Single Electron Tunneling

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Single Charge Tunneling, Coulomb Blockade Phenomena in Nanostructures
NATO ASI Series, Vol. B edited by H Grabert and M H Devoret,
Plenum Press New York, 1991

Single Electron Tunneling

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Chapter of the book on
Quantum Transport and Dissipation
T Dittrich P Hanggi G Ingold B Kramer G Schon W Zwerger
VCH Verlag
revised version October, 1997

Single-Electron Devices and Their Applications, KONSTANTIN K. LIKHAREV,
PROCEEDINGS OF THE IEEE, VOL. 87, NO. 4, APRIL 1999

Electron transmission through a barrier

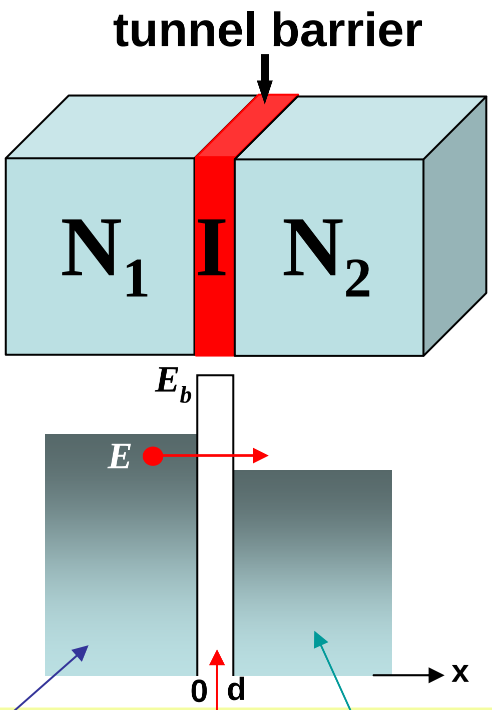
$$i\frac{\partial}{\partial t}\Psi(xt) = H\Psi(xt)$$

$$E\Psi(xt) = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + E_b(x)\right)\Psi(xt)$$

Separation of variables $\Psi(xt) = \Psi_x(x)\Psi_t(t)$

$$\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi_x(x) + (E - E_b)\Psi_x(x) = 0$$

$$T(E) = |S(E)|^2 = \left[1 + \frac{\sinh^2(kd)}{4(E/E_b)(1-E/E_b)}\right]^{-1}$$



$$\Psi_x(x)\begin{cases} Ae^{ikx} + Be^{-ikx} \\ Ce^{-kx} + De^{kx} \\ AS(E)e^{ik(x-d)} \end{cases}$$

tunneling matrix element

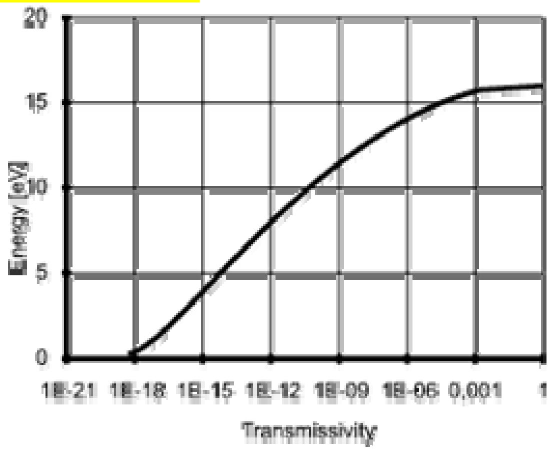
for $\lambda = 2\pi/k << d$

$$T(E) \approx 16\frac{E}{E_b}\left(1 - \frac{E}{E_b}\right)\exp(-2kd)$$

$$k = \sqrt{2m(E_b - E)}/\hbar$$

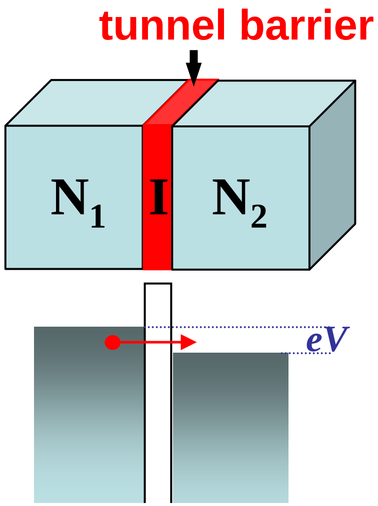
For a typical case, *E*=12V, barrier height=4V (*i.e.* E_b=16V), *d*=1nm

→ *T*(*E*)≈10^{-9}



D:\CCD\Articles\from web\
The Tunneling Current - A Simple Theory.htm

Tunneling



Rules: 1. occupied state to empty state

2. No associated energy change

Current from N_1 to N_2

$$I_{1\to 2} = A\int_{-\infty}^{\infty} |T|^2 N_1(E) f(E) N_2(E+eV)[1-f(E+eV)]dE$$

$$I_{2\to 1} = A\int_{-\infty}^{\infty} |T|^2 N_1(E)[1-f(E)]N_2(E+eV) f(E+eV)dE$$

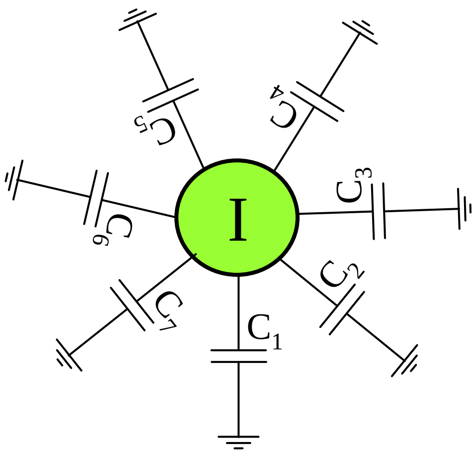
$$I = I_{1\to 2} - I_{2\to 1}$$

$$= A\int_{-\infty}^{\infty} |T|^2 N_1(E) N_2(E+eV)[f(E)-f(E+eV)]dE$$

$$\approx \frac{1}{eR}\int_{-\infty}^{\infty} [f(E)-f(E+eV)]dE$$

$$I = \frac{1}{eR}\frac{-\Delta E}{1-\exp\left(\Delta E / k_B T\right)}$$

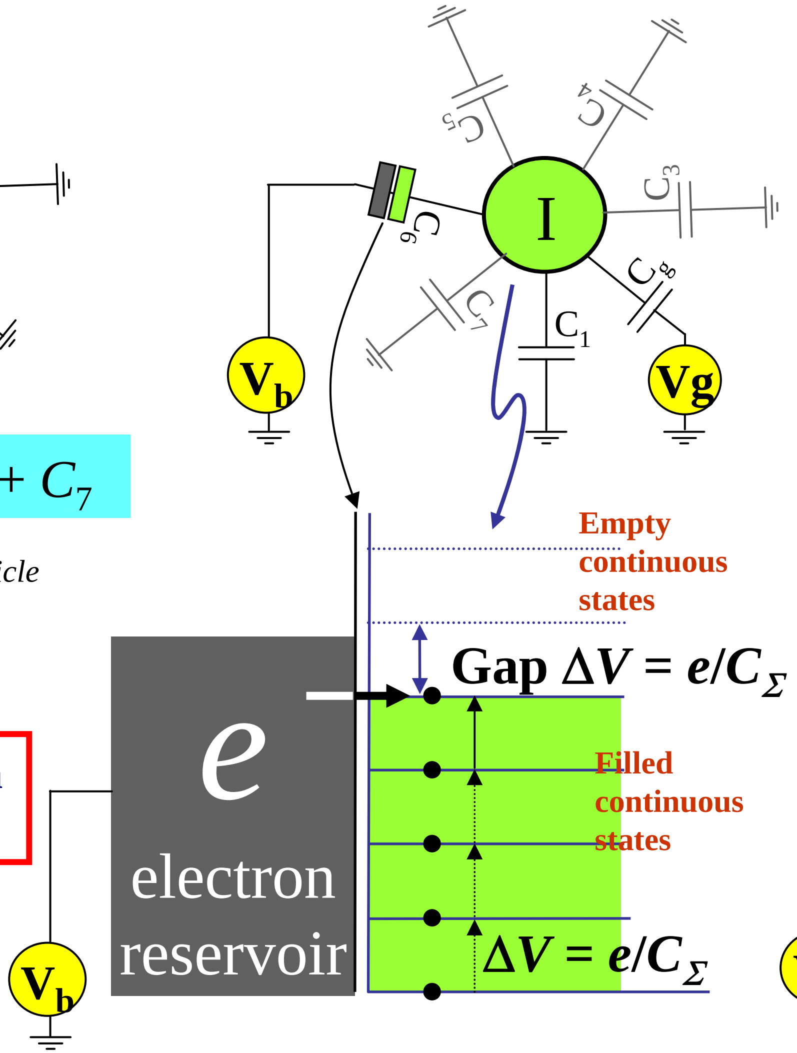
A neutral isolated dot



$$C_\Sigma \equiv C_1 + C_2 + \bullet\bullet\bullet + C_7$$

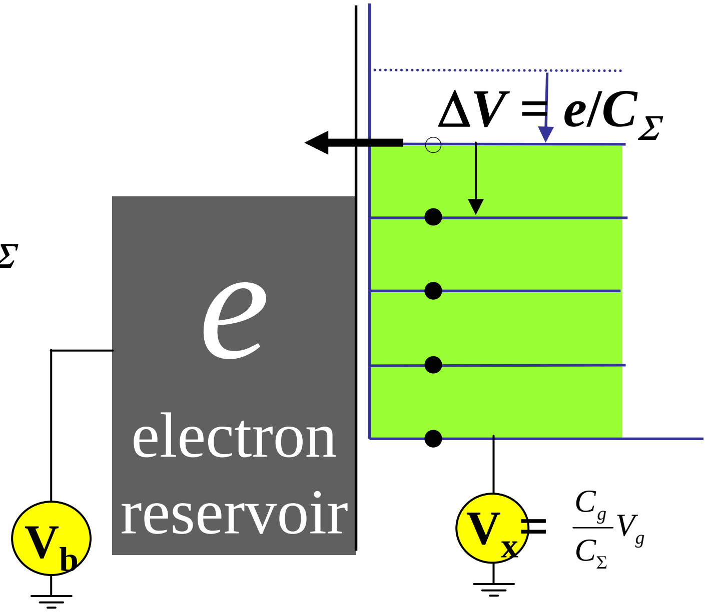
$$C_{particle} = 2\pi\varepsilon_0\varepsilon_r r_{particle}$$

Electron Box

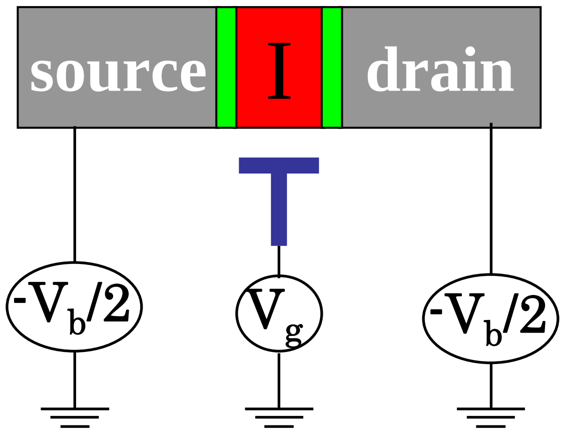


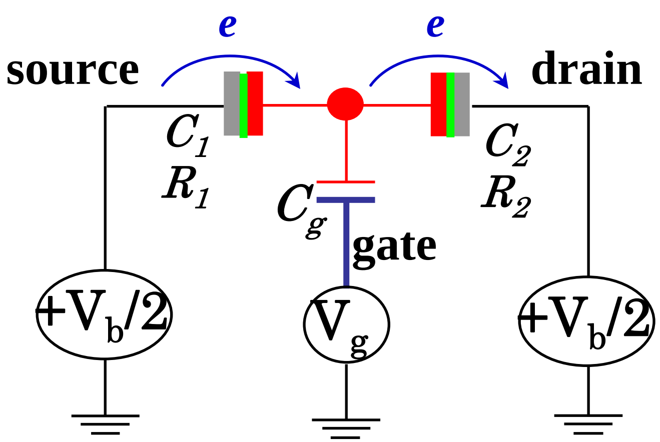
Inject electrons via a tunnel junction

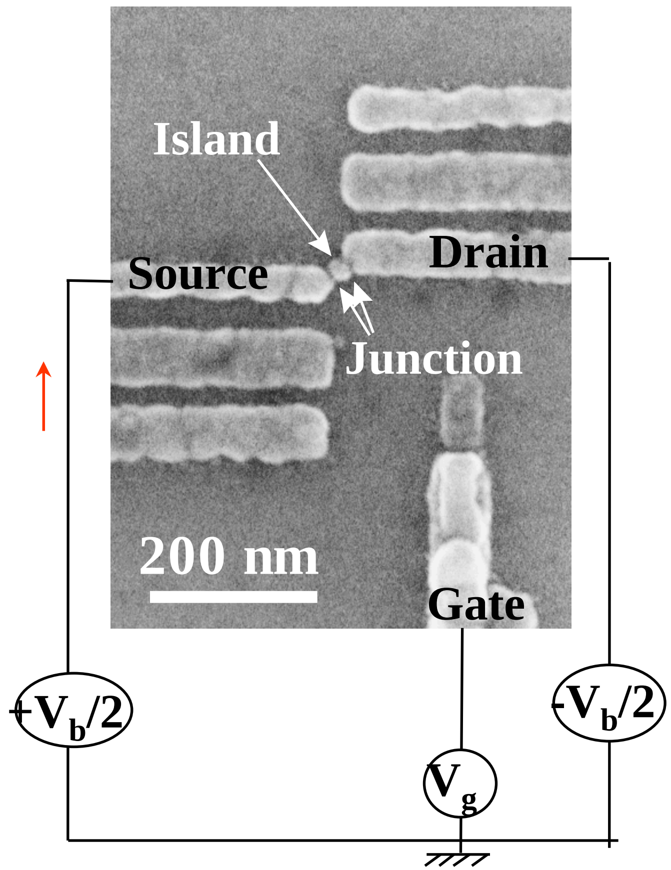
An applied gate voltage can lift up island potential



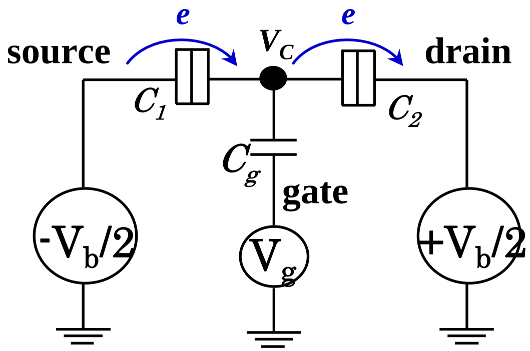
Circuit and a real sample







Single Electron Transistor



$$Q=\sum Q_i=(V_c-V_b/2)C_1+(V_c+V_b/2)C_2+(V_c-V_g)C_g$$

for C_1=C_2 and V_b=0

$$V_c=\frac{Q+C_gV_g}{C_\Sigma}\equiv\frac{-ne+C_gV_g}{C_\Sigma}$$

$$E=\sum\frac{Q_i^2}{2C_i}+\sum Q_iV_i$$

$$=\frac{e^2}{2C_\Sigma}\left[n+\frac{C_gV_g}{e}\right]^2+\text{terms independent of } n$$

$$\equiv E_n$$

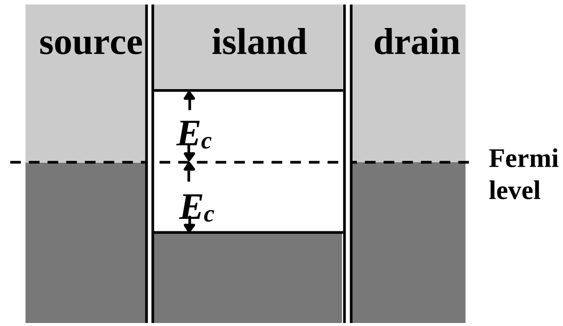
$$C_i=C_1,C_2,C_g$$
$$V_i=\pm V_b/2,V_g$$
$$Q_i=Q_1,Q_2,Q_g$$

$$C_\Sigma\equiv C_1+C_2+C_g$$

The characteristic energy scale
E_c=$e^2/2C_\Sigma$

Coulomb Blockade: $V_b=0;V_g=0$

occupied states
empty states
gap

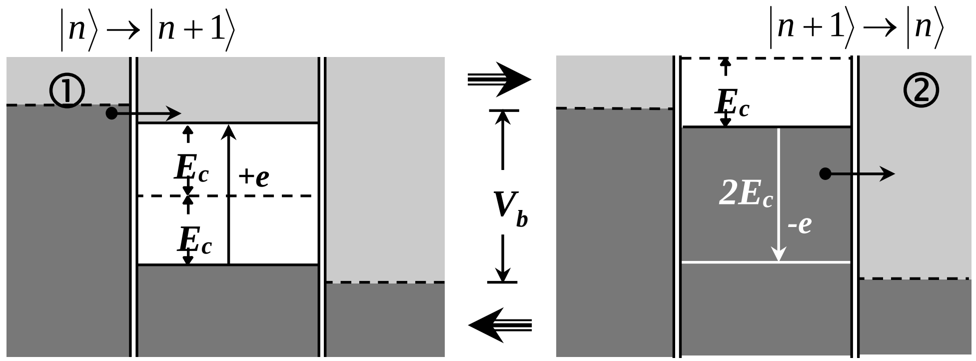


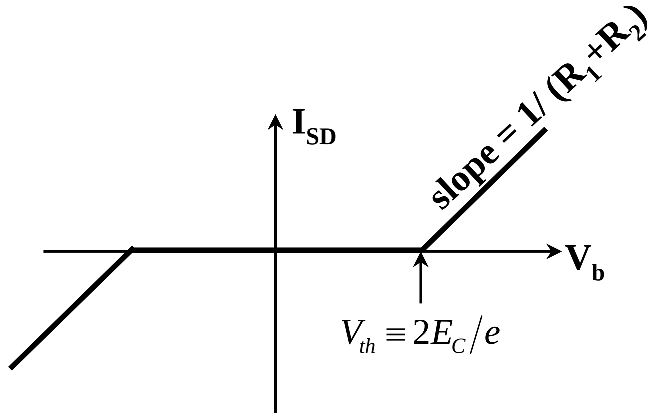
Rule for tunneling:
from occupied states to empty states

Effect of a bias voltage: $V_b \geq 2E_c/e; \quad V_g = 0$

Two possible tunneling sequences:

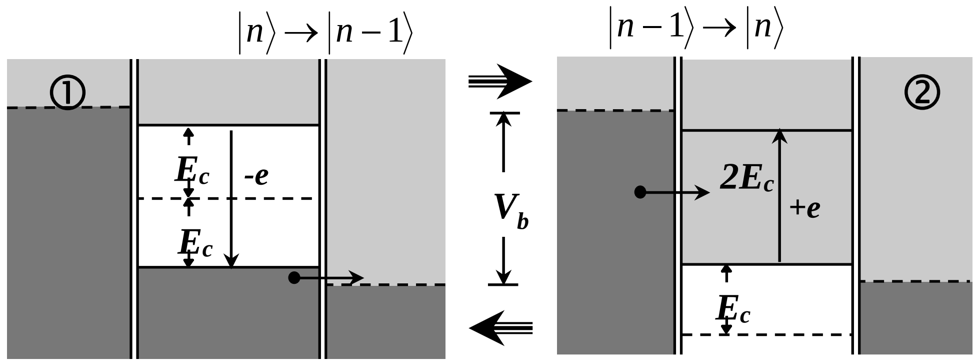
(1) $|n\rangle \rightarrow |n+1\rangle \rightarrow |n\rangle \rightarrow |n+1\rangle \rightarrow ....$





note $V_c = 2n\,E_C/e + \left(C_g/C_\Sigma\right)V_g$

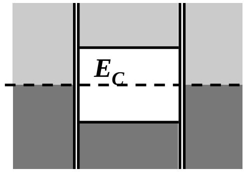
(2) $|n\rangle \rightarrow |n-1\rangle \rightarrow |n\rangle \rightarrow |n-1\rangle \rightarrow ....$



Effect of a gate voltage:

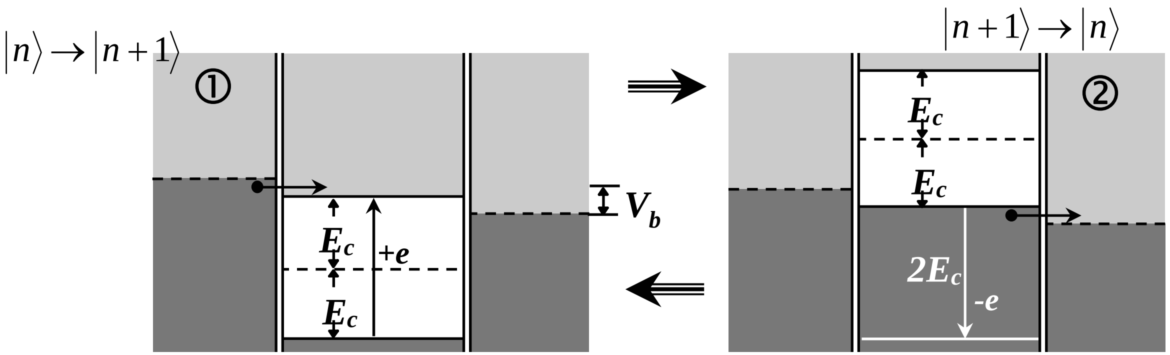
$$V_c = 2n E_C/e + \left(C_g/C_\Sigma\right)V_g$$

$$n = 0,\ \text{if}\ V_c = \pm\frac{E_C}{e} = \pm\frac{e}{2C_\Sigma}\ \text{then}\ V_g = \pm\frac{e}{2C_g}$$



measured
IV_b at various V_g

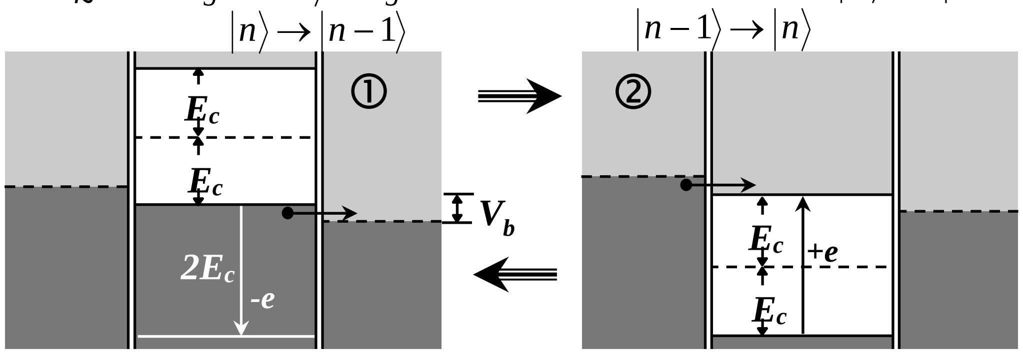
when $V_b \gtrsim 0$; $V_g = +e/2C_g$; tunneling sequence $= |n\rangle \to |n+1\rangle \to |n\rangle \to \ldots$

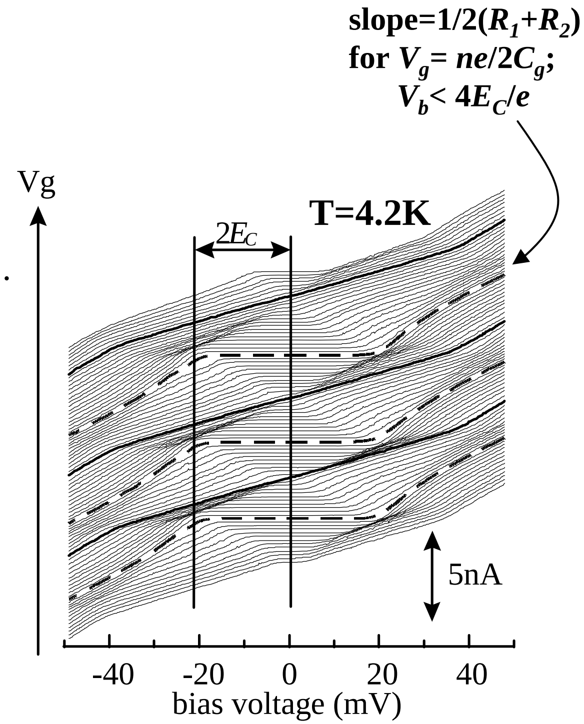


— IV_b for $V_g = \pm(2n+1)e/2C_g$

– – IV_b for $V_g = \pm ne/C_g$

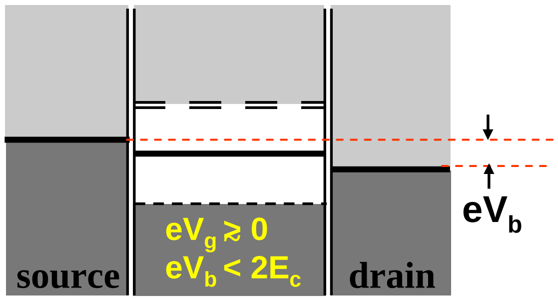
when $V_b \gtrsim 0$; $V_g = -e/2C_g$; tunneling sequence $= |n\rangle \to |n-1\rangle \to |n\rangle \to \ldots$

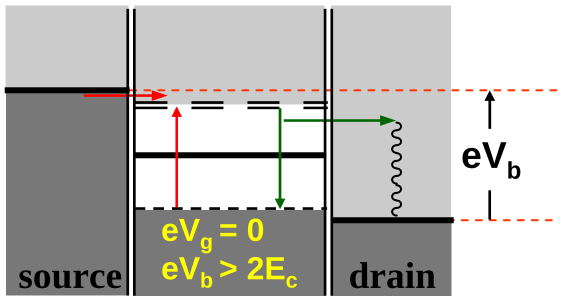


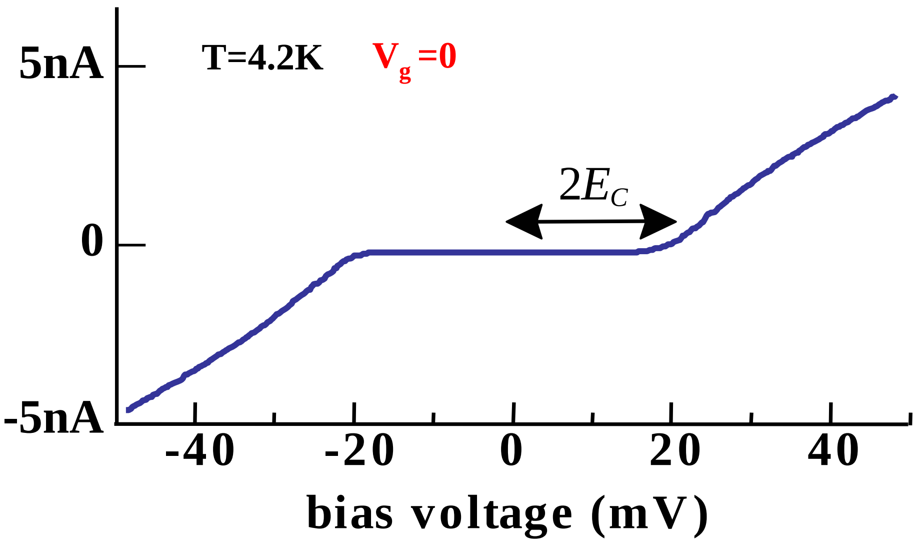


Summary

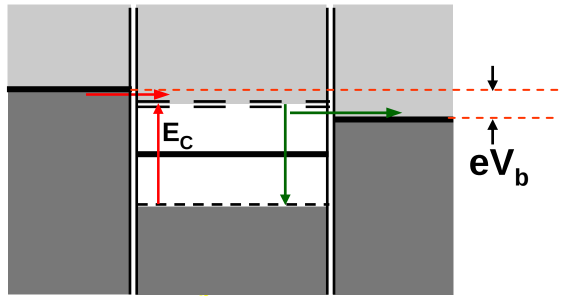
$V_b \neq 0$, $V_g = 0$

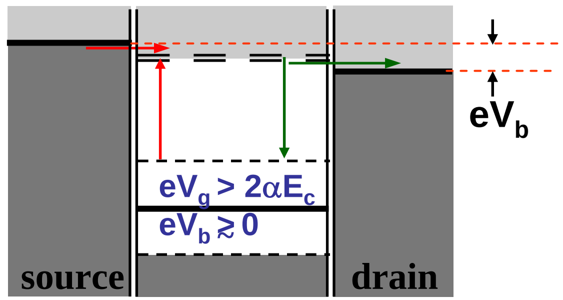


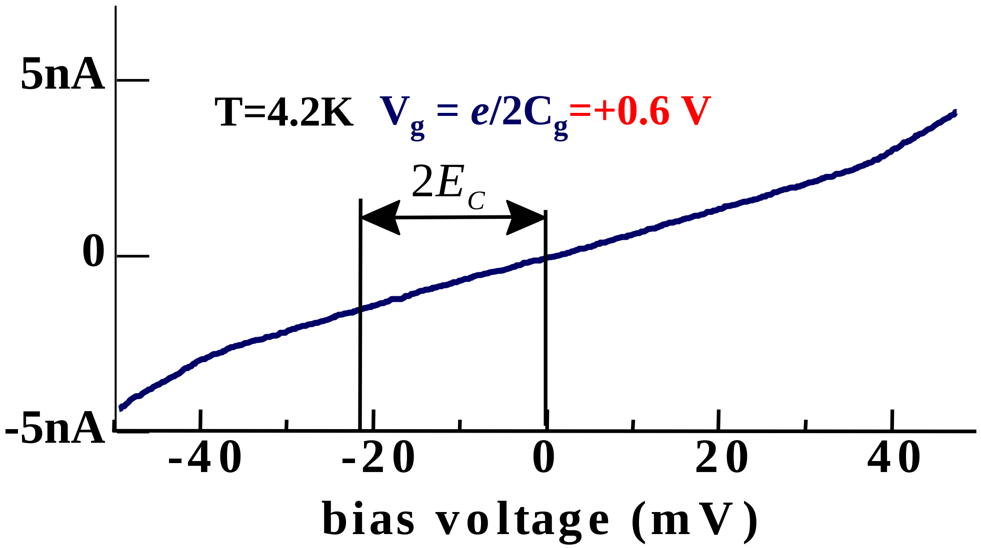




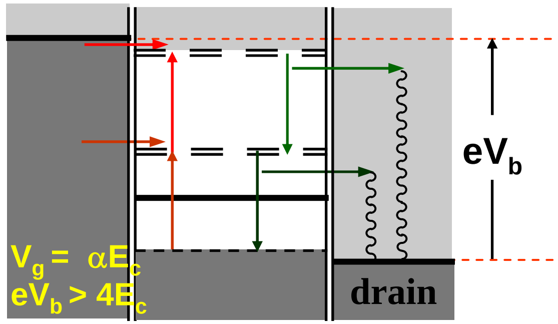
$V_b \approx 0$, $V_g \rightarrow E_C$

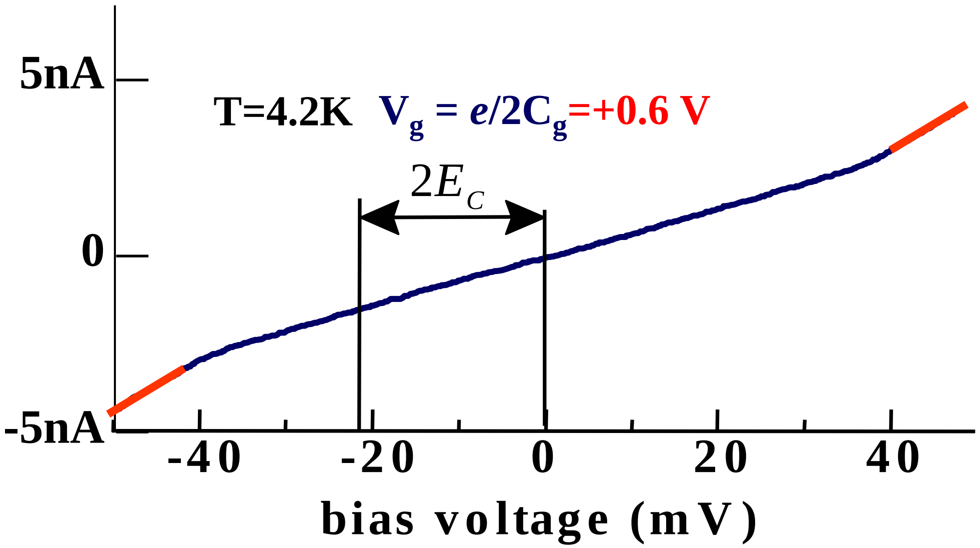




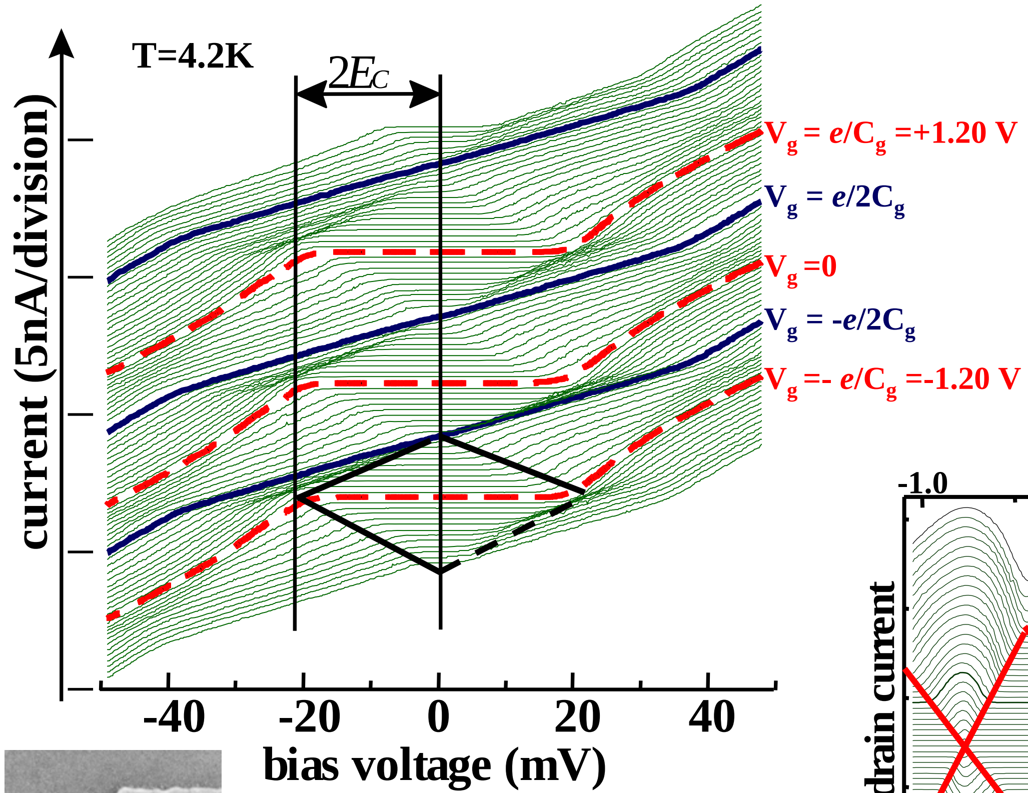


at $V_b > 4E_C/e$ and $V_g = e/2C_g$

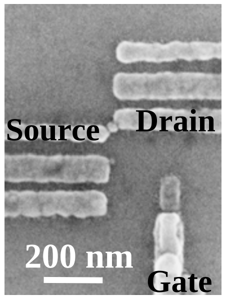




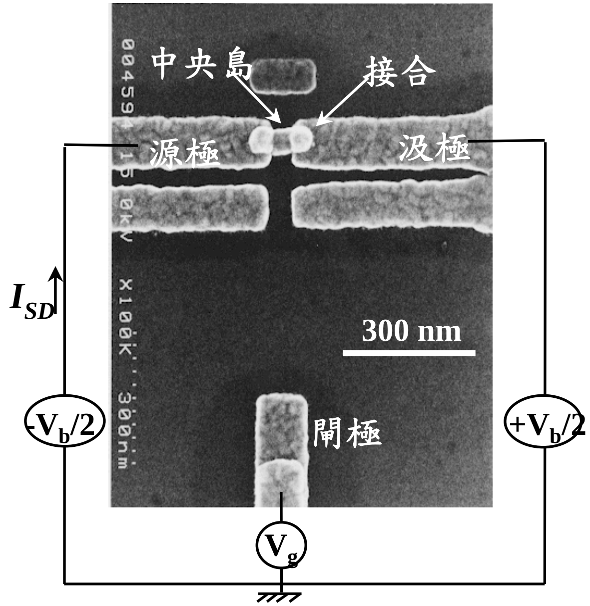
$I(V_b)$ vs. $I(V_g)$



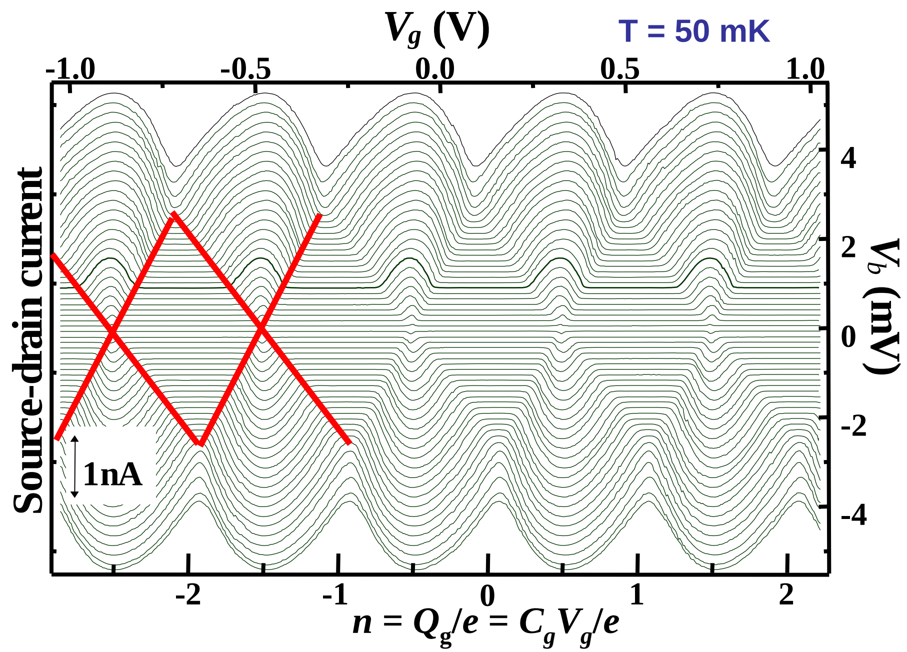
T=4.2K
2Ec
current (5nA/division)
bias voltage (mV)
-40
-20
0
20
40
Vg = e/Cg =+1.20 V
Vg = e/2Cg
Vg =0
Vg = -e/2Cg
Vg =- e/Cg =-1.20 V



Source
Drain
200 nm
Gate

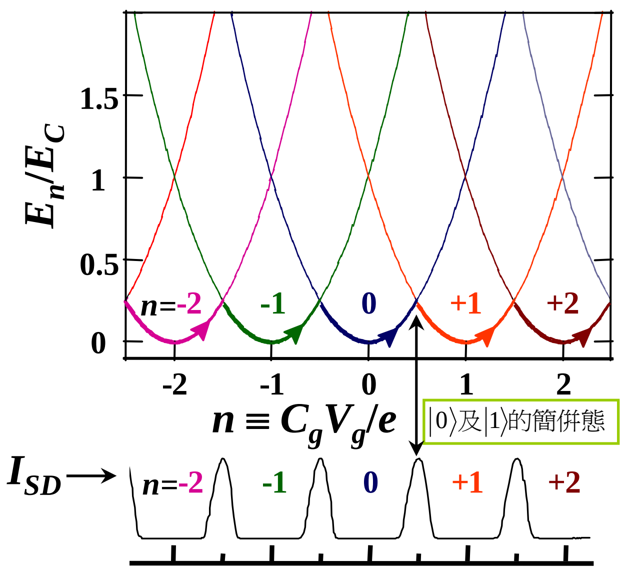


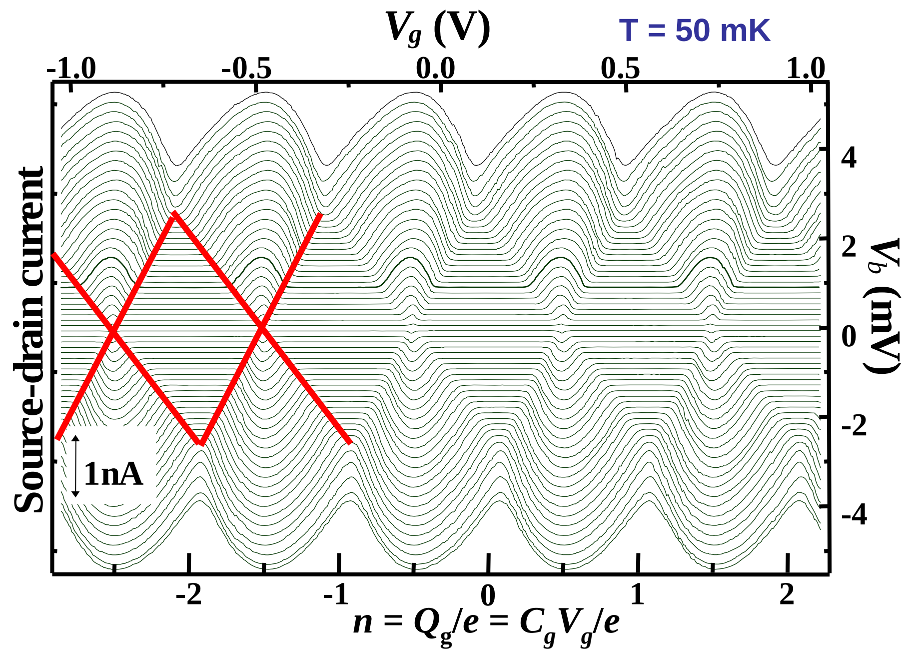
中央島
接合
源極
汲極
閘極
300 nm
ISD
-Vb/2
+Vb/2
Vg

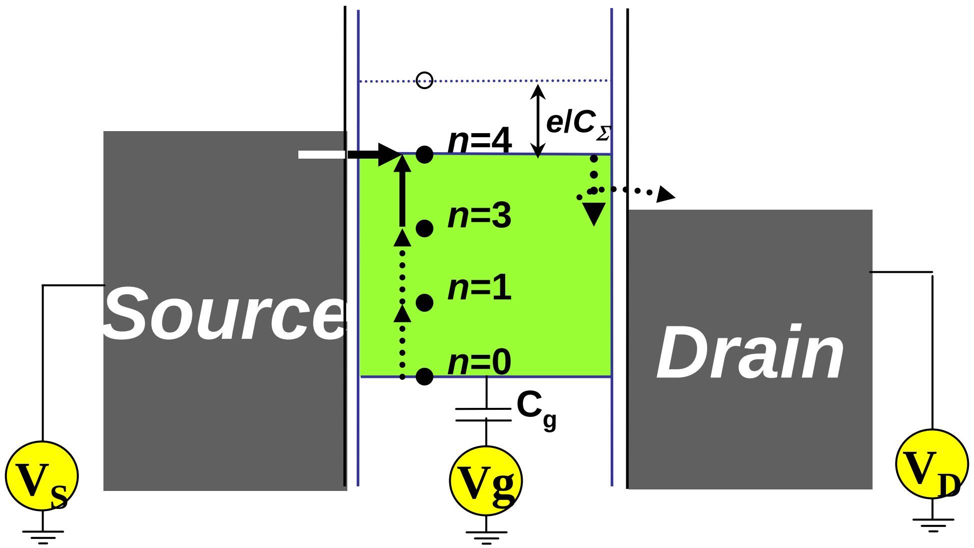


Vg (V)
T = 50 mK
-1.0
-0.5
0.0
0.5
1.0
Source-drain current
1nA
Vb (mV)
4
2
0
-2
-4
-2
-1
0
1
2
n = Qg/e = CgVg/e

Coulomb Oscillation







Each oscillation corresponds to a shift in the chemical potential of the island by an amount of e/C_Σ

Calculation of single electron tunneling current

Assumptions: 1. Sequential tunneling (no co-tunneling process)
2. Global rule: Fast charge redistribution
3. Fast energy relaxation

①

Calculate energy difference between two charge states

$$\Delta E_{n_i \to n_f} \equiv E(n_f) - E(n_i) - (n_f - n_i)\frac{V_b}{2}$$

$$\text{Tunneling rate } \Gamma^l_{n_i \to n_f} = \frac{1}{e^2 R_l}\frac{-\Delta E_{n_i \to n_f}}{1 - \exp\left(\Delta E_{n_i \to n_f} / k_B T\right)}$$

$$l = \begin{cases} S, \text{ source} \\ D, \text{ drain} \end{cases}$$

②

Find out P_n :the probability of having n excess electrons in the island

Master equation

$$\frac{dP_n}{dt} = \sum_{l=S,D}\Gamma^l_{n\pm1 \to n}P_{n\pm1} - \sum_{l=S,D}\Gamma^l_{n \to n\pm1}P_n = 0 \text{ in equilibrium states}$$

③

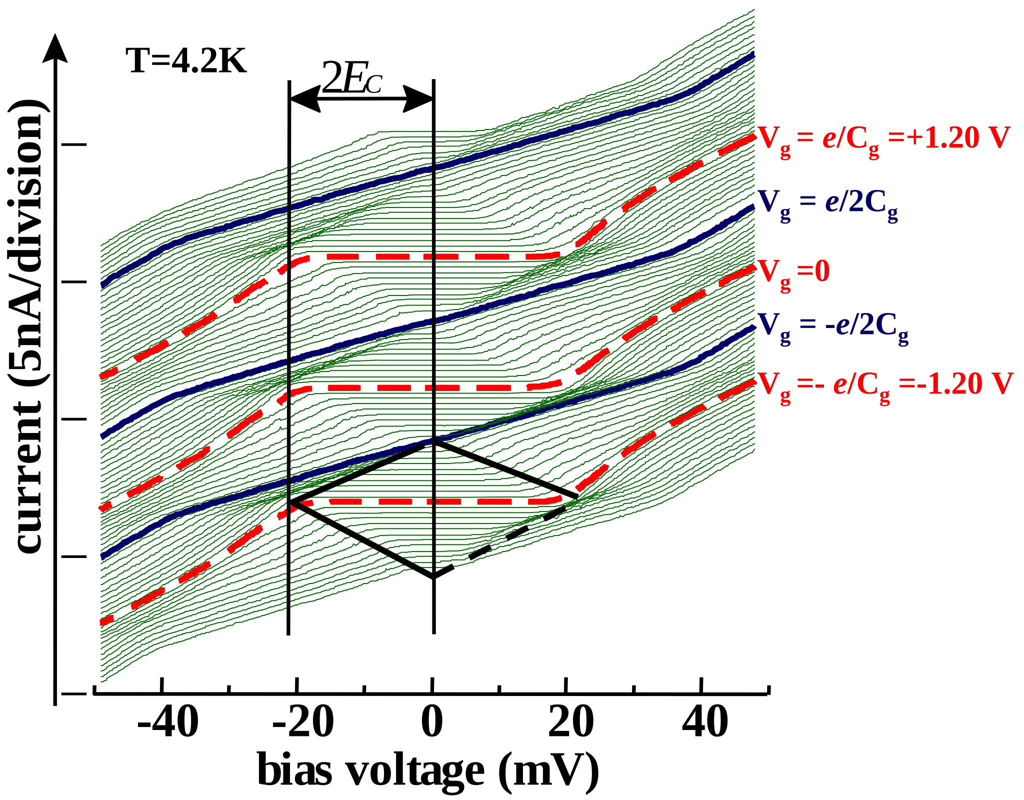
Current $\text{Current } I = e\hat{P} \cdot \hat{G}$ $\hat{P} \equiv \langle \cdots, P_{-2}, P_{-1}, P_0, P_{+1}, P_{+2}, \cdots \rangle$

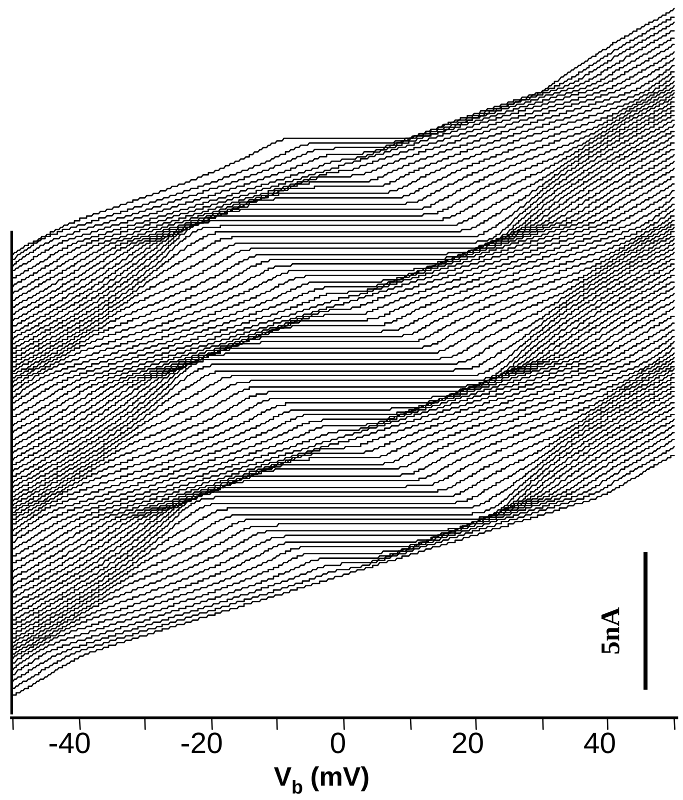
$$\tilde{\hat{G}} \equiv \langle \cdots, \Gamma_{-2}, \Gamma_{-1}, \Gamma_0, \Gamma_{+1}, \Gamma_{+2}, \cdots \rangle,$$

$$\Gamma_{n_f} \equiv \sum_{n_i}\Gamma_{n_i \to n_f}$$

$$I^l = e\sum_n \left[\Gamma^l_{n \to n+1} - \Gamma^l_{n \to n-1}\right] p_n$$

Measurement vs. Simulation





Asymptotic IV_b

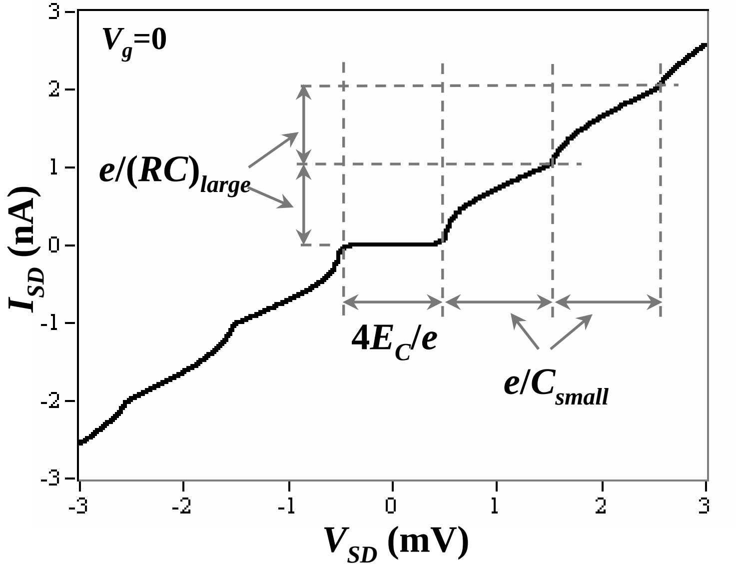
$$I \rightarrow \frac{\left(V_b - sign(V_b)\, e/2C_\Sigma\right)}{R_1 + R_2}$$

Asymmetric SET (simulations)

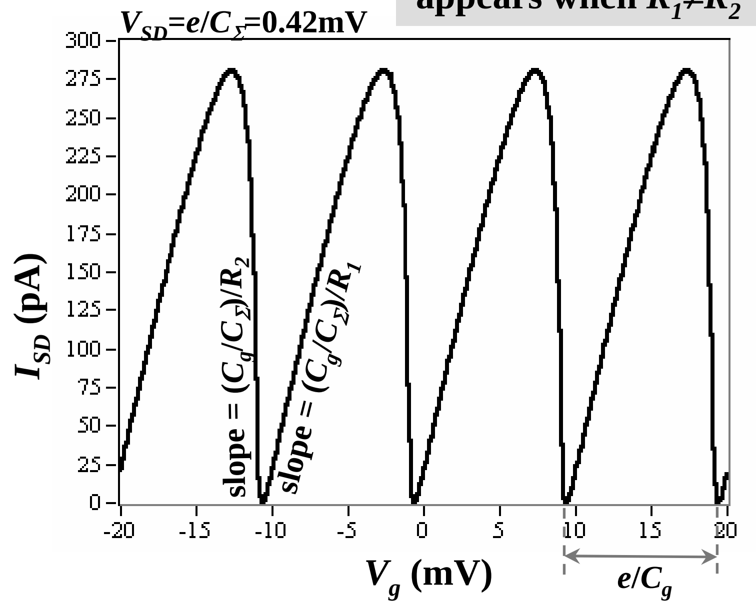
Coulomb Staircase

R_1= 50 kΩ, R_2= 1 MΩ, C_1=0.2fF, C_2=0.15fF, C_g=16aF,
E_C=2.54K, T=20mK

appears when $R_1C_1 \neq R_2C_2$

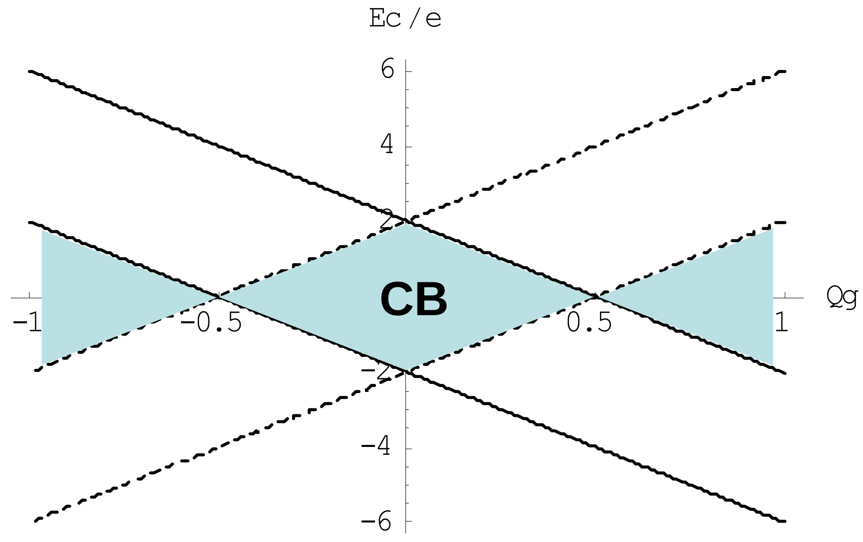


appears when $R_1 \neq R_2$

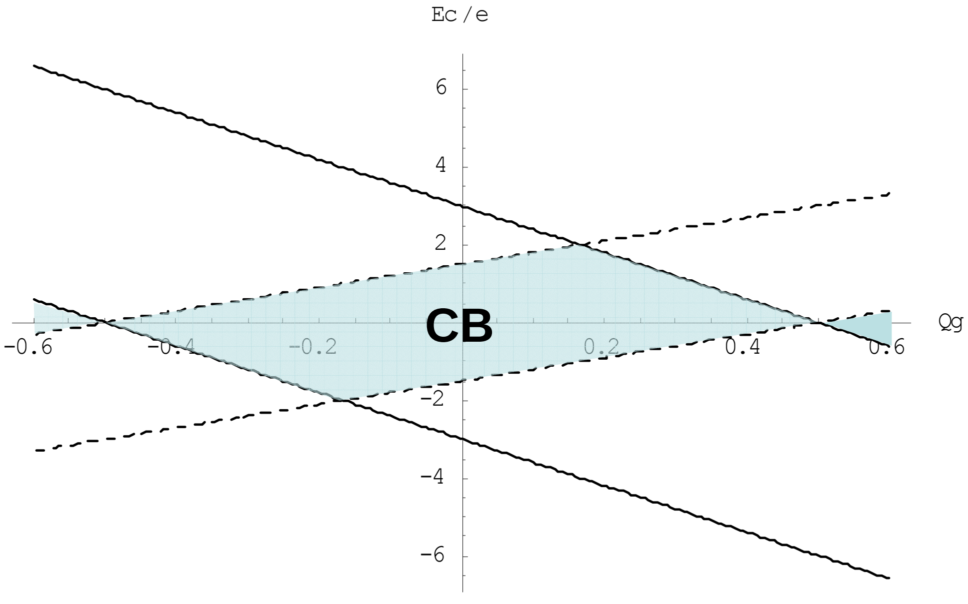


Stability diagram:

Symmetrical Junctions

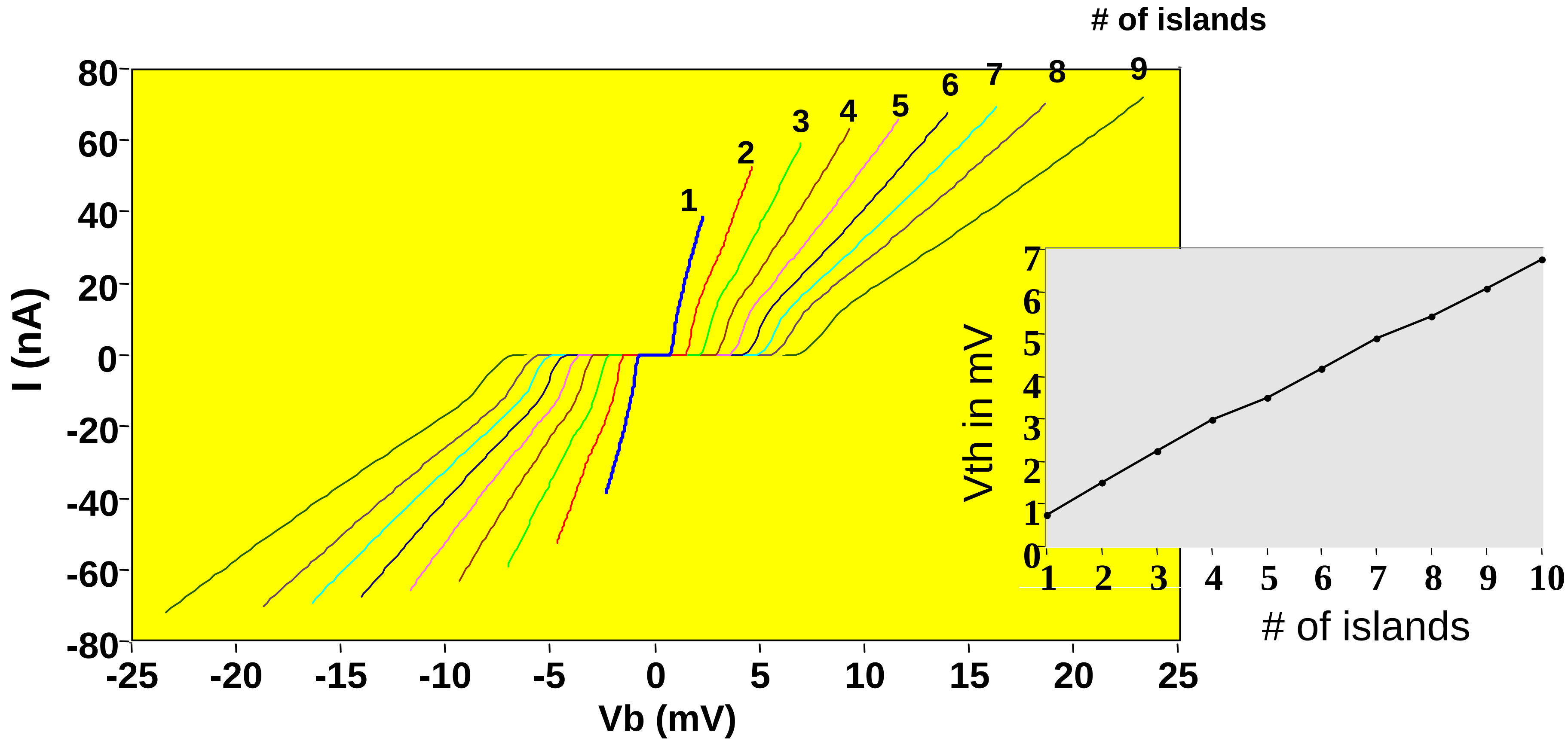


$C_r = 200C_g$
$C_l = 100C_g$



ref “NewTrans.nb”

IV characteristics for 1D array with C=100aF, R=20kΩ



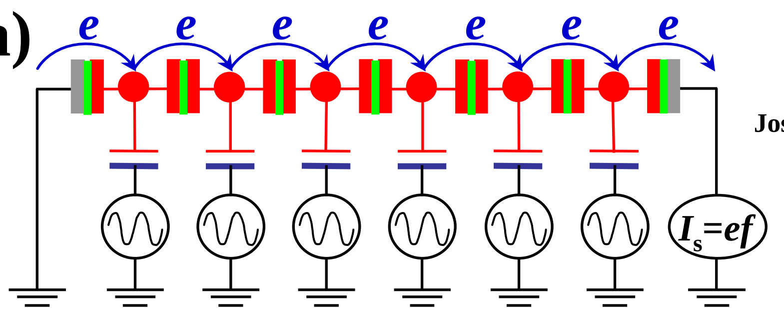
e/C=0.8mV

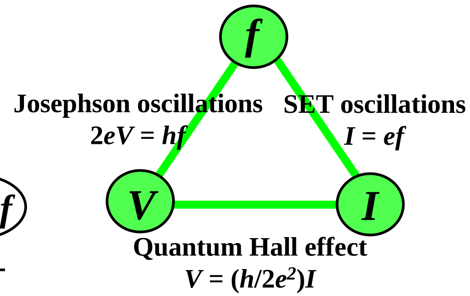
Vth in mV

0.747	1.495	2.243	2.991	3.497	4.197	4.890	5.403	6.078	6.753

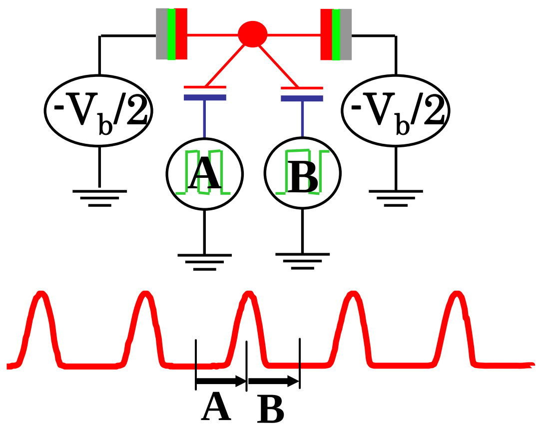
Applications: Summary

(a)



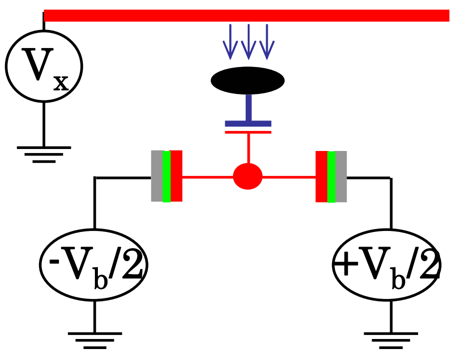


(b)

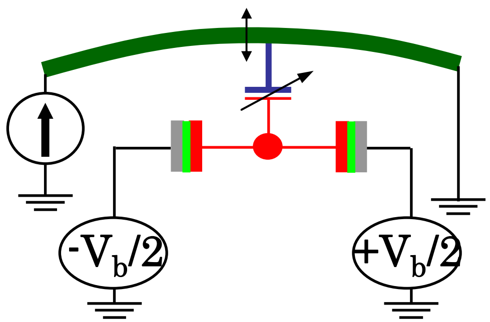


A	B	C
0	0	0
1	0	1
0	1	1
1	1	0

(c)



(d)



other applications:

- **ultra sensitive charge detector,**

  a small change in the island charge causes large change in current,

  sensitivity ~ 10^{-5} $e/\sqrt{\text{Hz}}$ (R.J.Schoelkopf *et al*, 98)

- **memory cell,**

  use number of charge stored in the island as an information bit,

  8×8 bit prototype room temperature memory cell array (K. Yano *et al*. 96)

  a non-volatile storage cell (C. D. Chen *et al*, 97)

- **Current Standard,**

  pumping charges one by one with an accuracy rf source, $I = e \times f$

  error = 15 parts in 10^9, or 15 ppb (John Martinis *et al*, 96)

- **Primary thermometry,**

  an universal constant

  the full width at half maximum of the $dI/dV|_{V\sim 0} = \alpha T$, α = 10.878.. $\times(k_B/e)$

  Good for $K_BT >> E_C$, experiment: T = 0.1~8 K, (J.P.Pekola *et al*, 94)

- **logic gate element,**

  forming AND, OR, NAND gates

  Computers: Making Single Electrons Compute, Science, v275, p. 303 (1997)

Coulomb blockade $\Longleftrightarrow$ isolated object

Charging energy (*$e^2/2C$*):
Electrostatic energy associated with charging/discharging an isolated object

Criteria (for a well defined charge number) :

1. to surmount thermal fluctuations

$$\frac{e^2}{2C} >> k_B T \Rightarrow \text{small } C \text{ or low } T$$

C **= total capacitance seen from the object**

2. to surmount quantum fluctuations

electrical paths to charge/discharge the object

$$\underbrace{\left(\frac{e^2}{2C}\right)}_{\Delta E}\underbrace{(RC)}_{\Delta t} \geq h \quad \Rightarrow \quad R \geq R_K \equiv \frac{h}{e^2} \approx 26\ \text{k}\Omega$$

(tunneling transport -- not diffusion)

R **= total resistance seen from the object**
(two resistance in parallel)

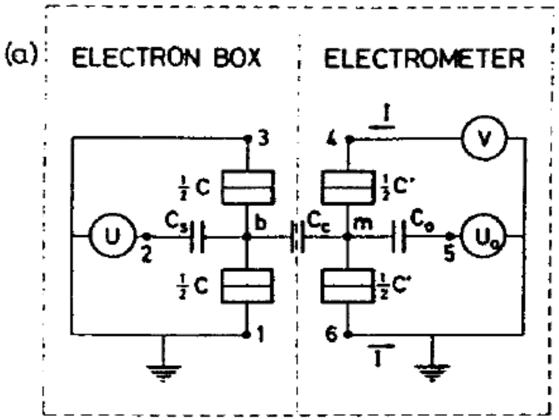
Material:
any conducting materials, including metal, semiconductor, conducting polymer, carbon nanotube, ...

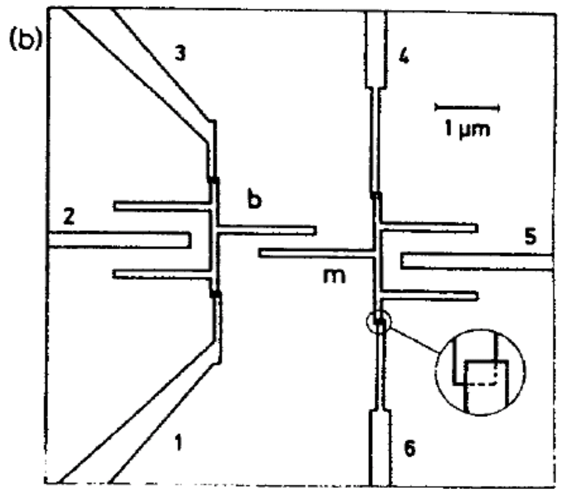
Structure:
any structures with isolated objects accessible through tunnel junctions

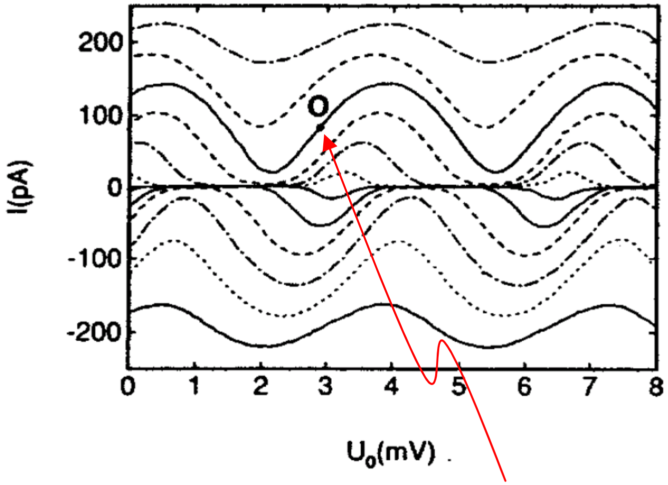
The simplest structure:

Single Electron Transistor (SET)

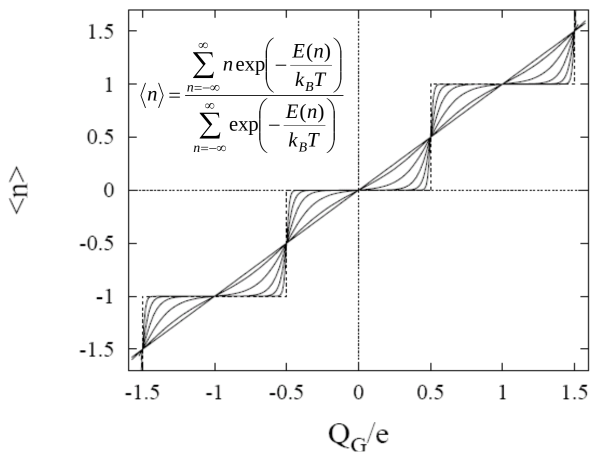
Controlling and Detection of Charge number in a Box







charge noise $10^{-4} e/\sqrt{Hz}$ at 1Hz

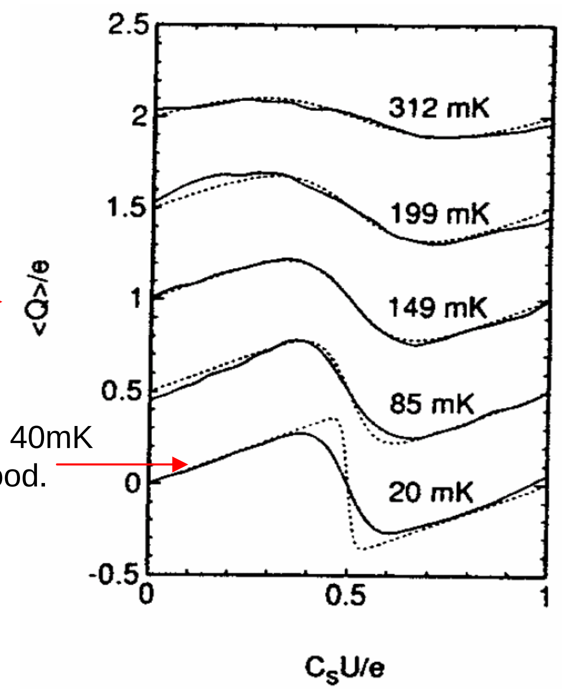


T/E_C=0(dashed steps), 0.02, 0.05, 0.1, 0.2, 0.4 and 1(nearly linear)

$$E(n) = \frac{(ne - \tilde{Q})^2}{2(C_S + C)}$$

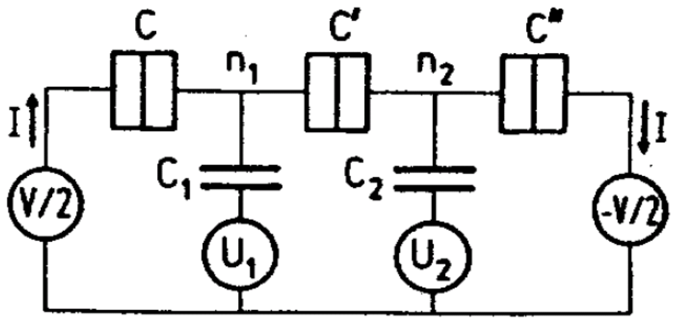
$$\langle Q\rangle = \frac{C}{C+C_S}\left(\tilde{Q} - \langle n\rangle e\right)$$

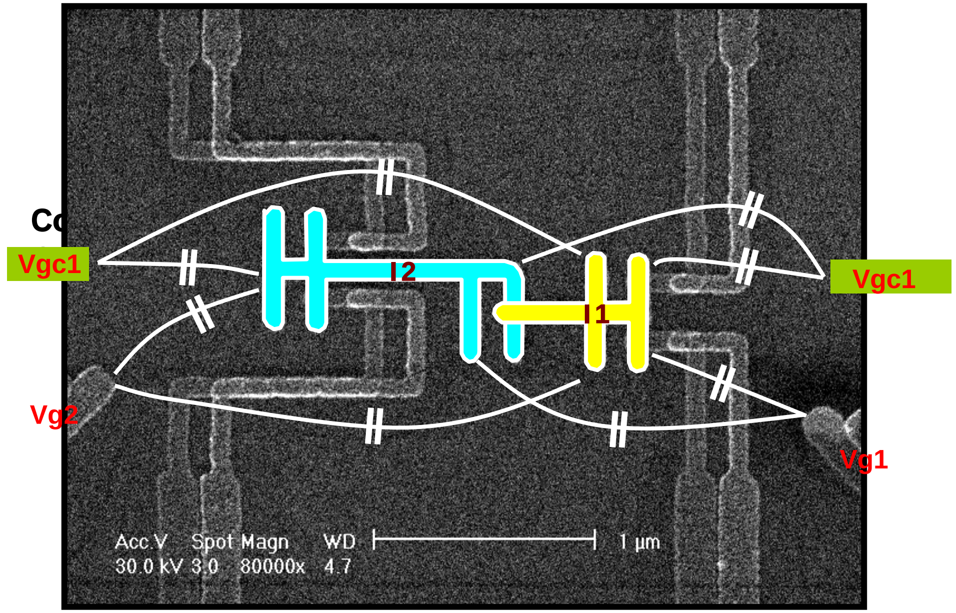
At 20mK, this discrepancy is 40mK and is only partially understood.

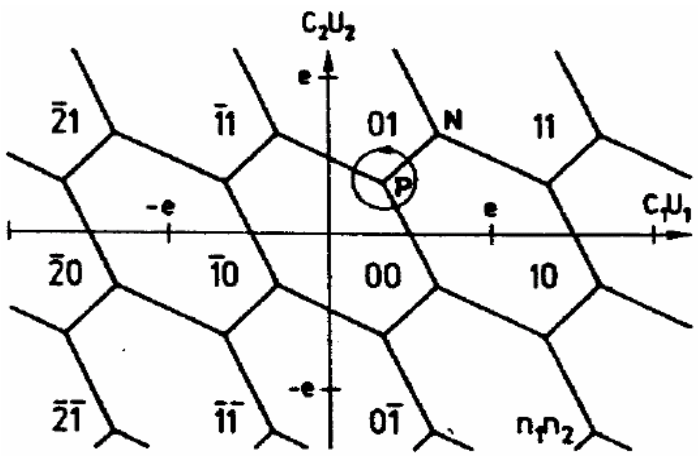


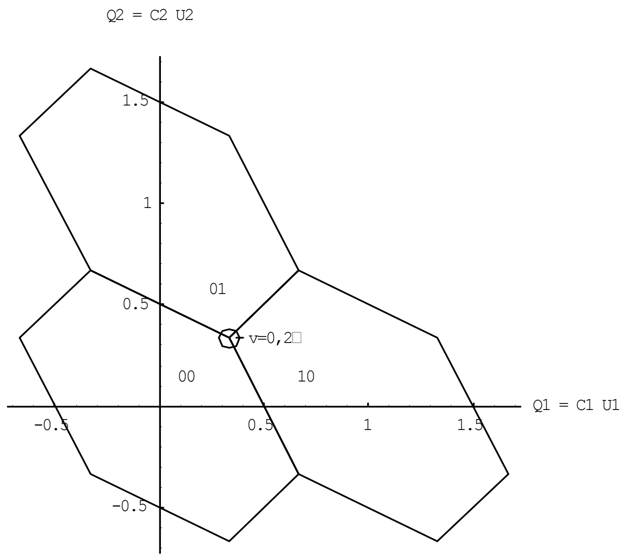
NATO, Single Charge Tunneling, p121

Single Electron Pump





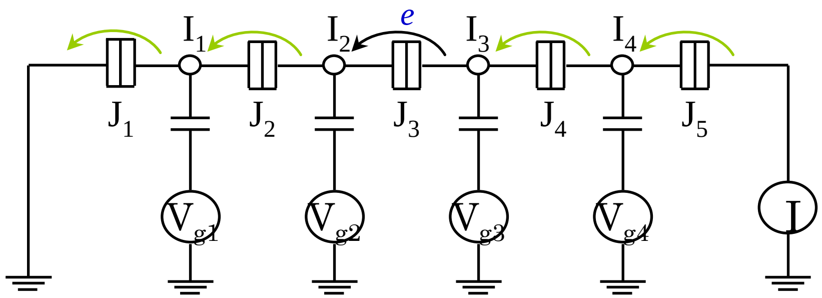


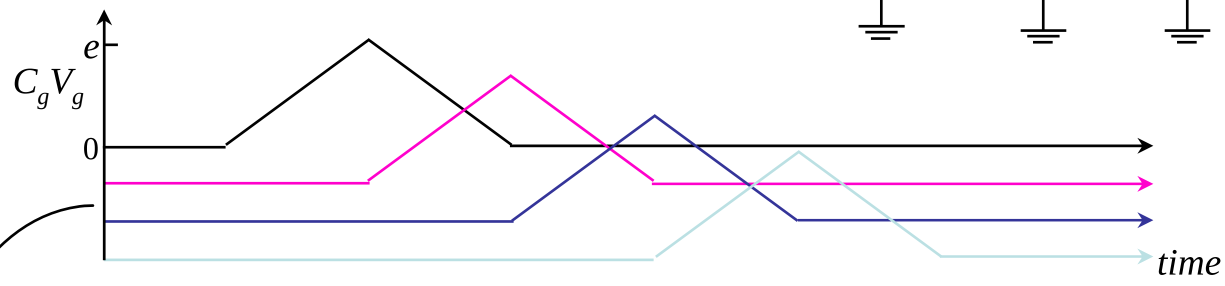


ref “Q8Final&.nb”

NATO, Single Charge Tunneling, p128

N-junction Electron Pump





Tunneling from $\mathbf{I_2}$ to $\mathbf{I_3}$ via $\mathbf{J_3}$

before tunneling: Q3 = - e/2, the rest = (+e/2)/4

after tunneling: Q3 = +e/2, the rest = (- e/2)/4

$Q_C = e/2$

optimal operation condition

$$C_{g2}V_{g2} + C_{g3}V_{g3} = e$$

Example:

$C_{g2}V_{g2} = +e - Q(t)$

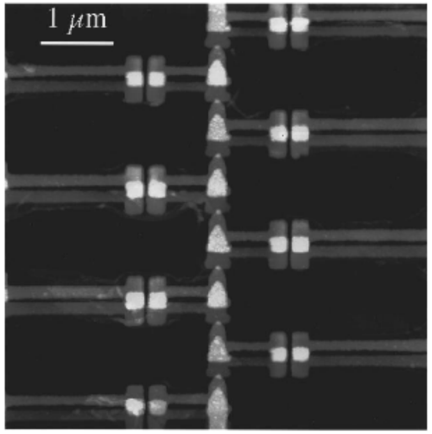
$C_{g3}V_{g3} = Q(t)$

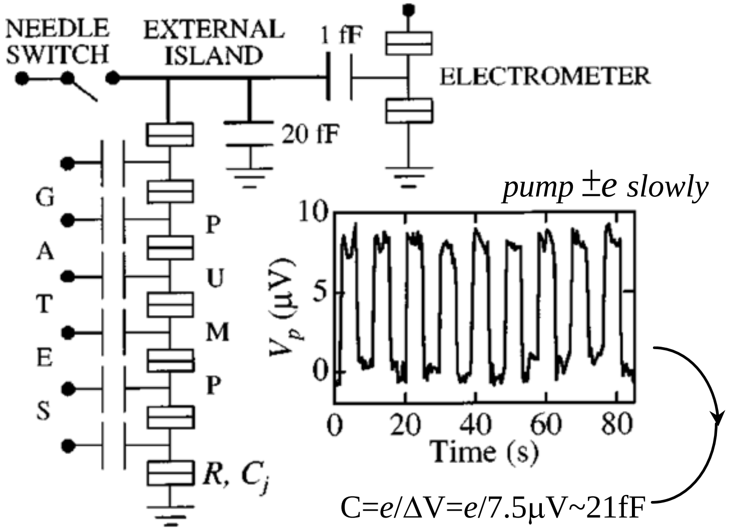
Ref: H. Dalsgaard Jensen and John M. Martinis PRB 46, 13407 (1992)

Accuracy of electron counting using a 7-junction electron pump

Appl. Phys. Lett. 69 (12), 1804 (1996)

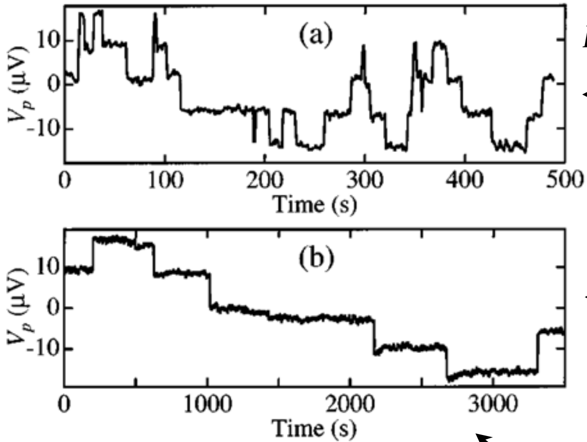
Mark W. Keller and John M. Martinis
National Institute of Standards and Technology, Boulder, Colorado 80303
Neil M. Zimmerman
National Institute of Standards and Technology, Gaithersburg, Maryland 20899
Andrew H. Steinbach
National Institute of Standards and Technology, Boulder, Colorado 80303





pump $\pm e$ *slowly*

C=*e*/ΔV=*e*/7.5μV~21fF



pump $\pm e$ *fast*

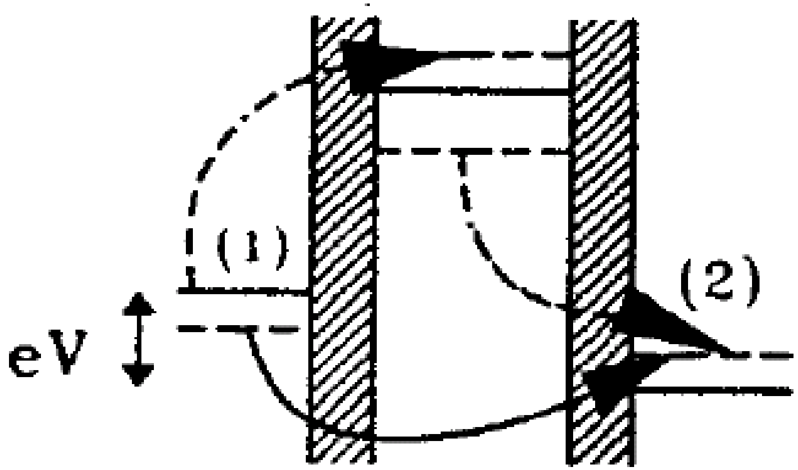
← pump $\pm e$ rate = 5.05MHz

← static

Average time per error ~ 13 sec.

1) Current standard, ($I=ef$)
error rate 15/10^9 (15 ppb)
2) Capacitance standard (C=*e*/ΔV)
3) hold time = 600 sec.

Co-tunneling



NATO, Single Charge Tunneling, p218

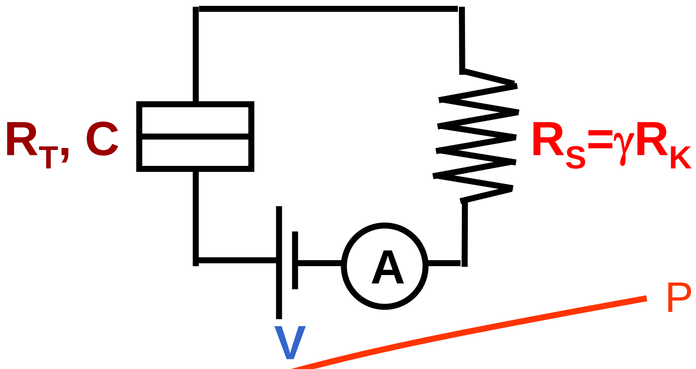
$$\Gamma_c(N,eV) = \frac{2\pi}{\hbar}\frac{e^2}{2C}\frac{N^{2N}}{(2N-1)!(N-1)!^2}\left(\frac{R_K}{4\pi^2 R_T}\right)^N \left(\frac{eV}{e^2/2C}\right)^{2N-1}$$

For N=4

$$\Gamma_c[Hz](4,eV) = 2.5\times 10^{-3}\frac{V^7}{R_T^4}\frac{[\mu V]}{[k\Omega]}$$

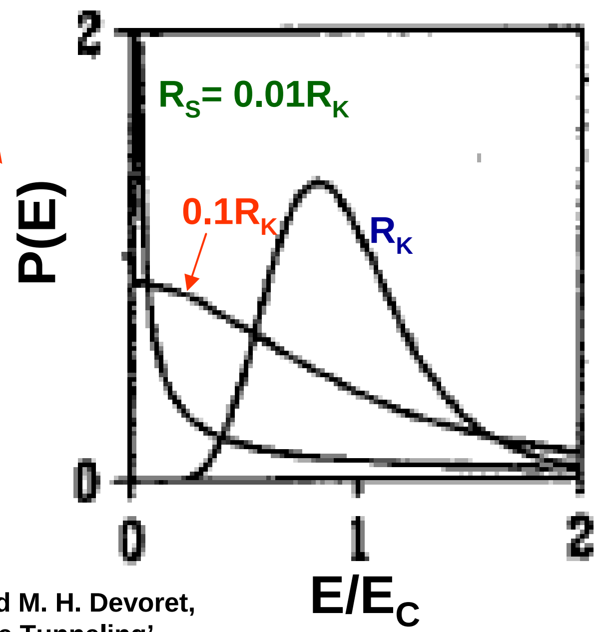
NATO, Single Charge Tunneling, p115

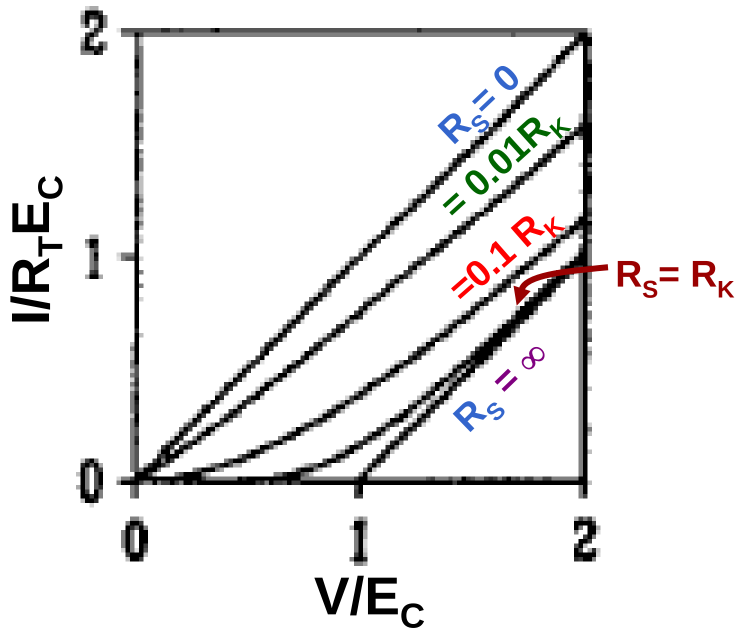
Effect of shunt resistor on a small tunnel junction:



$$E_C \equiv \frac{e^2}{2C}$$ $$R_K \equiv \frac{e^2}{h} \approx 26k\Omega$$

P(E) ---- Probability for the tunneling electron to emit energy E to the environment.

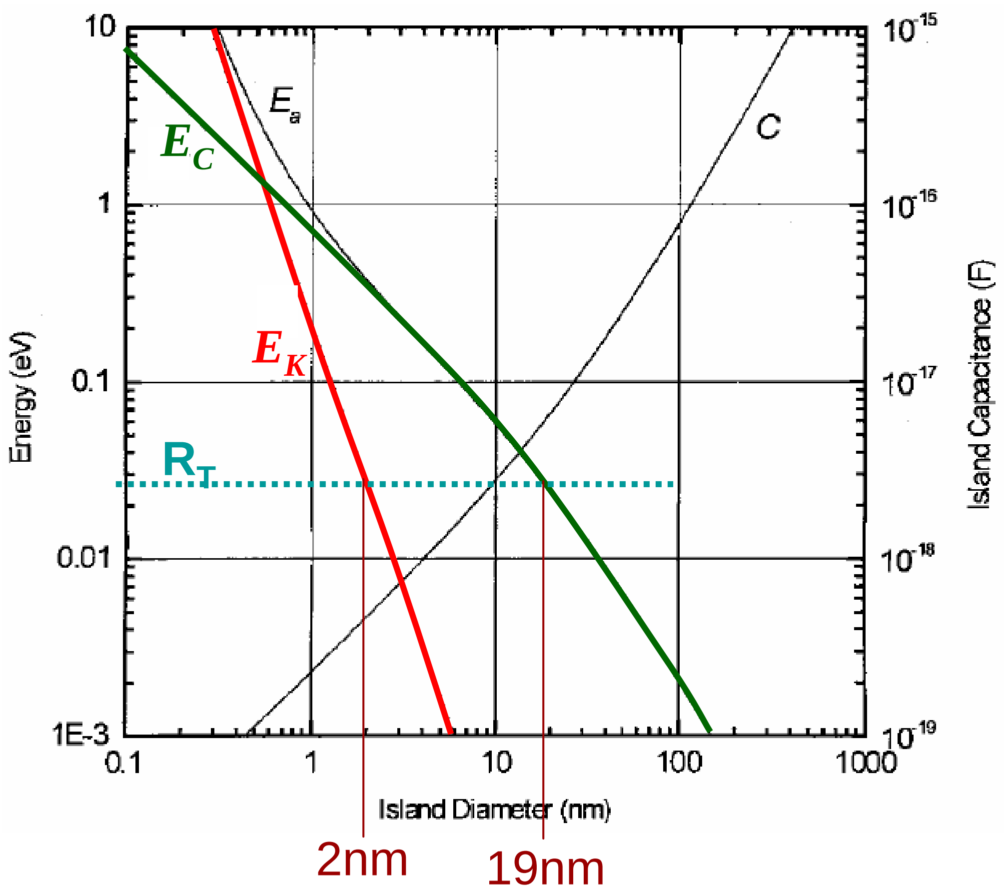




H. Grabert and M. H. Devoret, 'Single Charge Tunneling', (Plenum Press, New York, 1992)

Lower shunt resistance → lower junction resistance

Additive energy for an oxide-encapsulated nanoparticle



$E_C = e^2/2C$

E_K = level spacing = $1/D(E_F)V$

$n = 10^{22}\ \mathrm{cm}^{-3}$

$m = m_e$

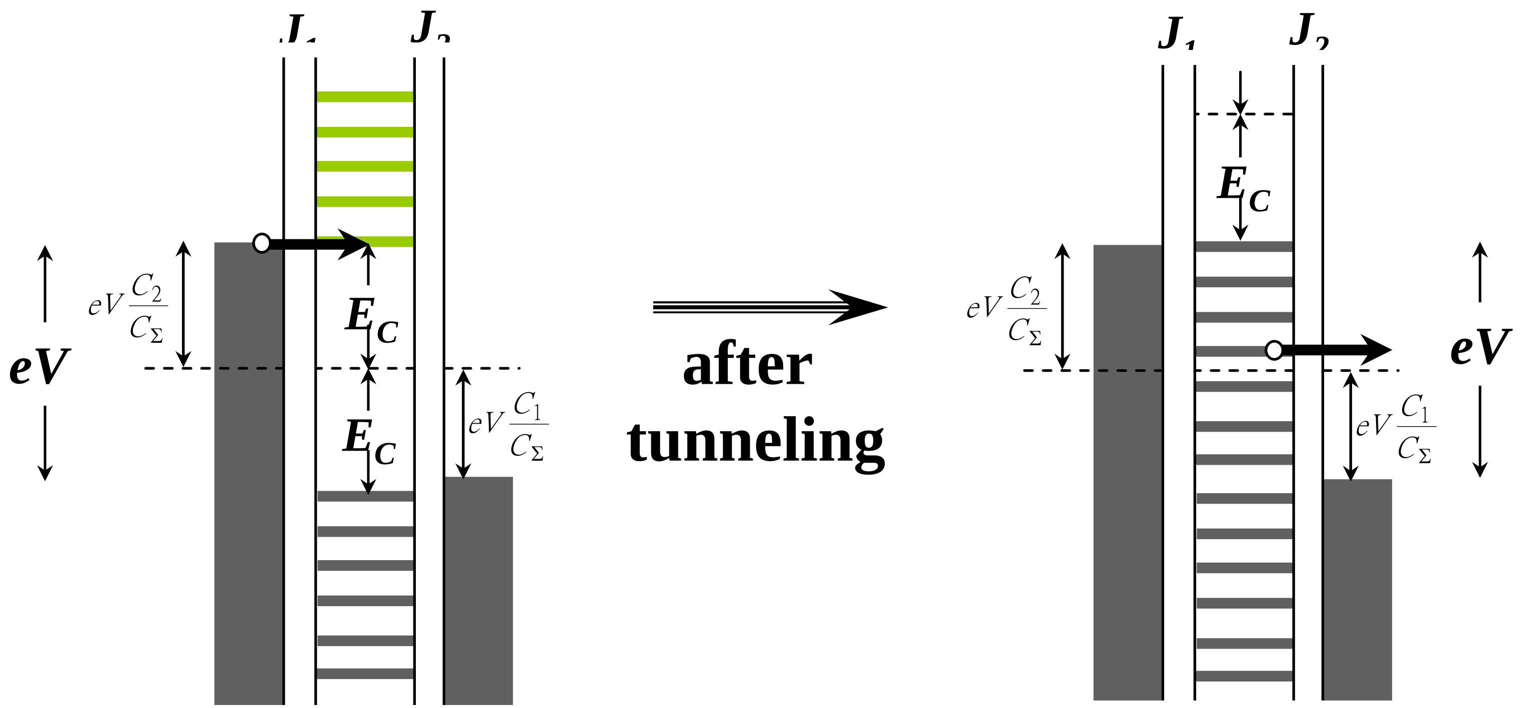
ε_r=4

10% tunnel barrier=2nm

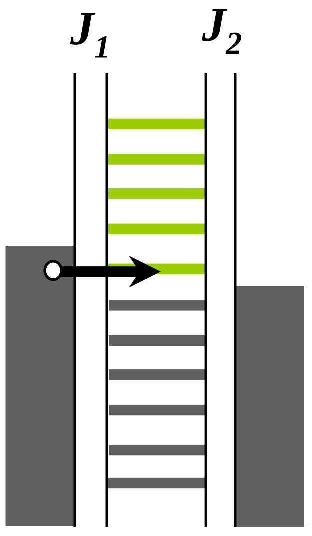
Single-Electron Devices and Their Applications, KONSTANTIN K. LIKHAREV, PROCEEDINGS OF THE IEEE, VOL. 87, NO. 4, APRIL 1999

Likharev review article, Fig. 2

Electron transfer through a quantum dot with charging energy E_C



What makes electrons flow?



biasing: $\mu_1 - \mu_2 = qV_D$

Fermi function:

$$f_1(E) \equiv \frac{1}{1+\exp[(E-\mu_1)/k_B T]} = f_0(E-\mu_1)$$

$$f_2(E) \equiv \frac{1}{1+\exp[(E-\mu_2)/k_B T]} = f_0(E-\mu_2)$$

Coupling strength for J_1 and J_2 = γ_1 and γ_2

$$I_1 = e\left(\frac{\gamma_1}{\hbar}\right)\left(f_1(\varepsilon_1) - p\right) \qquad I_2 = e\left(\frac{\gamma_2}{\hbar}\right)\left(f_2(\varepsilon_2) - p\right)$$

p = average number of electron in the QD $\quad 0 \le p \le 1$

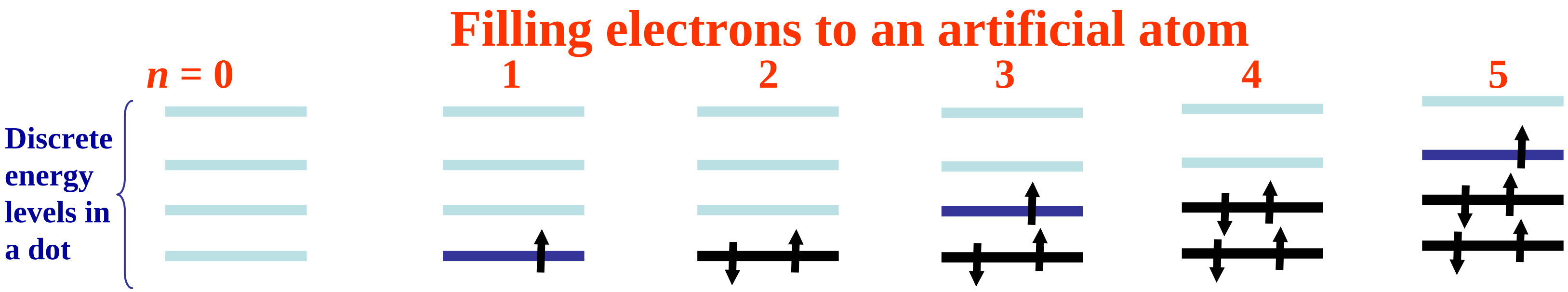
At steady state, $I_1 = I_2$ $\quad p = \dfrac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2}$

$$\Rightarrow I = I_1 = -I_2 = \frac{e}{\hbar}\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}[f_1(\varepsilon) - f_2(\varepsilon)]$$

Handbook of Nanoscience, Engineering, and Technology
Section: 12.2.2 Current flow as a balancing act
By Magnus Paulsson, Ferdows Zahid and Supriyo Datta,
Edited by William A. Goddard,III et al. CRC Press, 2003

Quantum transport: atom to transistor
Sec. 1.2 What makes electrons flow

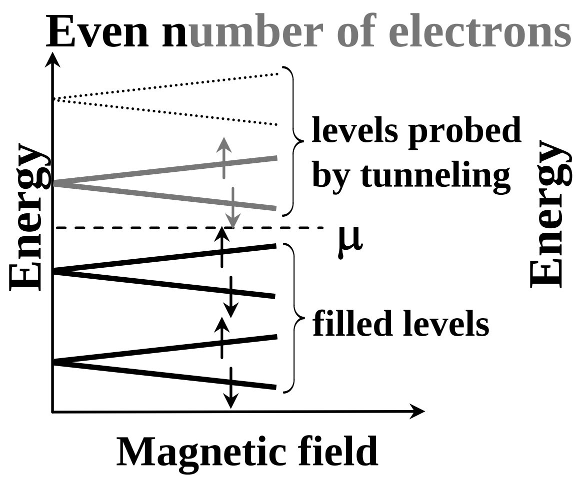
Electronic Spins in nano-particles: Zeeman Effect

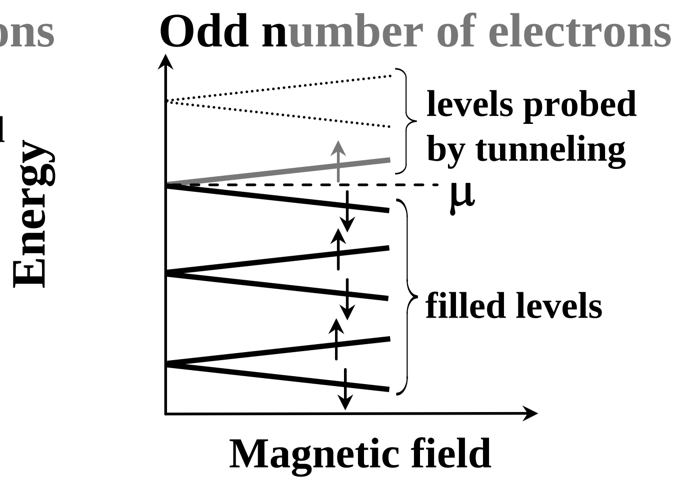


Hamiltonian:

$H_z = \sigma_z (g_o/2)\, \mu_B H$

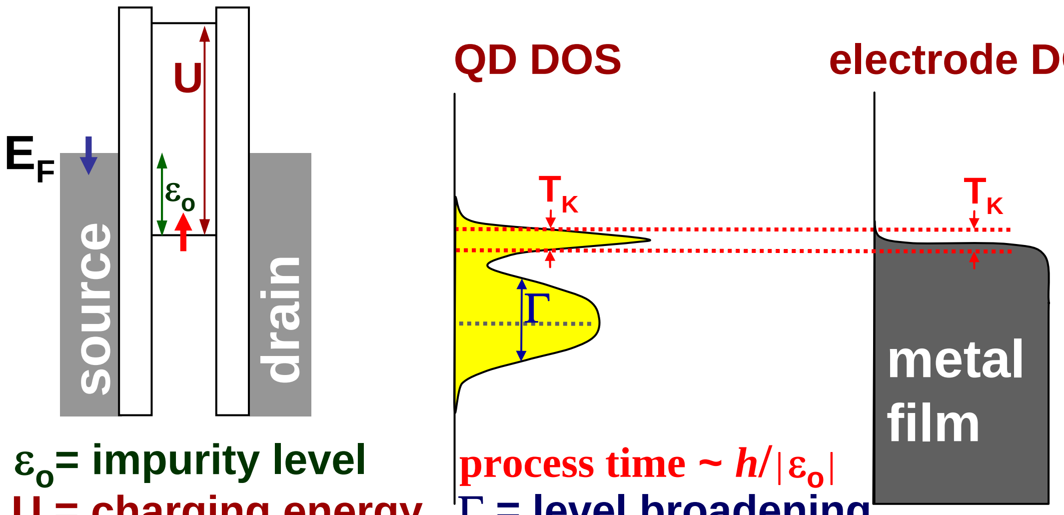
$\sigma_z = \pm 1/2$





von Delft et al.
PRL, 77, 3189 (96)

Kondo resonance in Quantum Dot systems



ε_o= impurity level
U = charging energy

process time ~ $h/|\varepsilon_o|$
Γ = level broadening

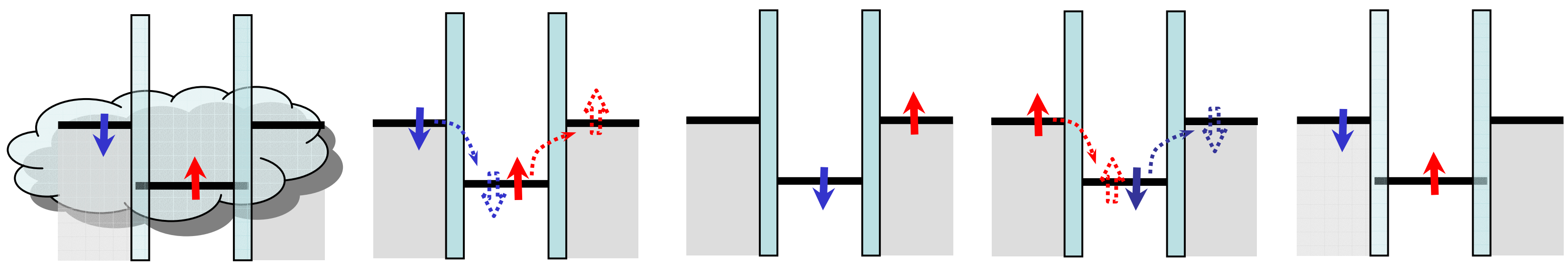
Kondo temperature

$$T_K = \sqrt{\Gamma U} \exp\left(\frac{\pi\varepsilon_0(\varepsilon_0 + U)}{\Gamma U}\right)$$

Duncan Haldane1978

in Quantum dots with odd number of electrons

Single channel resonance



Provide a new conduction channel in Coulomb blockade regime