Chapter 10: Superconductivity

References:

1. C. Kittel ; Introduction to solid state physics
2. M. Tinkham: Introduction to superconductivity
3. Paul Hansma, Tunneling Spectroscopy

Table of Contents

Periodic Table of Superconductivity

ambient pressure superconductor: T_c(K), T_c^{max}(K), P(GPa)

high pressure superconductor: T_c^{max}(K), P(GPa)

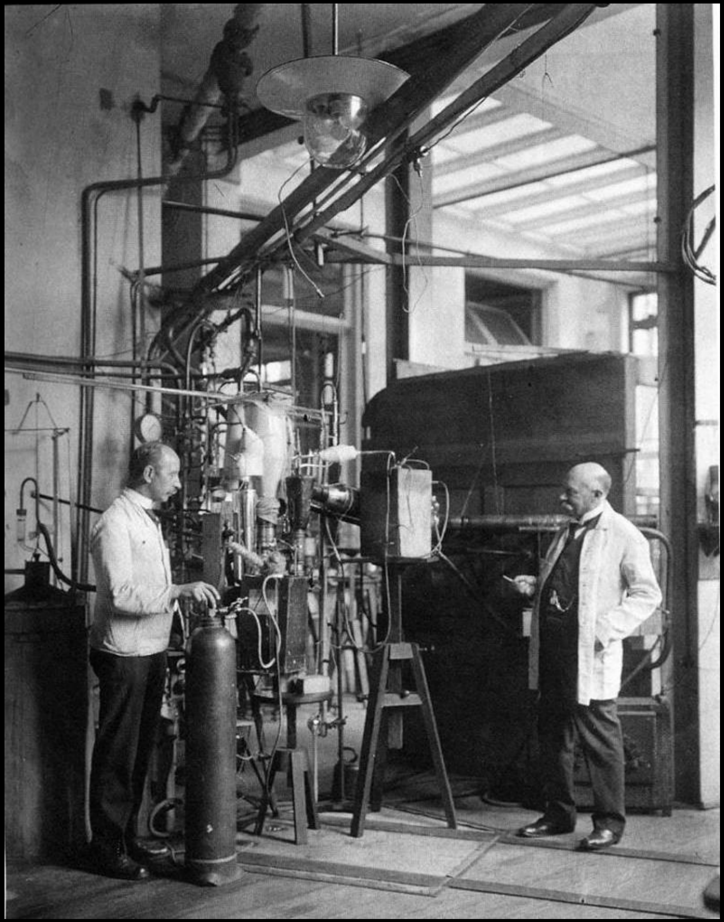
H																	He
Li 0.0004 14 30	Be 0.026 3.7 30											B 11 250	C	N	O 0.6 100	F	Ne
Na	Mg											Al 1.14	Si 8.2 15.2	P 13 30	S 17.3 190	Cl	Ar
K	Ca 29 217	Sc 19.6 106	Ti 0.39 3.35 56.0	V 5.38 16.5 120	Cr	Mn	Fe 2.1 21	Co	Ni	Cu	Zn 0.875	Ga 1.091 7 1.4	Ge 5.35 11.5	As 2.4 32	Se 8 150	Br 1.4 100	Kr
Rb	Sr 7 50	Y 19.5 115	Zr 0.546 11 30	Nb 9.50 9.9 10	Mo 0.92	Tc 7.77	Ru 0.51	Rh .00033	Pd	Ag	Cd 0.56	In 3.404	Sn 3.722 5.3 11.3	Sb 3.9 25	Te 7.5 35	I 1.2 25	Xe
Cs 1.3 12	Ba 5 18	insert La-Lu	Hf 0.12 8.6 62	Ta 4.483 4.5 43	W 0.012	Re 1.4	Os 0.655	Ir 0.14	Pt	Au	Hg-α 4.153	Tl 2.39	Pb 7.193	Bi 8.5 9.1	Po	At	Rn
Fr	Ra	insert Ac-Lr	Rf	Ha													
		La-fcc 6.00 13 15	Ce 1.7 5	Pr	Nd	Pm	Sm	Eu 2.75 142	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu 12.4 174	
		Ac	Th 1.368	Pa 1.4	U 0.8(β) 2.4(α) 1.2	Np	Pu	Am 0.79 2.2 6	Cm	Bk	Cf	Es	Fm	Md	No	Lr	

M. Debessai *et al.*, J. Phys.: Conf. Series 215, 012034 (2010).

Experimental Survey of Superconductivity Phenomenon

Helium Liquefaction in 1908

July 10, 1908



Heike Kamerlingh Onnes

Nobel Prize, 1913

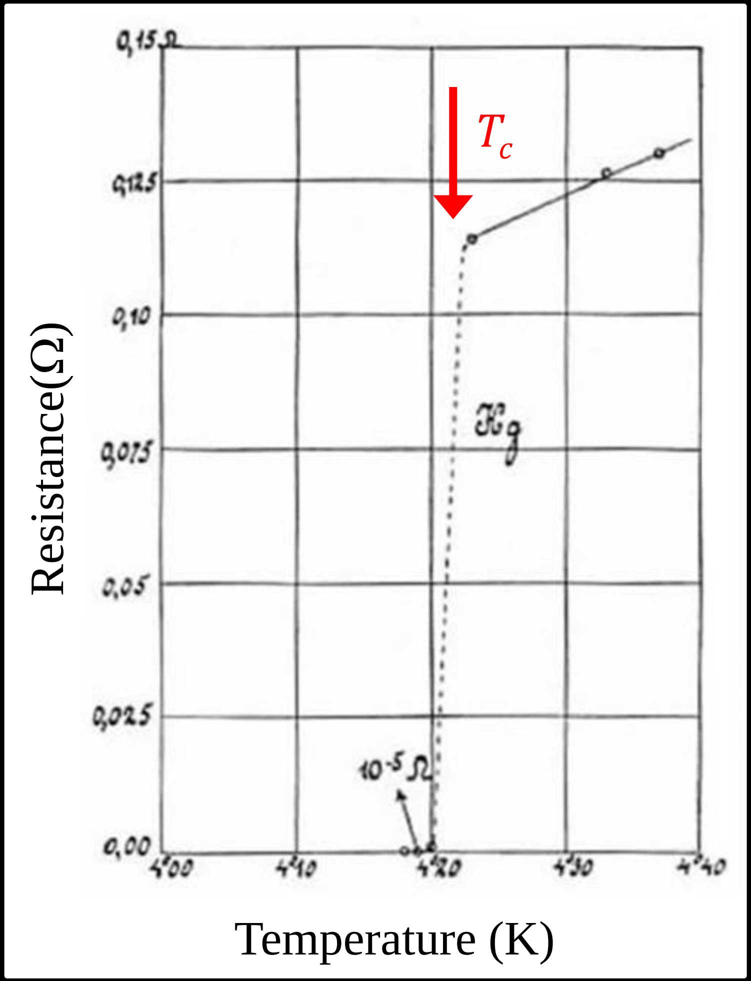
"Door meten tot weten"
(Knowledge through measurement)

WHAT IS A SUPERCONDUCTOR?

1. **Zero resistance**
2. **Complete expulsion of magnetic flux**

Discovery of Superconductivity in 1911

April 8, 1911



Heike Kamerlingh Onnes

Nobel Prize, 1913

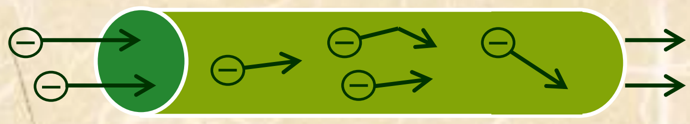
“Door meten tot weten”
(Knowledge through measurement)

H. Kamerlingh Onnes, *Commun. Phys. Lab. Univ. Leiden. Suppl.* **29** (Nov. 1911).

SUPERCONDUCTIVITY

Type of material | **What happens in a wire?** | **Result**

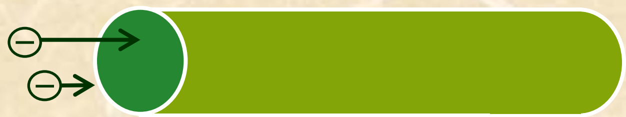
Conductor



Electrons flow easily (like water through a garden hose)

Collisions cause dissipation (heat)

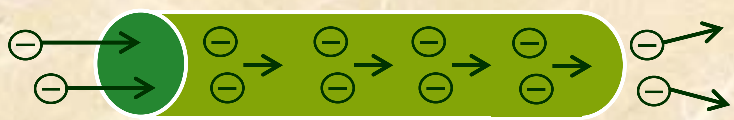
Insulator



Electrons are tightly bound no flow (like a hose plugged with cement)

No current flow at all

Superconductor



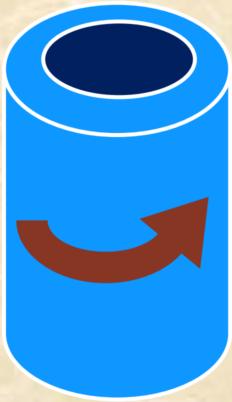
Electrons bind into pairs and cannot collide (a frictionless hose)

No collisions
No dissipation
No heat
No resistance

HOW SMALL IS THE RESISTANCE?

Copper Cylinder

1) Induce current
2) Current decays in about 1/1000 second

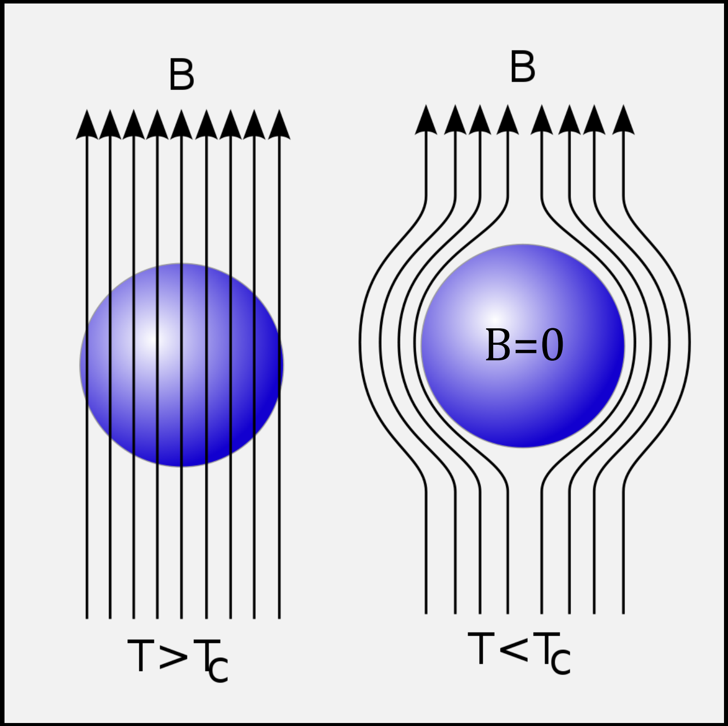


Superconducting Cylinder

1) Induce current
2) Current does not decay
   (less than 0.1% in a year)
   so, resistance is smaller than copper
   by $\dfrac{\text{1000 years}}{\text{1/1000 second}}$
   i.e., at least 1 trillion times!

The Meissner Effect in 1933

Perfect diamagnetism



Walther Meißner

Robert Ochsenfeld

Meissner Effect

$$\boxed{B = Ba + 4\pi M = 0\ ;} \qquad \boxed{\frac{M}{Ba} = -\frac{1}{4\pi}} \qquad \text{Eq.(1)}$$

Perfect Diamagnetism

The magnetic properties cannot be accounted for by the assumption that a superconductor is a normal conductor with zero electrical resistivity.

The result $B = 0$ cannot be derived from the characterization of a super-conductor as a medium of zero resistivity.

From Ohm's law, $\mathbf{E} = \rho\mathbf{j}$, we see that if the resistivity ρ goes to zero, while $\mathbf{j}$ is held finite, then $\mathbf{E}$ must be zero. By a Maxwell equation $d\mathbf{B}/dt$ is proportional to curl $\mathbf{E}$, so that zero resistivity implies $d\mathbf{B}/dt = 0$. This argument is not entirely transparent, but the result predicts that the flux through the metal cannot change on cooling through the transition. **The Meissner effect contradicts this result, and suggests that perfect diamagnetism is an essential property of the superconducting state.**

Type I superconductor Type II superconductor

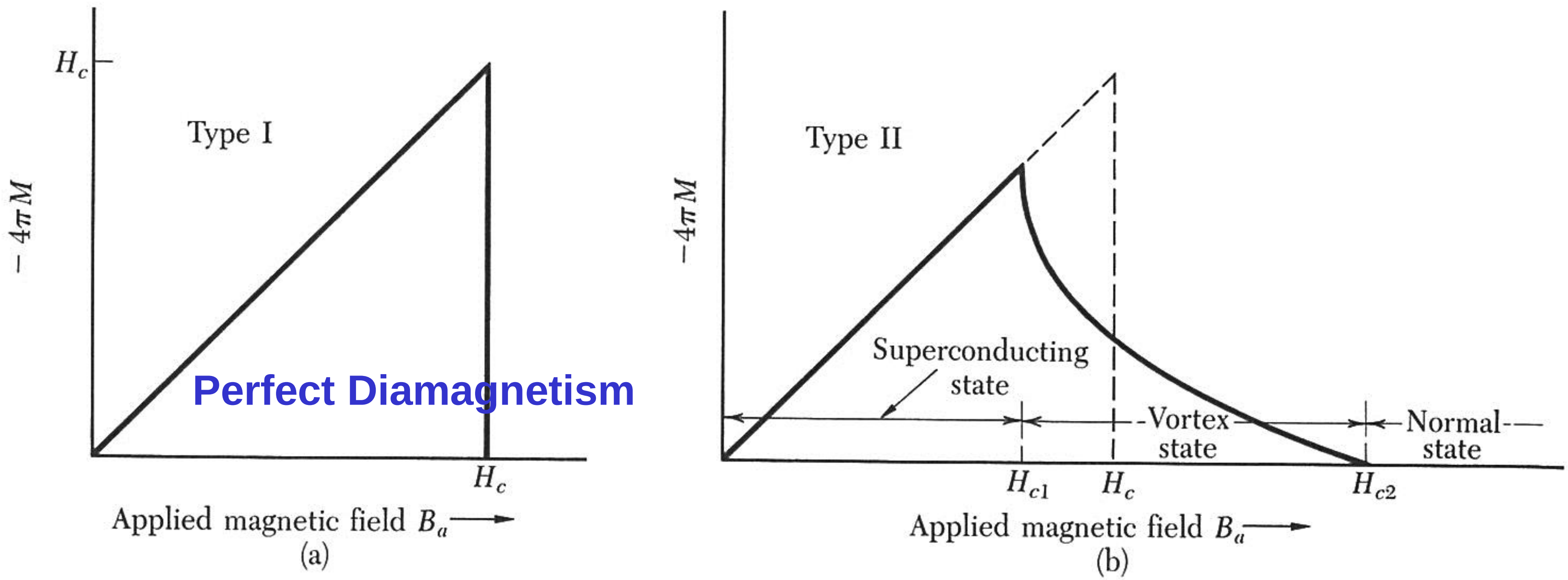
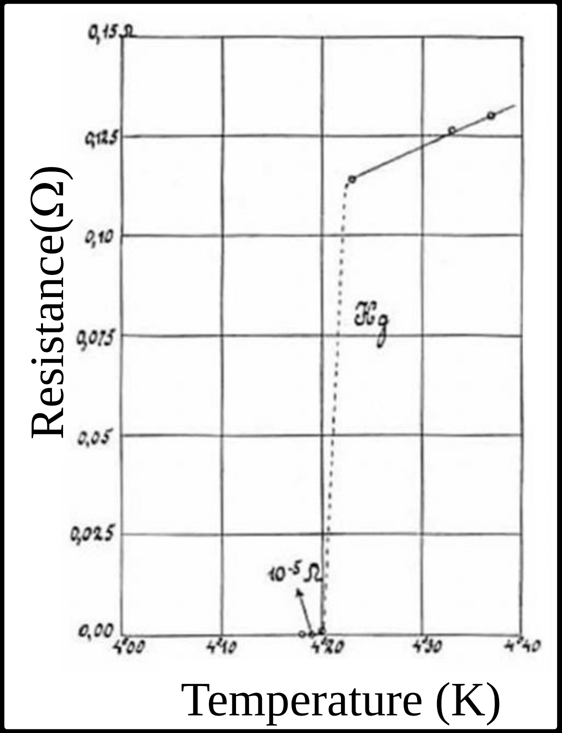
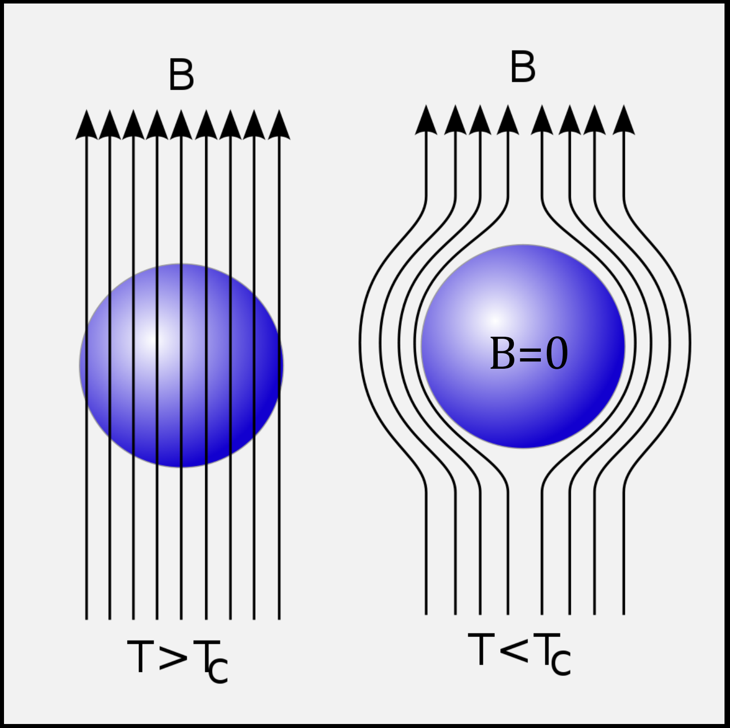


Figure 4 (a) Magnetization versus applied magnetic field for a bulk superconductor exhibiting a complete Meissner effect (perfect diamagnetism). A superconductor with this behavior is called a type I superconductor. Above the critical field H_c the specimen is a normal conductor and the magnetization is too small to be seen on this scale. Note that minus $4\pi M$ is plotted on the vertical scale: the negative value of M corresponds to diamagnetism. (b) Superconducting magnetization curve of a type II superconductor. The flux starts to penetrate the specimen at a field H_{c1} lower than the thermodynamic critical field H_c. The specimen is in a vortex state between H_{c1} and H_{c2}, and it has superconducting electrical properties up to H_{c2}. Above H_{c2} the specimen is a normal conductor in every respect, except for possible surface effects. For given H_c the area under the magnetization curve is the same for a type II superconductor as for a type I. (CGS units in all parts of this figure.)

Basic Properties of Superconductors

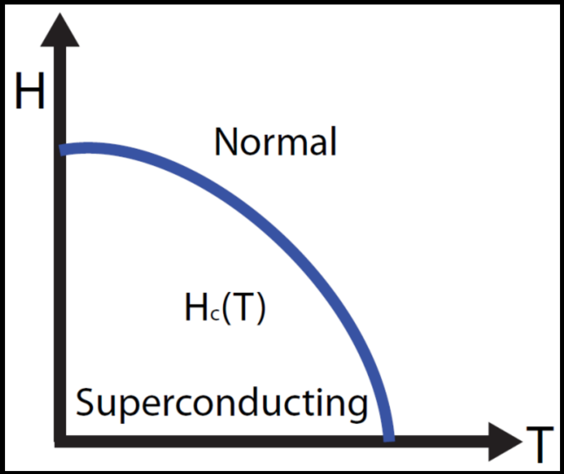
Zero electrical resistance + Meissner effect



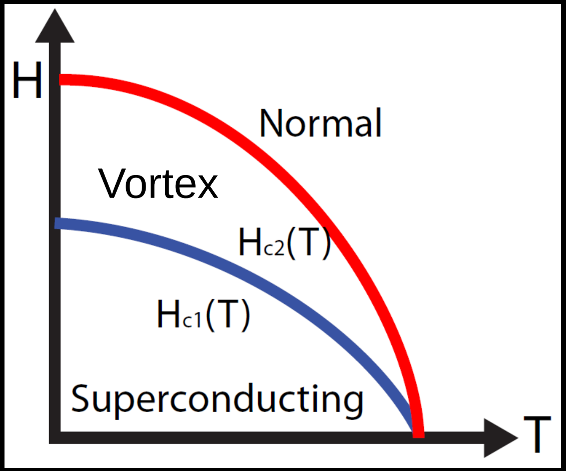


Type I & II Superconductors

Type I : Al, Pb…



Type II : Nb, NbTi, Nb_3Sn and HTSC



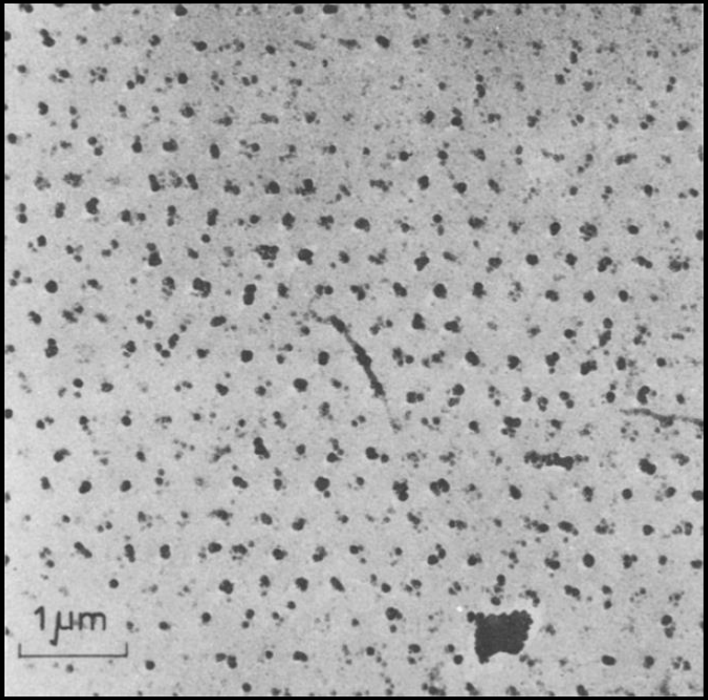
Lev V. Shubnikov

found type-II SC in Pb-Bi alloy in 1935.

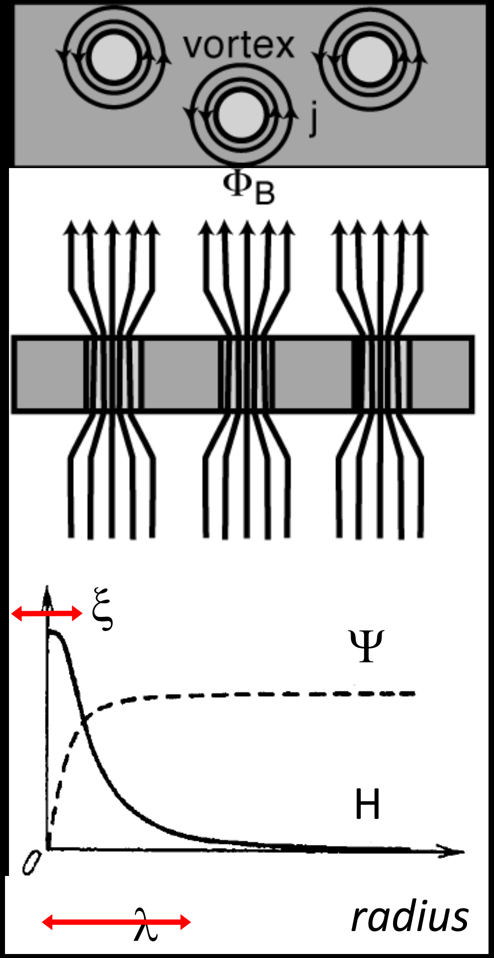
J. N. Rjabinin, L.W. Schubnikow, Physikalische Zeitschrift der Sowjetunion 7, 122 (1935)

Superconducting Vortices in type II SC

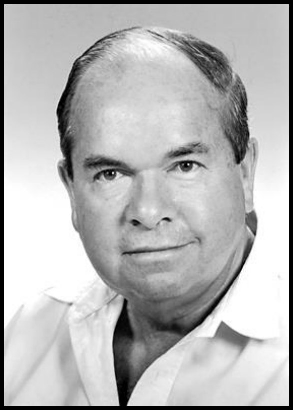
Decoration image of vortex lattice



U. Essmann and H. Trauble, Physics Letters 24A, 526 (1967)



Alexei A. Abrikosov



Nobel Prize 2003

A. A. Abrikosov, Doklady Akademii Nauk SSSR 86, 489 (1952)
A. A. Abrikosov, Sov. Phys. JETP **5**,1174 (1957)

Why Superconductivity is so fascinating ?

- Fundamental SC mechanism
- Novel collective phenomenon at low temp
- Applications

Bulk: - Persistent current, power storage
- Magnetic levitation
- High field magnet, MRI

Electronics:
- SQUID magnetometer
- Josephson junction electronics

POSSIBLE IMPACT OF SUPERCONDUCTIVITY

- **Energy**
  - Superconductivity generators & motors
  - Power transmission & distribution
  - Energy storage systems
  - Magnets for fusion power
  - Magnets for magneto-hydrodynamic power

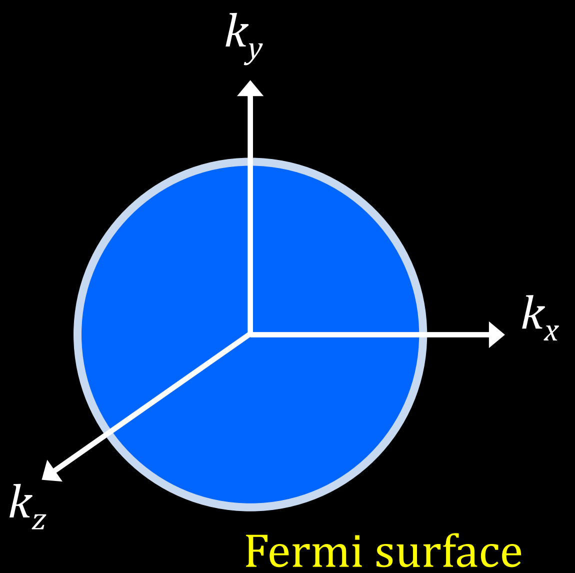
- **Transportation**
  - Magnets for levitated trains
  - Electro-magnetic powered ships
  - Magnets for automobiles

- **Health care**
  - Magnetic resonance imaging

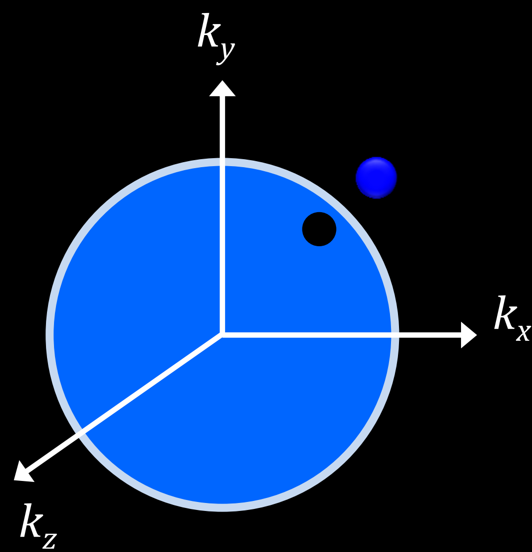
Normal Metallic State

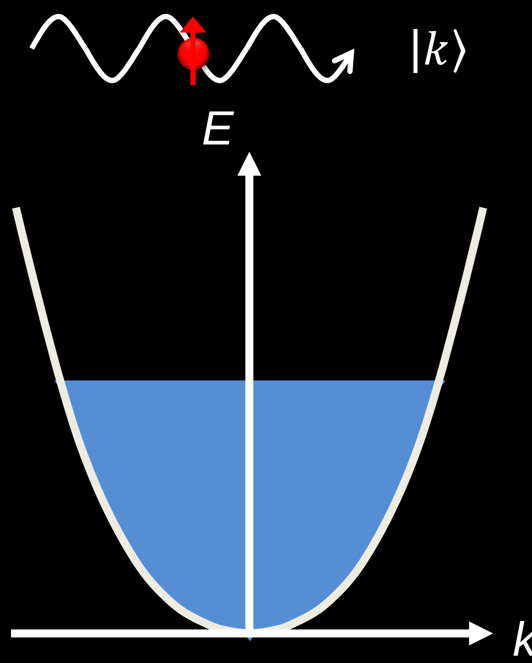
Electrons in wave-like states in momentum-space (k-space)

Free electron gas



Fermi liquid



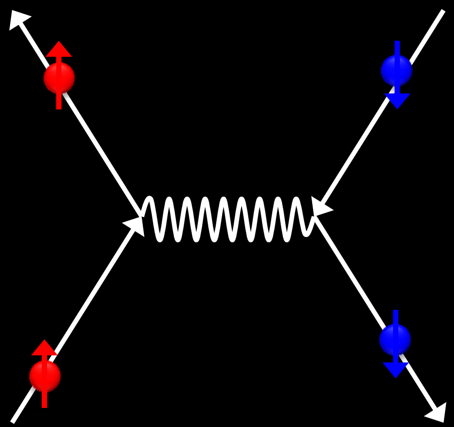


$$p = \hbar k = h/\lambda$$

$$E = \frac{\hbar^2 k^2}{2m}$$

BCS Theory in 1957 for Low T_c Superconductivity

Cooper Pairs



Exchange boson:
Lattice Vibration Mode

Microscopic theory for SC

John Bardeen

Leon Cooper

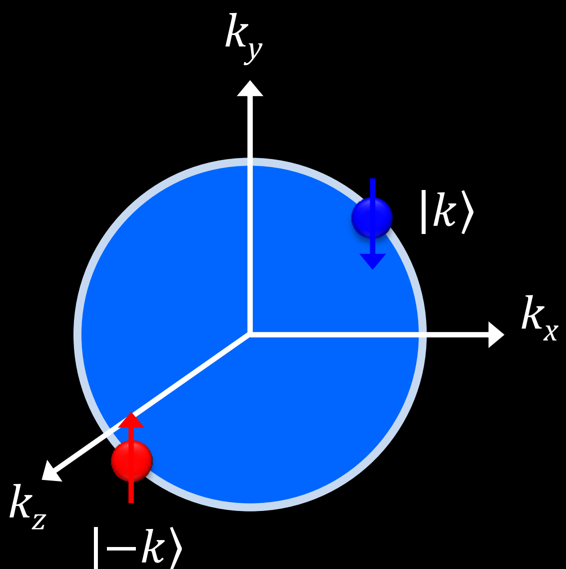
Robert Schrieffer

Nobel Prize 1972

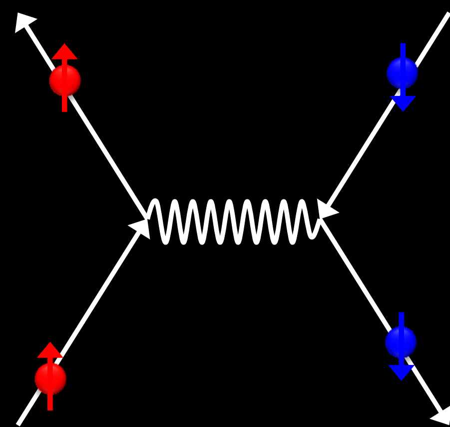
J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957)

Superconducting Ground State

Normal state

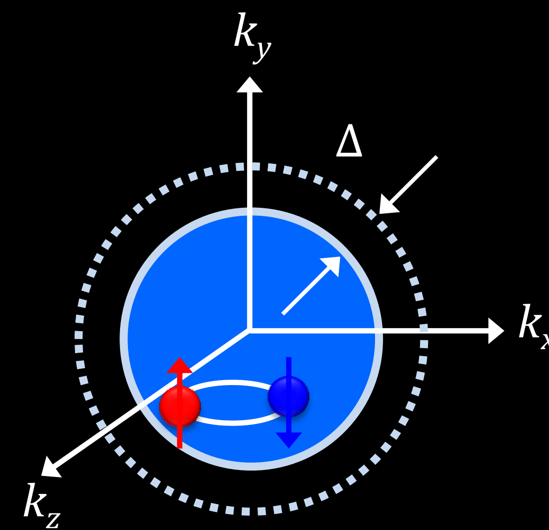


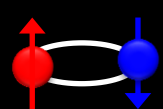
Cooper Pairs



Exchange boson:
Lattice Vibration Mode

Superconducting
ground state





- Spin singlet
- L=0; S=0
- Binding energy: Δ

Fundamental Mechanism

The superconducting state is an ordered state of the conduction electrons of the metal.

Electron-Phonon Coupling

Cooper Pair formed by two electrons ***k*****,** and ***–k*** with opposite spins near the Fermi level, as coupled through **phonons** of the lattice

The nature and origin of the ordering was explained by Bardeen, Cooper, and Schrieffer.[3]

BCS Theory, 1957

J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **106**, 162 (1957); **108**, 1175 (1957).

The Discovery of Superconductivity

- Early 90's -- elemental *SP* metals like *Hg, Pb, Al, Sn, Ga*, etc.
- Middle 90's -- transitional metals, alloys, and compounds like Nb, NbN, Nb_3Sn, etc.
- Late 90's -- perovskite oxides

Table 2 Superconductivity of selected compounds

Compound		T_c, in K	Compound		T_c, in K
Nb_3Sn	A-15	18.05	V_3Ga		16.5
Nb_3Ge		23.2	V_3Si		17.1
Nb_3Al		17.5	$YBa_2Cu_3O_{6.9}$	HTSC	90.0
NbN	B1	16.0	Rb_2CsC_{60}		31.3
K_3C_6O		19.2	La_3In		10.4

A-15 compound A_3B, with T_c = 15-23 K

In the so called β–W structure
With three perpendicular linear chains of ***A*** atoms on the cubic face,
and B atoms are at body centered cubic site,
With the presence of a sharp peak of *N(E) at* E_F

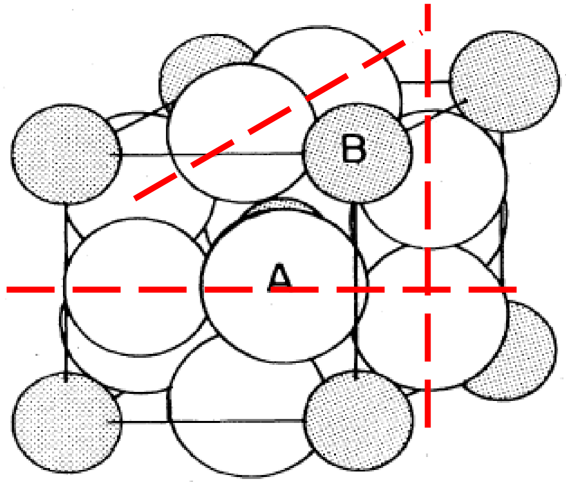


FIG. 1. The A-15 (or β-W) crystal structure for the compound formula A_3B. For the high T_c superconductors, A is a transition metal (usually V or Nb) and B is usually (but not always) a nontransition metal (e.g., Si, Ge, Sn, Al, Ga).

1973 discovery of Nb_3Ge, 23K !
how about Nb_3Si ??

Low temperature Superconductors

-- Mediated by Electron phonon coupling

-- McMillian formula for T_c

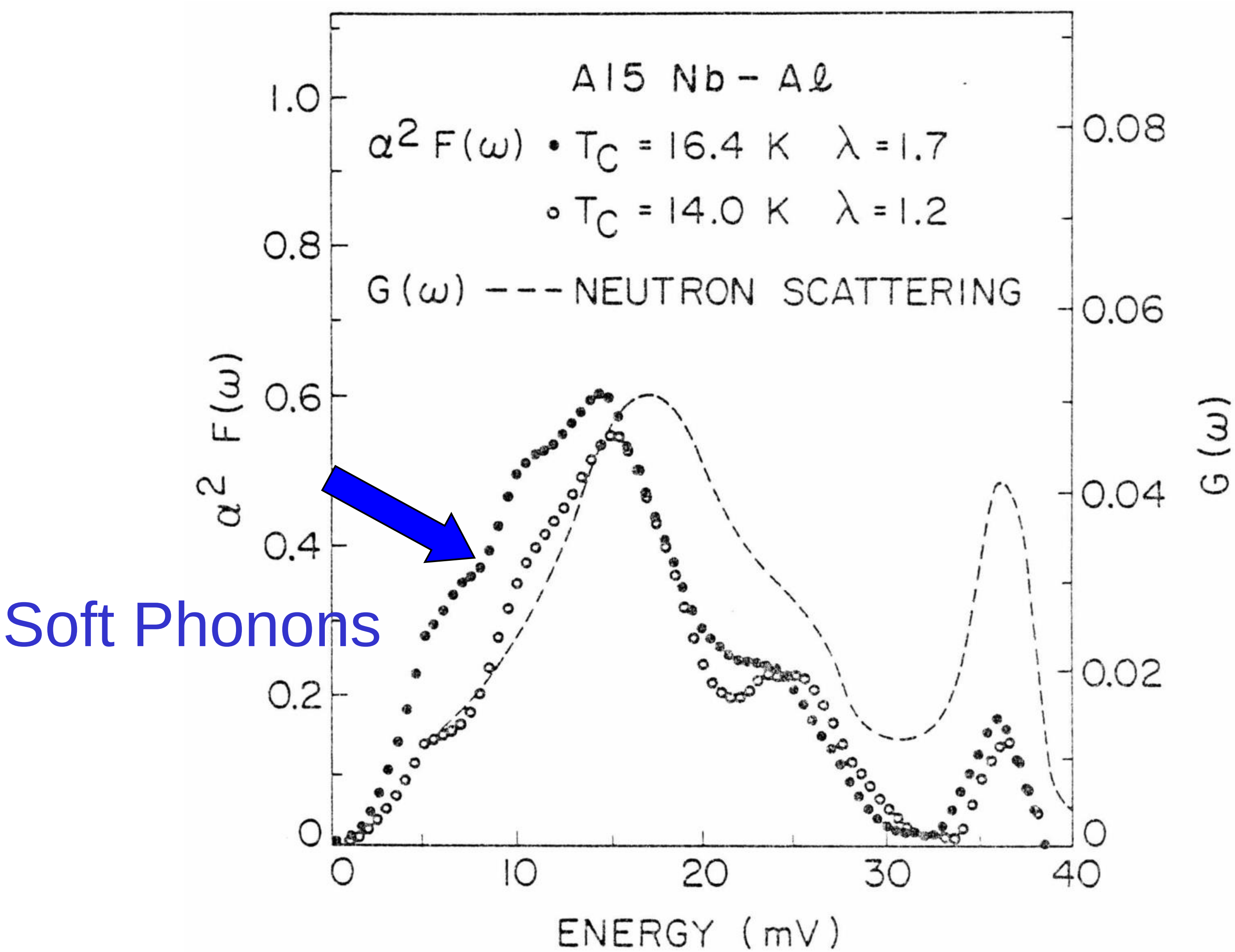
$$T_c = \frac{\Theta_D}{1.45} \exp\left\{ - \left[\frac{(1+\lambda_{ep})}{\lambda_{ep} - \mu^*(1+0.62\lambda_{ep})} \right] \right\}$$

λ : electron phonon coupling constant
μ^* : Coulomb repulsion of electrons

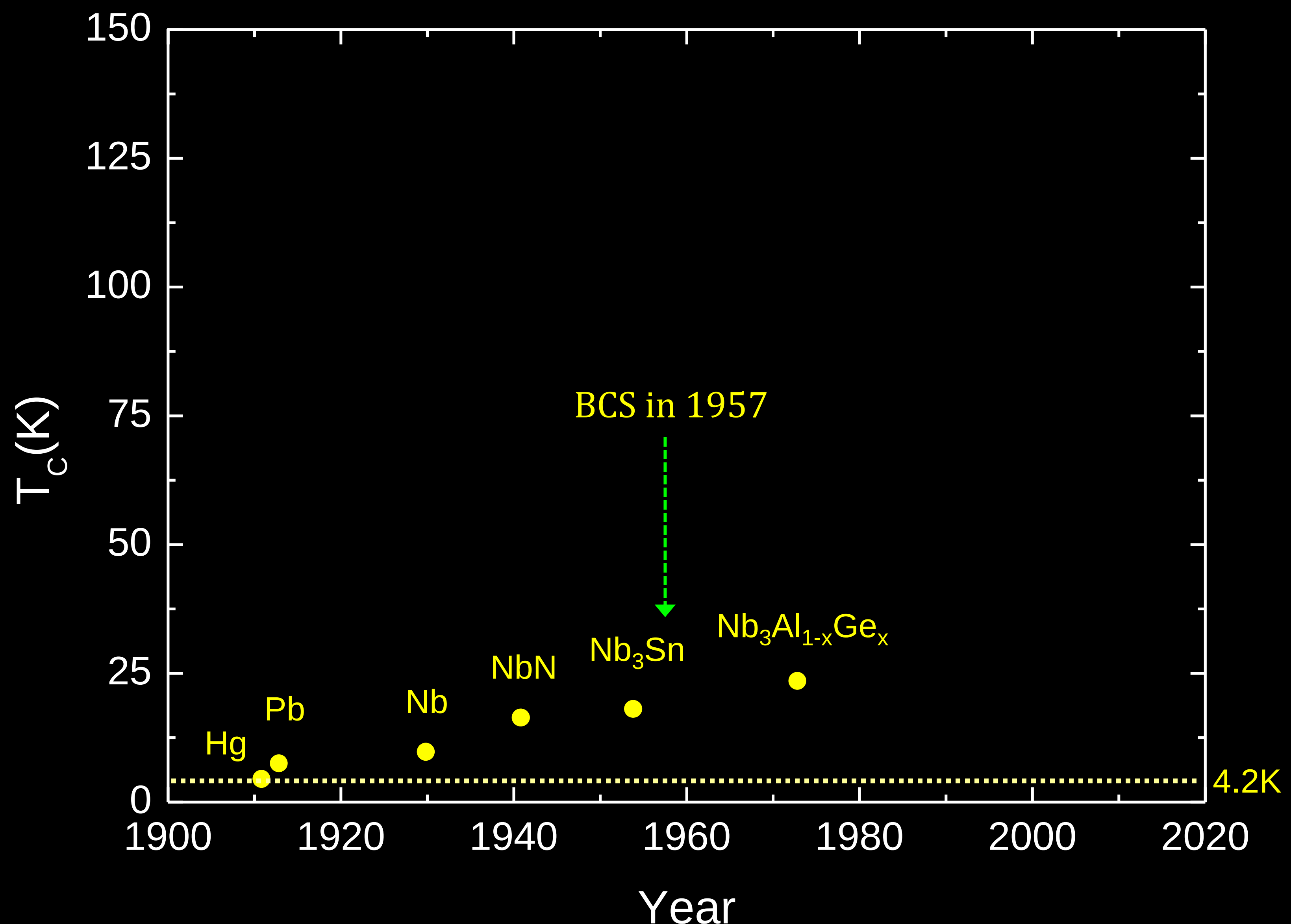
$\lambda \propto N(0) < I^2 > / \omega^2$

Are electrons or phonons more important?

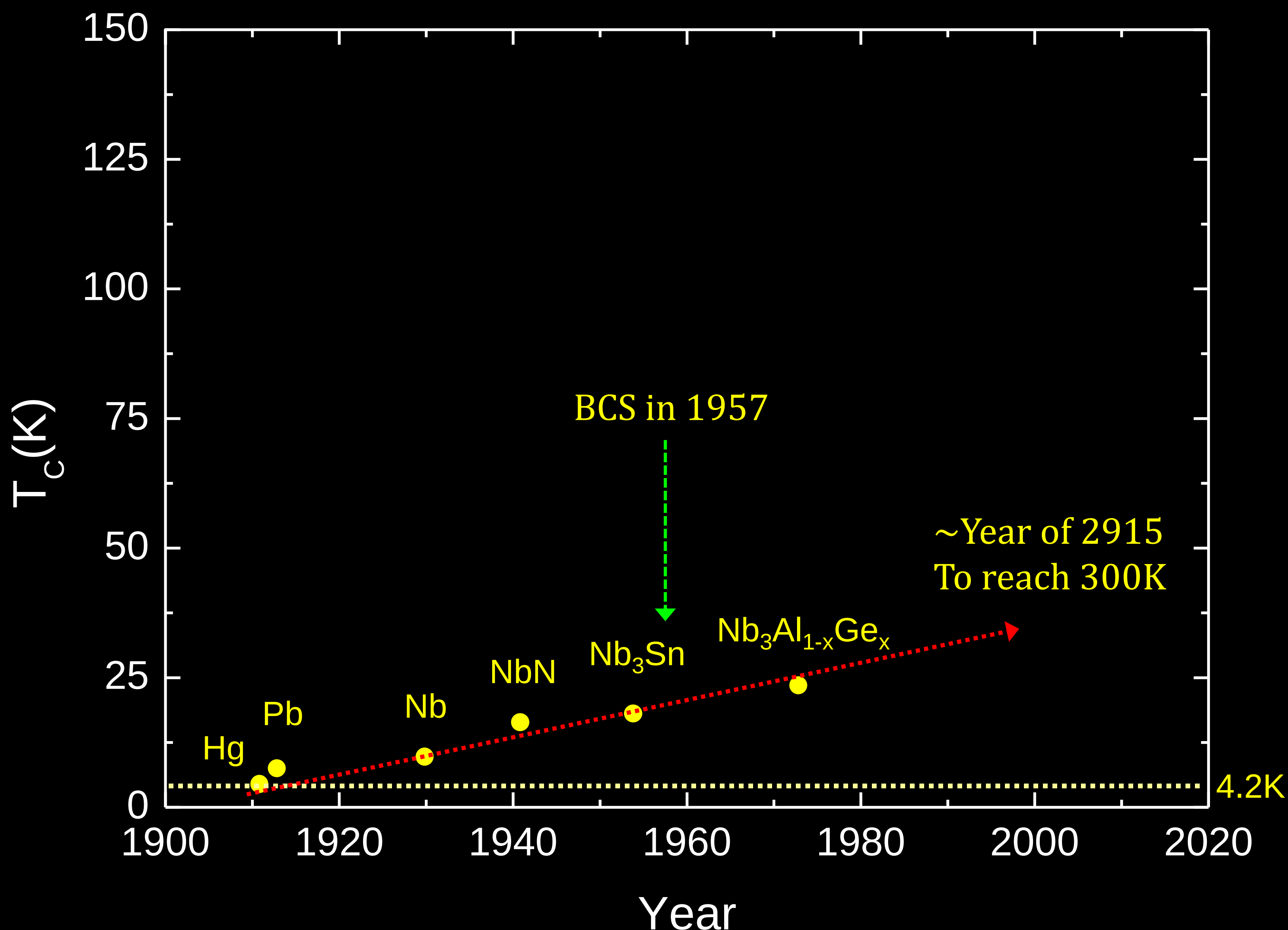
The Phonon Spectrum of the low T_c A-15 compound Nb_3Al



History of Conventional SC



History of Conventional SC



Can we raise the T_c higher than 30K?

Are we reaching the limitation of the BCS Theory ?

Matthias's Rules for Searching High T_C SC

Bernd Matthias

1. Stay away from insulators; transition metals are better.
2. There are favorable electron/atom ratios.
3. High symmetry is good; cubic symmetry is best.
4. Stay away from Oxygen
5. Stay away from magnetism
6. Stay away from theorists.

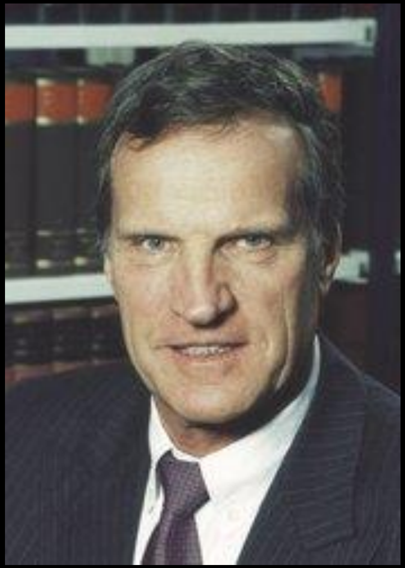
W. E. Pickett , Physica B **296**, 112 (2001)
I. I. Mazin, Nature **464**, 183 (2010)

A legacy of Superconductivity

Stanford, April, 2015

The Beginning of Unconventional SC: Heavy Fermion SC

Enormous effective mass of their charge carriers. This is achieved by a sharp spike in the DOS at the Fermi surface, to as much as 1000 times the density of states in Cu.

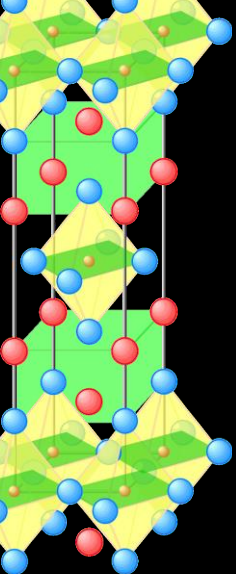


Frank Steglich

F. Steglich *et al.*, Phys. Rev. Lett. 43, 1892 (1979)

Breakthrough in late 1986
By Bednorz and Muller

Start the HTSC Era !



Discovery of High T_c Cuprates

Possible High T_c Superconductivity in the Ba – La – Cu – O System

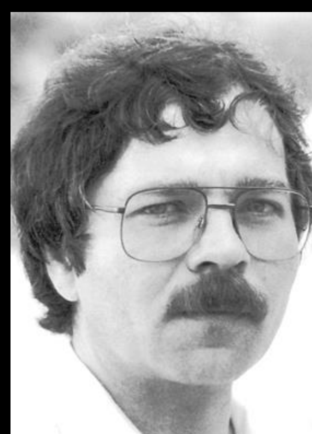
J.G. Bednorz and K.A. Müller
IBM Zürich Research Laboratory, Rüschlikon, Switzerland

Received April 17, 1986

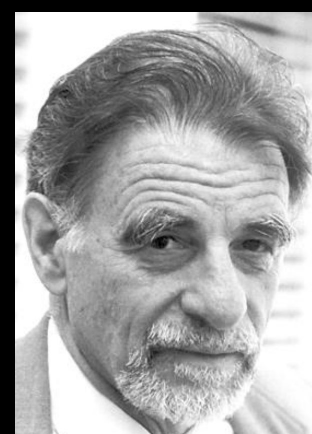
Z. Phys. B – Condensed Matter 64,189 (1986)

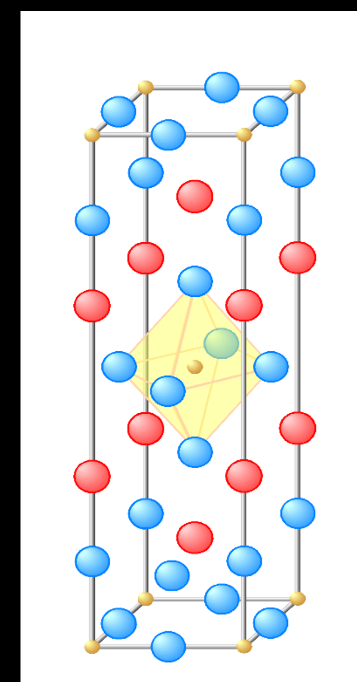
$La_{2-x}Ba_xCuO_4$, T_c=30K

J. Georg Bednorz



K. Alex Müller





Nobel Prize 1987

Discovery of High T_c Cuprates

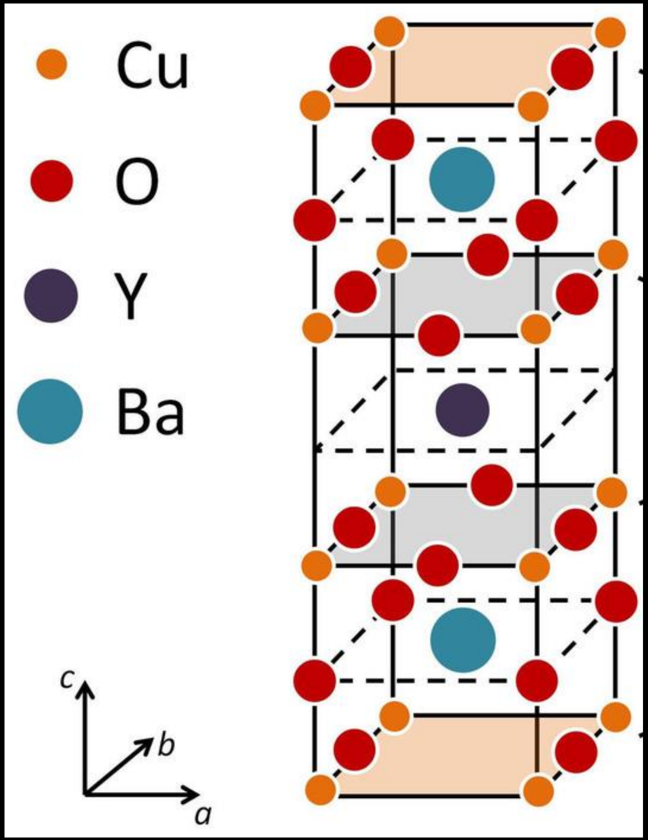
$T_c > 77K$!

朱經武

吳茂昆

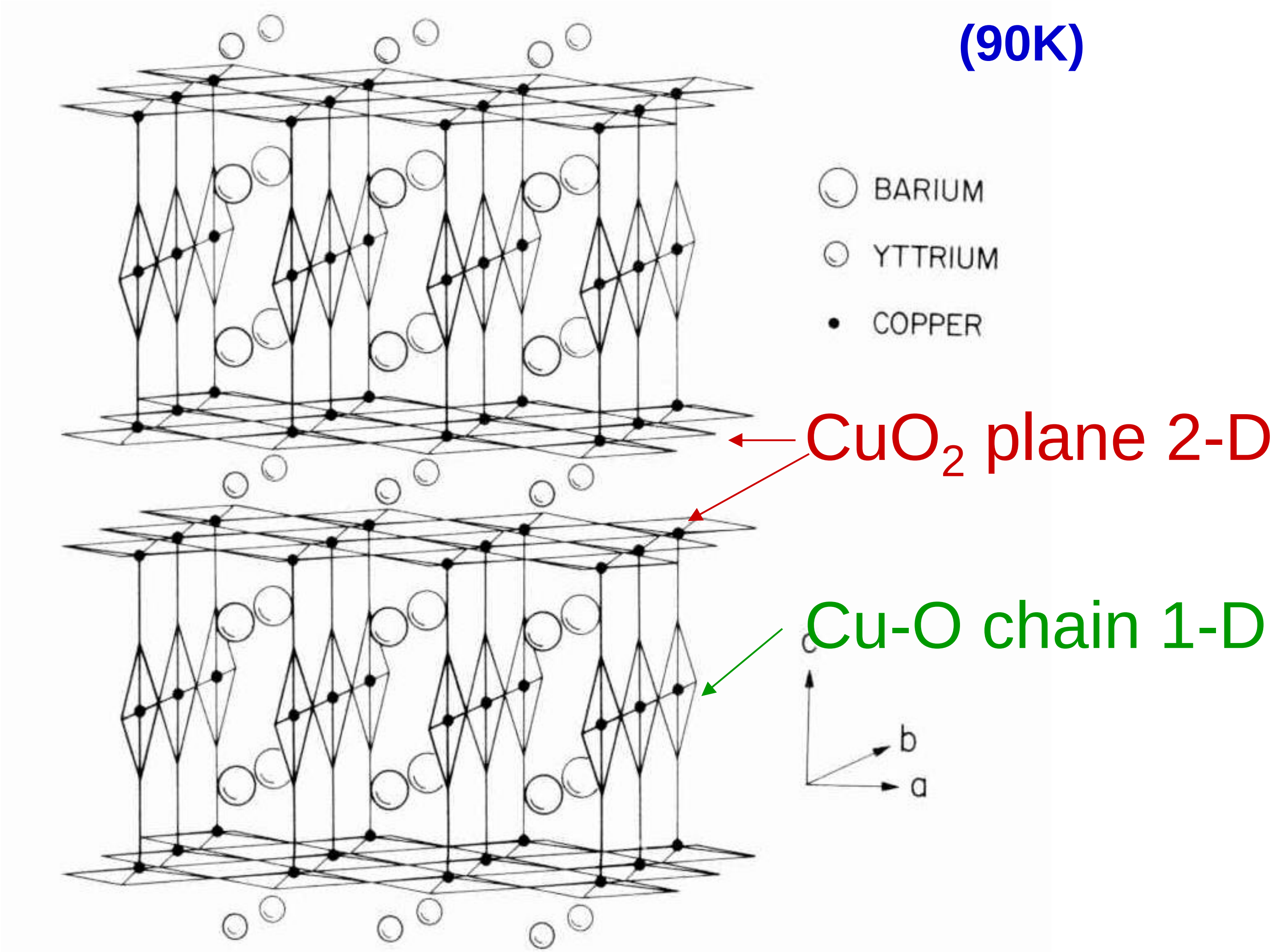
M. K. Wu *et al.*, PRL **58**, 908 (1987)

$YBa_2Cu_3O_{7-\delta}$, $T_c \sim 93K$



High Temperature Superconductor $YBa_2Cu_3O_7$

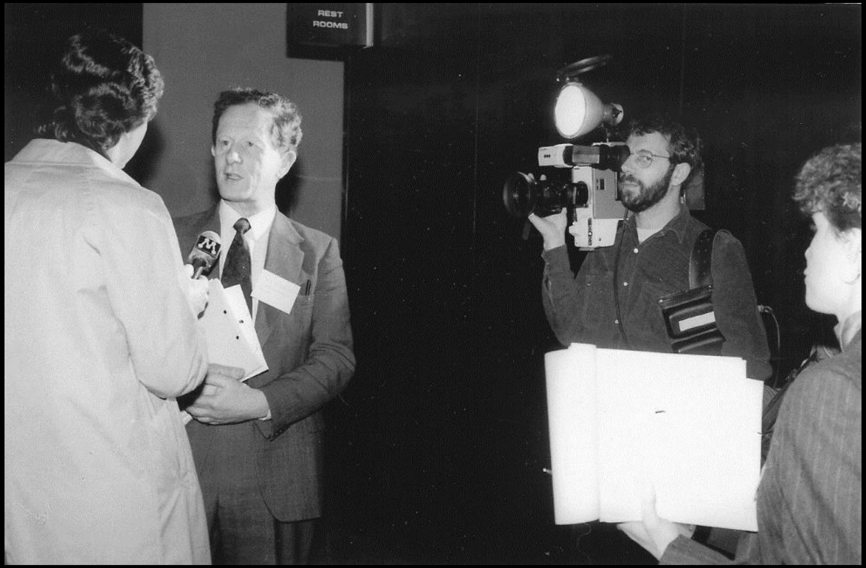
(90K)



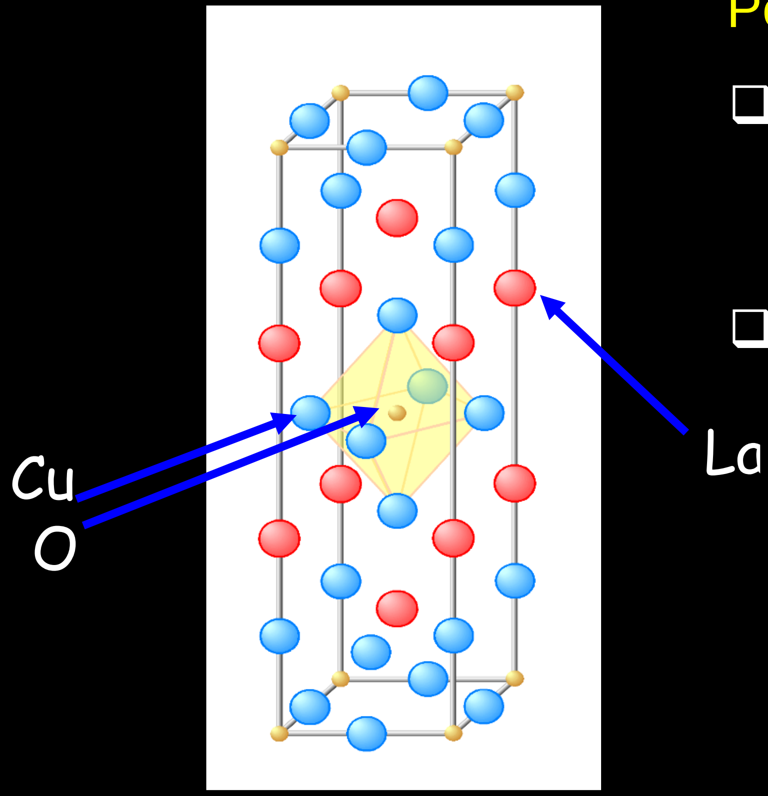
Invention of Oxide Molecular Beam Epitaxy
For HTSC Single Crystal Films.

Woodstock of Physics - March Meeting 1987

“The stores and the bars were all ‘Physicists welcome,’ ” said Paul M. Grant, who headed the superconductivity research at I.B.M.’s Almaden Research Center in San Jose. He recalled a discotheque in Chelsea with a long line of people waiting to get in. “The bouncers took anybody that had a physical society badge on to the front,” Dr. Grant recalled, “and we got in gratis. Can you imagine what a culture shift? We had a hell of a good time.” – NY Times



$La_{2-x}Ba_xCuO_4$

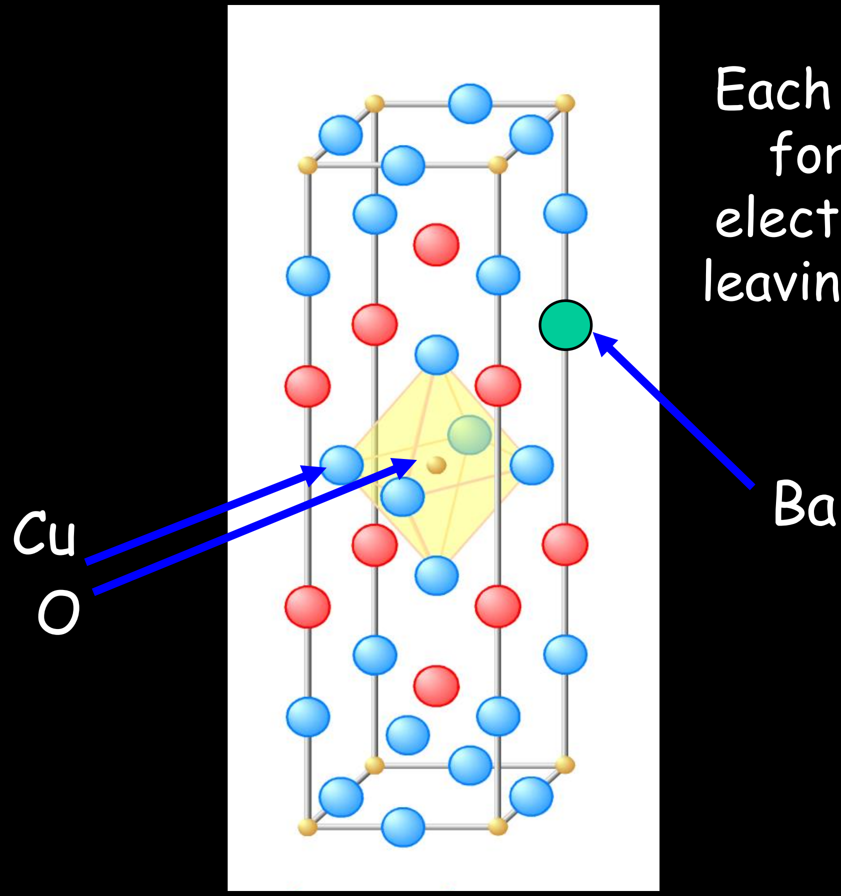


Perovskite oxide structure

- At small x cation doping, Antiferromagnetic Mott Insulator
- For SC state, the Tc is maximum at x = 0.15

Z. Phys. Rev. **B 64** 189 (1986)

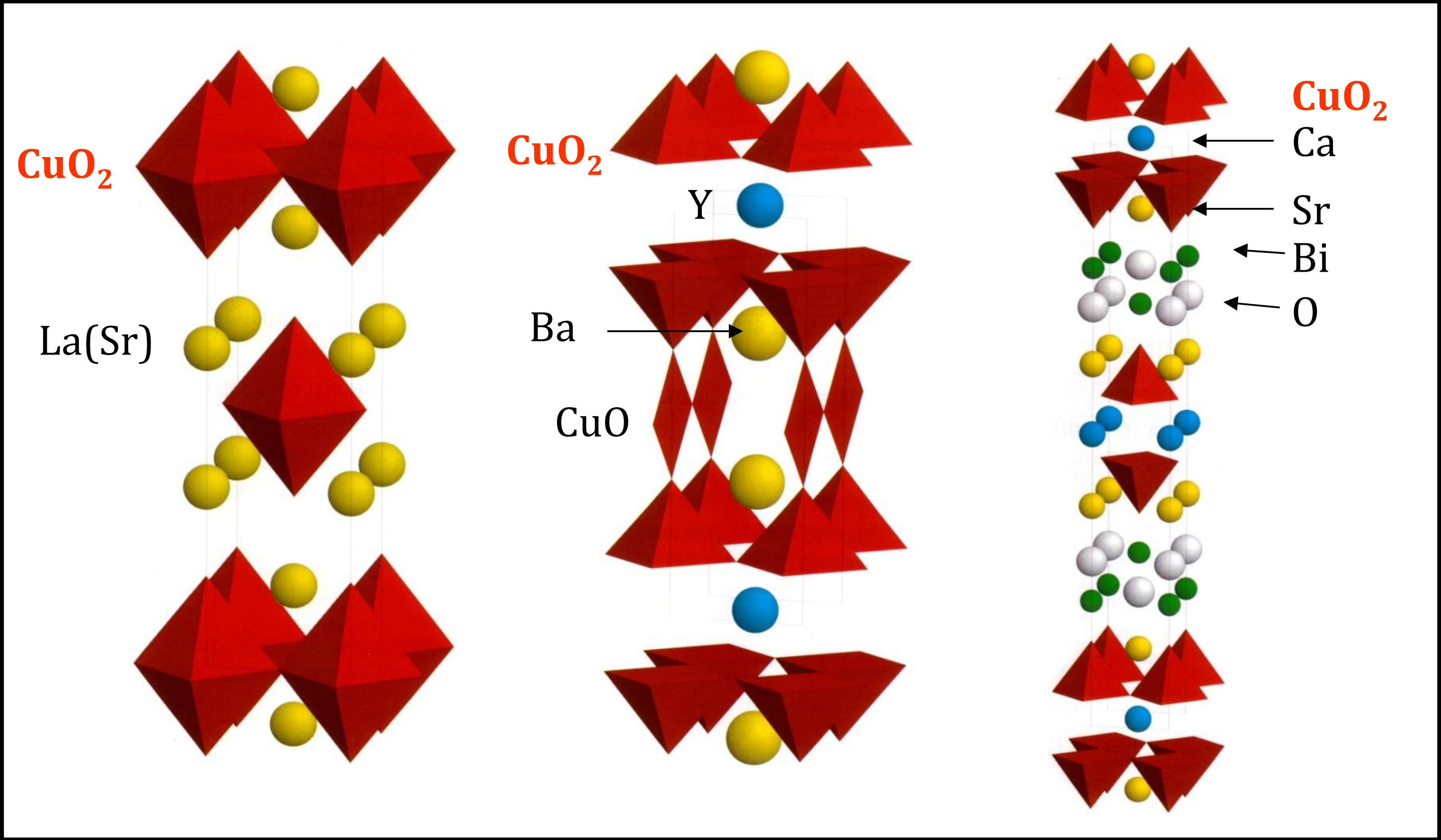
$La_{2-x}Ba_xCuO_4$



Each Ba atom substituted for the captures, and electron from CuO_2 plane leaving *p* holes per unit cell

Z. Phys. Rev. **B 64** 189 (1986)

High T_c Cuprate Superconductors (CuSC)



$La_{2-x}Sr_xCuO_4$
(LSCO)

($T_c^{max} \sim 40$ K)

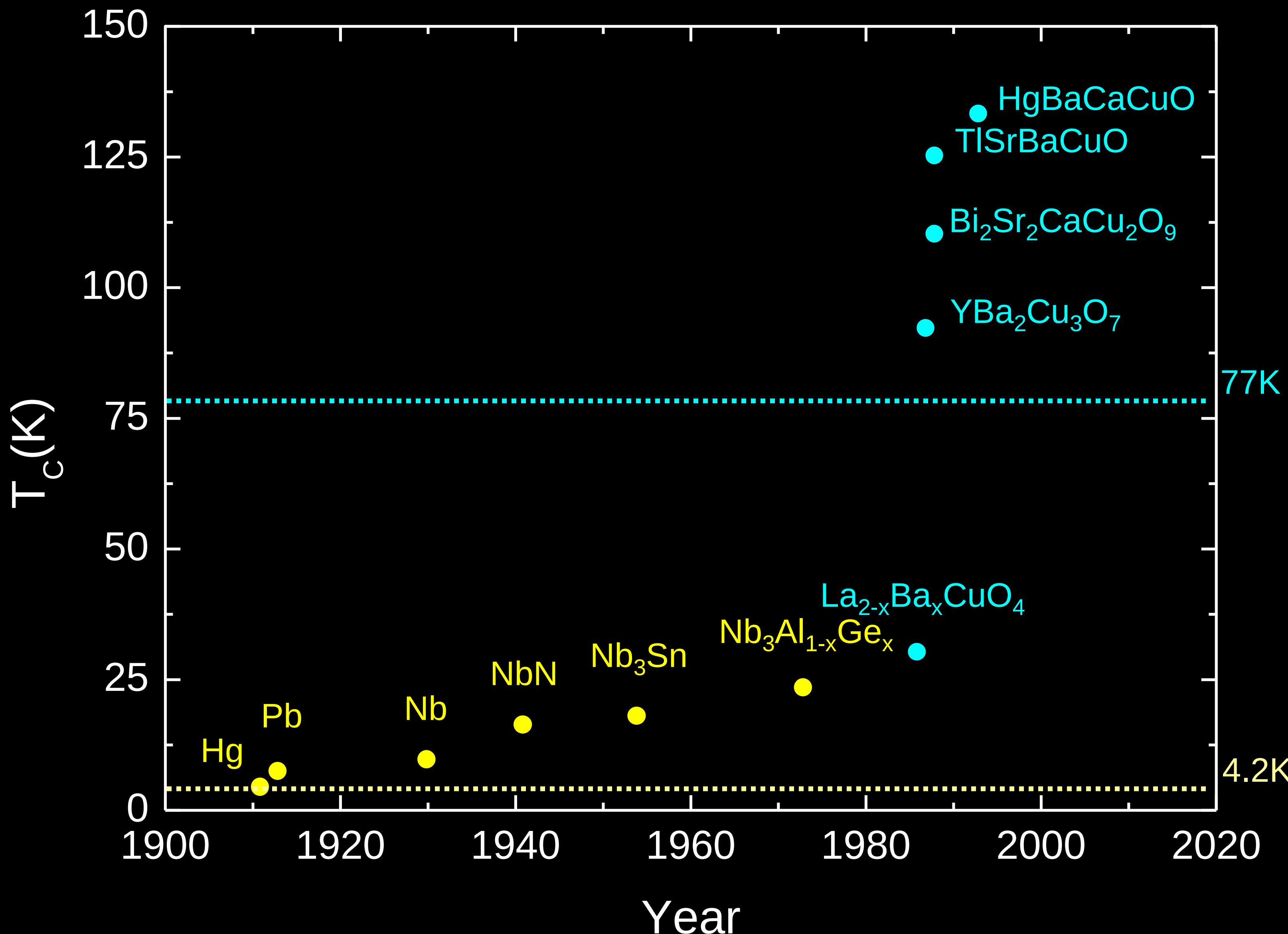
$YBa_2Cu_3O_y$
(YBCO)

($T_c^{max} \sim 93$ K)

$Bi_2Sr_2CaCu_2O_y$
(Bi2212 or BSCCO)

($T_c^{max} \sim 95$ K)

History of Superconductors



Honorable Mention : MgB_2 in 2001

T_c=39K
Two superconducting gaps
Strong sp^2 bonding and hybridization
E_{2g} phonon and σ bond coupling leads to high T_c

Jun Akimitsu
秋光純

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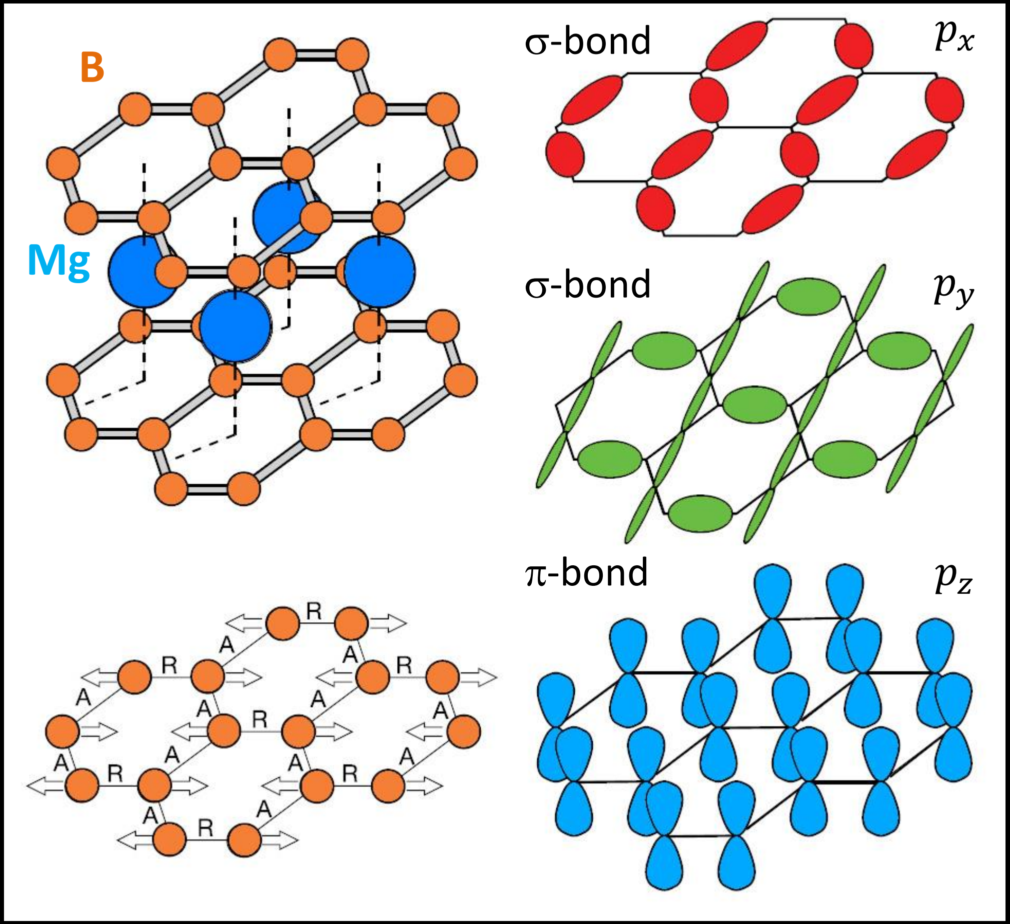
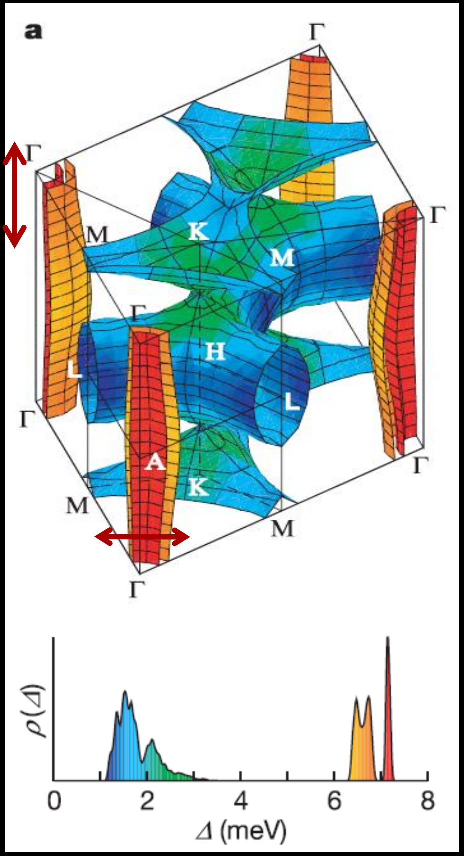


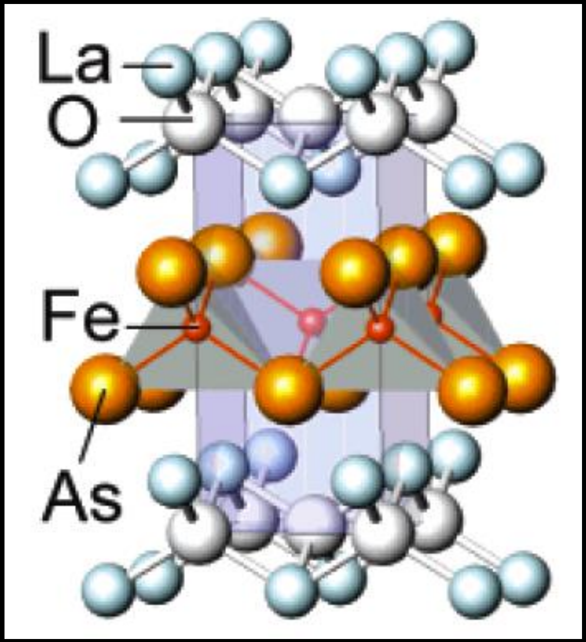


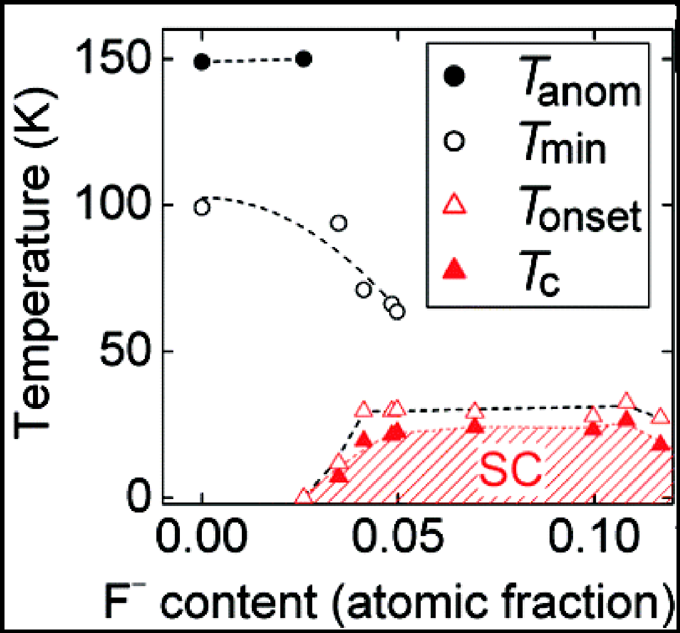
J. Nagamatsu *et al*., Nature 410, 63 (2001)
Amy Liu *et al*., PRL 87, 087005 (2001)
H.J. Choi *et al*., Nature 418, 758 (2002)

The Discovery of Fe-based Superconductors (FeSC) in 2006

2006 : $LaFeP(O_{1-x}F_x)$: T_C~5K
2007 : LaNiPO: T_C~3K
2008 : $LaFeAs(O_{1-x}F_x)$, T_C~26K

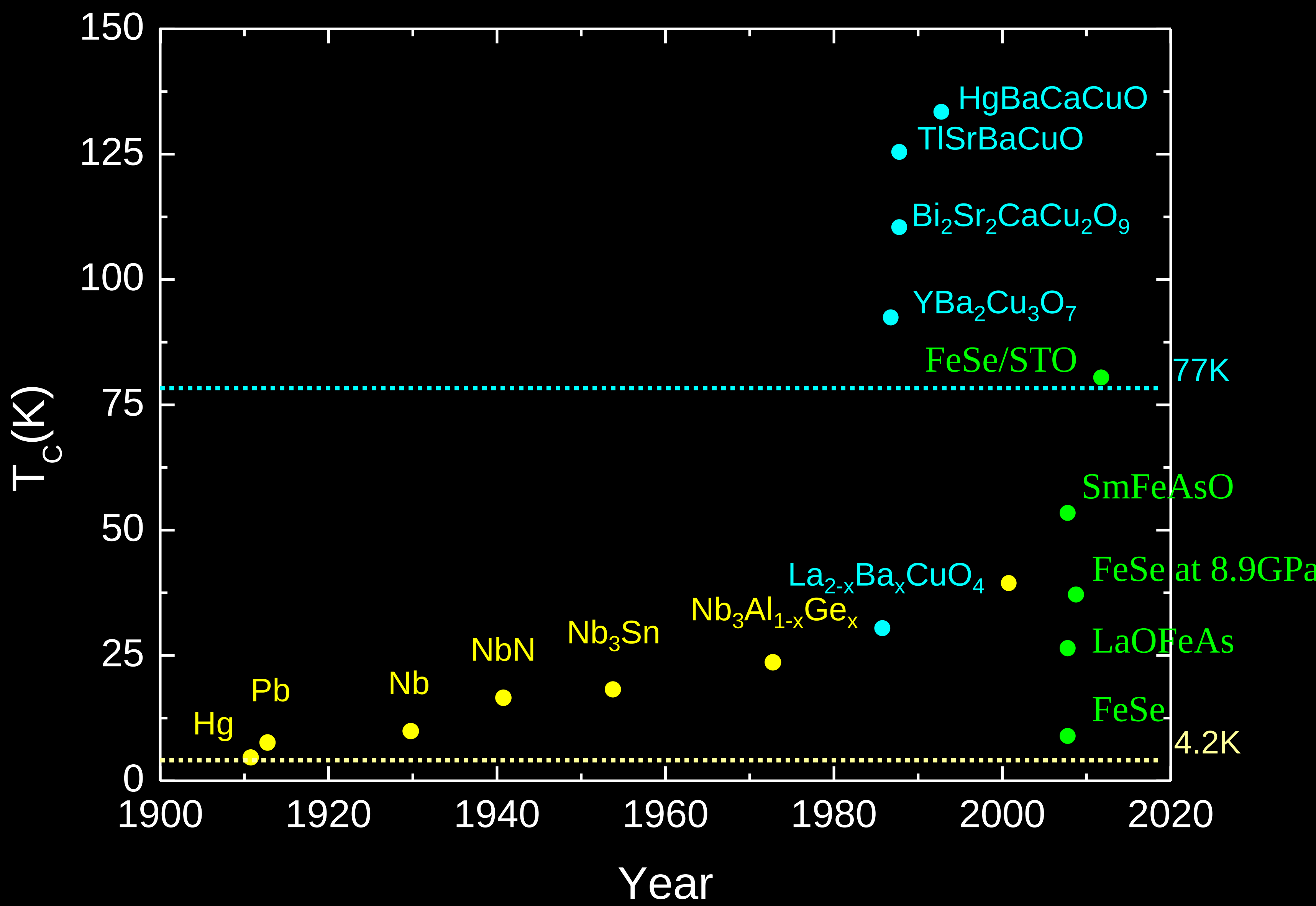
Hideo Hosono





Y. Kamihara *et al.*, JACS. **128**, 10012 (2006)
T. Watanabe *et al.*, JACS. **46**, 7719 (2007)
Y. Kamihara *et al.*, JACS. **130**, 3296 (2008)

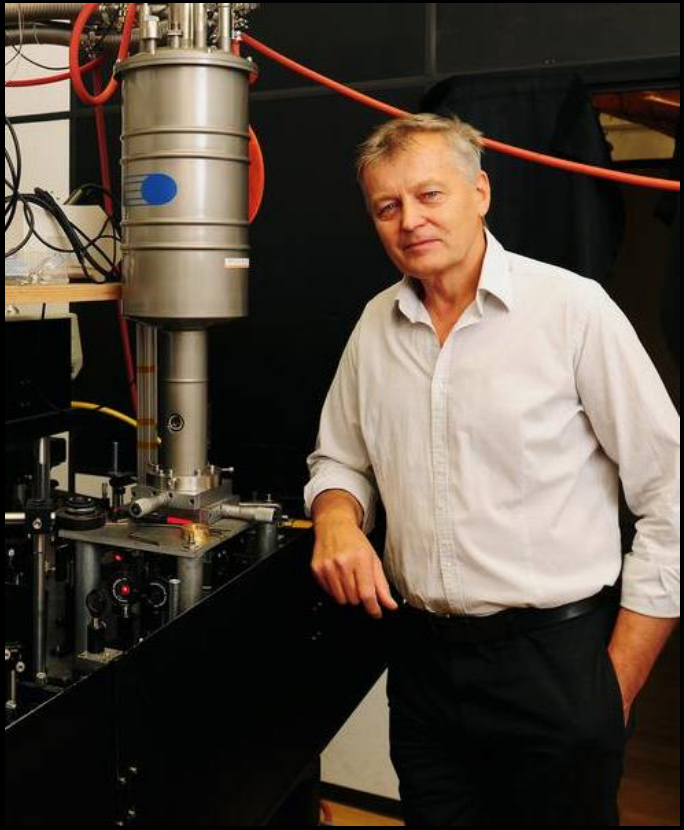
History of Conventional and High T_C Superconductors

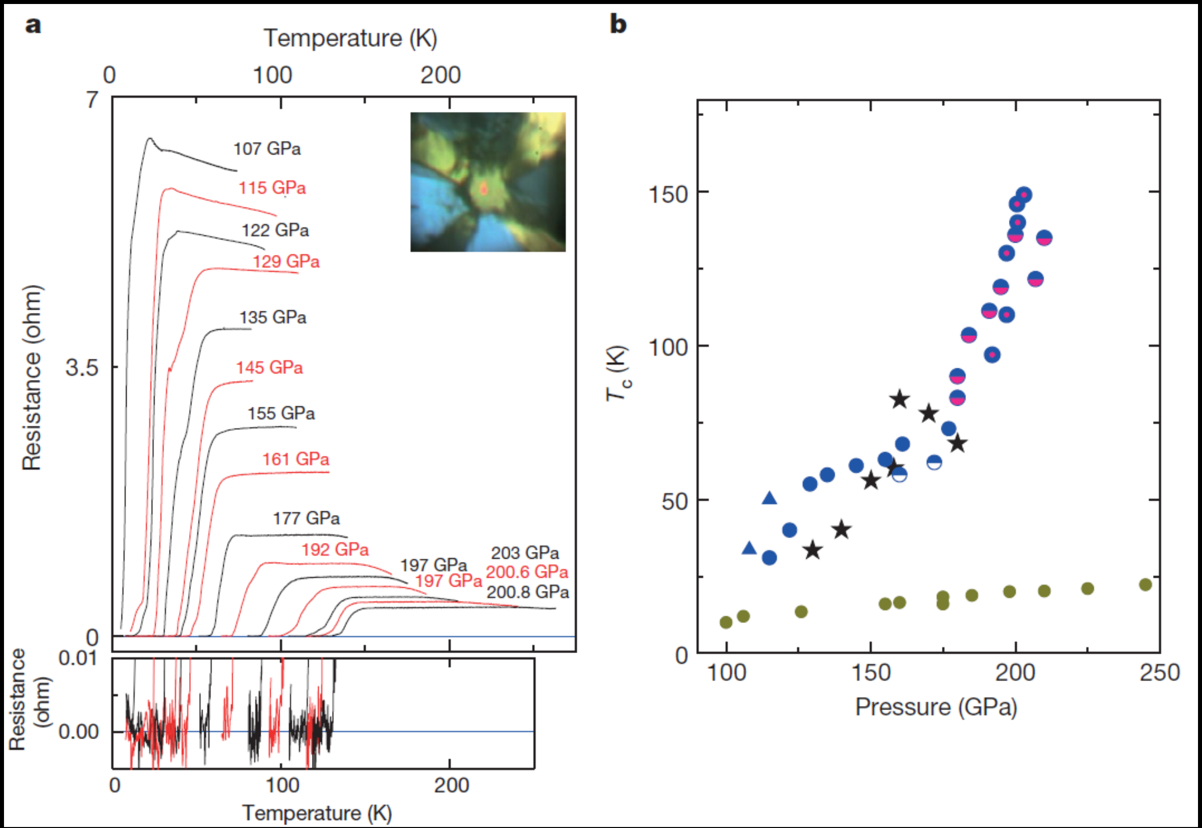


Honorable Mention : H_3S in 2015

T_C=203K under High Pressure
Likely H-rich H_3S
Conventional BCS superconductor ?

Mikhail Eremets





A. P. Drozdov *et al.*, Nature 525, 73 (2015)

- The results are the work of Mikhail Eremets, Alexander Drozdov and their colleagues at the Max Planck Institute for Chemistry in Mainz, Germany in 2015. They find that when they subject samples of hydrogen sulfide to extremely high pressures — around 1.5 million atmospheres (150 Gigapascals) — and cool them below 203 K, the samples display the classic hallmarks of superconductivity: zero electrical resistance and the Meissner effect.

- Other hydrogen compounds may be good candidates for high T_c too. For instance, compounds that pair hydrogen with *Pt, K, Se, Te,* instead of sulfur.

- Zhang in Dallas and Yugui Yao of the Beijing Institute of Technology in China predict that substituting 7.5% of the sulfur atoms in hydrogen sulfide with phosphorus, and upping the pressure to 2.5 million atmospheres (250 GPa) could raise the superconducting transition temperature all the way to 280 K, above water's freezing point.

Will all non magnetic metal become SC at low T?

(I) Destruction of Superconductivity by Magnetic Impurities

It is important to eliminate from the specimen even trace quantities of foreign paramagnetic elements

(II) Destruction of Superconductivity by Magnetic fields

At the critical temperature the critical field is zero: $H_c(T_c)=0$

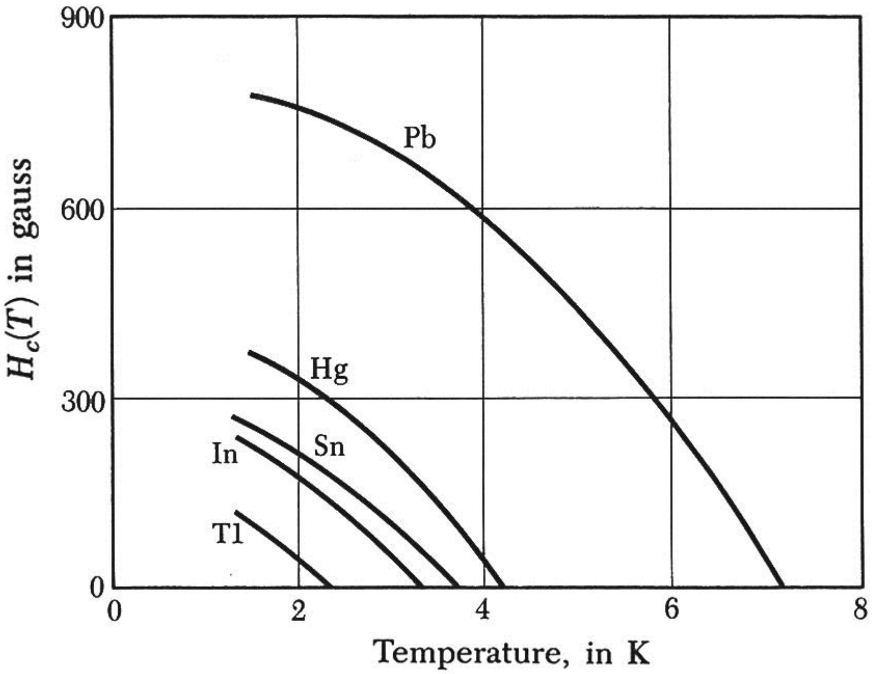


Figure 3 Experimental threshold curves of the critical field $H_c(T)$ versus temperature for several superconductors. A specimen is superconducting below the curve and normal above the curve.

Type I superconductor | Type II superconductor

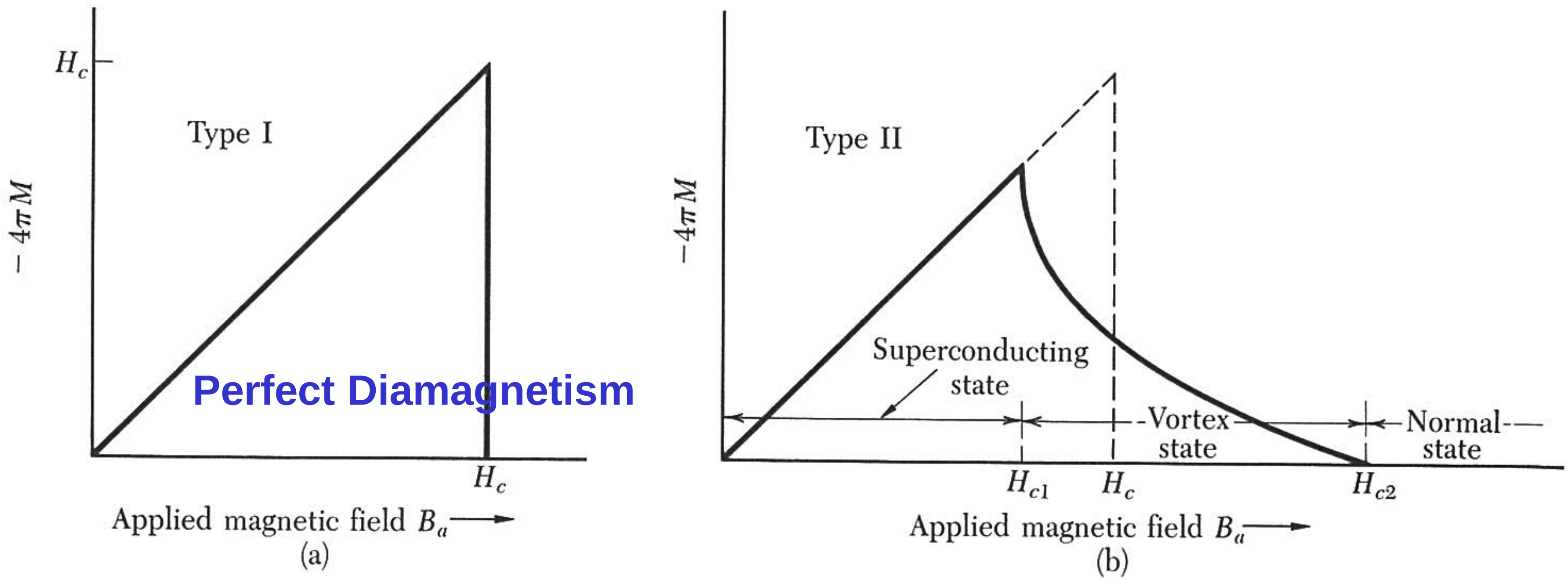


Figure 4 (a) Magnetization versus applied magnetic field for a bulk superconductor exhibiting a complete Meissner effect (perfect diamagnetism). A superconductor with this behavior is called a type I superconductor. Above the critical field H_c the specimen is a normal conductor and the magnetization is too small to be seen on this scale. Note that minus $4\pi M$ is plotted on the vertical scale: the negative value of M corresponds to diamagnetism. (b) Superconducting magnetization curve of a type II superconductor. The flux starts to penetrate the specimen at a field H_{c1} lower than the thermodynamic critical field H_c. The specimen is in a vortex state between H_{c1} and H_{c2}, and it has superconducting electrical properties up to H_{c2}. Above H_{c2} the specimen is a normal conductor in every respect, except for possible surface effects. For given H_c the area under the magnetization curve is the same for a type II superconductor as for a type I. (CGS units in all parts of this figure.)

Type II Superconductors

1. A good type **I** superconductor excludes a magnetic field until superconductivity is destroyed suddenly, and then the field penetrates completely.

2. (a) A good type **II** superconductor excludes the field completely up to a field $\boldsymbol{H}_{c1}$.

   (b) Above $\boldsymbol{H}_{c1}$ the field is partially excluded, but the specimen remains electrically superconducting.

   (c) At a much higher field, $\boldsymbol{H}_{c2}$, the flux penetrates completely and superconductivity vanishes.

   (d) An outer surface layer of the specimen may remain superconducting up to a still higher field $\boldsymbol{H}_{c3}$.

3. An important difference in a type **I** and a type **II** superconductor is in the mean free path of the conduction electrons in the normal state. are type **I**, with $\boldsymbol{\kappa} < 1$, will be type **II**. is the situation when $\boldsymbol{\kappa} = \lambda / \xi > 1$.

1. A superconductor is type **I** if the surface energy is always positive as the magnetic field is increased, For $H < H_c$

2. And type **II** if the surface energy becomes negative as the magnetic field is increased. For $Hc_1 < H < Hc_2$

The free energy of a bulk superconductor is increased when the magnetic field is expelled. However, a parallel field can penetrate a very thin film nearly uniformly (Fig. 17), only a part of the flux is expelled, and the energy of the superconducting film will increase only slowly as the external magnetic field is increased.

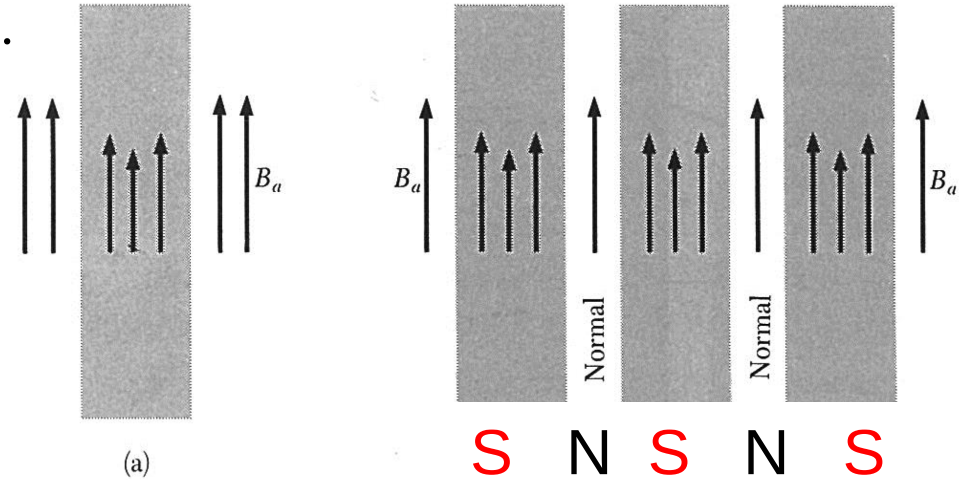


Figure 17 (a) Magnetic field penetration into a thin film of thickness equal to the penetration depth λ. The arrows indicate the intensity of the magnetic field. (b) Magnetic field penetration in a homogeneous bulk structure in the mixed or vortex state, with alternate layers in normal and superconducting states. The superconducting layers are thin in comparison with λ. The laminar structure is shown for convenience; the actual structure consists of rods of the normal state surrounded by the superconducting state. (The *N* regions in the vortex state are not exactly normal, but are described by low values of the stabilization energy density.)

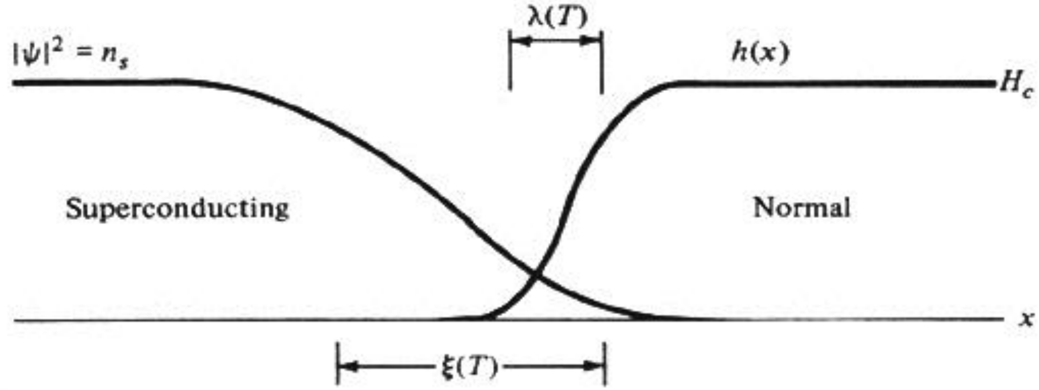
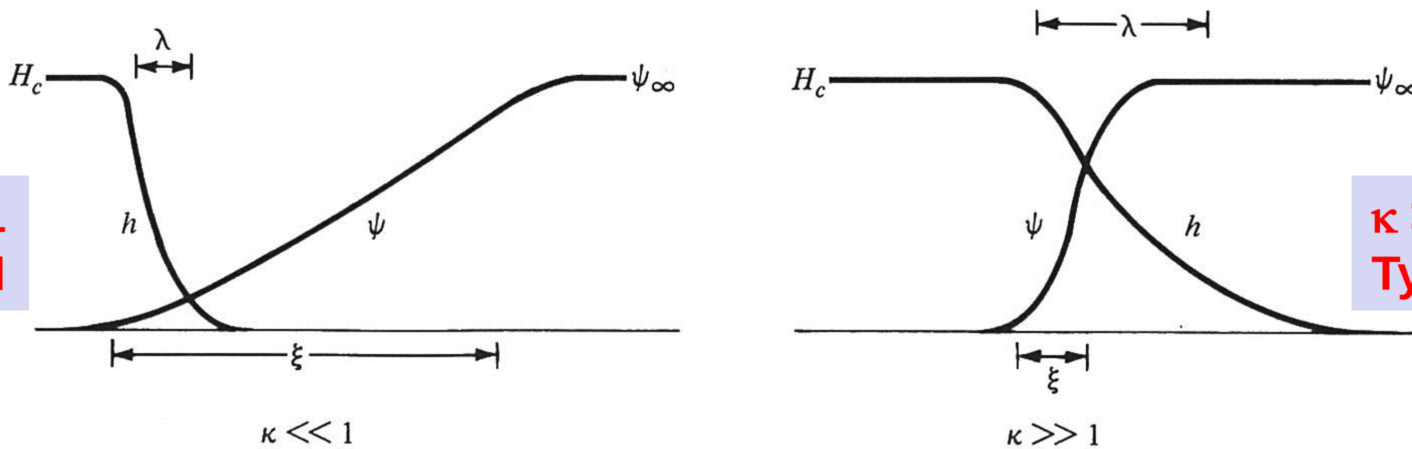


FIGURE 1-4
Interface between superconducting and normal domains in the intermediate state.

$$\kappa = \frac{\lambda_{\text{eff}}(T)}{\xi(T)} = \frac{2\sqrt{2}\,\pi H_c(T)\lambda_{\text{eff}}^2(T)}{\Phi_0}$$

Ginsburg Landau Parameter

κ << 1
Type I



κ >> 1
Type II

FIGURE 4-2
Schematic diagram of variation of h and ψ in a domain wall. The case $\kappa \ll 1$ refers to a type I superconductor (positive wall energy); the case $\kappa \gg 1$ refers to a type II superconductor (negative wall energy).

Vortex State

In such mixed state, called the vortex state, the external magnetic field will penetrate the thin normal regions uniformly, and the field will also penetrate somewhat into the surrounding superconducting materials

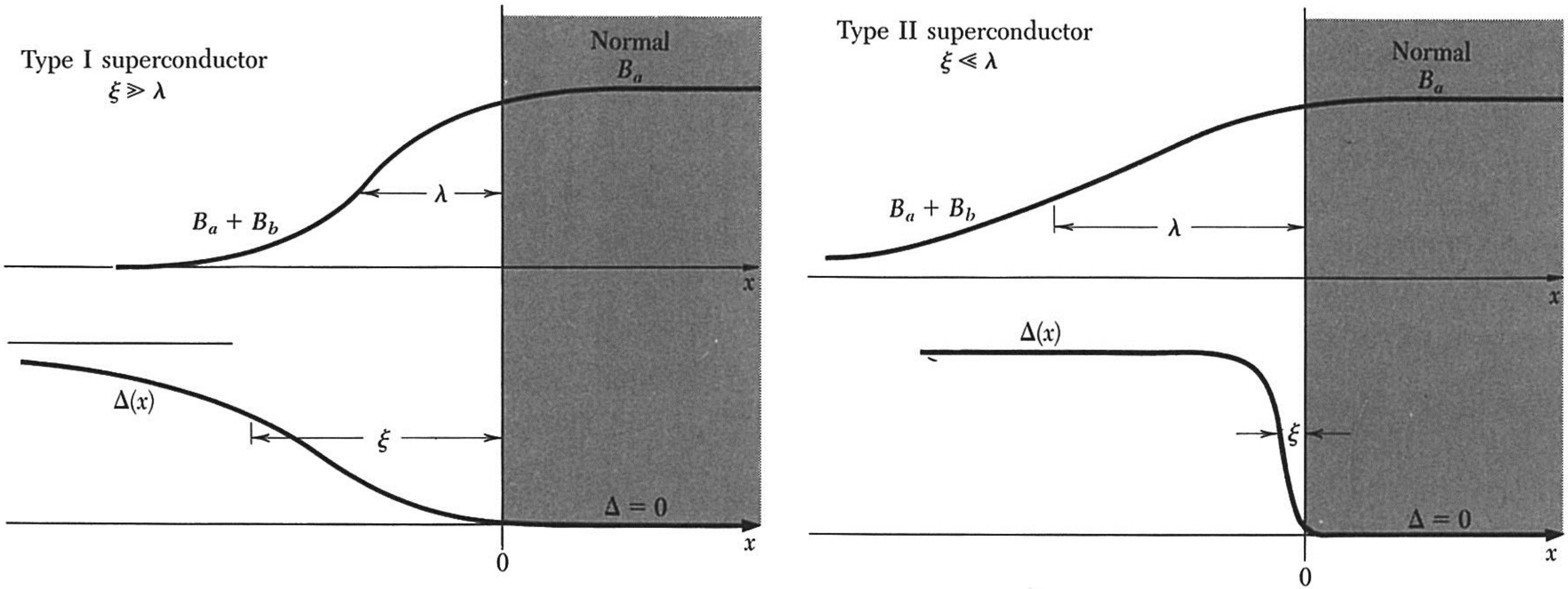


Figure 18 Variation of the magnetic field and energy gap parameter $\Delta(x)$ at the interface of superconducting and normal regions, for type I and type II superconductors. The energy gap parameter is a measure of the stabilization energy density of the superconducting state.

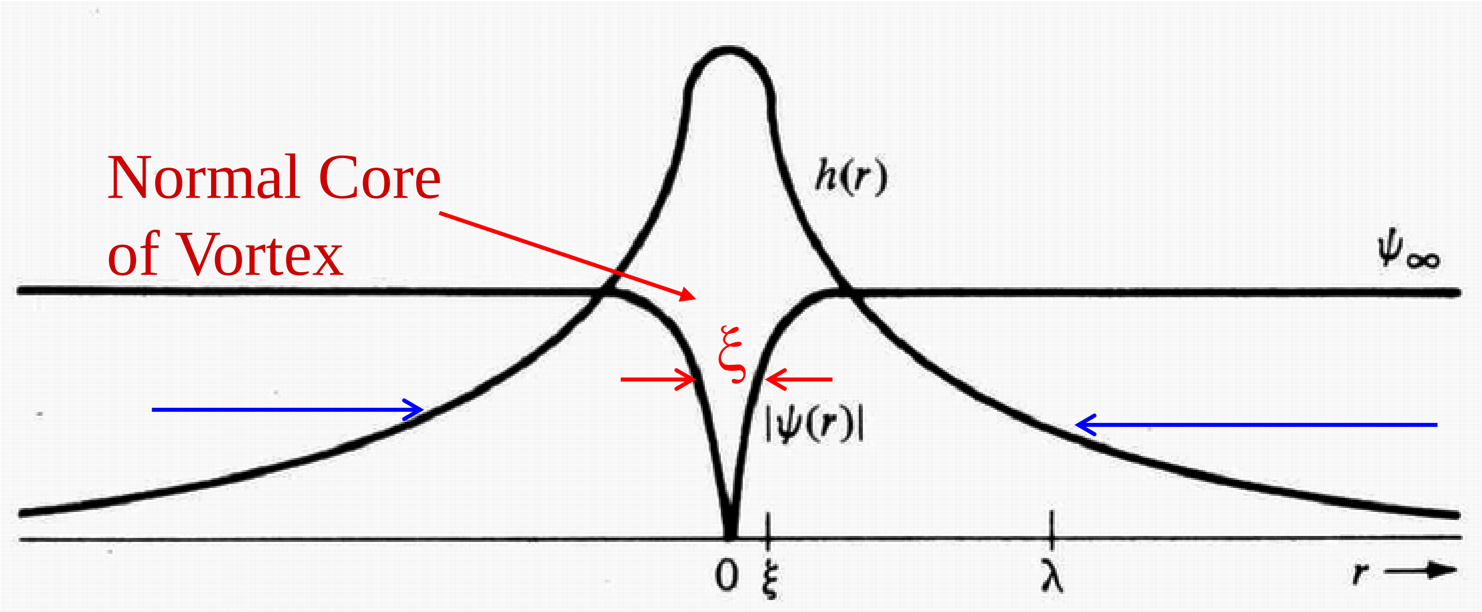
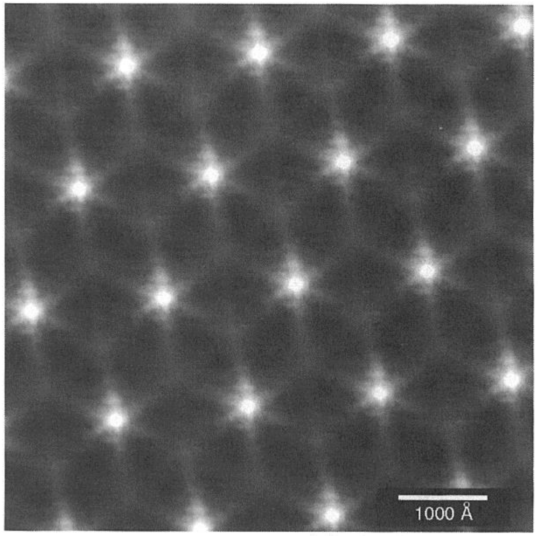


FIGURE 5-1
Structure of an isolated Abrikosov vortex in a material with $\kappa \approx 8$. The maximum value of $h(r)$ is approximately $2H_{c1}$.

$$\kappa = \lambda / \xi > 1$$

The term vortex state describes the circulation of superconducting currents in vortices throughout the bulk specimen,

Flux lattice at 0.2K of $NbSe_2$



Abrikosov triangular lattice as imaged by LT-STM, H. Hess et al

Figure 19 Flux lattice in $NbSe_2$ at 1,000 gauss at 0.2K, as viewed with a scanning tunneling microscope. The photo shows the density of states at the Fermi level, as in Figure 23. The vortex cores have a high density of states and are shaded white; the superconducting regions are dark, with no states at the Fermi level. The amplitude and spatial extent of these states is determined by a potential well formed by $\Delta(x)$ as in Figure 18 for a Type II superconductor. The potential well confines the core state wavefunctions in the image here. The star shape is a finer feature, a result special to $NbSe_2$ of the sixfold disturbance of the charge density at the Fermi surface. Photo courtesy of H. F. Hess, AT&T Bell Laboratories.

The vortex state is stable when the penetration of the applied field into the superconducting material causes the surface energy become negative. A type II superconductor is characterized bv a vortex state stable over a certain range of magnetic field strength; namely, between $\boldsymbol{H}_{c1}$ and $\boldsymbol{H}_{c2}$.

Vortex Imaging of $NbSe_2$ by LT-STM

Harald F. Hess

2H-$NbSe_2$: T_c = 7.1 K, T_{CDW} = 29 K

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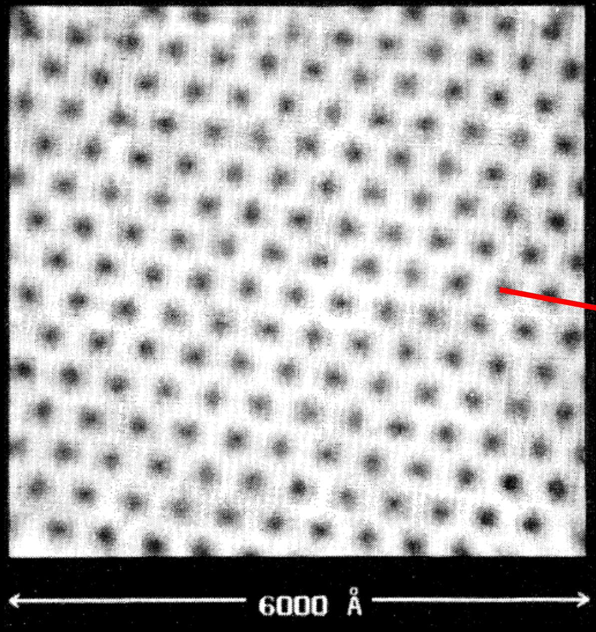
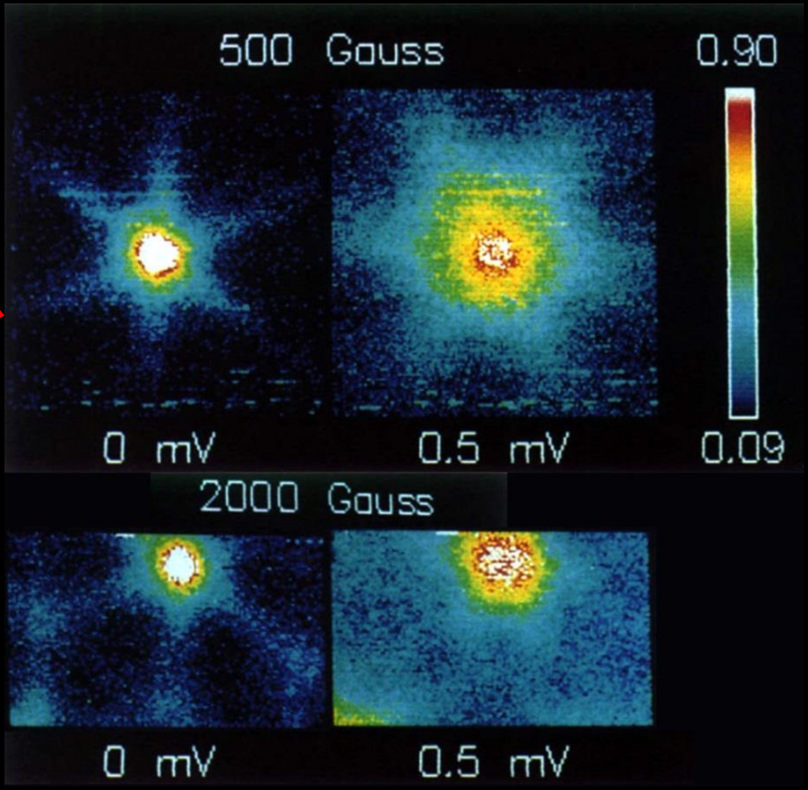




Abrikosov triangular lattice

H. F. Hess *et al.*, PRL 62, 214 (1989).
H. F. Hess *et al.*, PRL 64, 2711 (1990).

Doping *Pb* with some *In*

Type I SC becomes type II SC

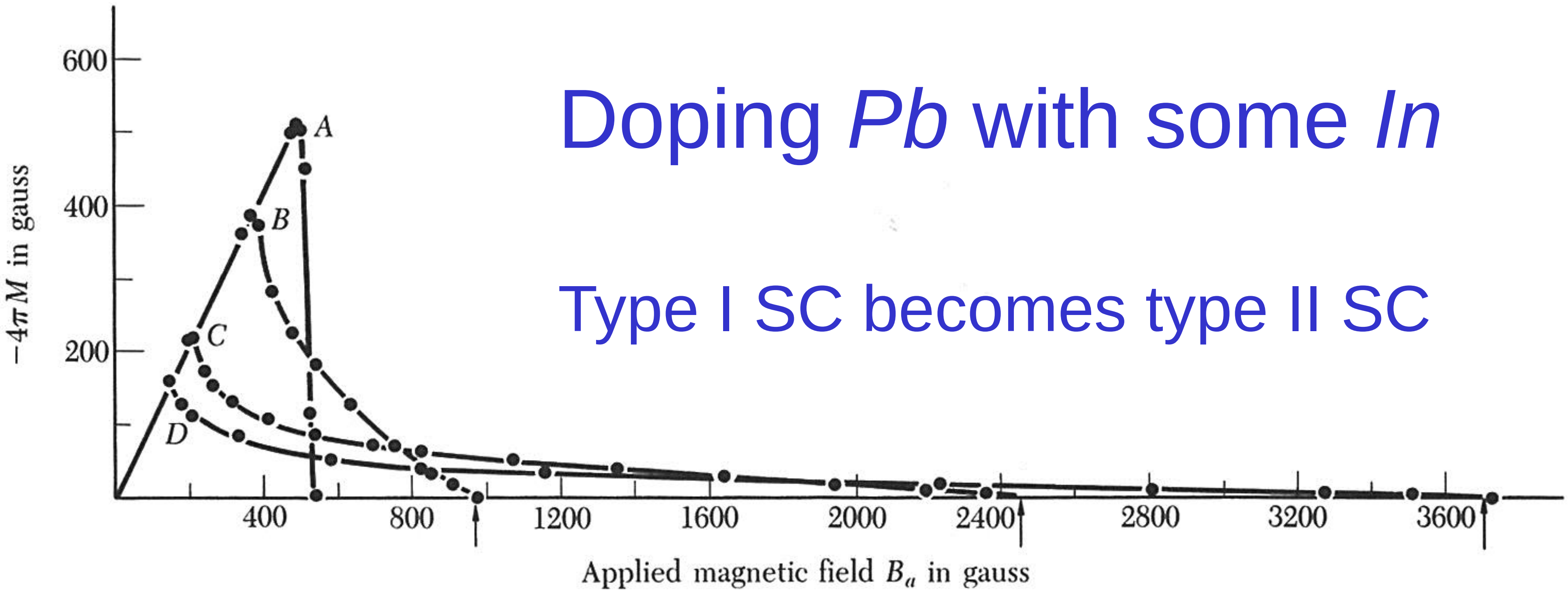


Figure 5a Superconducting magnetization curves of annealed polycrystalline lead and lead-indium alloys at 4.2 K. (*A*) lead; (*B*) lead–2.08 wt. percent indium; (*C*) lead–8.23 wt. percent indium; (*D*) lead–20.4 wt. percent indium. (After Livingston.)

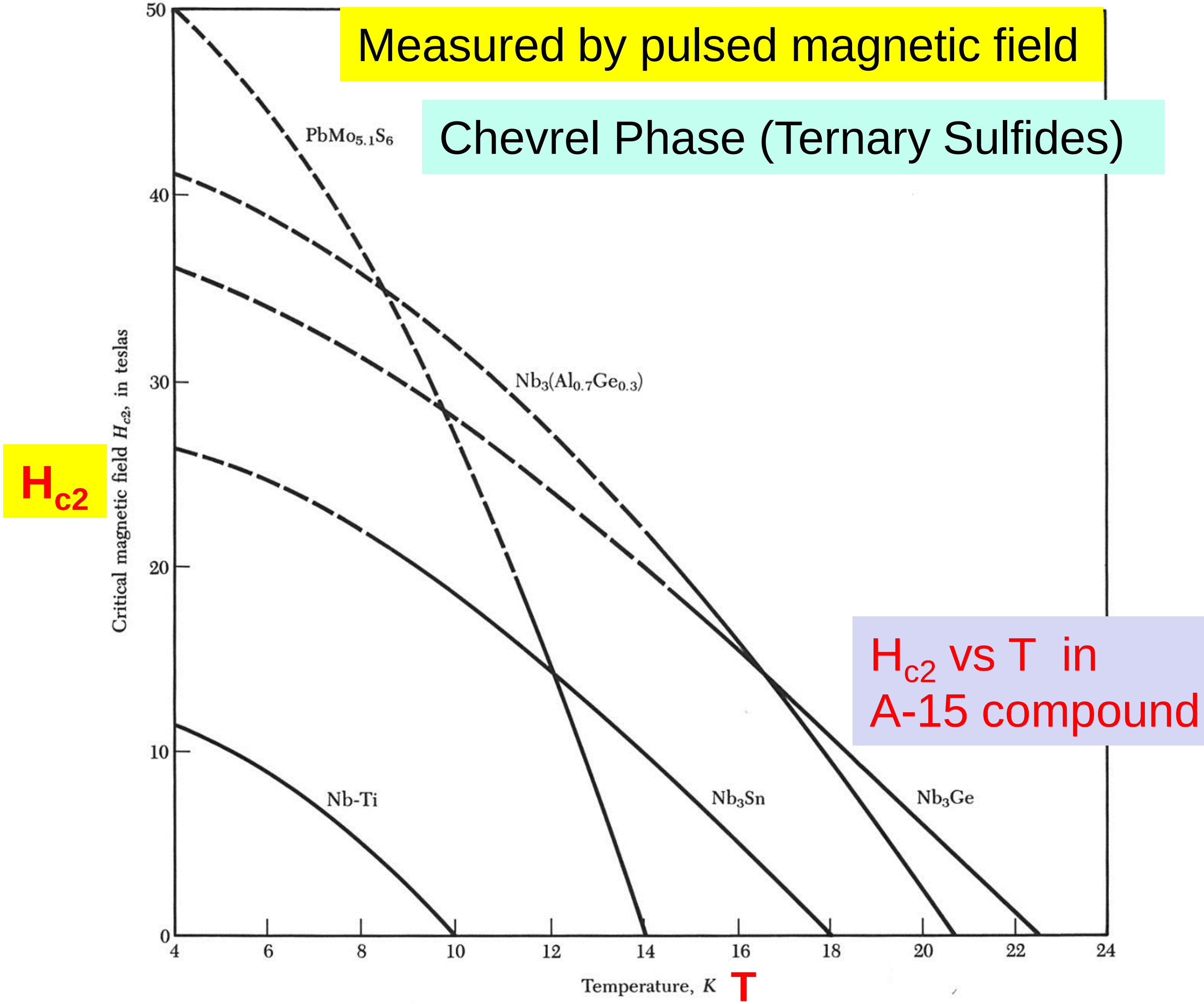


Figure 5b Stronger magnetic fields than any now contemplated in practical superconducting devices are within the capability of certain Type II materials. These materials cannot be exploited, however, until their critical current density can be raised and until they can be fabricated as finely divided conductors. (Magnetic fields of more than about 20 teslas can be generated only in pulses, and so portions of the curves shown as broken lines were measured in that way.)

Entropy S vs T for Aluminum

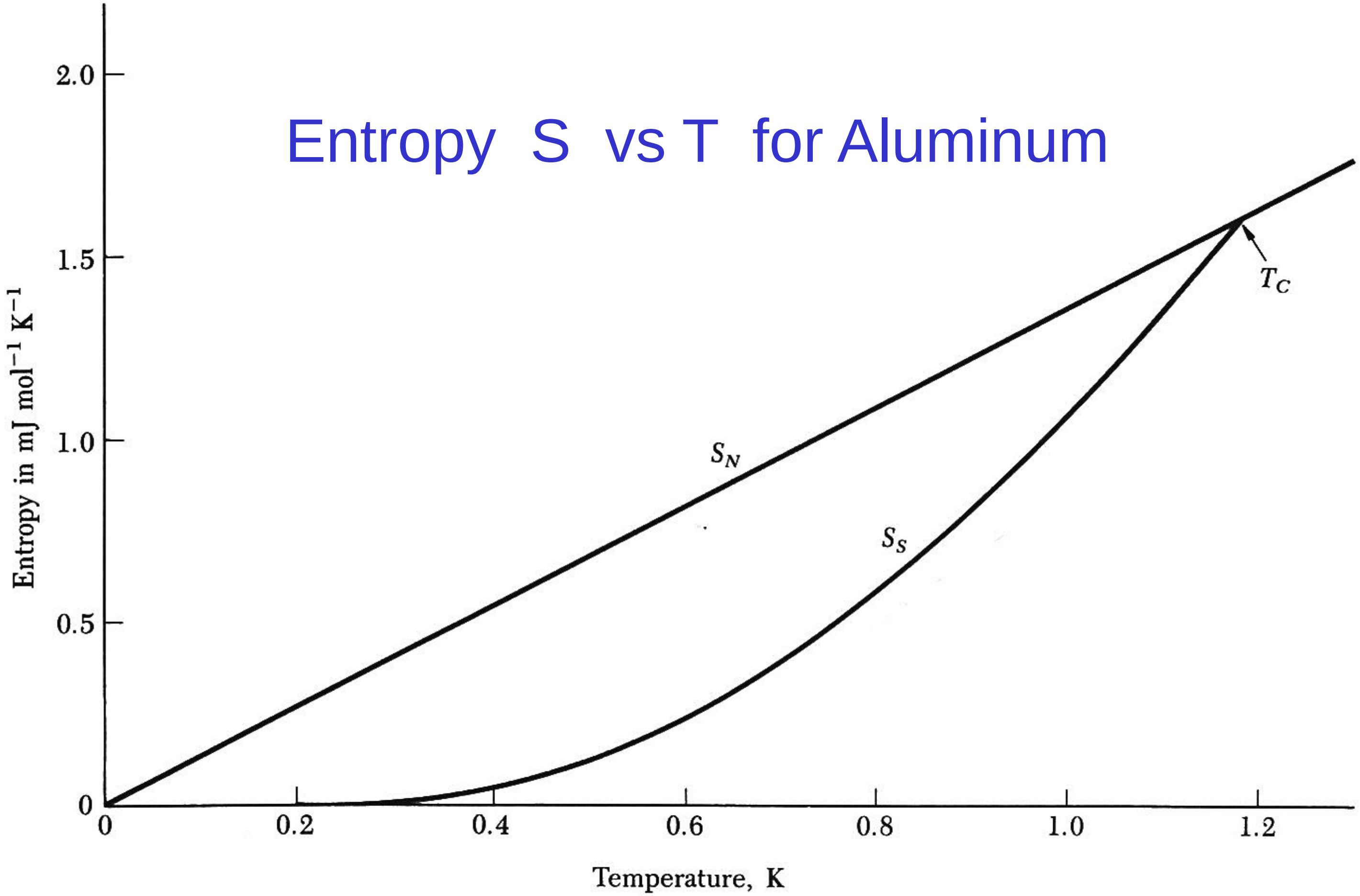
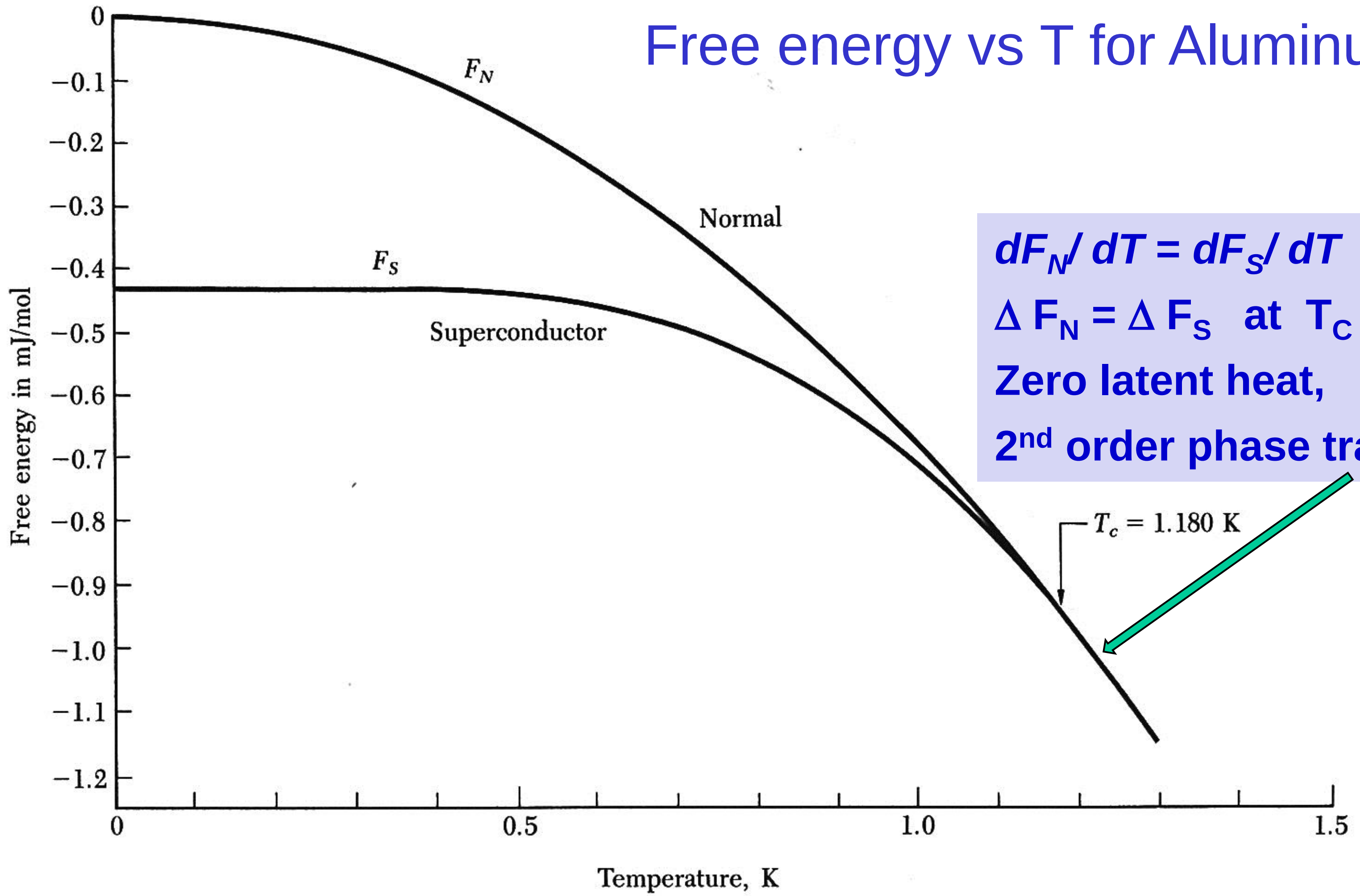


Figure 6 Entropy S of aluminum in the normal and superconducting states as a function of the temperature. The entropy is lower in the superconducting state because the electrons are more ordered here than in the normal state. At any temperature below the critical temperature T_c the specimen can be put in the normal state by application of a magnetic field stronger than the critical field.

The small entropy change must mean that only a small fraction (of the order of 10^{-4}) of the conduction electrons participate in the transition to the ordered superconducting state.

Free energy vs T for Aluminum



$dF_N/dT = dF_S/dT$ at T_C

$\Delta F_N = \Delta F_S$ at T_C

Zero latent heat,

2nd order phase transition

So that the phase transition is second order (there is no latent heat of transition at T_c).

heat capacity of an electron gas is

$$C_{el} = \frac{1}{3}\pi^2 D(\epsilon_F)\, k_B^{\,2}\, T \qquad (34)$$

$$D(\epsilon_F) = 3N/2\epsilon_F = 3N/2\, k_B T_F \qquad (35)$$

$$\boxed{C_{el} = \frac{1}{2}\pi^2 N k_B T/T_F.} \qquad (36)$$

Compare with $C_V = 2Nk_BT/T_F$

where $\mathcal{E}_F = k_B T_F$

T_F is called the Fermi temperature,

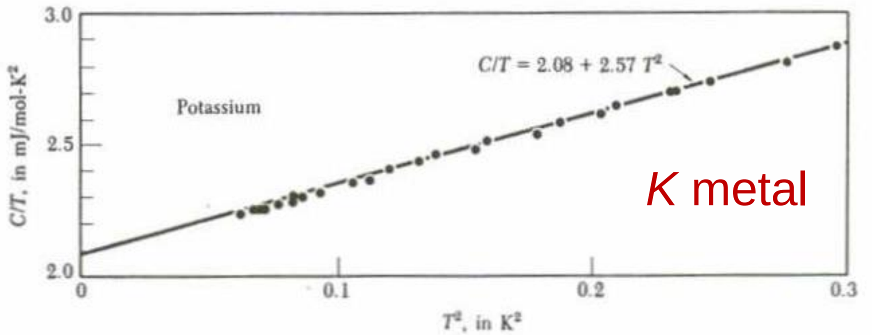


Figure 9 Experimental heat capacity values for potassium, plotted as C/T versus T^2. (After W. H. Lien and N. E. Phillips.)

$$\gamma = \frac{1}{2}\pi^2 N k_B T/T_F \quad \text{Since } \epsilon_F \propto T_F \propto 1/\text{m} \qquad \therefore \gamma \propto m$$ (See Eq. 17)

At temperatures much below both the Debye temperature and the Fermi temperature, the heat capacity of metals may be written as the sum of electron and phonon contributions: $C = \gamma T + AT^3$

$$\boxed{C/T = \gamma + AT^2} \qquad (37)$$

γ , called the Sommerfeld parameter

At low T, the electronic term dominates.

Heat Capacity of Ga at low T

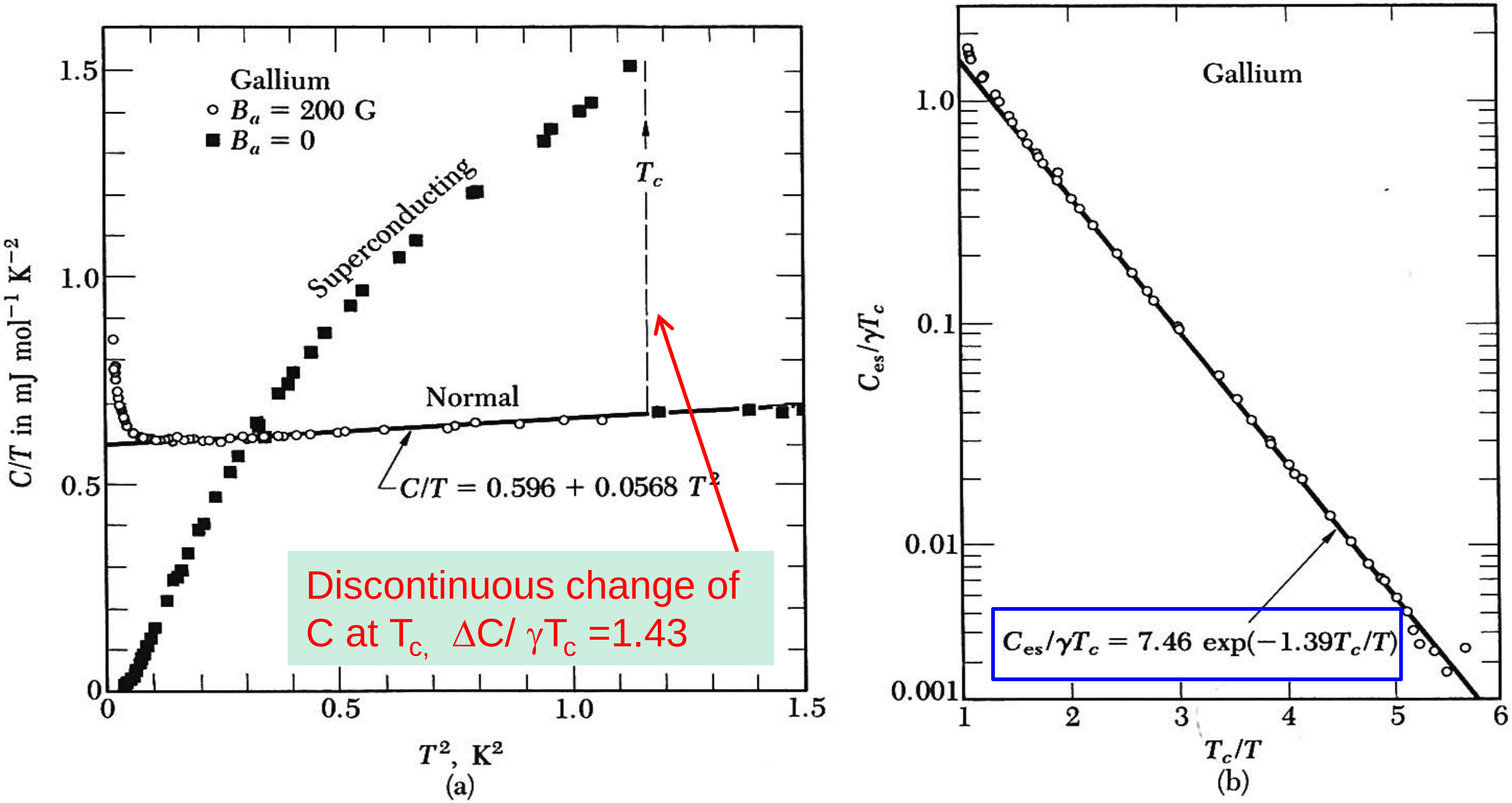


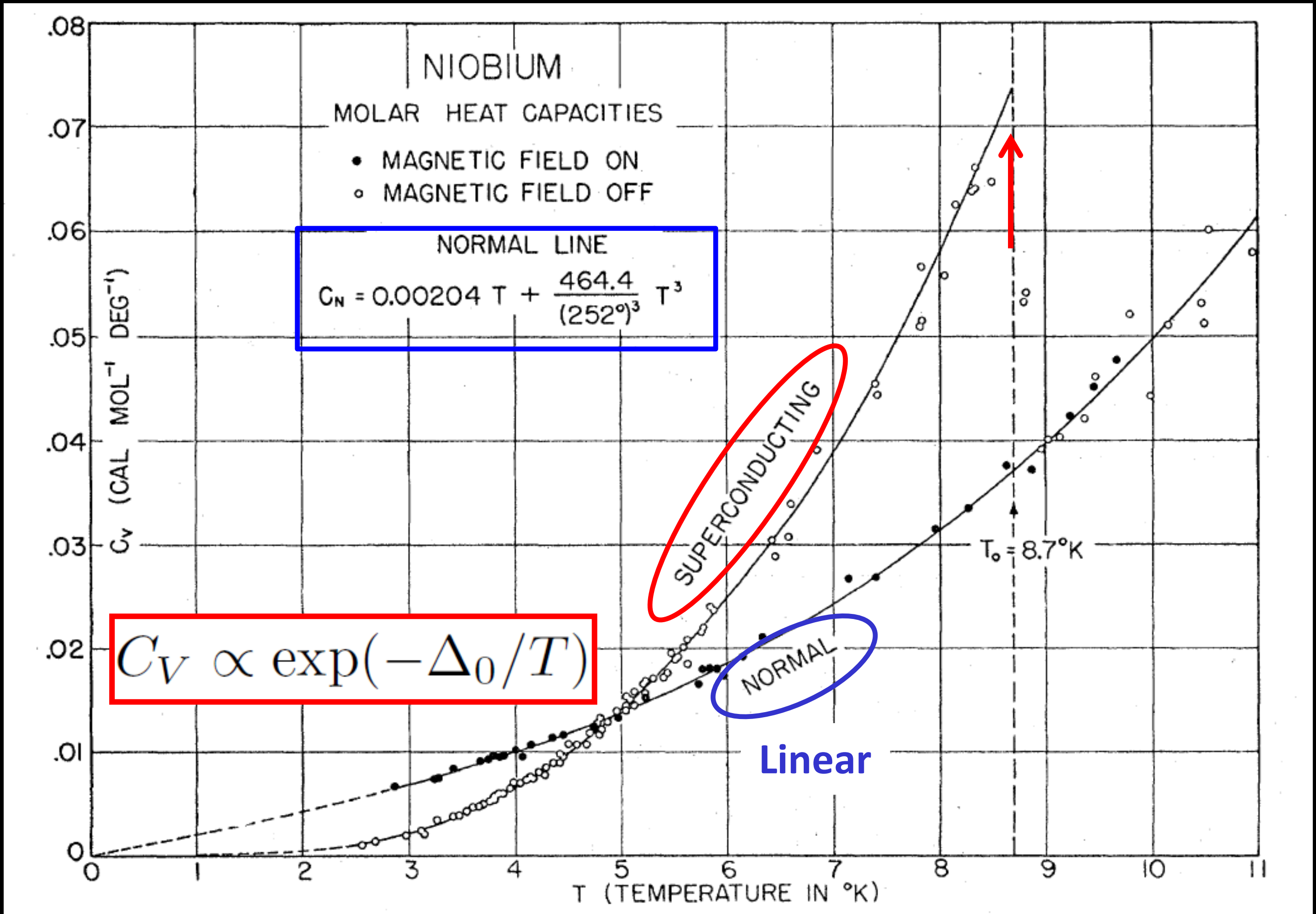
Figure 8 (a) The heat capacity of gallium in the normal and superconducting states. The normal state (which is restored by a 200 G field) has electronic, lattice, and (at low temperatures) nuclear quadrupole contributions. In (b) the electronic part C_{es} of the heat capacity in the superconducting state is plotted on a log scale versus T_c/T: the exponential dependence on $1/T$ is evident. Here $\gamma = 0.60$ mJ mol^{-1} deg^{-2}. (After N. E. Phillips.)

Electronic part of heat capacity in SC state: ***$C_{es}/\gamma T_c \propto a\exp(-bT_c/T)$***

Proportional to -1/T, suggestive of excitation of electrons across an energy gap.

Evidence for Energy Gap in 1953

Another motivation for the BCS theory of superconductivity.



A. Brown, M. W. Zemansky, and H. A. Boorse, Phys. Rev. 92, 52 (1953)
B. B. Goodman, Proc. Phys. Soc. (London) **A66**, 217 (1953)

Energy Gap

In a superconductor the important interaction is the electron-electron interaction via phonons, which orders the electron in the k space with respect to the Fermi gas of electrons.

The exponential factor in the electron heat capacity of a superconductor Is found to be $-E_g/2k_BT$

$$C_{es} = \gamma T_c \exp(-1.76\, T_c/T)$$

The transition in zero magnetic field from the superconducting state to the normal state is observed to be a second-order phase transition.

Energy Gap of superconductors in **Table 3**

$E_g(0)/k_BT_c = 3.52$ Weak electron-phonon coupling

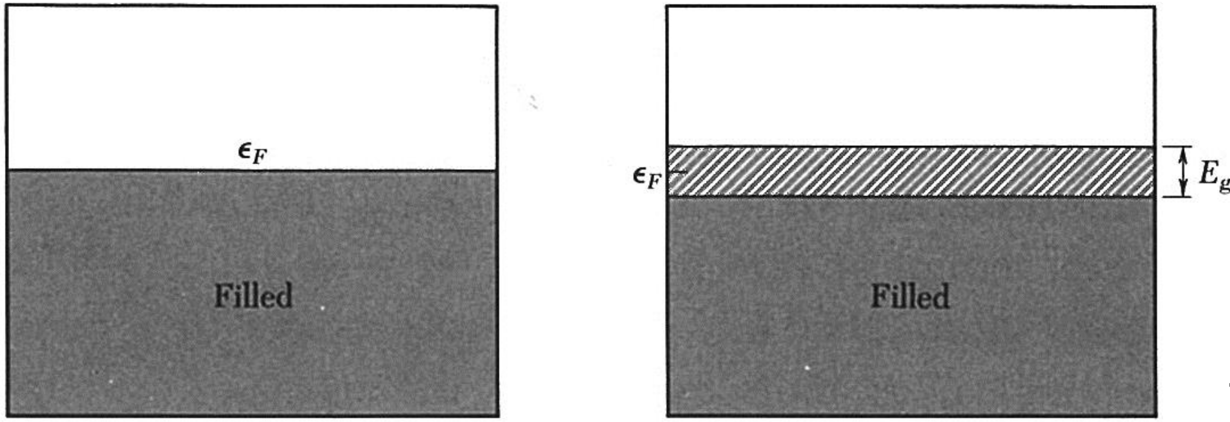
$E_g(0)/k_BT_c > 3.52$ Strong electron-phonon coupling

Table 3 Energy gaps in superconductors, at $T = 0$

$E_g(0)$ in 10^{-4}eV.
$E_g(0)/k_B T_c$.

$= 2\Delta$

										Al 3.4 3.3	Si
Sc	Ti	V 16. 3.4	Cr	Mn	Fe	Co	Ni	Cu	Zn 2.4 3.2	Ga 3.3 3.5	Ge
Y	Zr	Nb 30.5 3.80	Mo 2.7 3.4	Tc	Ru	Rh	Pd	Ag	Cd 1.5 3.2	In 10.5 3.6	Sn (w) 11.5 3.5
La fcc 19. 3.7	Hf	Ta 14. 3.60	W	Re	Os	Ir	Pt	Au	Hg (α) 16.5 4.6	Tl 7.35 3.57	Pb 27.3 4.38



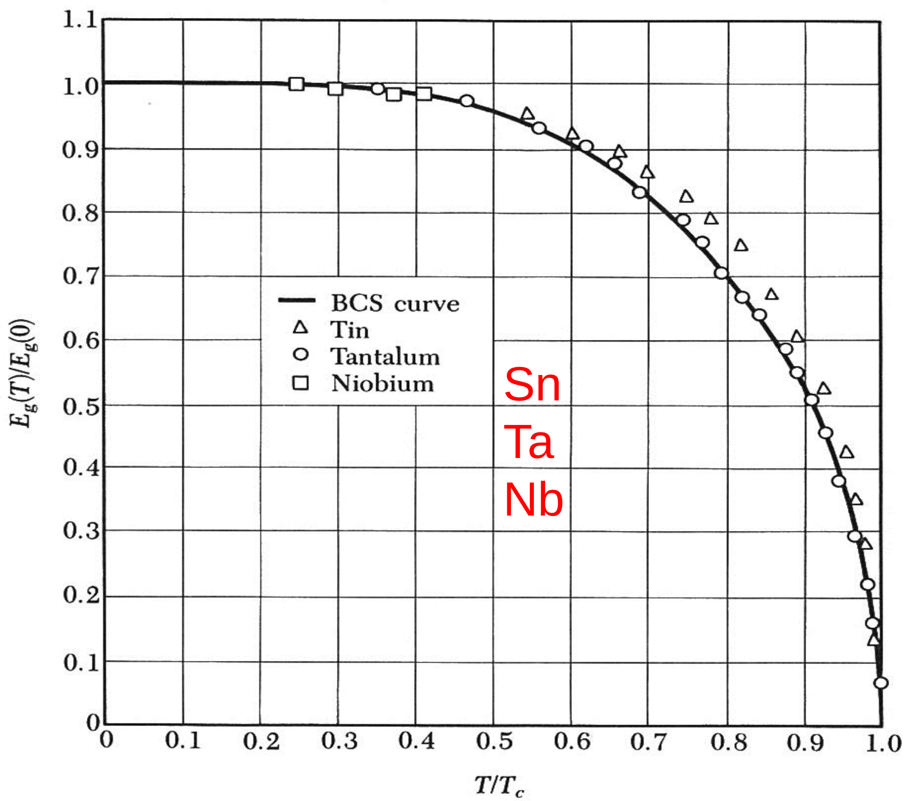
Normal
(a)

Superconductor
(b)

$E_g \sim 10^{-4}\ \varepsilon_F$

1-5 meV

3-10 eV



$\Delta(T)/\Delta(0) = (1 - T/T_c)^{1/2}$

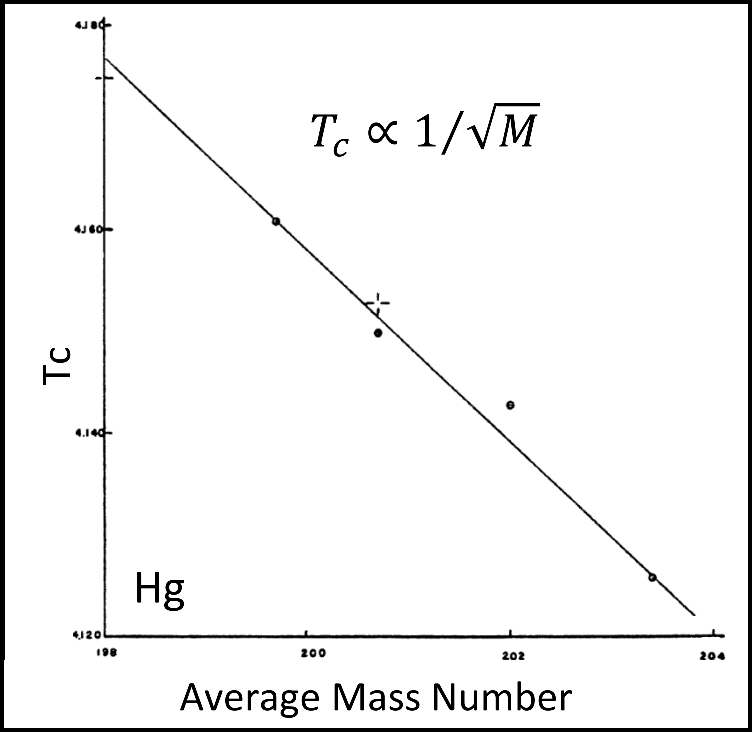
Mean field theory

Figure 10 Reduced values of the observed energy gap $E_g(T)/E_g(0)$ as a function of the reduced temperature T/T_c, after Townsend and Sutton. The solid curve is drawn for the BCS theory.

$E_g(T)$ as the order parameter,
goes smoothly to zero at T_c
-- second order phase transition

Isotope Effect in 1950

- Lattice **vibration** is a part of the SC process.
- A crucial step to a microscopic theory.



Emanuel Maxwell, Phys. Rev. 78, 477 (1950)
C.A. Reynolds et al., Phys. Rev. 78, 487 (1950)

Emanuel Maxwell

Bernard Serin & Charles Reynolds

Isotope Effect

It has been observed that the critical temperature of superconductors varies with isotopic mass.

The experimental results within each series of isotopes may be fitted by a relation of the form

$$\boxed{M^\alpha T_c = \text{constant} \qquad \alpha \sim 0.5}$$

Table 4 Isotope effect in superconductors

Experimental values of α in $M^\alpha T_c$ = constant, where M is the isotopic mass.

Substance	α	Substance	α
Zn	0.45 ± 0.05	Ru	0.00 ± 0.05
Cd	0.32 ± 0.07	Os	0.15 ± 0.05
Sn	0.47 ± 0.02	Mo	0.33
Hg	0.50 ± 0.03	Nb_3Sn	0.08 ± 0.02
Pb	0.49 ± 0.02	Zr	0.00 ± 0.05

From the dependence of T_c on the isotopic mass we learn that lattice vibrations and hence electron-lattice interactions are deeply involved in superconductivity.

$\theta \propto \upsilon \propto M^{-1/2}$

$$\boxed{T_c \propto \theta_{\text{Debye}} \propto M^{-1/2}, \text{ so that } \alpha = \tfrac{1}{2}}$$

3. The penetration depth and the coherence length emerge as natural consequence of the BCS theory. The London equation is obtained for magnetic fields that vary slowly in space. Thus, the central phenomenon in superconductivity, the Meissner effect, is obtained in a natural way.

penetration depth (λ) ; coherence length (ξ)

4. The electron density of orbitals ***$D(E_F)$*** of one spin at the Fermi level, and the electron –lattice interaction ***U***. For ***$UD(E_F)$*** << 1, the BCS theory predicts:

$$\boxed{T_c = 1.14\theta \exp[-1/UD(\epsilon_F)] \quad , \quad 2\Delta / k_B T_c = 3.52}$$

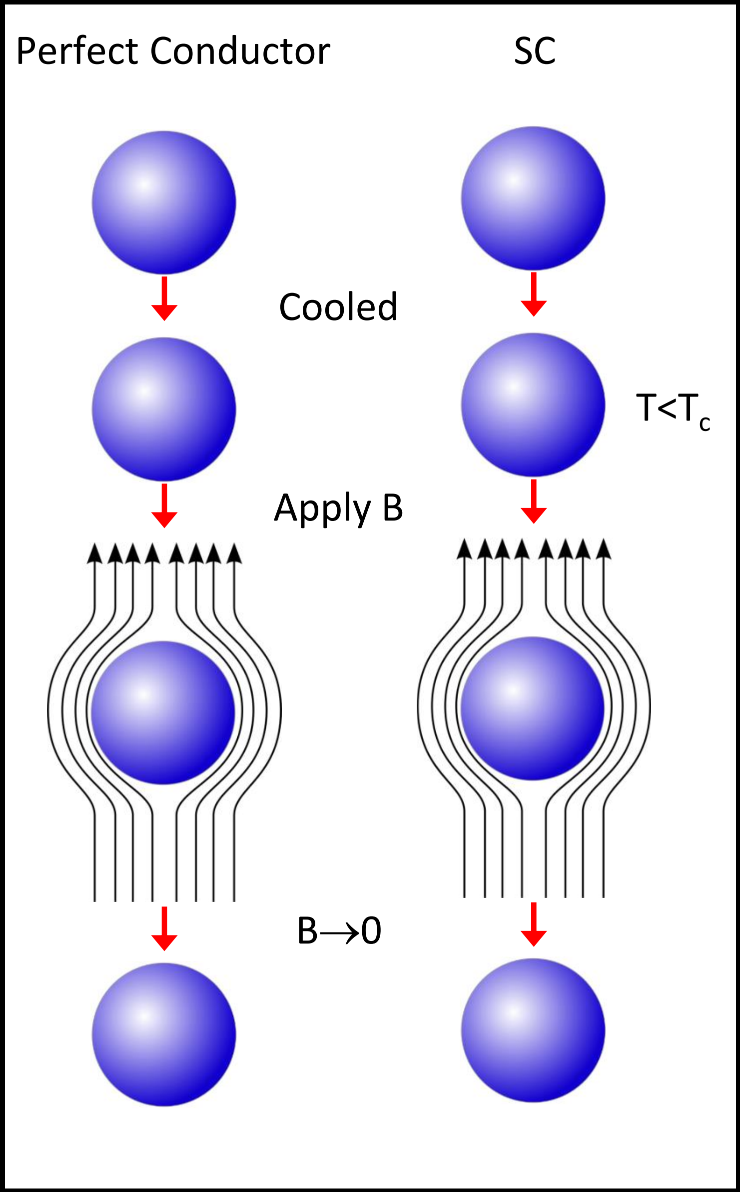
Where θ is the Debye temperature, and ***U*** is an attractive interaction (electron-phonon interaction).

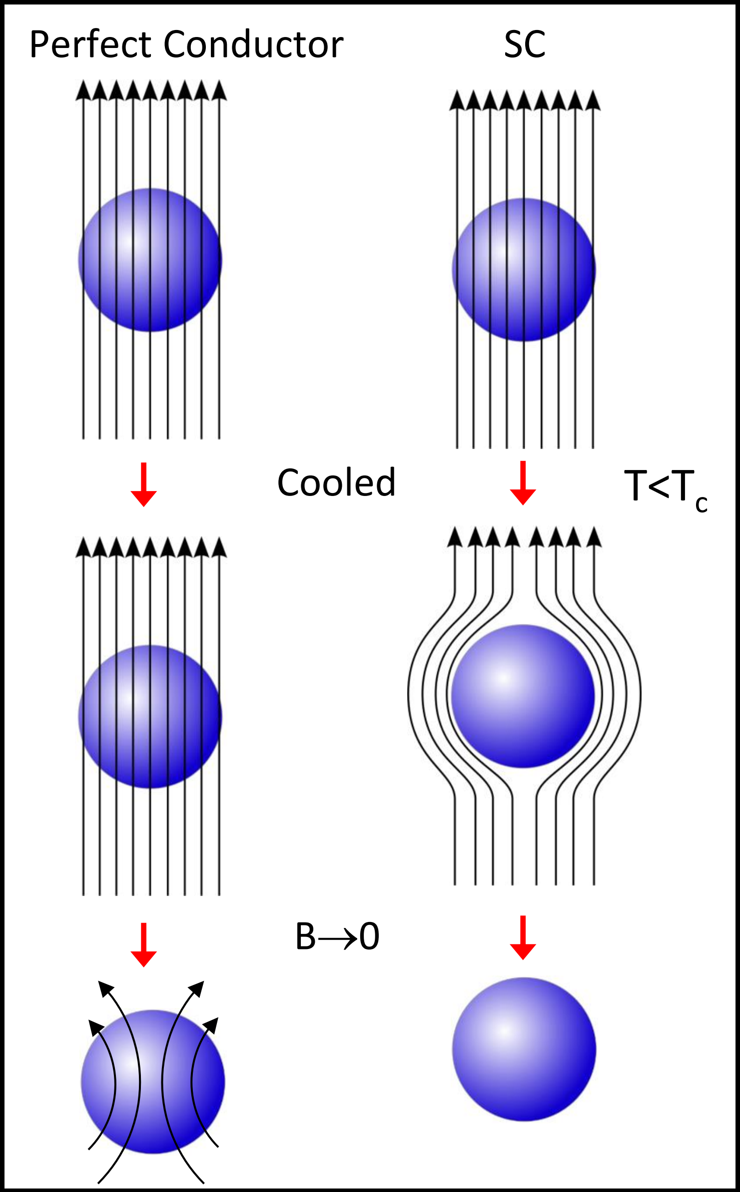
For dirty metal (a poor conductor) $\rightarrow$ $\rho(300)\uparrow$, $U\uparrow$, $T_c\uparrow$ (but a good SC)

5. Magnetic flux through a superconducting ring is quantized and the effective unit of charge is ***2e*** rather than ***e***.

Evidence of pairing of electrons

Perfect Conductor vs Superconductor





Vortex-Current Interaction

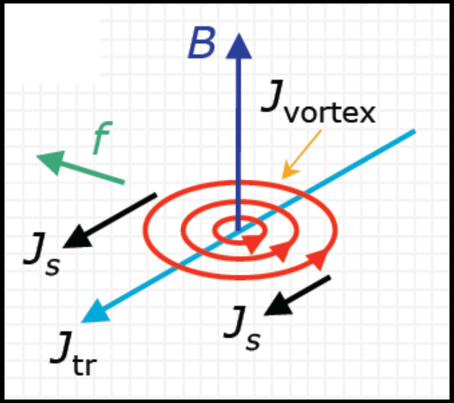
- Lorentz force on J_S due to the interaction between J_S and B.

$$f = \int J_s \times B \, d^2r = J_{tr} \times \int B \, d^2r = J_{tr} \times (\phi_0 \hat{B})$$

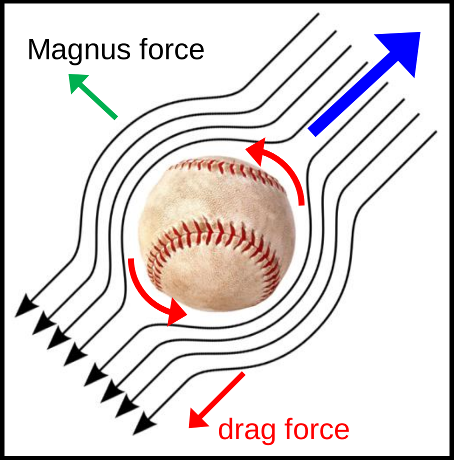
- Vortex motion implies that the vortex is subject to a power input per unit volume of vortex of characteristic radius r_B

$$P = \frac{f\,v}{\pi r_B^2} = J_{tr} \frac{\phi_0}{\pi r_B^2} v = \underbrace{J_{tr}\,B}_{\text{Lorentz force per unit volume}} v$$

- Vortex motion leads to dissipation! $R \neq 0$!
- Vortex pinning is crucial for applications.

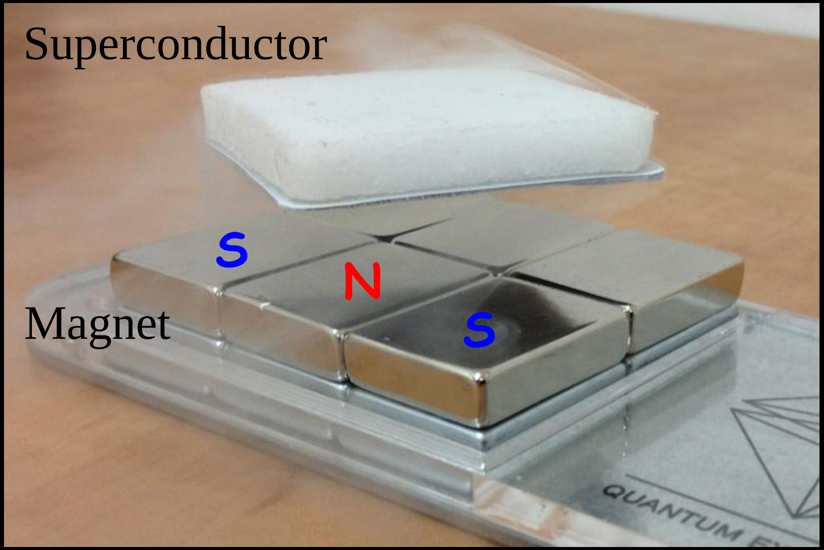


Magnus force



Quantum Levitation

Magnetic flux pinning is key.
Unstable for type I superconductors.



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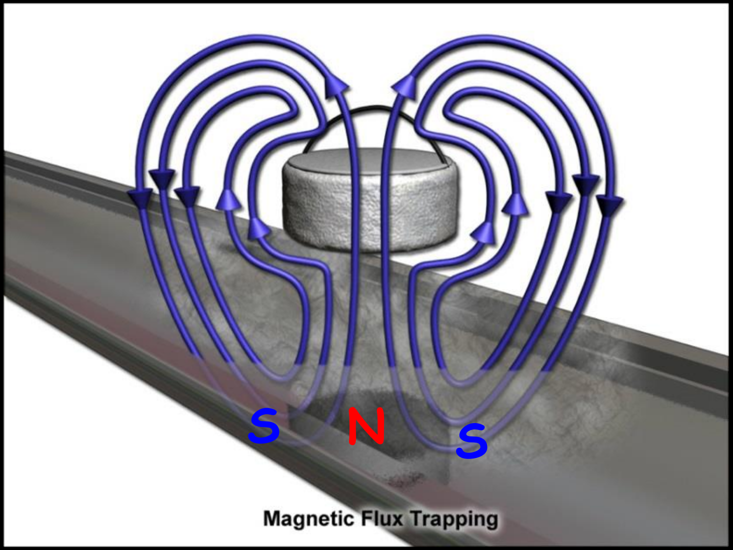

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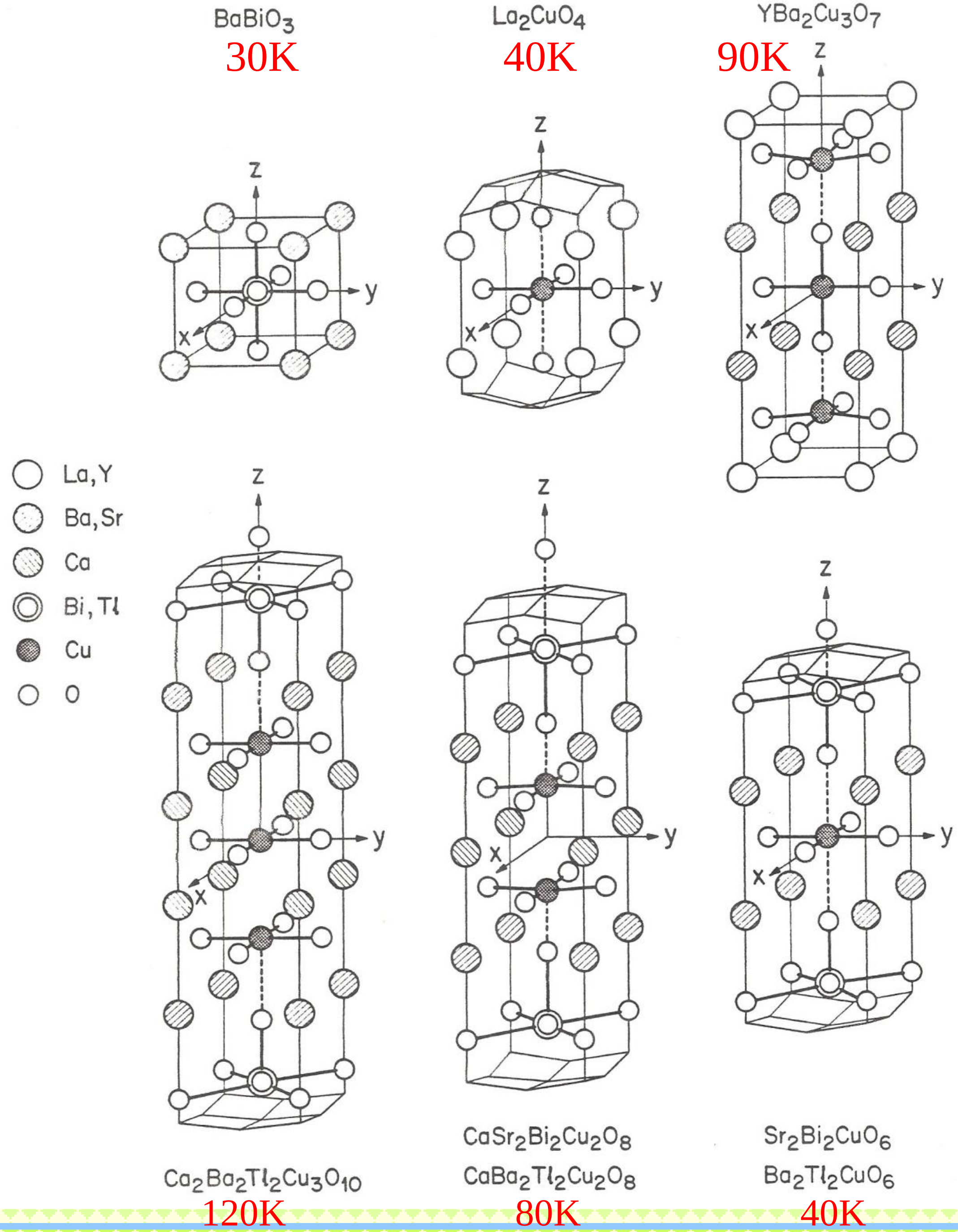
A legacy of Superconductivity



$BaBiO_3$
30K
La_2CuO_4
40K
$YBa_2Cu_3O_7$
90K
La,Y
Ba,Sr
Ca
Bi,Tl
Cu
O
z
y
x
$Ca_2Ba_2Tl_2Cu_3O_{10}$
120K
$CaSr_2Bi_2Cu_2O_8$
$CaBa_2Tl_2Cu_2O_8$
80K
$Sr_2Bi_2CuO_6$
$Ba_2Tl_2CuO_6$
40K