磁學 exchange interaction

李尚凡

$$\chi = \frac{\partial M}{\partial H} = -\frac{1}{V}\frac{\partial^2 E_d(H)}{\partial H^2}$$

$$E = M\cdot H \Rightarrow M = \frac{\partial E}{\partial H} \Rightarrow \frac{\partial M}{\partial H} = \frac{\partial^2 E}{\partial H^2}$$

Langevin Diamagnetism Eq.

Larmor theorm, pression in $\vec{H}$, $\omega = \frac{eB}{2mc}$ $\quad (\vec{H} = B\hat{z})$

dipolemoment $\mu = I\cdot A = \pi\cdot\rho^2\cdot\left(-\frac{Ze}{c}\right)\left(\frac{1}{2\pi}\frac{eB}{2mc}\right)$

$$= -\frac{Ze^2B}{4mc^2}\langle\rho^2\rangle$$

$$\langle\rho^2\rangle = \langle x^2\rangle + \langle y^2\rangle$$
$$\langle r^2\rangle = \langle x^2\rangle + \langle y^2\rangle + \langle z^2\rangle = 3\langle x^2\rangle$$
$$\langle\rho^2\rangle = \frac{2}{3}\langle r^2\rangle$$

單位體積的磁矩大小

$$\chi = \frac{\partial M}{\partial H} = \frac{N\mu}{B} = -\frac{NZe^2}{6mc^2}\langle r^2\rangle$$

體積有幾個 1個磁矩大小

Maxwell eq.

$$\vec{\nabla}\cdot\vec{B} = 0 \Rightarrow \vec{B} = \vec{\nabla}\times\vec{A} \Rightarrow \vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$$

$$H = \frac{1}{2m}\left(\vec{p} - \frac{e}{c}\vec{A}\right)^2 + Q\varphi$$

$$= \underbrace{\frac{p^2}{2m} + Q\varphi}_{H_0} + \overbrace{\boxed{\frac{ie\hbar}{2mc}(\vec{\nabla}\cdot\vec{A} + \vec{A}\cdot\vec{\nabla}) + \frac{e^2}{2mc^2}A^2}}^{=\Delta T}$$

e.g. $\vec{A} = -\frac{1}{2}\vec{r}\times\vec{B}$ $\begin{cases} A_x = -\frac{1}{2}yB \\ A_y = \frac{1}{2}xB \\ A_z = 0 \end{cases}$ $\quad (x,y,z)\times(0,0,B)$

$$H = H_0 + \Delta T + \Delta H_{spin} \quad \left(\Delta T = \frac{ie\hbar}{2mc}\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) + \frac{e^2B^2}{8mc^2}(x^2+y^2)\right)$$

$\Delta H_{spin} = \underset{\downarrow\, 2}{g_0} \mu_B B S_z \qquad S_z = \sum_i S_z^i$

$\rightarrow \langle \rho^2 \rangle = \frac{2}{3}\langle r^2 \rangle$

$\therefore \Delta T = \mu_B \vec{L} \cdot \vec{B} + \frac{e^2 B^2}{8mc^2}(x^2+y^2)$

system at ground state with filled e^- shell

$J|0\rangle = L|0\rangle = S|0\rangle = 0$

$\Delta E_0 = \frac{e^2}{12mc^2} B^2 \langle 0 | \sum_i r_i^2 | 0 \rangle$

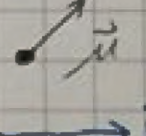
$\chi = -\frac{N}{V}\frac{\partial^2 E_0}{\partial B^2} = \boxed{-\frac{e^2}{6mc^2}\frac{N}{V}\langle 0 | \sum_i r_i^2 | 0 \rangle} < 0$

∴此是 Dia magnetism
逆

$H = H_0 + \mu_B(\vec{L} + g_0 \vec{S})\vec{B} + \frac{e^2 B^2}{8mc^2}(x^2+y^2)$

Ⅲ magnetism (paramagnetism) 順磁

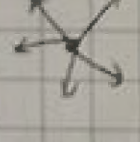
Classical (superparamag.)



$\underset{\searrow \text{energy}}{E} = -\underset{\searrow \text{dipole moment} = IA}{\vec{\mu}} \cdot \vec{B} = -\mu B \cos\theta$

$\longrightarrow \vec{H} = B\hat{z}$

很多



Boltzmenn factor $e^{-E/k_BT} = e^{\frac{\mu B}{k_BT}\cos\theta}$

$$P(\theta)d\theta = \frac{\exp\left[\frac{\mu B}{k_BT}\cos\theta\right]\sin\theta\, d\theta}{\int_0^{\pi}}$$

李尚凡

$$M = N\mu \langle\cos\theta\rangle$$

$$= N\mu \int_0^\pi \cos\theta\, P(\theta)\, d\theta$$

$$= N\mu \frac{\int_0^\pi \exp\left[\frac{\mu B}{k_B T}\cos\theta\right]\cos\theta \sin\theta\, d\theta}{\int_0^\pi \exp\left[\frac{\mu B}{k_B T}\cos\theta\right]\sin\theta\, d\theta}$$

$$\frac{\mu B}{k_B T} \equiv \alpha \quad \cos\theta \equiv x$$

$$= N\mu \frac{\int_{-1}^{1} \exp[\alpha x]\, x\, dx}{\int_{-1}^{1} \exp[\alpha x]\, dx}$$

数学技巧

$$\int_{-1}^{1} e^{\alpha x} dx = \frac{1}{\alpha}\left[e^{\alpha x}\right]_{-1}^{1} = \frac{1}{\alpha}\left(e^{\alpha} - e^{-\alpha}\right)$$

$$\frac{d}{d\alpha}(\quad) = \frac{1}{\alpha}\left(e^{\alpha}+e^{-\alpha}\right) - \frac{1}{\alpha^2}\left(e^{\alpha}-e^{-\alpha}\right)$$

$$\parallel$$

$$\int_{-1}^{1} e^{\alpha x} x\, dx$$

$$= N\mu \left(\frac{e^{\alpha}+e^{-\alpha}}{e^{\alpha}-e^{-\alpha}} - \frac{1}{\alpha}\right)$$

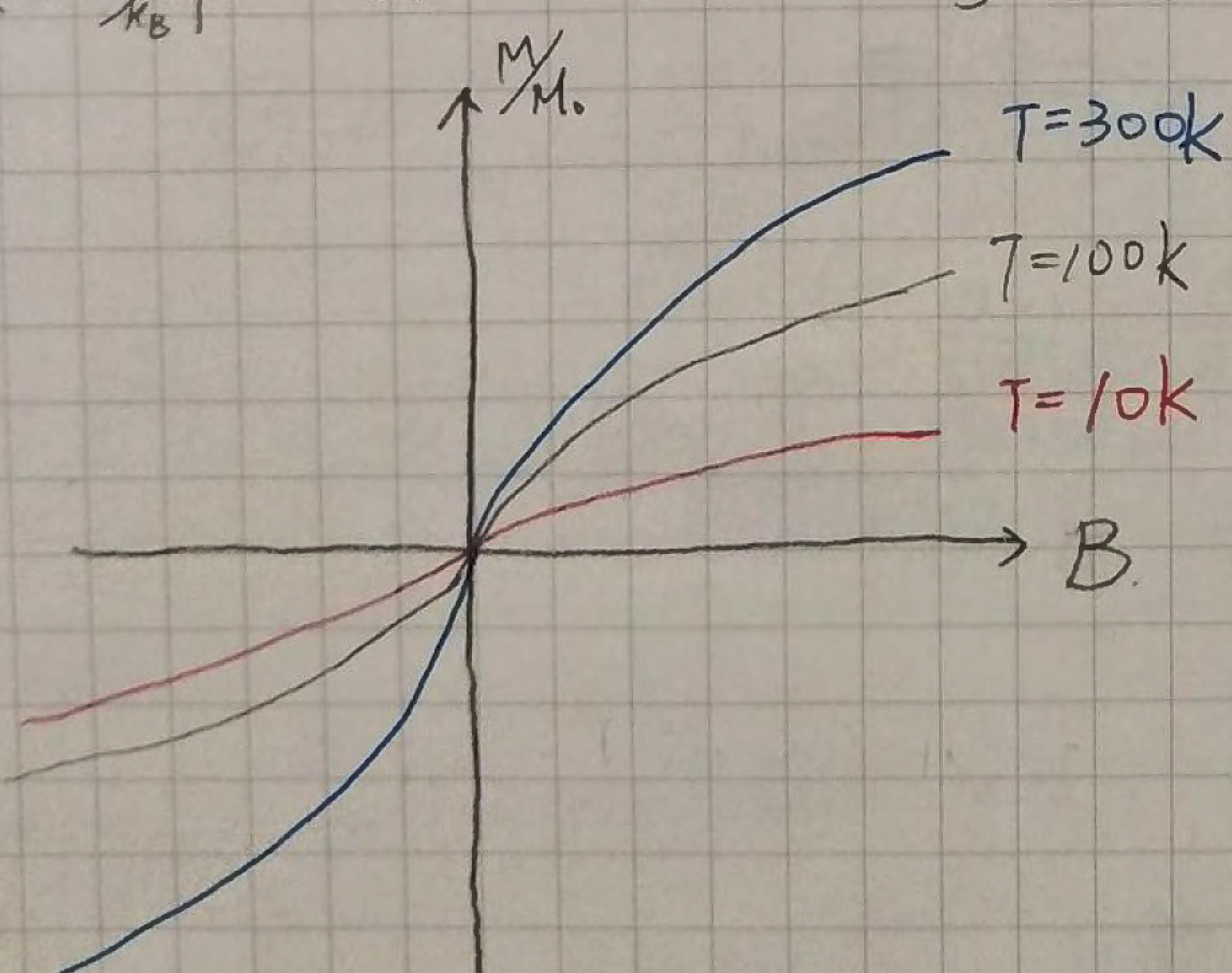
$$= N\mu \left(\boxed{\coth\alpha - \frac{1}{\alpha}}\right)$$

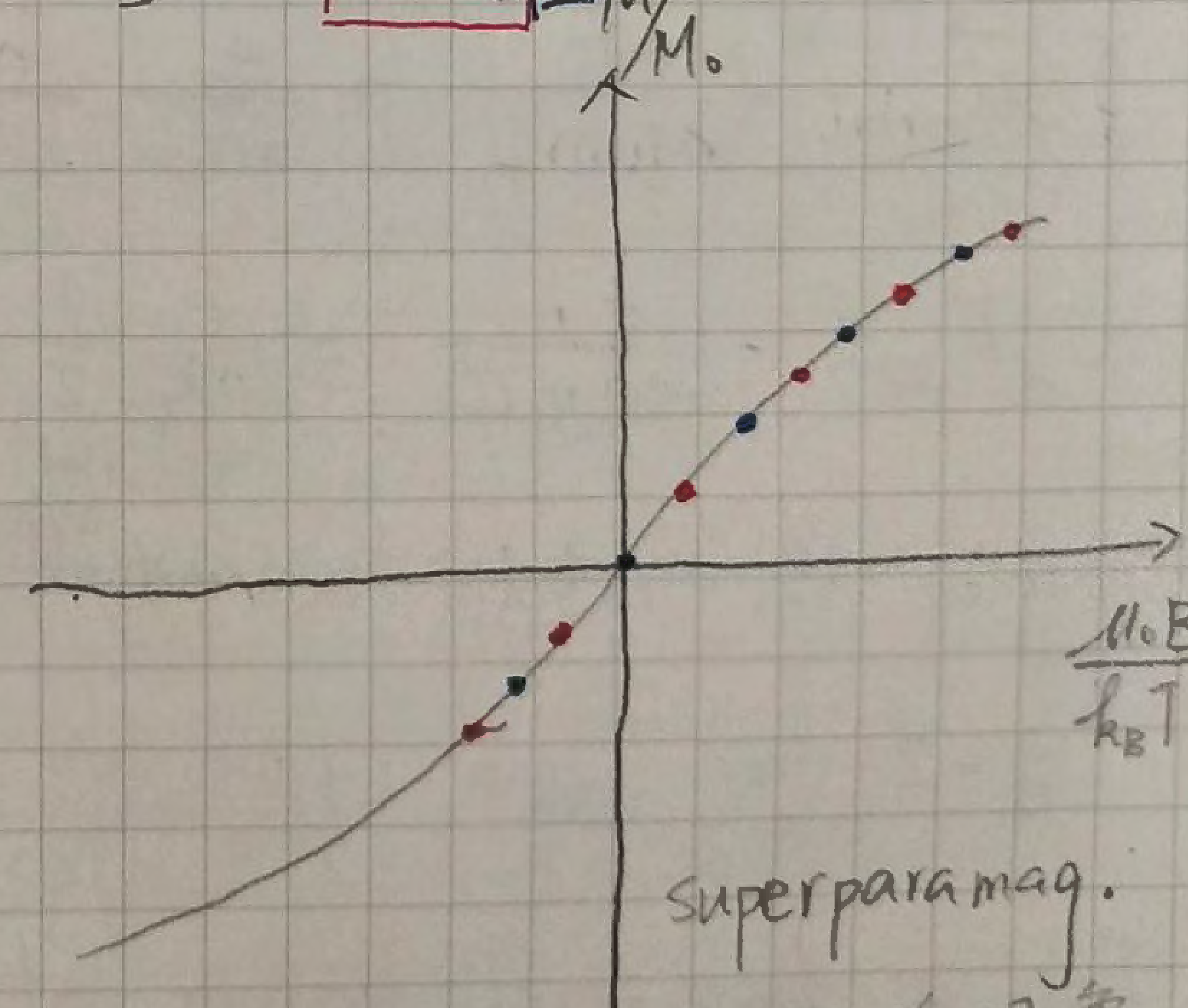
$\equiv L(\alpha)$ Langevin func.

$$L(\alpha) = \frac{\alpha}{3} - \frac{\alpha^3}{45} + \frac{2\alpha^5}{945} + \cdots\cdots$$

$$\alpha \equiv \frac{\mu B}{k_B T} \qquad \alpha \ll 1 \Rightarrow L(\alpha) = \frac{\alpha}{3} \quad \Rightarrow M = \frac{N\mu\alpha}{3} = \frac{\overset{\text{const}}{\boxed{\frac{N\mu^2}{3k_B}}}\,B}{\boxed{T}}$$

居禮溫度.





superparamag.

超順磁現象

$$\chi = \frac{M}{B} = \frac{N\mu^2}{3k_B}\frac{1}{T} = \frac{C}{T}$$ → 居禮常数

Quantum theory in free space

$$\vec{\mu} = \gamma \hbar \vec{J} = -g \mu_B \vec{J} \qquad \mu_B \equiv \frac{e\hbar}{2mc} \text{ in cgs}$$

Bohr magneton

$$\hbar \vec{J} = \hbar \vec{L} + \hbar \vec{S}$$

gyromag. ratio

magnetogyric "

g factor, spectroscopic splitting factor

$$g \mu_B \equiv -\gamma \hbar \qquad g = 2.0023$$

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$ Landé eq.

$$U = -\vec{\mu} \cdot \vec{B} = m_J g \mu_B B \qquad m_J = -J, -J+1, \cdots, J$$

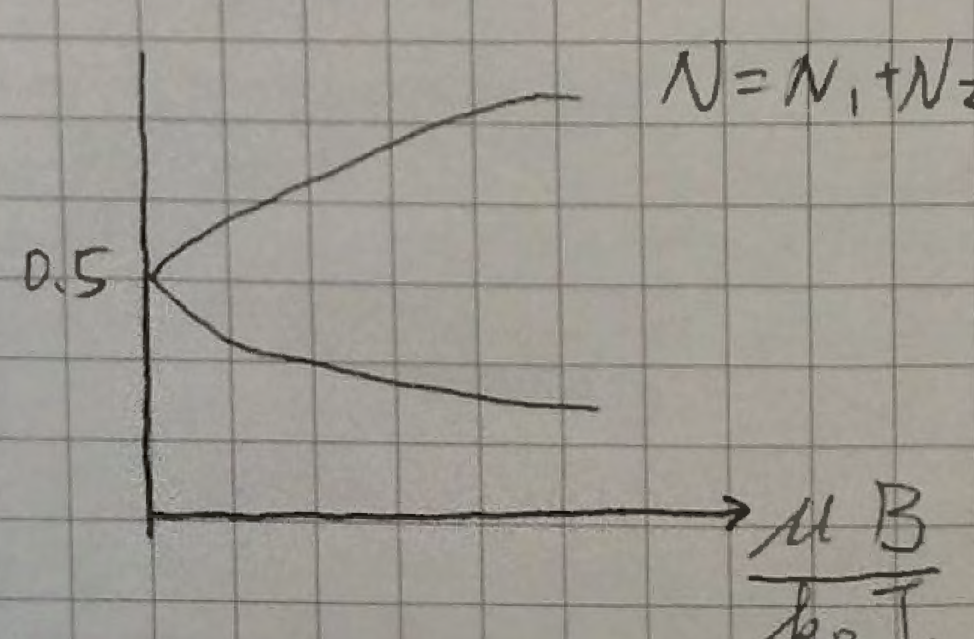
for $\boxed{\text{an } e^-}$

$$\begin{cases} \vec{\mu}_{orbit} = \dfrac{eh}{4\pi mc} = \dfrac{e}{2mc}\dfrac{h}{2\pi} = \dfrac{-e}{2mc}\vec{P}_{orbit} \\ \vec{\mu}_{spin} = \dfrac{eh}{4\pi mc} = \dfrac{e}{mc}\dfrac{1}{2}\dfrac{h}{2\pi} = \dfrac{-e}{mc}\vec{P}_{spin} \end{cases}$$

$g = 1$ for orbital moment

$g = 2$ for spin "

For a two level system at eq.



$$\frac{N_1}{N} = \frac{\exp[\alpha]}{\exp[\alpha] + \exp[-\alpha]} \qquad \alpha \equiv \frac{\mu B}{k_B T}$$

$$\frac{N_2}{N} = \frac{e^{-\alpha}}{e^{-\alpha} + e^{-\alpha}}$$

李尚凡

$$M = (N_1 - N_2)\mu$$

$$= N\mu \frac{e^x - e^{-x}}{e^x + e^{-x}} \qquad x = \alpha$$

$$= N\mu \tanh(x)$$

for $x \ll 1$ $\quad M \simeq \boxed{\frac{N\mu^2}{k_B}}\frac{B}{T}$

For an atom with quantum number J. 總角动量

$$E = -m_J g \mu_B B$$

Boltzmann factor

$$M = N \frac{\sum_{m_J} m_J g \mu_B \exp\left[\frac{m_J g \mu B}{k_B T}\right]}{\sum_{m_J} \exp\left[\frac{m_J g \mu B}{k_B T}\right]}$$

$$= Ng\mu \left[\frac{2J+1}{2J} \coth\left(\frac{2J+1}{2J} x\right) - \frac{1}{2J} \coth\left(\frac{x}{2J}\right)\right]$$

$$= Ng\mu J \boxed{B_J(x)}$$ Brilbuin function

Curie - Brilbuin law.

$$\frac{M}{Ng\mu_B J} = B_J(x) \quad \text{for } x \ll 1 \qquad \coth = \frac{1}{x} + \frac{x}{3} - \frac{x^3}{45}$$

when $J \to \infty$ $\quad B_J(x) = \coth x - \frac{1}{2J} - \frac{2J}{x} = \coth x - \frac{1}{x}$

back to Langevin func.

$$\chi = \frac{M}{B} = \frac{NJ(J+1) g^2 \mu^2}{3kT} = \frac{N P^2 \mu^2}{3k} \cdot \frac{1}{T} = \frac{C}{T}$$

→ 餘結 e⁻ 数

$P \equiv g[J(J+1)]^{1/2}$ effective number of Bohr magnetons.

Magtetic State $^{2S+1}L_J$

L : 0 , 1 , 2 , 3
s p d f 軌域

e.g full shells $L=0$, $S=0$, $J=0$
滿的軌域

Gd^{3+} : $4f^7 5S^2 5p^6$, $n=4$ $l=3$

half fill 半滿

$L=0$, $S=\frac{1}{2}(2l+1)=\frac{7}{2}$ $J=L+S=\frac{7}{2}$

$\Rightarrow$ $^8S_{7/2}$

Ising model $E=-\sum_{ij} S_i \cdot S_j \cdot A$ or $-\sum_{ij} \vec{S}_i \cdot \vec{S}_j \cdot A$

$|1\rangle = \frac{x+iy}{\sqrt{2}}$ $|0\rangle = z$ $|-1\rangle = \frac{x-iy}{\sqrt{2}}$ (second quantization)

For Iron group ions

$\chi = \frac{C}{T}$ doesn't work

effective number of Bohr magneton

$p \equiv g\sqrt{J(J+1)}$

$p' = g\sqrt{S(S+1)}$ $L=0$ why L is quenched ?

4f shell is deep inside the ions

3d shell is the outer in homogeneous $\boxed{\text{crystall field}}$

$\vec{L}\cdot\vec{S}$ spin orbital coupling

Quenching of the orbital angular momentum L_z

In a free atom with one e^- in p-state

$$R_{nl}(r)\,Y_{11}(\theta,\phi) \propto f(r)\left[\frac{x+iy}{\sqrt{2}}\right]$$

$$R_{nl}(r)\,Y_{10}(\theta,\phi) \propto f(r)\, z$$

$$R_{nl}(r)\,Y_{1-1}(\theta,\phi) \propto f(r)\left[\frac{x-iy}{\sqrt{2}}\right]$$

I guess second quan z

Consider the e^- in $L=1$ orbit

the atom in an inhomogeneous crystal field

For 簡化計算, 設計 $S=0$

take $S=0$ in orthorhombic symmetry (斜方晶系)

$e\phi = Ax^2 + By^2 - (A+B)z^2$ A,B are constant

$\nabla^2\phi = 0$

Consider the e^- in $L=1$ orbit 李尚凡

the atom in an inhomogeneous crystal field

take $S=0$ in orthorhombic symmetry

斜方晶系

$e\phi = Ax^2 + By^2 - (A+B)y^2$, A, B are Const.

$\nabla^2\phi = 0$

the unperturbed ground state of the ion

$U_x = x f(r)$ $U_y = y f(r)$ $U_z = f(z)$ normalized orthoganal to each other.

$\mathcal{L}^2 U_i = L(L+1) U_i = 2U_i$

$\mathcal{L}$: angular momentum operator

U_i are diagonal w.r.t. $e\phi$

off-diagonal terms

$\langle U_x | e\phi | U_y \rangle = \langle U_y | e\phi | U_z \rangle = \langle U_z | e\phi | U_x \rangle = 0$

Consider the e^- in $L=1$ orbit

$$\langle U_x | e\phi | U_y \rangle = \int \underbrace{xy\,|f(r)|^2}_{\text{odd in } x \cdot y} \{A x^2 + By^2 - (A+B)z^2\}\, dr dy.$$

$= 0$

$$\langle U_x | e\phi | U_x \rangle = \int |f(r)|^2 \{ Ax^4 + Bx^2y^2 - (A+B)x^2y^2 \} dxdy$$

$$= A(I_1 - I_2) \quad \text{with} \quad I_1 = \int |f(r)|^2 x^4 dxdy$$

$$I_2 = \int |f(r)|^2 x^2 y^2 dxdy.$$

$\langle U_x | L_z | U_x \rangle$

$$\underbrace{|1\rangle = \frac{x+iy}{\sqrt{2}} \qquad |0\rangle = z \qquad |-1\rangle = \frac{x-iy}{\sqrt{2}}}$$

$$x = \frac{1}{\sqrt{2}}(|1\rangle + |-1\rangle)$$

$$y = \frac{1}{\sqrt{2}i}(|1\rangle - |-1\rangle)$$

$$\left.\begin{array}{l} \langle U_x | L_z | U_x \rangle \\ \langle U_y | L_z | U_y \rangle \\ \langle U_z | L_z | U_z \rangle \end{array}\right\} = 0$$

<u>Spectroscopic splitting factor</u> consider crystal field $e\phi$

orbital wavefunc. $U_x = x f(r)$

$S = \frac{1}{2} \quad S_z = \pm\frac{1}{2}$

spin func. α β

$\lambda \vec{L} \cdot \vec{S}$

$$|\psi\rangle = \frac{\langle \psi_0 | \lambda \vec{L} \cdot \vec{S} | \psi_0 \rangle}{E_0 - E_n} \qquad \psi_0 = U_x \alpha = x f(r) \alpha$$

李尚凡

$$\psi = \left[U_x - i\left(\frac{\lambda}{2\Delta_1}\right) U_y \right]\alpha - i\left(\frac{\lambda}{2\Delta_2}\right) U_z \beta$$

$$\begin{cases} \Delta_1 \equiv E(U_y) - E(U_x) \\ \Delta_2 \equiv E(U_z) - E(U_x) \end{cases} \qquad \langle \psi | L_z | \psi \rangle = \frac{-\lambda}{\Delta_1}$$

magnetic moment in $\hat{z}$

$$\mu_B \langle \psi | L_z + 2S_z | \psi \rangle = \left[-\frac{\lambda}{\Delta_1} + 1 \right] \mu_B \qquad S_z = 1\,\frac{1}{2}$$

magnetic field B. $\Delta E = g \mu_B B = \boxed{2\left[1 - \frac{\lambda}{\Delta_1}\right]} \mu_0 B$

通常寫成 $\vec{S} = \frac{\hbar}{2}\vec{\sigma}$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \uparrow \text{ (spin up)}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \beta = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \downarrow \text{ (spin down)}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

用矩陣的形式表達

$$\hat{S}_z \alpha = \frac{\hbar}{2}\alpha$$

$$U_x = x f(r)$$

VanVleck temperature-indep. paramagnetism

只講結果

Curie' law $\chi \equiv \frac{C}{T}$

paramagentic susceptibility of conducting e^-
順磁

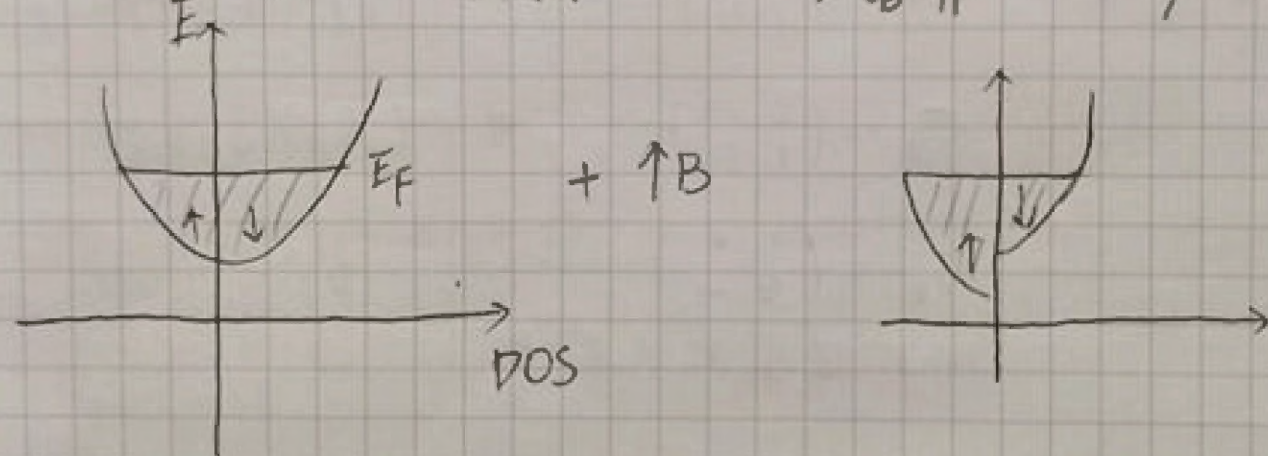
$$M_{e^-} = \frac{N\mu_B^2}{k_B}\frac{B}{T} \quad ?? \quad (不是)$$

Pauli : Fermi-Dirac distribution

only e^- near ε_F ($\pm k_B T$) can rotate

$$\frac{k_B T}{\varepsilon_F} = \frac{T}{T_F} \quad \text{fermi temperature}$$

$$M = \left(n\frac{T}{T_F}\right)\left(\frac{\mu_B^2 B}{k_B T}\right) = \frac{n\mu_B^2 B}{k_B T_F} \quad \text{indep of temp.}$$



parallel to B

$$N_\uparrow = \frac{1}{2}\int_{-\mu B}^{\varepsilon_F} d\varepsilon \; D(\varepsilon+\mu B)$$

$$\simeq \frac{1}{2}\int_0^{\varepsilon_F} d\varepsilon \; D(\varepsilon) + \frac{1}{2}D(\varepsilon)\mu B$$

$$N_\downarrow \simeq \frac{1}{2}\int_0^{\varepsilon_F} d\varepsilon \; D(\varepsilon) - \frac{1}{2}D(\varepsilon)\mu B$$

$$M = \mu(N_\uparrow - N_\downarrow) = \mu^2 D(\varepsilon)\, B$$

$$D(\varepsilon_F) = \frac{3}{2}\frac{N}{\varepsilon_F}$$

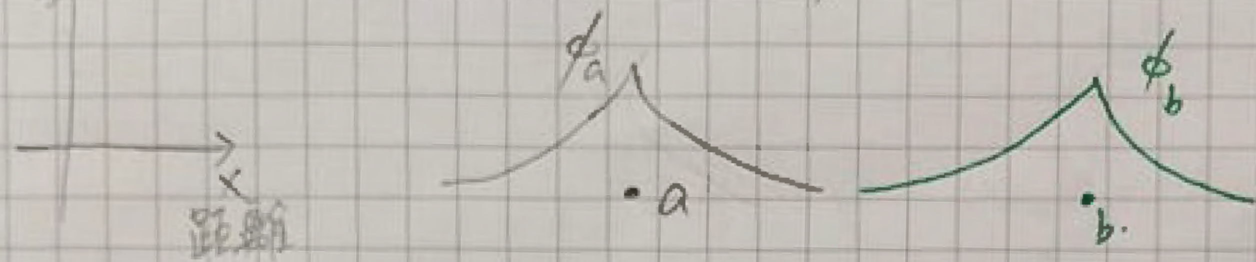
$$M = \frac{3}{2}\frac{N\mu^2}{k_B T_F} B \quad \text{Pauli spin magnetization}$$

李尚凡

Ferromagnetism

Exchange interaction !!

ex: 2-e^- system H_2



$$H\Psi = \left[\sum_{i=1}^{2} -\frac{\hbar^2}{2m}\nabla_i^2 - \frac{e^2}{|\vec{r}_i - \vec{R}_a|} - \frac{e^2}{|\vec{r}_i - \vec{R}_b|} + \frac{e^2}{|\vec{r}_i - \vec{r}_j|} \right] \Psi(\vec{r}_1, \vec{r}_2)$$

$= E\Psi(\vec{r}_1, \vec{r}_2)$ spin-indep Schr. eq $\xRightarrow{\text{也可以得到}}$ M

spin states $|\uparrow,\uparrow\rangle$, $|\uparrow\downarrow\rangle$, $|\downarrow,\uparrow\rangle$, $|\downarrow,\downarrow\rangle$

ψ_s $\uparrow\uparrow$ $\xrightarrow{\text{若变}}$ $-\psi_s$ $\quad \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

exchange spins

表示它是 antisymmetry $\quad S=0 \quad S_z=0$

spin singlet.

ψ_s $\uparrow\uparrow$ $\longrightarrow$ ψ_s

$$\begin{cases} |\uparrow\uparrow\rangle & S=1 \ S_z=1 \\ \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) & \text{spin triplet. } S=1 \ S_z=0 \\ |\downarrow\downarrow\rangle & S=1 \ S_z=-1 \end{cases}$$

$\boxed{\text{2}e^- \text{ system}}$: $E_s < E_t$ ① $h\phi(\vec{r}) = \varepsilon\,\phi(\vec{r})$ one e^- schr. eq

ϕ_0 ϕ_1 ϕ_2 ... with ε_0, ε_1 ...

② two e^-

singlet symmetric $\psi(\vec{r}_1, \vec{r}_2) = \phi_0(\vec{r}_1)\,\phi_0(\vec{r}_2)$

是anti ∴ 空間上是 $\qquad E_s = 2\varepsilon_0$

triplet, antisymmetric $\psi(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}}[\phi_0(\vec{r}_1)\,\phi_1(\vec{r}_2)$
$- \phi_0(\vec{r}_2)\,\phi_1(\vec{r}_1)]$
??

$$E_t = \varepsilon_0 + \varepsilon_1$$

$\therefore\ E_s - E_t < 0$

If $\phi_0(r) = \varphi_a(r) + \varphi_b(r)$ symmetric

$\phi_1(r) = \varphi_a(r) - \varphi_b(r)$ antisym.

ignore $e^- - e^-$ interaction

$J = E_s - E_t \neq 0$ $\boxed{\text{exchange interaction splitting}}$

$J < 0$ singlet ground state, $S = 0$ non mag.

$J > 0$ $S \neq 0$ magnetic

$$E = \sum_{ij} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

若 $J<0$ 則 ↑↑ or ↓↓ 能量是負的,較低,∴材料是鐵磁

若 $J>0$ 則 ↓↑ or ↑↓ 能量才是負的 ∴材料是反鐵磁

◎ spin Hamiltonian and Heisenberg Model

Map the H problem to a spin system with 4 eigen states

Spin Operator

$\vec{S}_i^{\,2} = \frac{1}{2}(\frac{1}{2}+1) = \frac{3}{4}$ individual spin

考慮 $\vec{e}^-$

$$\vec{S}^2 = (\vec{S}_1 + \vec{S}_2)^2 = \vec{S}_1^{\,2} + \vec{S}_2^{\,2} + 2\vec{S}_1 \cdot \vec{S}_2$$

$$= \frac{3}{2} + 2\, \vec{S}_1 \cdot \vec{S}_2$$

For singlet, $S = (+\frac{1}{2} - \frac{1}{2}) = 0 \Rightarrow \vec{S}_1 \cdot \vec{S}_2 |\phi_s\rangle = \frac{1}{2}[\vec{S}^2 - \vec{S}_1^{\,2} - \vec{S}_2^{\,2}] |\phi_s\rangle$

$$= \frac{1}{2}[0(0+1) - \frac{3}{4} - \frac{3}{4}] |\phi_s\rangle$$

$$= -\frac{3}{4} |\phi_s\rangle$$

For triplet, $S = 1 \Rightarrow \vec{S}_1 \cdot \vec{S}_2 |\phi_t\rangle = \frac{1}{2}[1(1+1) - \frac{3}{4} - \frac{3}{4}] |\phi_t\rangle$

$$= \frac{1}{4} |\phi_t\rangle$$

Trial spin Hamiltonian

$$H^{spin} = \frac{1}{4}(E_s + 3E_t) - \underbrace{(E_s - E_t)}_{J} \vec{S}_1 \cdot \vec{S}_2 \quad \text{(湊出來)}$$

J 系数 不是 coupling 的 J S.L

$$H^{spin} |\phi_s\rangle = E_s |\phi_s\rangle$$

$$H^{spin} |\phi_t\rangle = E_t |\phi_t\rangle$$

redefine energy reference point

$$H^{spin} = -J\, \vec{S}_1 \cdot \vec{S}_2 \qquad J \equiv E_s - E_t$$

For a N-ion system

$$H^{spin} = -\sum_{i,j} \underbrace{\boxed{J_{ij}}}_{\text{exchange coupling const.}} \vec{S}_i \cdot \vec{S}_j$$

Heisenberg $J > 0$ ferro

$J < 0$ antiferro.

Molecular-field approximation
(internal)

$$H^{spin} = -\sum_{ij} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

$$\approx -\sum_{k} \Big[\sum_{i<k} J_{ik} \langle \vec{S}_i \rangle \cdot \vec{S}_k + \sum_{j>k} J_{kj}\, \vec{S}_k \cdot \langle \vec{S}_j \rangle \Big]$$

考慮第k個spin.
跟其它的交互作用

$$= -\sum_{k} \vec{S}_k \cdot \overbrace{\Big(\sum_{i\neq k} J_{ik} \langle \vec{S}_i \rangle \Big)}^{\text{molecular field or Wiess field}}$$

$$E = \vec{\mu} \cdot \vec{B} \qquad \vec{\mu} = g \cdot \mu_0 \vec{S}_i$$

B↑
內在
裡外加

$$H^{spin} = -\sum_{i} (g \mu_0 S_i^z) B_i^z$$

它感受到的內場

$$= -\sum_{i} \mu_i^z \cdot B_i^z$$

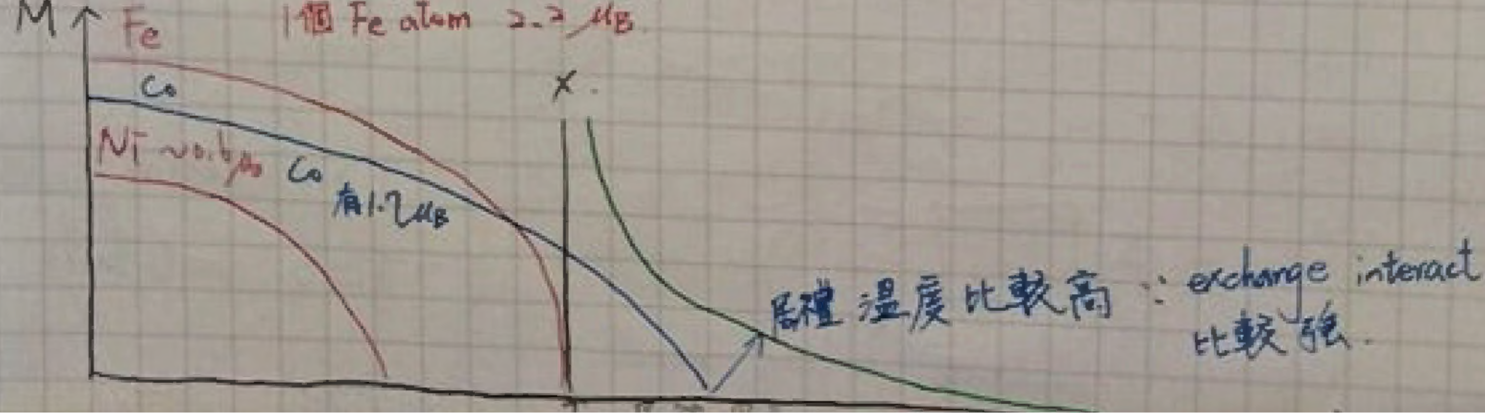
$$B_i^z = \frac{1}{g\mu_0} \sum_{i\neq j} J_{ij} \langle S_j^z \rangle$$

$$\vec{B}_i = \lambda \vec{M}$$

$$B^z = \lambda M^z = \frac{\lambda g \mu_0 \langle S_j^z \rangle}{V}$$

$\boxed{\text{單位体積有多少 dipole moment.} = M}$

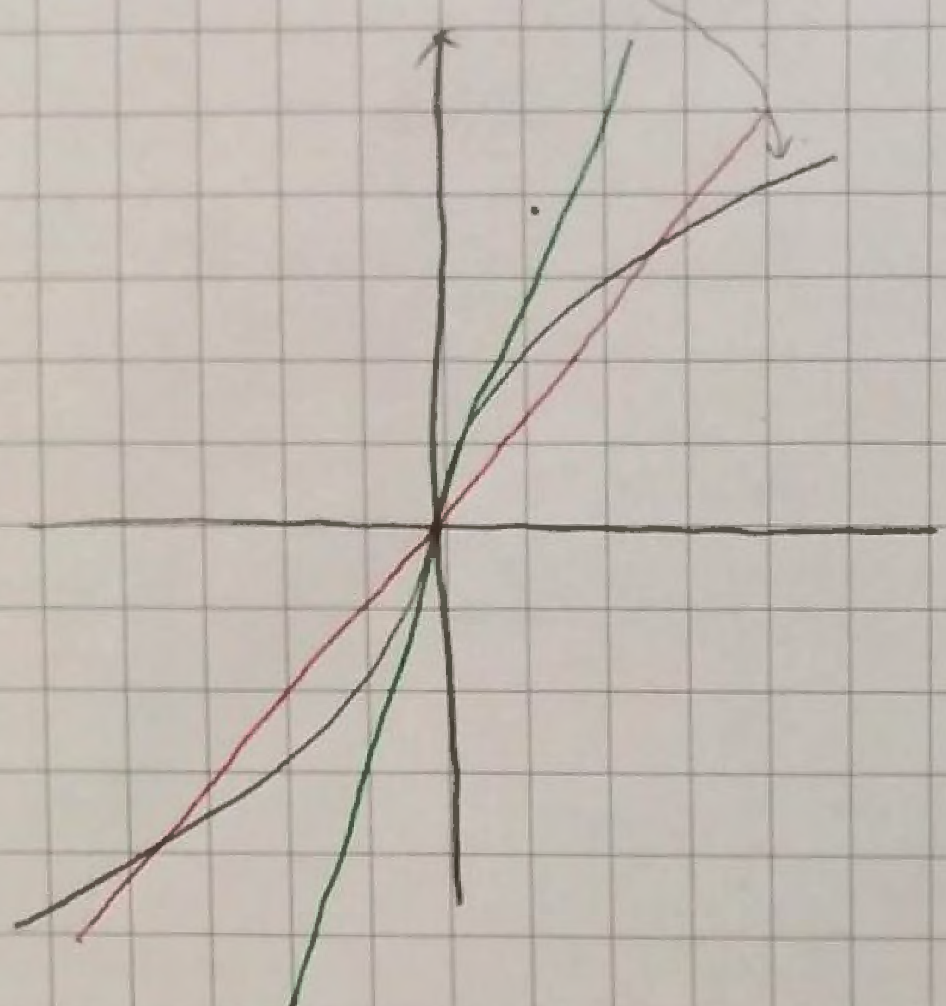
λ: molecular field coefficient



$T<T_c \quad \vec{B} = B_z\hat{z} = 0$

$$M^z = N\mu \tanh \frac{\lambda\mu_0 M^z}{k_B T}$$

單体的 dipole moment



$M \propto (T-T_c)^{1/2}$

$\chi \propto (T-T_c)^{-1}$

由上張圖可知 $M\uparrow$ T_c不一定$\uparrow$

under external magnetic field $\vec{B} = B_z\hat{z}$

$$H^{tot} = \sum_i \mu_i^z (\underbrace{B_i^z}_{\text{internal}} + \underbrace{B^z}_{\text{external}})$$

magnetization $M^z = \sum_i \mu_i^z = N\langle\mu^z\rangle$

↳ 之前算过 two level sys

$$\langle\mu_z\rangle = \mu_0 \tanh \frac{\mu_0(B_i^z + B^z)}{k_B T}$$

a) $B_i^z = 0$ 假設 total

$$M^z = N\langle\mu_z\rangle = N\mu_0 \tanh \frac{\mu_0 B^z}{k_B T}$$

for $B \to 0$ $\approx \dfrac{N\mu_0^2 B^z}{k_B T}$, $\chi = \dfrac{M}{B} = \dfrac{N\mu_0^2}{k_B}\dfrac{1}{T}$

use $\langle\mu^2\rangle = (\underbrace{g}_{2}\mu_0)^2 \underbrace{\langle \vec{S}^2\rangle}_{=\frac{3}{4}} = 3\mu_0^2$

$\therefore$ $\boxed{\chi = \dfrac{N\langle\mu^2\rangle}{3k_B T}}$ Wiess law.

b) $B_i^z \neq 0$

$$M^z = N\mu_0 \tanh \frac{\mu_0 (B^z + \lambda M^z)}{k_B T}$$

移項 $M^z\left(1 - \frac{N\mu_0^2 \lambda}{k_B T}\right) = \frac{N\mu_0^2 B^z}{k_B T}$ $M^z \to 0,\ B^z \to 0$

$$\chi = \frac{M^z}{B^z} = \frac{N\mu_0^2}{k_B T\left(1 - \frac{N\mu_0^2\lambda}{k_B T}\right)} = \frac{N\mu_0^2}{k_B T - N\mu_0^2\lambda}$$

$$= \frac{N\langle\mu^2\rangle/3}{k_B T - N\langle\mu^2\rangle\lambda/3} = \frac{N\langle\mu^2\rangle}{3k_B} \cdot \frac{1}{T - T_c}$$

with $\boxed{T_c \equiv \frac{N\langle\mu^2\rangle\lambda}{3k_B}}$ Curie-Weiss law.

Next week.

鉄磁材料為何是鉄磁？且 why 过渡金屬只有 Fe Co Ni