental Electrical Resistivity of Metals

The electrical resistivity of most metals is dominated at room temperature (300 K) by collisions of the conduction electrons with lattice phonons and at liquid helium temperature (4 K) by collisions with impurity atoms and mechanical imperfections in the lattice (Fig. 11).

Lattice phonons

$$\frac{1}{\tau} = \frac{1}{\tau_L} + \frac{1}{\tau_i} \ ,$$

Imperfections

To a good approximation the rates are often independent.
And can be summed together

where τ_L and τ_i are the collision times for scattering by phonons and by imperfections, respectively.

The net resistivity is given by

$$\boxed{\rho = \rho_L + \rho_i \ ,} \qquad \text{Since } \rho \sim 1/\tau \qquad (47)$$

Often ρ_L is independent of the number of defects when their concentration is small, and often ρ_i is independent of temperature.

This empirical observation expresses **Matthiessen's Rule.**

Potassium metal

At $T > \theta$

$\rho \propto T$

Different $\rho_i(0)$ but the same ρ_L

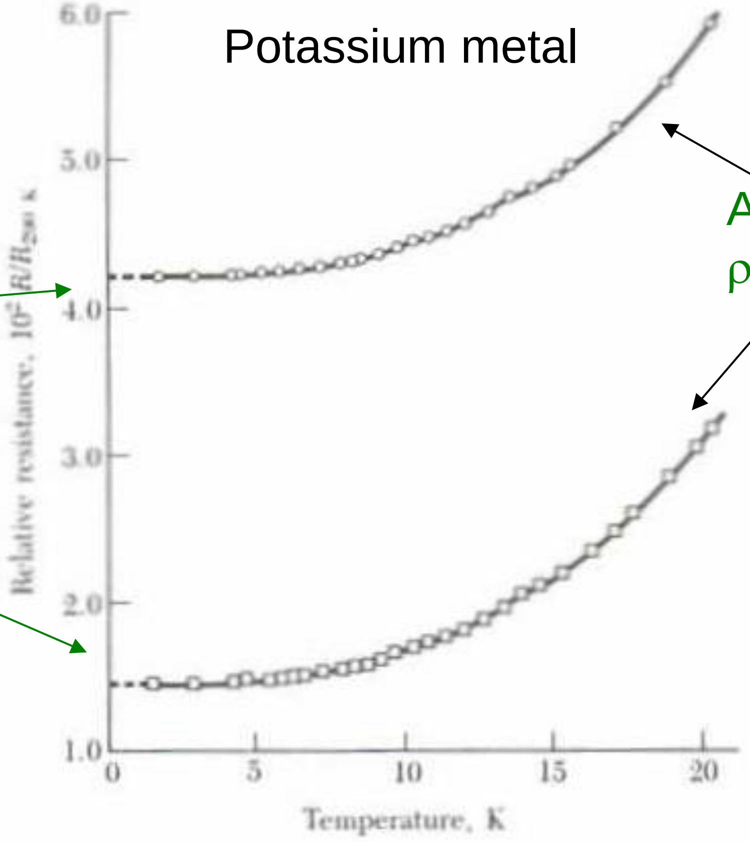
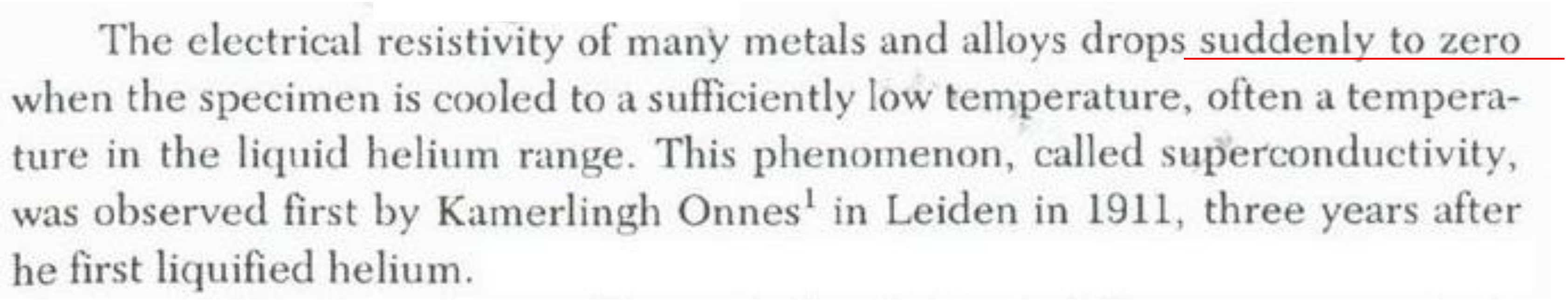


Figure 12 Resistance of potassium below 20 K, as measured on two specimens by D. MacDonald and K. Mendelssohn. The different intercepts at 0 K are attributed to different concentrations of impurities and static imperfections in the two specimens.

The temperature-dependent part of the electrical resistivity is proportional to the rate at which an electron collides with thermal phonons

One simple limit is at temperatures over the Debye temperature θ: here the phonon concentration is proportional to the temperature T, so that $\rho \propto T$ for $T > \theta$.

$N_{ph} \propto T$ hence $\rho \propto 1/\tau \propto N_{ph} \propto T$



The electrical resistivity of many metals and alloys drops suddenly to zero when the specimen is cooled to a sufficiently low temperature, often a temperature in the liquid helium range. This phenomenon, called superconductivity, was observed first by Kamerlingh Onnes[1] in Leiden in 1911, three years after he first liquified helium.

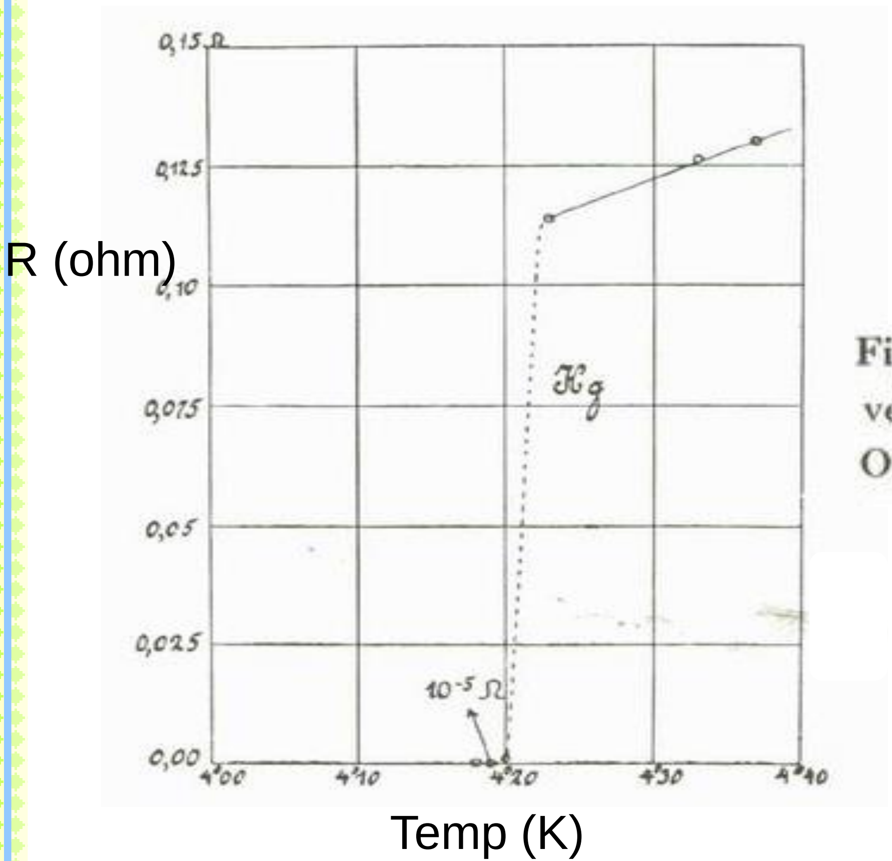
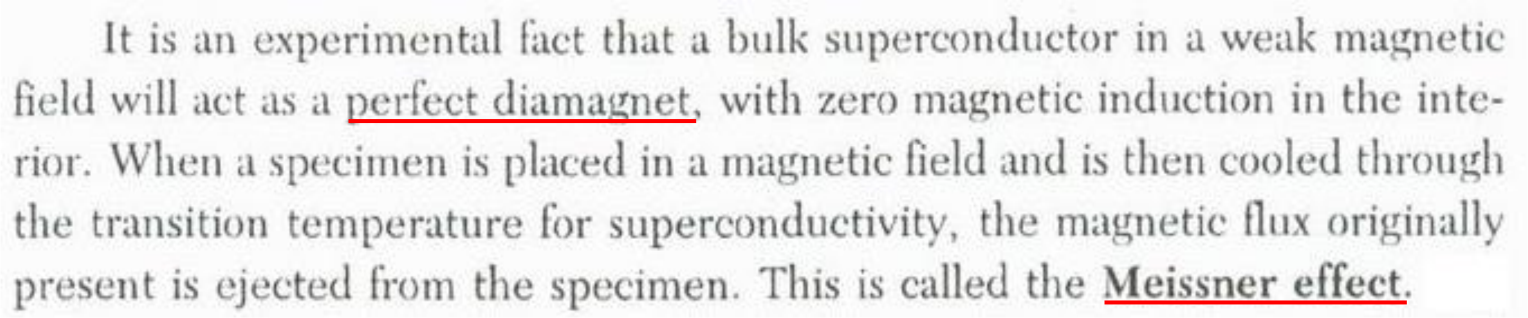


Figure 1 Resistance in ohms of a specimen of mercury versus absolute temperature. This plot by Kamerlingh Onnes marked the discovery of superconductivity.

First SC found in Hg by 1911 !

Decay of persistent current from 1 year up to 10^5 year

Meissner Effect



It is an experimental fact that a bulk superconductor in a weak magnetic field will act as a perfect diamagnet, with zero magnetic induction in the interior. When a specimen is placed in a magnetic field and is then cooled through the transition temperature for superconductivity, the magnetic flux originally present is ejected from the specimen. This is called the **Meissner effect**.

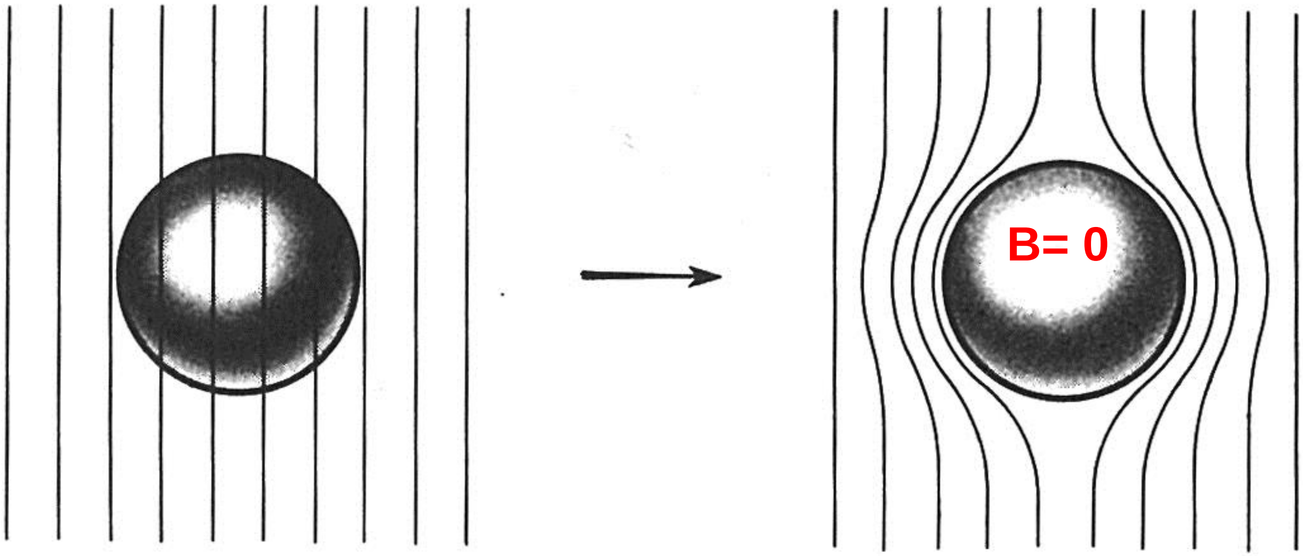


Figure 2 Meissner effect in a superconducting sphere cooled in a constant applied magnetic field; on passing below the transition temperature the lines of induction **B** are ejected from the sphere.

Fundamental Mechanism

The superconducting state is an ordered state of the conduction electrons of the metal.

Electron-Phonon Coupling

Cooper Pair formed by two electrons ***k*****,** and ***–k*** with opposite spins near the Fermi level, as coupled through **phonons** of the lattice

The nature and origin of the ordering was explained by Bardeen, Cooper, and Schrieffer.[3]

BCS Theory, 1957

[3] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **106**, 162 (1957); **108**, 1175 (1957).

The Discovery of Superconductivity

- Early 90’s -- elemental SP metals like Hg, Pb, Al, Sn, Ga, etc.
- Middle 90’s -- transitional metals, alloys, and compounds like Nb, NbN, Nb_3Sn, etc.
- Late 90’s -- in perovskite oxides

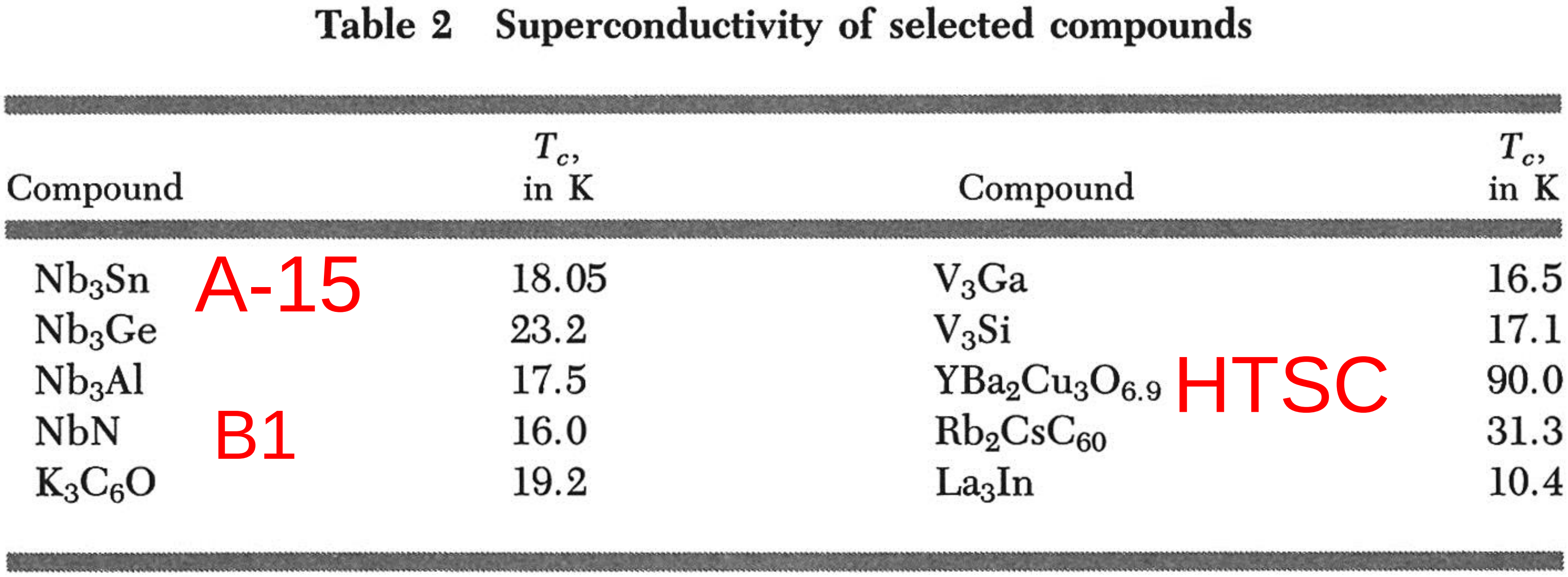


Table 2 Superconductivity of selected compounds

Compound	T_c, in K	Compound	T_c, in K
Nb_3Sn	18.05	V_3Ga	16.5
Nb_3Ge	23.2	V_3Si	17.1
Nb_3Al	17.5	$YBa_2Cu_3O_{6.9}$	90.0
NbN	16.0	Rb_2CsC_{60}	31.3
K_3C_6O	19.2	La_3In	10.4

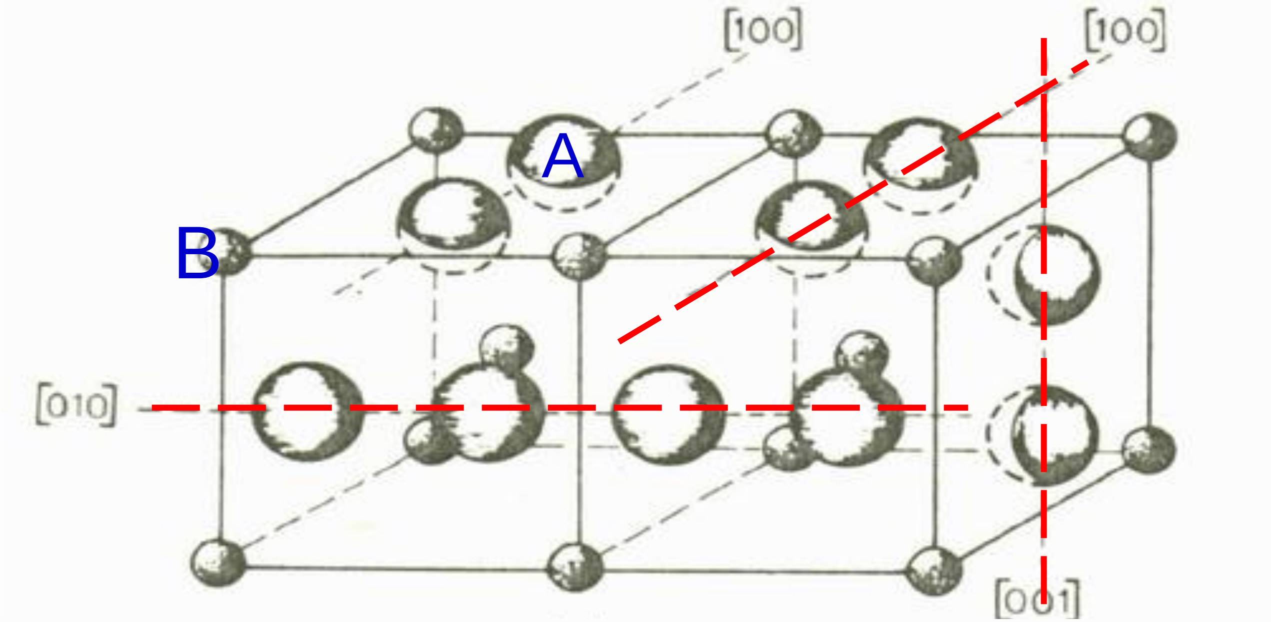
A-15

B1

HTSC

A-15 compound A_3B, with T_c = 15-23 K

With three perpendicular linear chains of ***A*** atoms on the cubic face, and
B atoms are at body centered cubic site



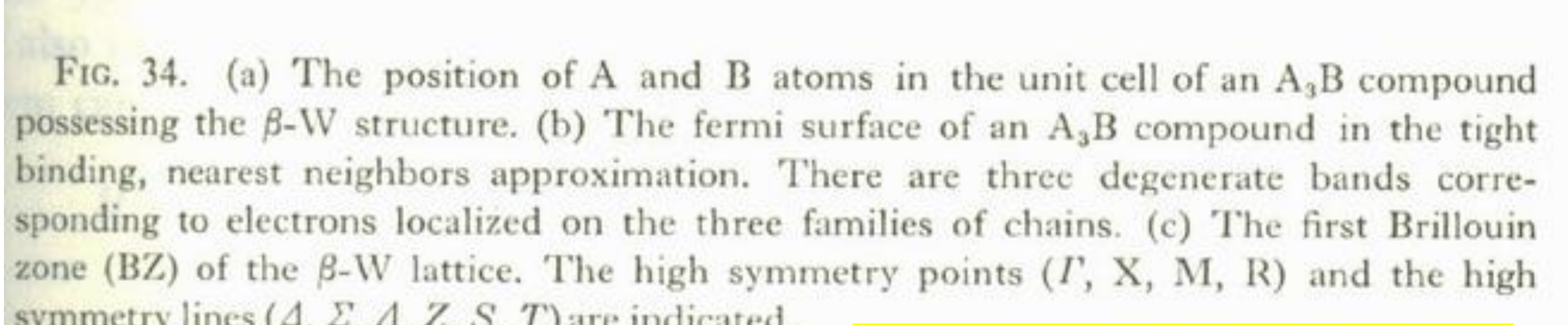


FIG. 34. (a) The position of A and B atoms in the unit cell of an A_3B compound possessing the β-W structure. (b) The fermi surface of an A_3B compound in the tight binding, nearest neighbors approximation. There are three degenerate bands corresponding to electrons localized on the three families of chains. (c) The first Brillouin zone (BZ) of the β-W lattice. The high symmetry points (Γ, X, M, R) and the high symmetry lines (Δ, Σ, Λ, Z, S, T) are indicated.

1973 Nb_3Ge, 23K !

Low temperature Superconductors

-- Mediated by Electron phonon coupling

-- McMillian formula for T_c

$$T_c = \frac{\Theta_D}{1.45} \exp\left\{-\left[\frac{(1+\lambda_{\mathrm{ep}})}{\lambda_{\mathrm{ep}} - \mu^*(1+0.62\lambda_{\mathrm{ep}})}\right]\right\}$$

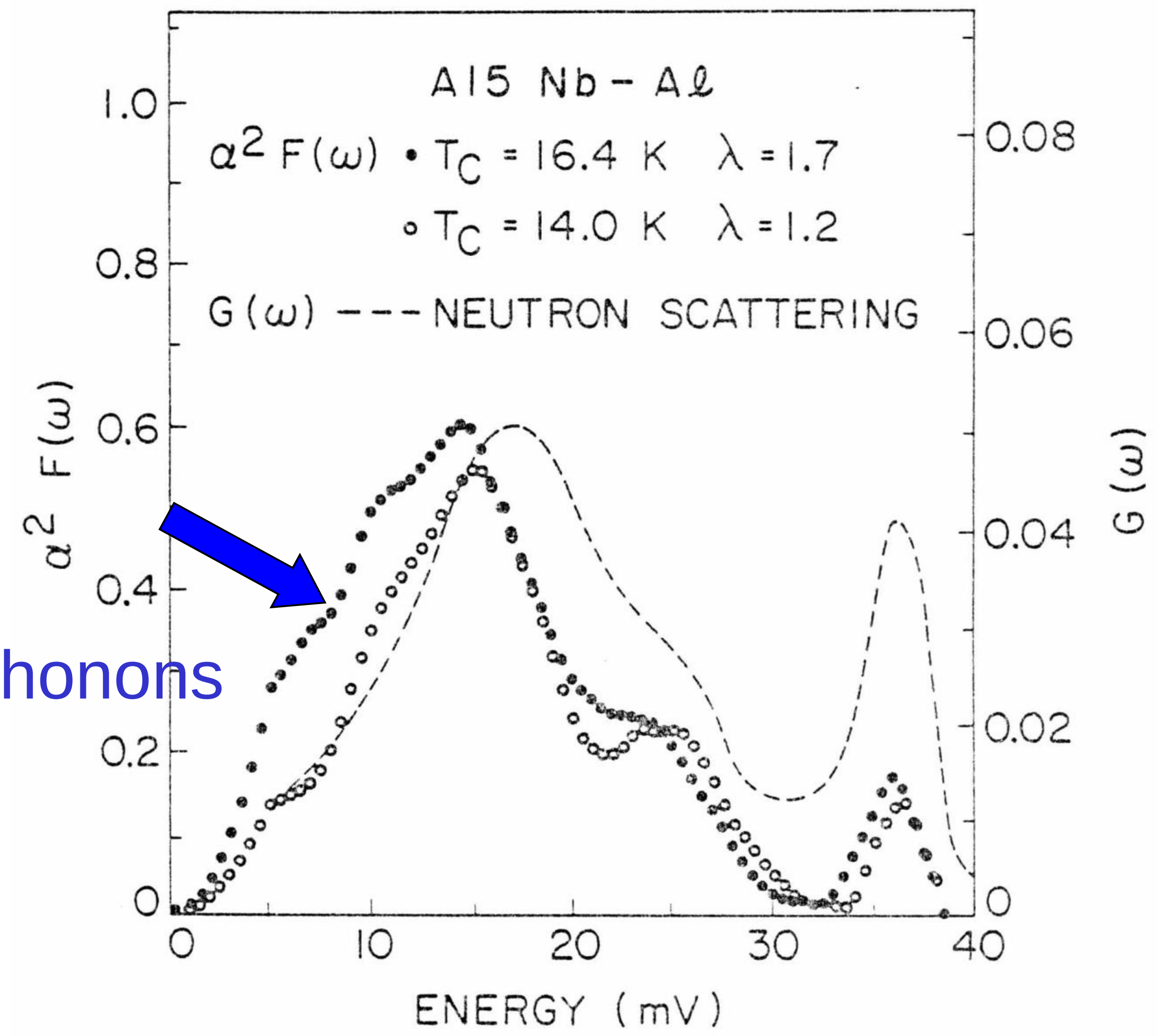
λ : electron phonon coupling constant

μ^* : Coulomb repulsion of electrons

$\lambda \propto N(0) < I^2 > / \omega^2$

Are electrons or phonons more important?

The Phonon Spectrum of the low T_c A-15 compound Nb_3Al



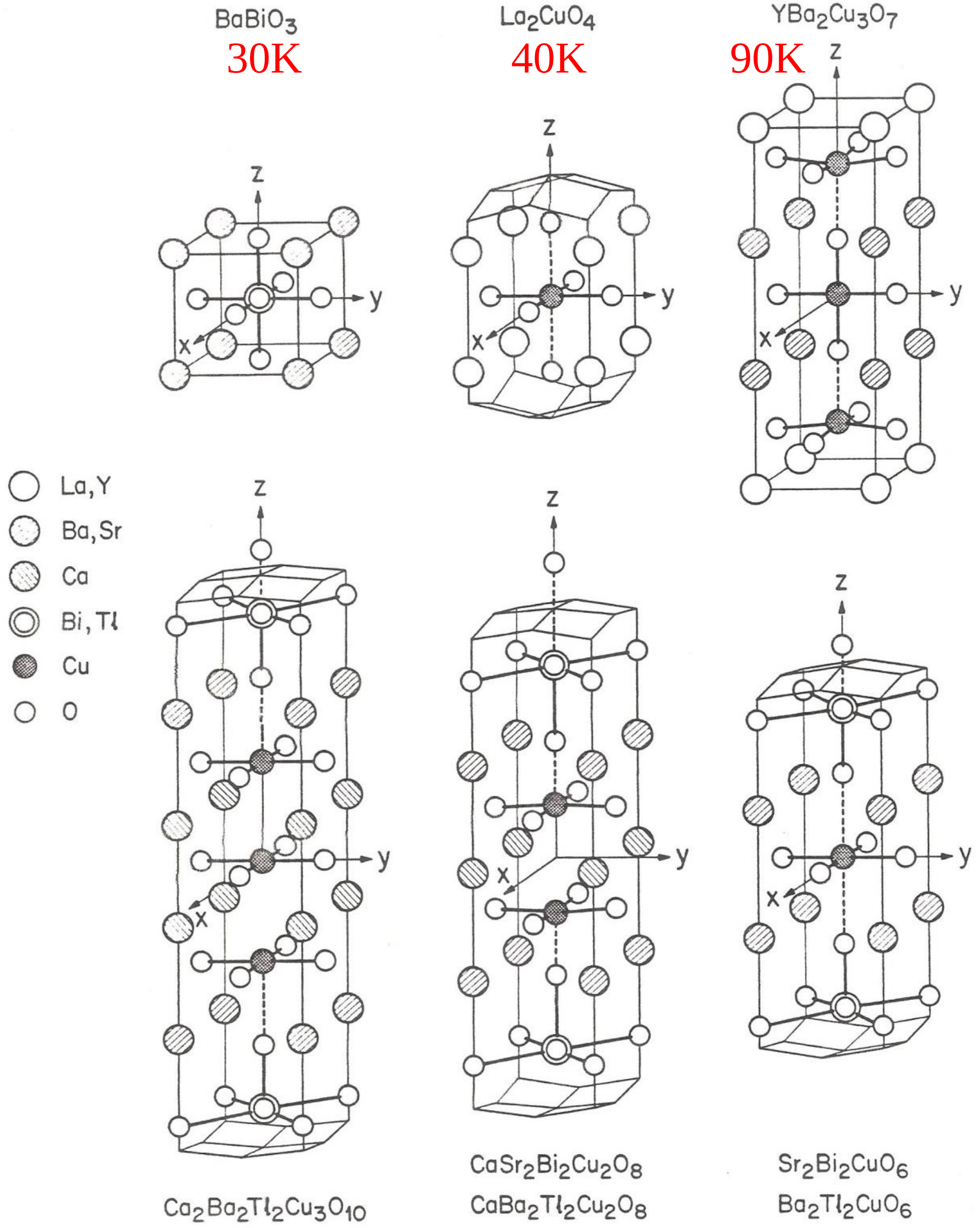
Soft Phonons

Can we raise the T_c higher than 30K?

Are we reaching the limitation of the BCS Theory ?

Breakthrough in late 1986
By Bednorz and Muller

Start the HTSC Era !



30K

40K

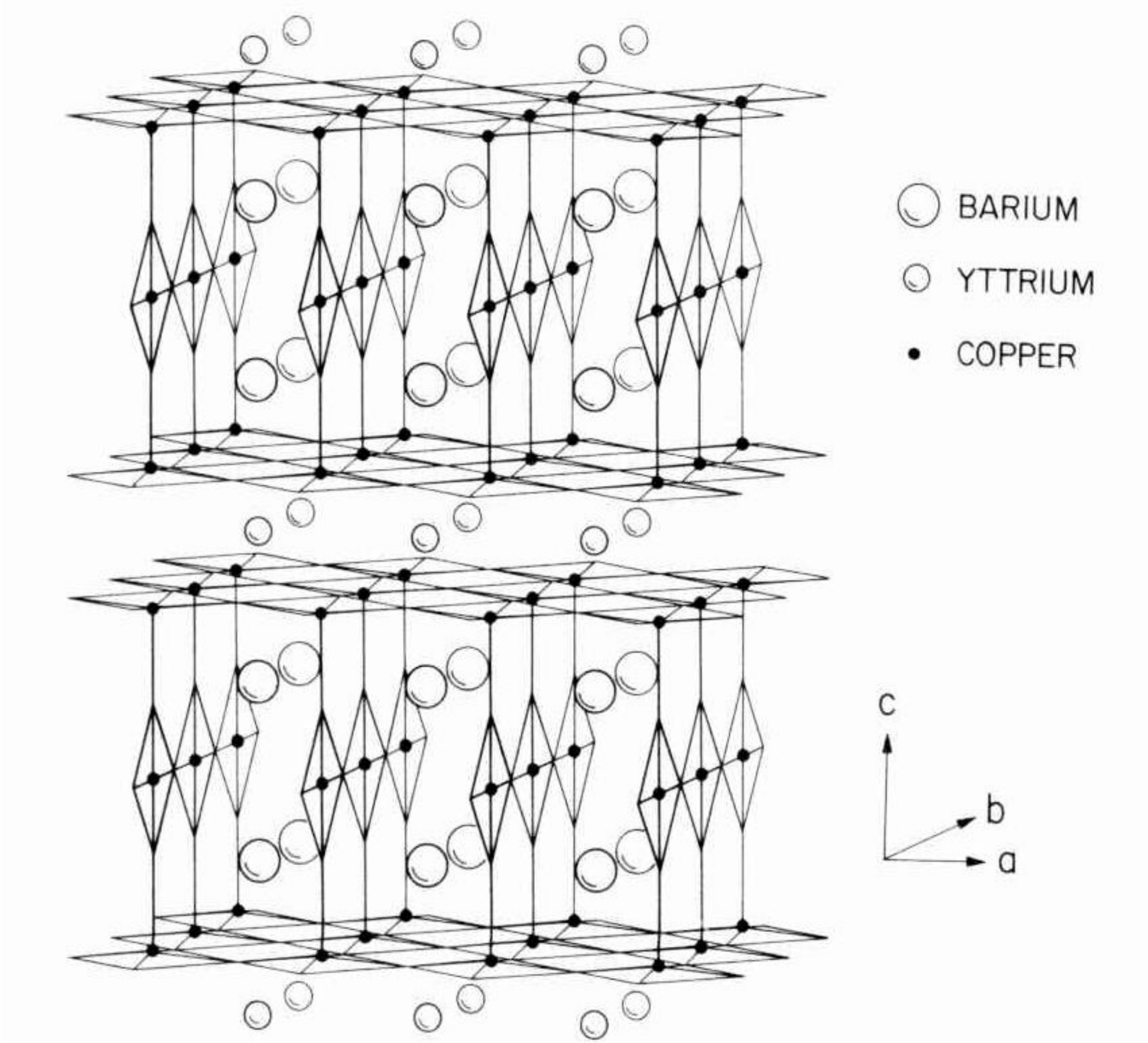
90K

120K

80K

40K

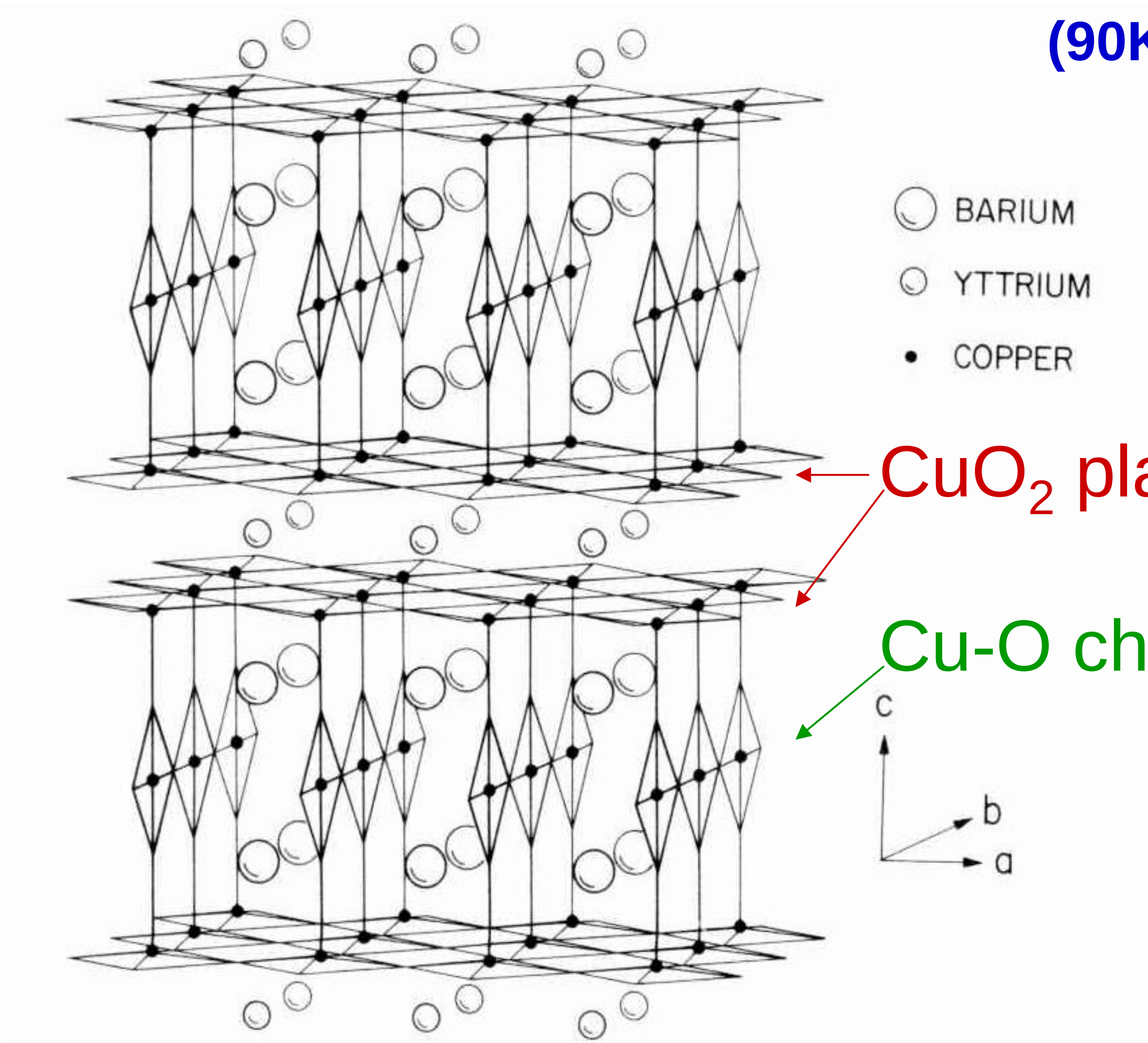
High Temperature Superconductor $YBa_2Cu_3O_7$



Invention of Oxide Molecular Beam Epitaxy in 1988 For HTSC Single Crystal Films.

High Temperature Superconductor $YBa_2Cu_3O_7$

(90K)



CuO_2 plane 2-D

Cu-O chain 1-D

**Invention of Oxide Molecular Beam Epitaxy
For HTSC Single Crystal Films.**

Will all non magnetic metal become SC at low T?

(I) Destruction of Superconductivity by Magnetic Impurities

It is important to eliminate from the specimen even trace quantities of foreign paramagnetic elements

(II) Destruction of Superconductivity by Magnetic fields

At the critical temperature the critical field is zero: $H_c(T_c)=0$

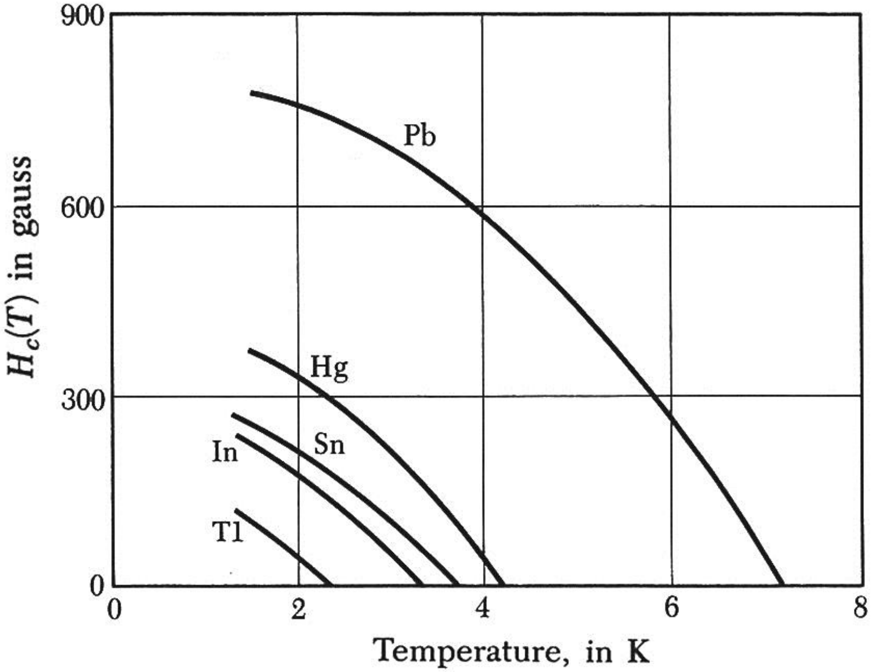
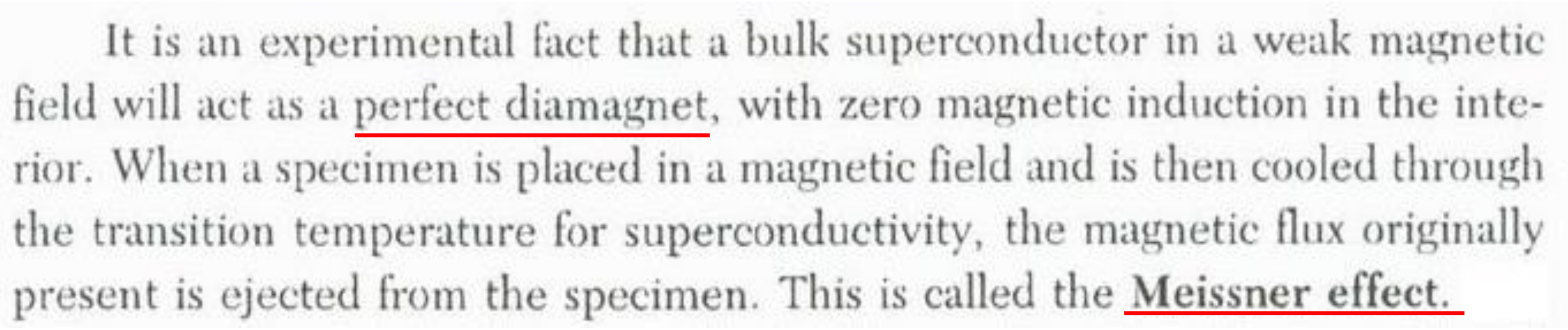


Figure 3 Experimental threshold curves of the critical field $H_c(T)$ versus temperature for several superconductors. A specimen is superconducting below the curve and normal above the curve.

Meissner Effect



It is an experimental fact that a bulk superconductor in a weak magnetic field will act as a perfect diamagnet, with zero magnetic induction in the interior. When a specimen is placed in a magnetic field and is then cooled through the transition temperature for superconductivity, the magnetic flux originally present is ejected from the specimen. This is called the **Meissner effect.**

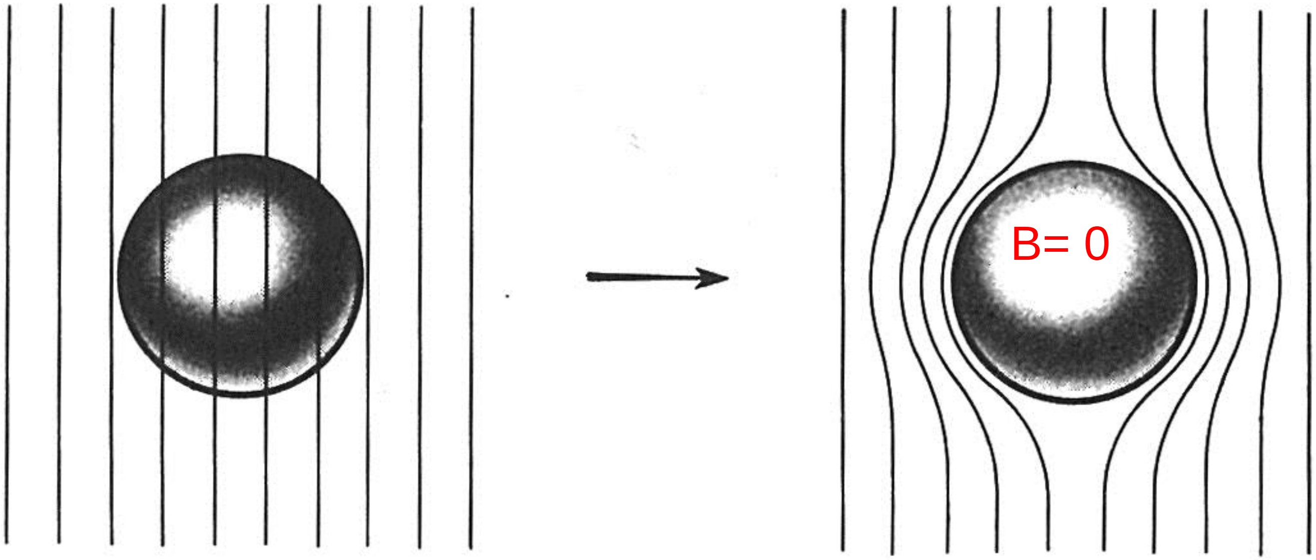
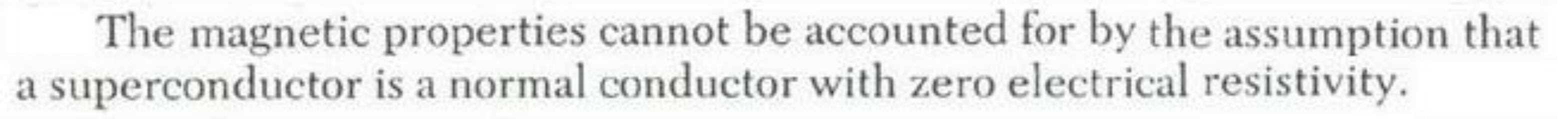


Figure 2 Meissner effect in a superconducting sphere cooled in a constant applied magnetic field; on passing below the transition temperature the lines of induction **B** are ejected from the sphere.

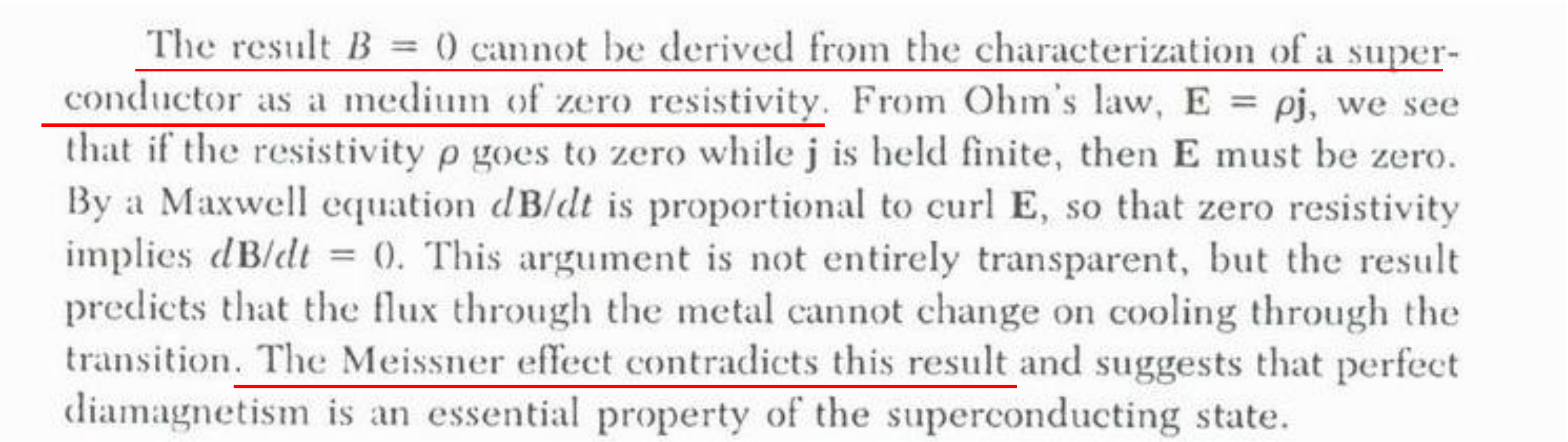
Meissner Effect

$$\boxed{B = B_a + 4\pi M = 0 \;;} \qquad \frac{M}{B_a} = -\frac{1}{4\pi} \qquad \text{Eq.(1)}$$

Perfect Diamagnetism



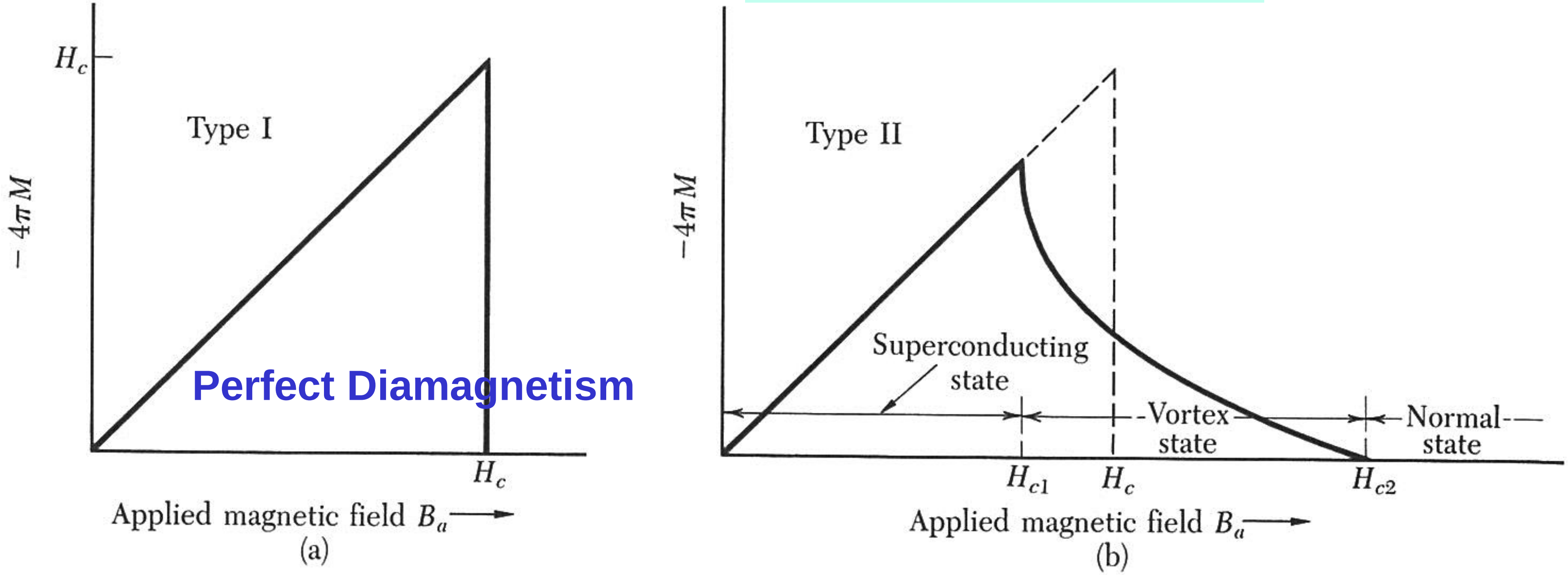
The magnetic properties cannot be accounted for by the assumption that a superconductor is a normal conductor with zero electrical resistivity.



The result $B = 0$ cannot be derived from the characterization of a superconductor as a medium of zero resistivity. From Ohm's law, $\mathbf{E} = \rho\mathbf{j}$, we see that if the resistivity ρ goes to zero while $\mathbf{j}$ is held finite, then $\mathbf{E}$ must be zero. By a Maxwell equation $d\mathbf{B}/dt$ is proportional to curl $\mathbf{E}$, so that zero resistivity implies $d\mathbf{B}/dt = 0$. This argument is not entirely transparent, but the result predicts that the flux through the metal cannot change on cooling through the transition. The Meissner effect contradicts this result and suggests that perfect diamagnetism is an essential property of the superconducting state.

Type I superconductor

Type II superconductor



Perfect Diamagnetism

Figure 4 (a) Magnetization versus applied magnetic field for a bulk superconductor exhibiting a complete Meissner effect (perfect diamagnetism). A superconductor with this behavior is called a type I superconductor. Above the critical field H_c the specimen is a normal conductor and the magnetization is too small to be seen on this scale. Note that minus $4\pi M$ is plotted on the vertical scale: the negative value of M corresponds to diamagnetism. (b) Superconducting magnetization curve of a type II superconductor. The flux starts to penetrate the specimen at a field H_{c1} lower than the thermodynamic critical field H_c. The specimen is in a vortex state between H_{c1} and H_{c2}, and it has superconducting electrical properties up to H_{c2}. Above H_{c2} the specimen is a normal conductor in every respect, except for possible surface effects. For given H_c the area under the magnetization curve is the same for a type II superconductor as for a type I. (CGS units in all parts of this figure.)

Type II Superconductors

1. A good type I superconductor excludes a magnetic field until superconductivity is destroyed suddenly, and then the field penetrates completely.

2.

   (a) A good type II superconductor excludes the field completely up to a field H_{c1}.

   (b) Above H_{c1} the field is partially excluded, but the specimen remains electrically superconducting.

   (c) At a much higher field, H_{c2}, the flux penetrates completely and superconductivity vanishes.

   (d) An outer surface layer of the specimen may remain superconducting up to a still higher field H_{c3}.

3. An important difference in a type I and a type II superconductor is in the mean free path of the conduction electrons in the normal state.

   are type I, with $\kappa < 1$

   will be type II. is the situation when $\kappa = \lambda/\xi > 1$,

1. A superconductor is type I if the surface energy is always positive as the magnetic field is increased, For $H < H_c$

2. and type II if the surface energy becomes negative as the magnetic field is increased. For $Hc_1 < H < Hc_2$

The free energy of a bulk superconductor is increased when the magnetic field is expelled. However, a parallel field can penetrate a very thin film nearly uniformly (Fig. 17), only a part of the flux is expelled, and the energy of the superconducting film will increase only slowly as the external magnetic field is increased.

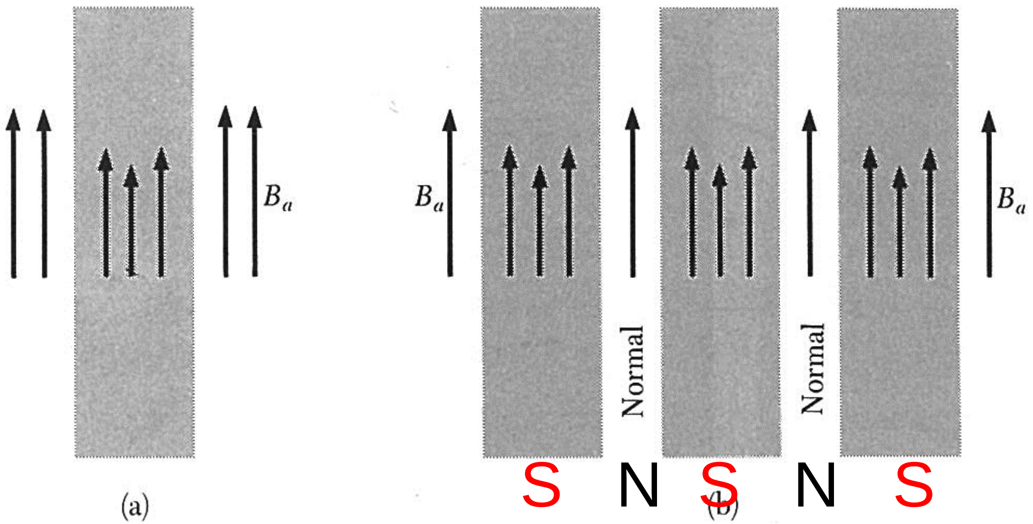


Figure 17 (a) Magnetic field penetration into a thin film of thickness equal to the penetration depth λ. The arrows indicate the intensity of the magnetic field. (b) Magnetic field penetration in a homogeneous bulk structure in the mixed or vortex state, with alternate layers in normal and superconducting states. The superconducting layers are thin in comparison with λ. The laminar structure is shown for convenience; the actual structure consists of rods of the normal state surrounded by the superconducting state. (The *N* regions in the vortex state are not exactly normal, but are described by low values of the stabilization energy density.)

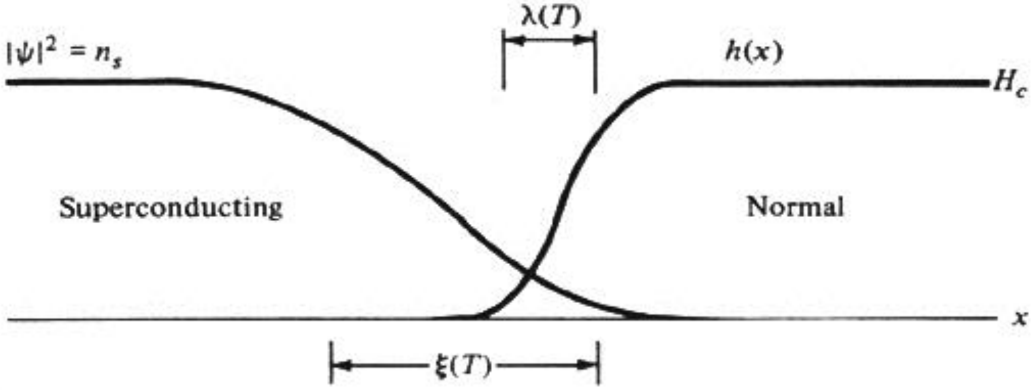
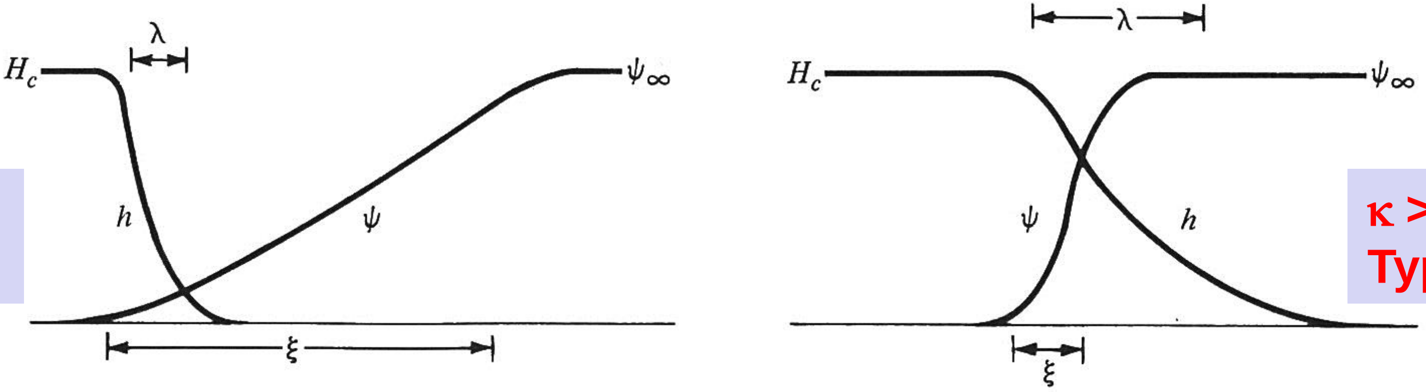


FIGURE 1-4
Interface between superconducting and normal domains in the intermediate state.

$$\kappa = \frac{\lambda_{\mathrm{eff}}(T)}{\xi(T)} = \frac{2\sqrt{2}\,\pi H_c(T)\lambda_{\mathrm{eff}}^2(T)}{\Phi_0}$$

Ginsburg Landau Parameter

κ << 1
Type I



$\kappa << 1$

$\kappa >> 1$

κ >> 1
Type II

FIGURE 4-2
Schematic diagram of variation of h and ψ in a domain wall. The case $\kappa \ll 1$ refers to a type I superconductor (positive wall energy); the case $\kappa \gg 1$ refers to a type II superconductor (negative wall energy).

Vortex State.

In such a mixed state, called the vortex state, the external magnetic field will penetrate the thin normal regions uniformly, and the field will also penetrate somewhat into the surrounding superconducting material

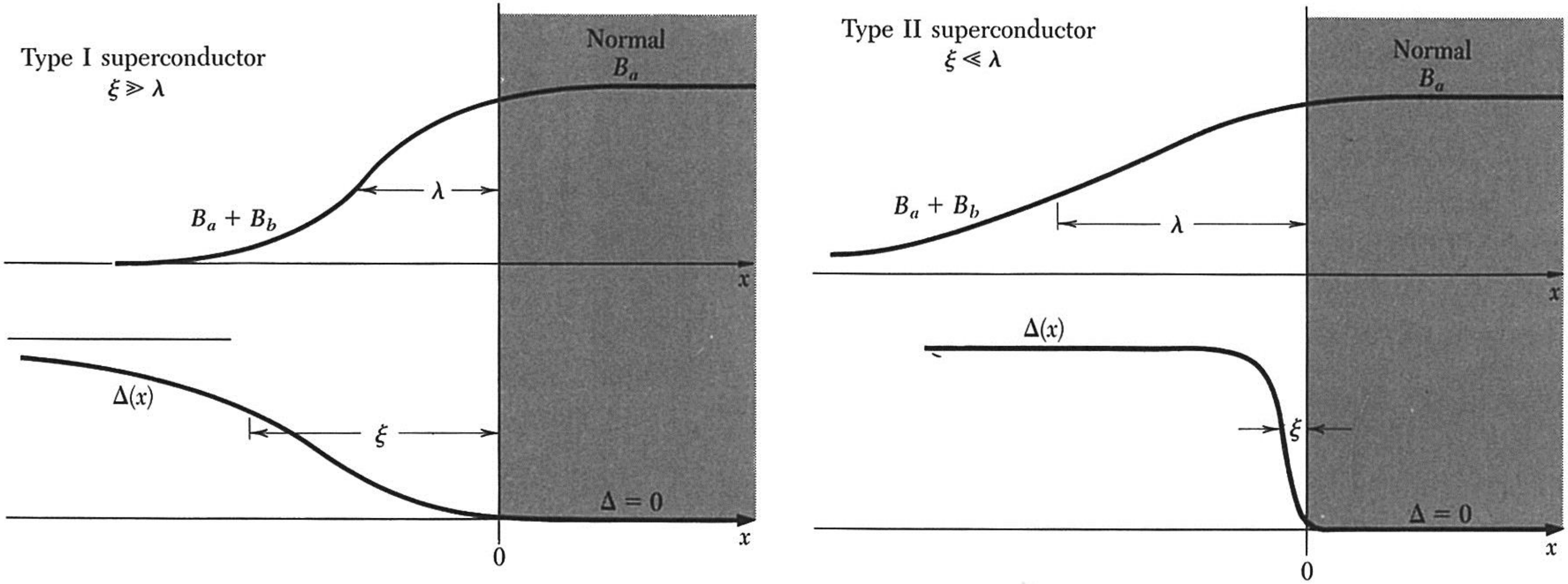
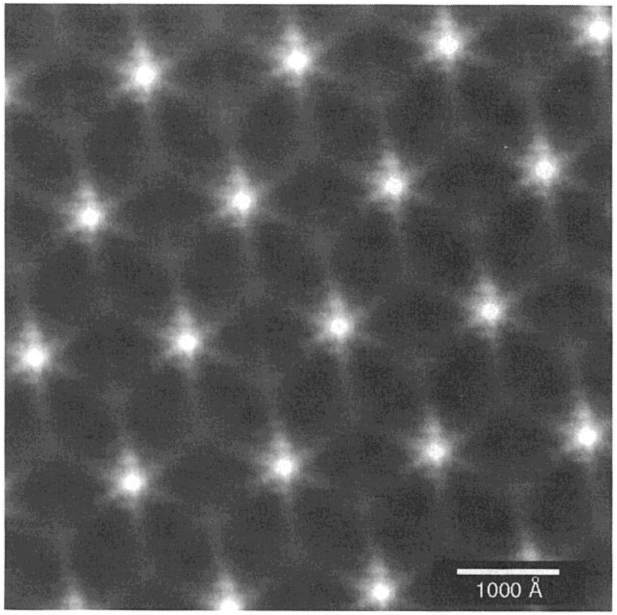


Figure 18 Variation of the magnetic field and energy gap parameter $\Delta(x)$ at the interface of superconducting and normal regions, for type I and type II superconductors. The energy gap parameter is a measure of the stabilization energy density of the superconducting state.

The term vortex state describes the circulation of superconducting currents in vortices throughout the bulk specimen,

Flux lattice
at 0.2K of $NbSe_2$



Abrikosov triangular lattice as imaged by LT-STM, H. Hess et al

Figure 19 Flux lattice in $NbSe_2$ at 1,000 gauss at 0.2K, as viewed with a scanning tunneling microscope. The photo shows the density of states at the Fermi level, as in Figure 23. The vortex cores have a high density of states and are shaded white; the superconducting regions are dark, with no states at the Fermi level. The amplitude and spatial extent of these states is determined by a potential well formed by $\Delta(x)$ as in Figure 18 for a Type II superconductor. The potential well confines the core state wavefunctions in the image here. The star shape is a finer feature, a result special to $NbSe_2$ of the sixfold disturbance of the charge density at the Fermi surface. Photo courtesy of H. F. Hess, AT&T Bell Laboratories.

The vortex state is stable when the penetration of the applied field into the superconducting material causes the surface energy become negative. A type II superconductor is characterized by a vortex state stable over a certain range of magnetic field strength; namely, between H_{c1} and H_{c2}.

Doping in Pb with In

Type I to become type II

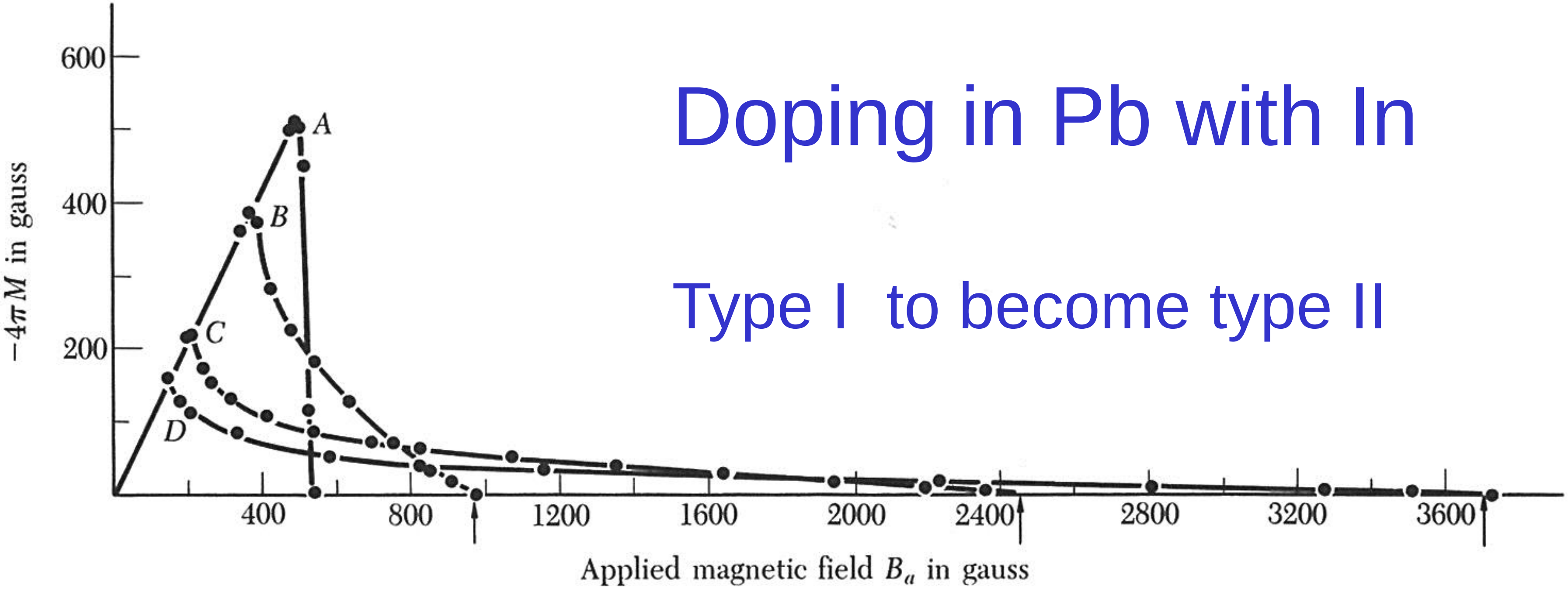


Figure 5a Superconducting magnetization curves of annealed polycrystalline lead and lead-indium alloys at 4.2 K. (*A*) lead; (*B*) lead–2.08 wt. percent indium; (*C*) lead–8.23 wt. percent indium; (*D*) lead–20.4 wt. percent indium. (After Livingston.)

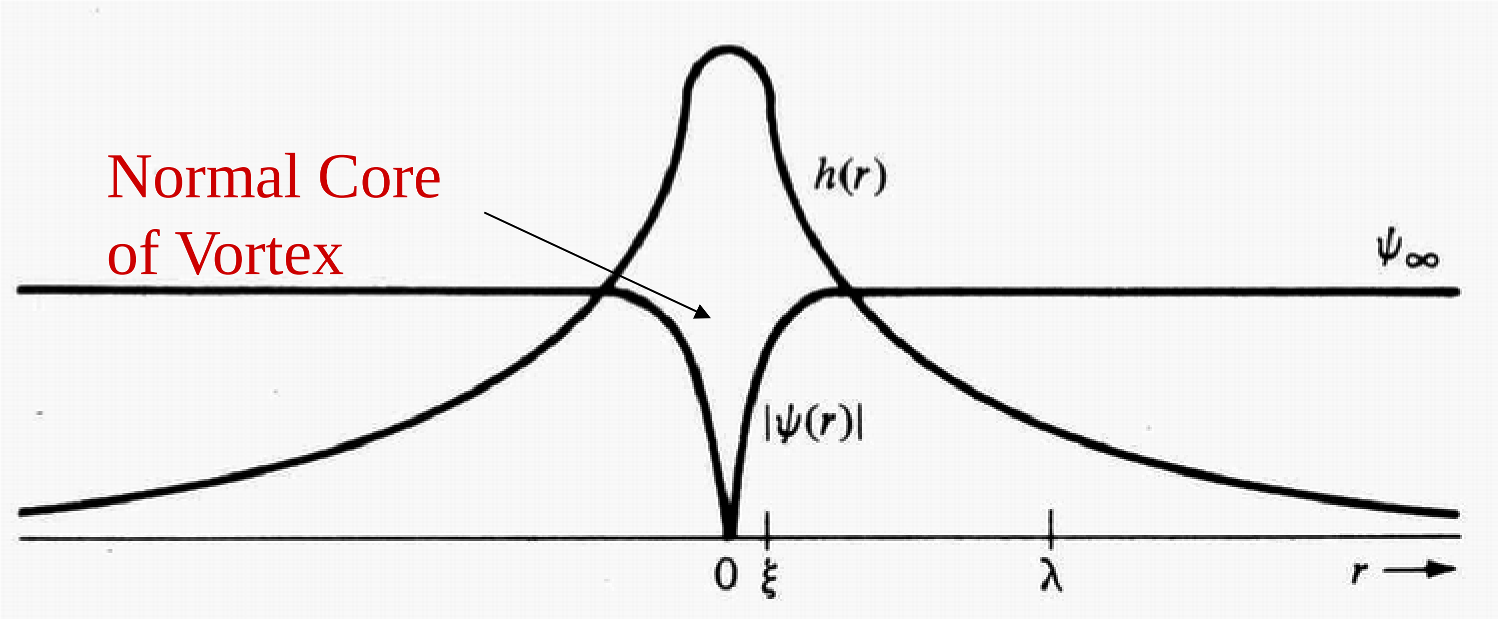
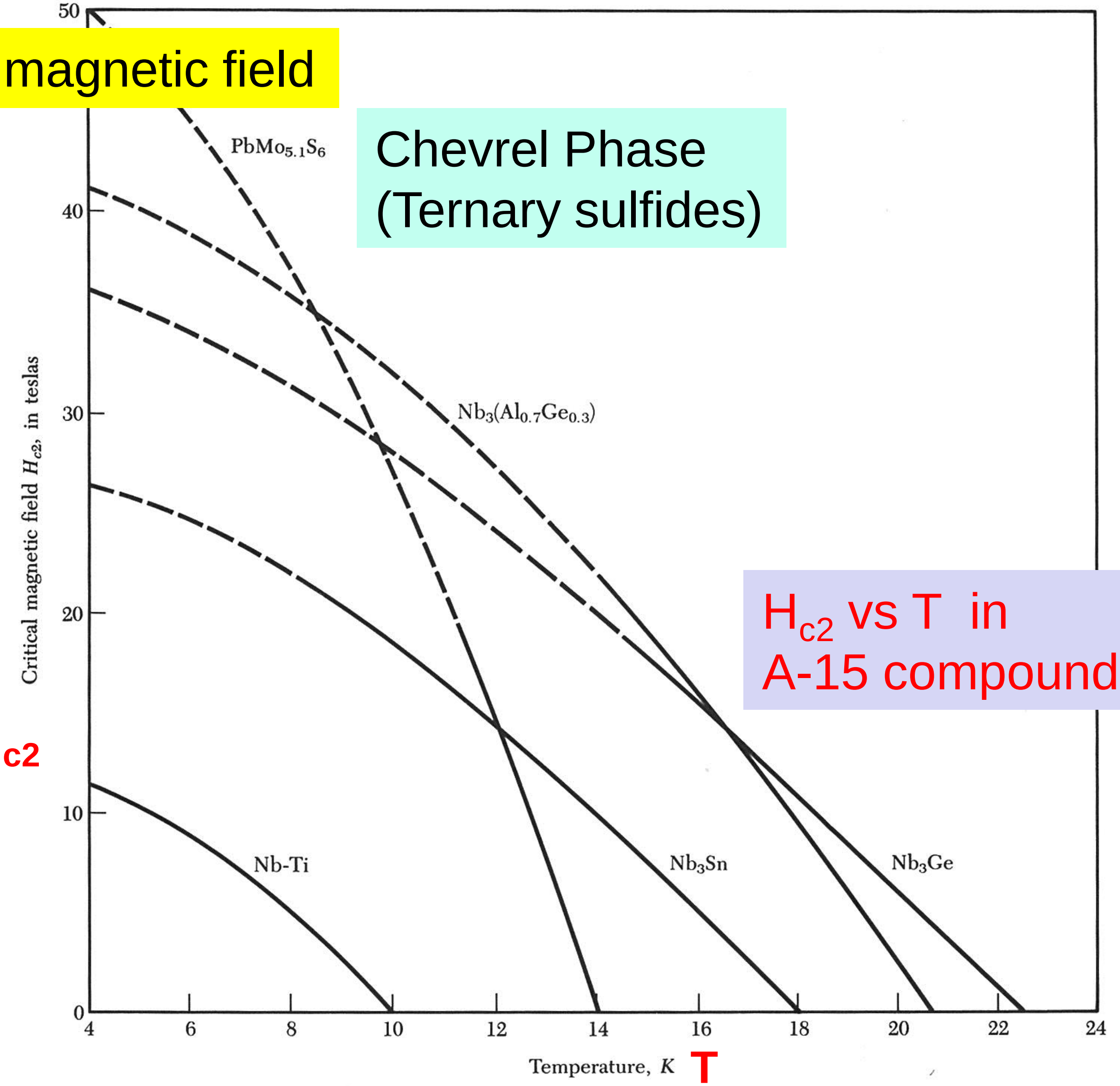


FIGURE 5-1
Structure of an isolated Abrikosov vortex in a material with $\kappa \approx 8$. The maximum value of $h(r)$ is approximately $2H_{c1}$.

by pulsed magnetic field

Chevrel Phase
(Ternary sulfides)

H_{c2} vs T in
A-15 compound



H_{c2}

T

Figure 5b Stronger magnetic fields than any now contemplated in practical superconducting devices are within the capability of certain Type II materials. These materials cannot be exploited, however, until their critical current density can be raised and until they can be fabricated as finely divided conductors. (Magnetic fields of more than about 20 teslas can be generated only in pulses, and so portions of the curves shown as broken lines were measured in that way.)

Entropy S vs T for Aluminum

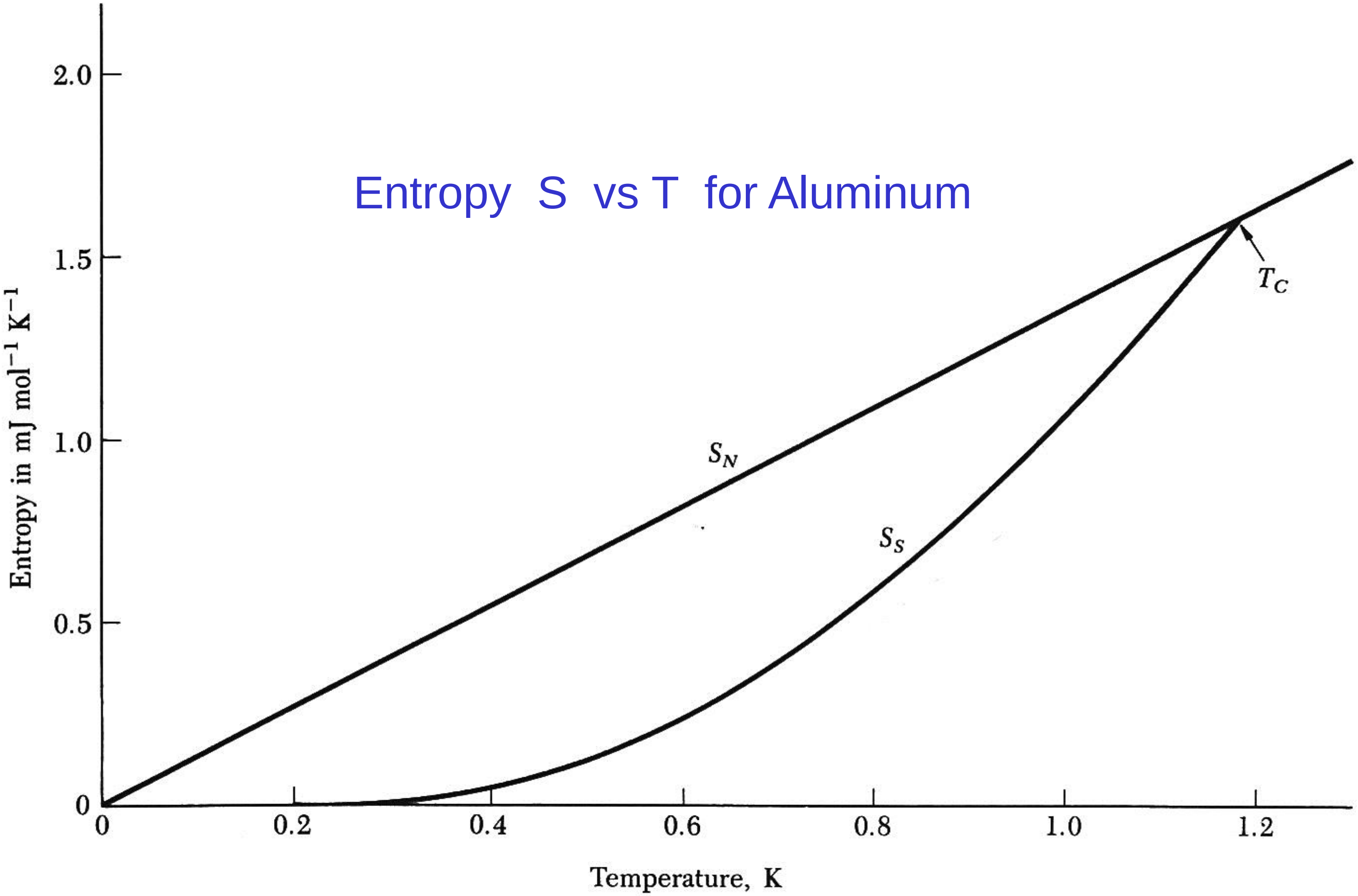
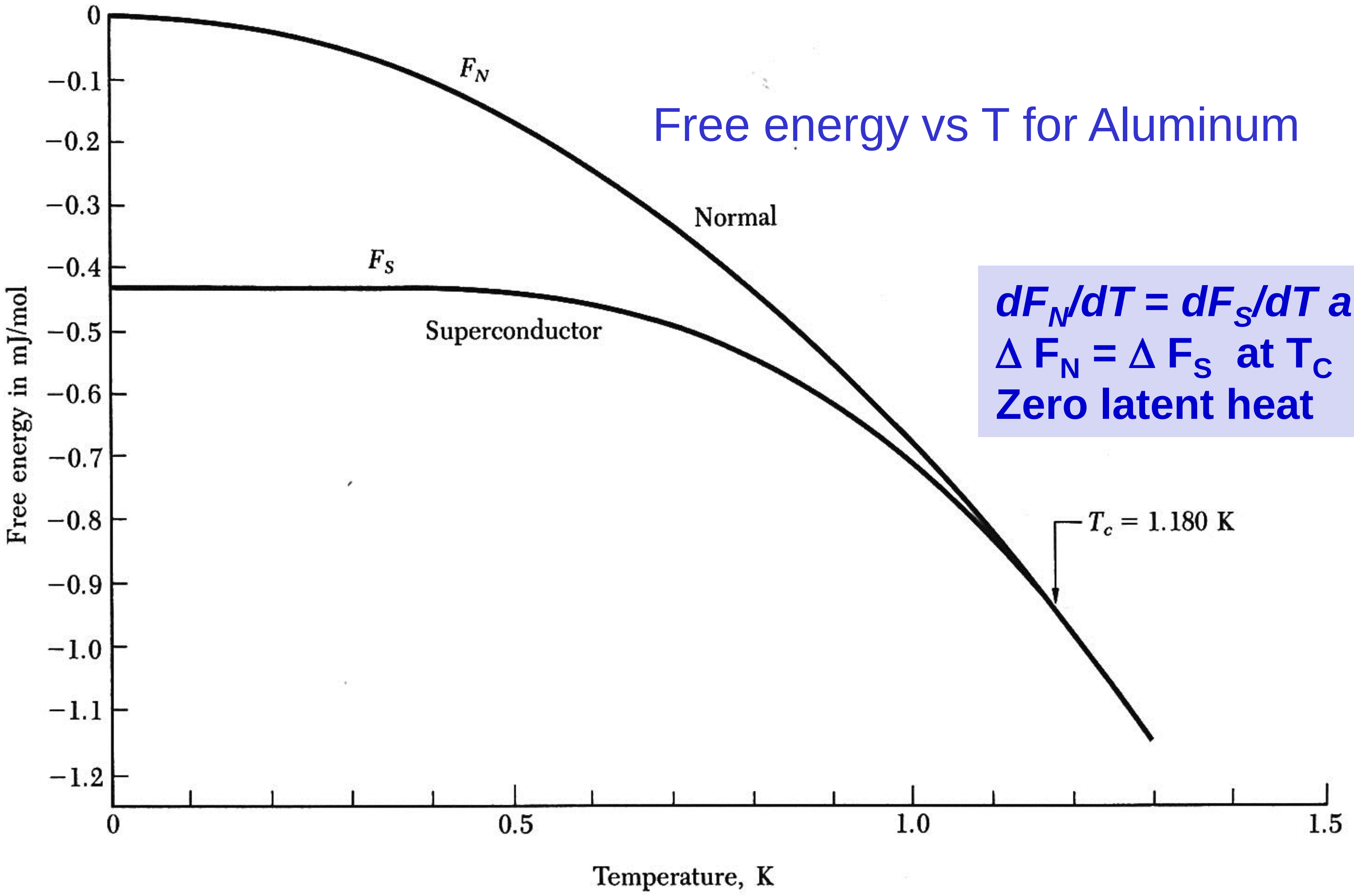


Figure 6 Entropy S of aluminum in the normal and superconducting states as a function of the temperature. The entropy is lower in the superconducting state because the electrons are more ordered here than in the normal state. At any temperature below the critical temperature T_c the specimen can be put in the normal state by application of a magnetic field stronger than the critical field.

The small entropy change must mean that only a small fraction (of the order of 10^{-4}) of the conduction electrons participate in the transition to the ordered superconducting state.

Free energy vs T for Aluminum



$dF_N/dT = dF_S/dT$ at T_C
$\Delta F_N = \Delta F_S$ at T_C
Zero latent heat

So that the phase transition is second order (there is no latent heat of transition at T_c).

heat capacity of an electron gas is

$$C_{el} = \frac{1}{3}\pi^2 D(\epsilon_F) k_B^2 T \quad . \qquad (34)$$

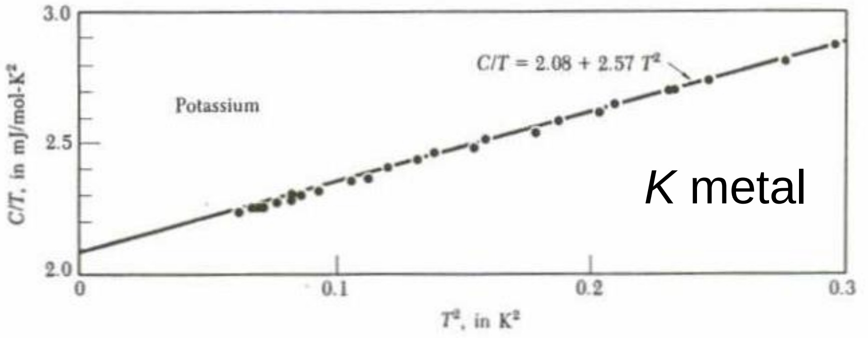
$$D(\epsilon_F) = 3N/2\epsilon_F = 3N/2k_BT_F \qquad (35)$$

$$\boxed{C_{el} = \frac{1}{2}\pi^2 N k_B T/T_F \quad .} \qquad (36)$$

Compare with $C_V = 2Nk_BT/T_F$

T_F is called the Fermi temperature,

where $\mathcal{E}_F = k_BT_F$



K metal

Figure 9 Experimental heat capacity values for potassium, plotted as C/T versus T^2. (After W. H. Lien and N. E. Phillips.)

$\gamma = \frac{1}{2}\pi^2 N k_B T/T_F$ Since $\epsilon_F \propto T_F \propto 1/m$ $\therefore\ \gamma \propto m$

(See Eq. 17)

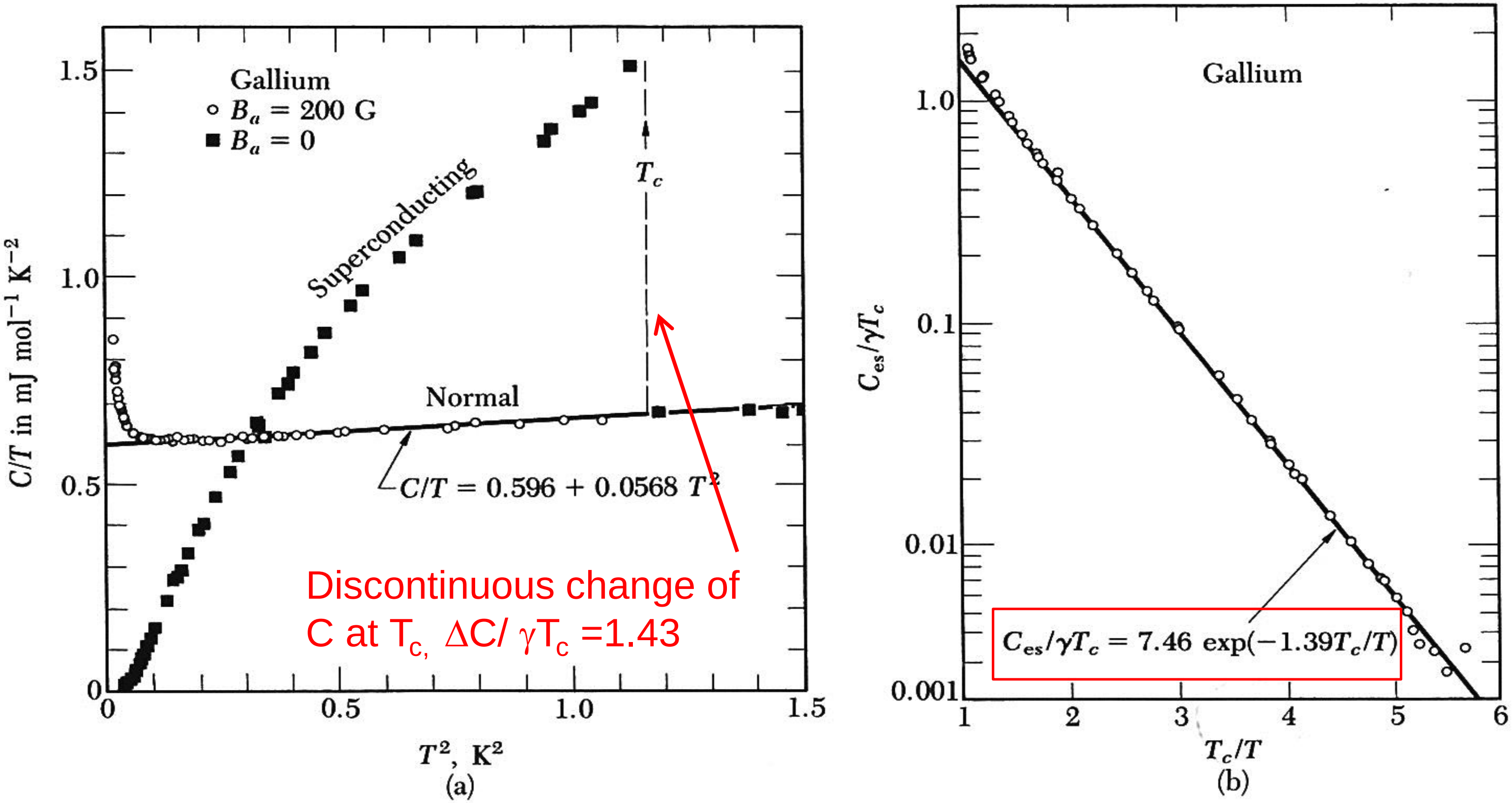
At temperatures much below both the Debye temperature and the Fermi temperature, the heat capacity of metals may be written as the sum of electron and phonon contributions: $C = \gamma T + AT^3$

$$\boxed{C/T = \gamma + AT^2 \quad ,} \qquad (37)$$

γ, called the Sommerfeld parameter

At low T, the electronic term dominates,

Heat Capacity of Ga at low T



Discontinuous change of C at $T_{c,}$ $\Delta C/\ \gamma T_c$ =1.43

$C_{es}/\gamma T_c = 7.46\ \exp(-1.39T_c/T)$

Figure 8 (a) The heat capacity of gallium in the normal and superconducting states. The normal state (which is restored by a 200 G field) has electronic, lattice, and (at low temperatures) nuclear quadrupole contributions. In (b) the electronic part C_{es} of the heat capacity in the superconducting state is plotted on a log scale versus T_c/T: the exponential dependence on $1/T$ is evident. Here $\gamma = 0.60$ mJ mol^{-1} deg^{-2}. (After N. E. Phillips.)

Electronic part of heat capacity in SC state: **$C_{es}/\gamma T_c \propto$ a exp (-b T_c/T)**

proportional to $-1/T$, suggestive of excitation of electrons across an en ergy gap

Energy Gap

In a superconductor the important interaction is the electron-electron interaction which orders the electrons in k space with respect to the Fermi gas of electrons.

the exponential factor in the electronic heat capacity of a superconductor is found to be $-E_g/2k_BT$

$$C_{es} = \gamma T_c \exp(-1.76\ T_c/T)$$

The transition in zero magnetic field from the superconducting state to the normal state is observed to be a second-order phase transition.

Energy Gap of superconductors in **Table 3**

$E_g(0)/k_BT_c = 3.52$ Weak electron-phonon coupling

$E_g(0)/k_BT_c > 3.52$ Strong electron-phonon coupling

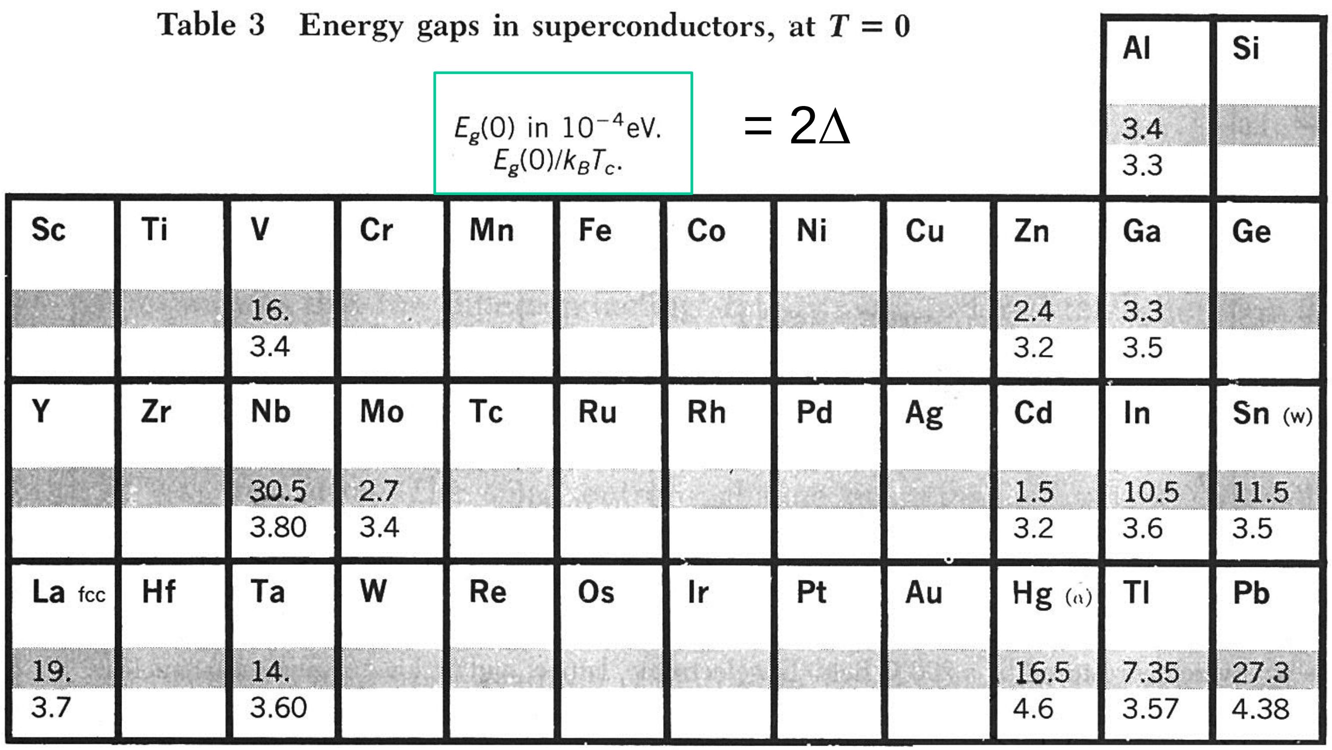
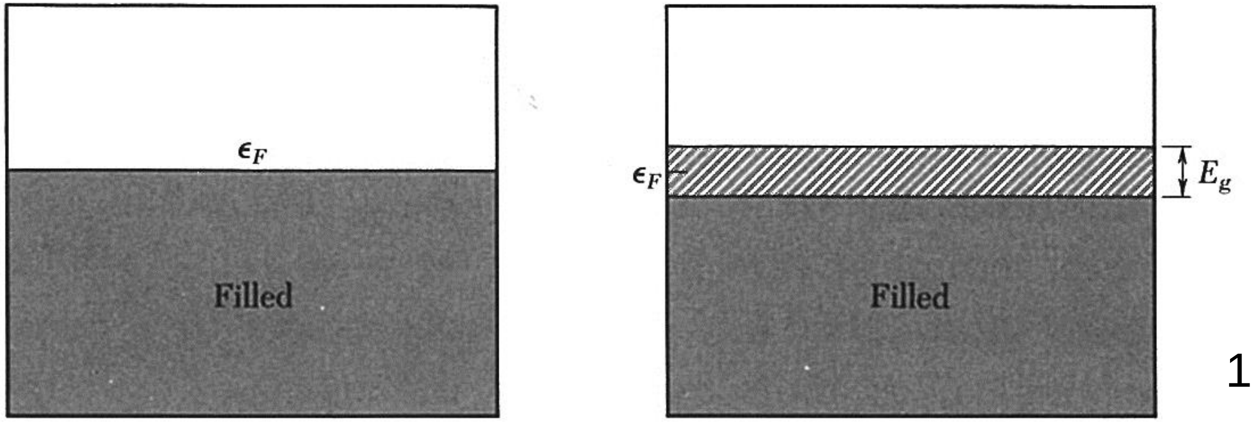


Table 3 Energy gaps in superconductors, at $T = 0$

$E_g(0)$ in 10^{-4}eV.
$E_g(0)/k_BT_c$.

$= 2\Delta$

										Al 3.4 3.3	**Si**
Sc	**Ti**	**V** 16. 3.4	**Cr**	**Mn**	**Fe**	**Co**	**Ni**	**Cu**	**Zn** 2.4 3.2	**Ga** 3.3 3.5	**Ge**
Y	**Zr**	**Nb** 30.5 3.80	**Mo** 2.7 3.4	**Tc**	**Ru**	**Rh**	**Pd**	**Ag**	**Cd** 1.5 3.2	**In** 10.5 3.6	**Sn** (w) 11.5 3.5
La fcc 19. 3.7	**Hf**	**Ta** 14. 3.60	**W**	**Re**	**Os**	**Ir**	**Pt**	**Au**	**Hg** (α) 16.5 4.6	**Tl** 7.35 3.57	**Pb** 27.3 4.38



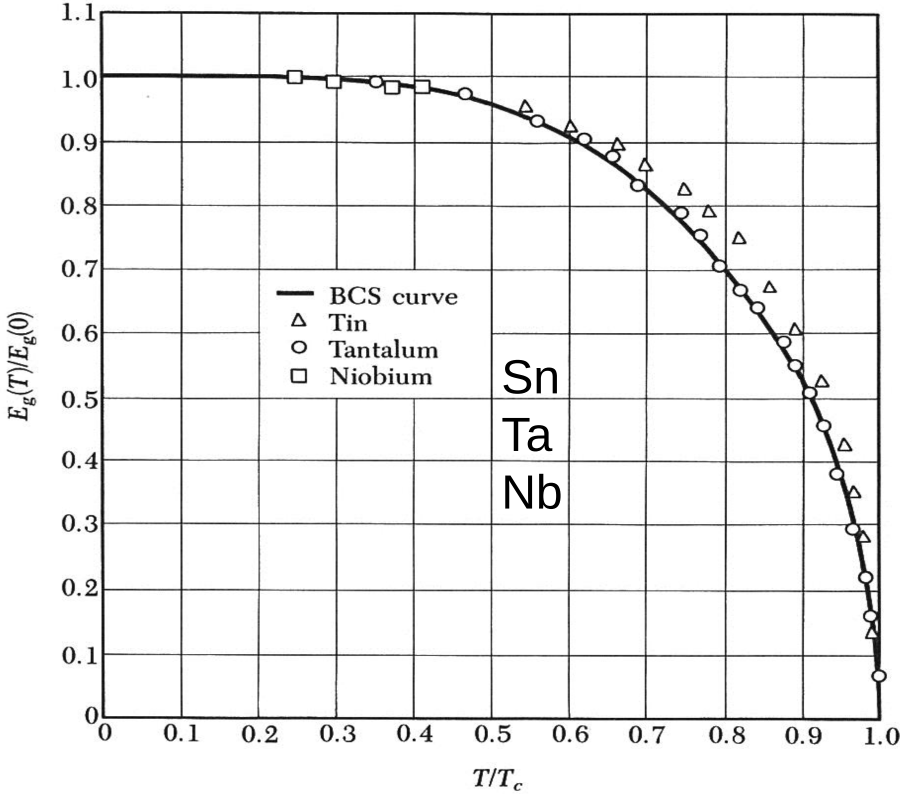
Normal
(a)

Superconductor
(b)

$E_g \sim 10^{-4}\ \varepsilon_F$

1-5 meV

3-10 eV



$$\Delta(T)/\Delta(0) = (1 - T/T_c)^{1/2}$$

Mean field theory

Figure 10 Reduced values of the observed energy gap $E_g(T)/E_g(0)$ as a function of the reduced temperature T/T_c, after Townsend and Sutton. The solid curve is drawn for the BCS theory.

$E_g(T)$ as the order parameter, goes smoothly to zero at T_c
 -- second order phase transition

Isotope Effect

It has been observed that the critical temperature of superconductors varies with isotopic mass.

The experimental results within each series of isotopes may be fitted by a relation of the form

$$\boxed{M^{\alpha}T_c = \text{constant} \qquad \alpha \sim 0.5}$$

Table 4 Isotope effect in superconductors

Experimental values of α in $M^{\alpha}T_c$ = constant, where M is the isotopic mass.

Substance	α	Substance	α
Zn	0.45 ± 0.05	Ru	0.00 ± 0.05
Cd	0.32 ± 0.07	Os	0.15 ± 0.05
Sn	0.47 ± 0.02	Mo	0.33
Hg	0.50 ± 0.03	Nb_3Sn	0.08 ± 0.02
Pb	0.49 ± 0.02	Zr	0.00 ± 0.05

From the dependence of T_c on the isotopic mass we learn that lattice vibrations and hence electron-lattice interactions are deeply involved in superconductivity.

$\theta \propto \upsilon \propto M^{-1/2}$ $\quad$ $T_c \propto \theta_{\text{Debye}} \propto M^{-1/2}$, so that $\alpha = \frac{1}{2}$