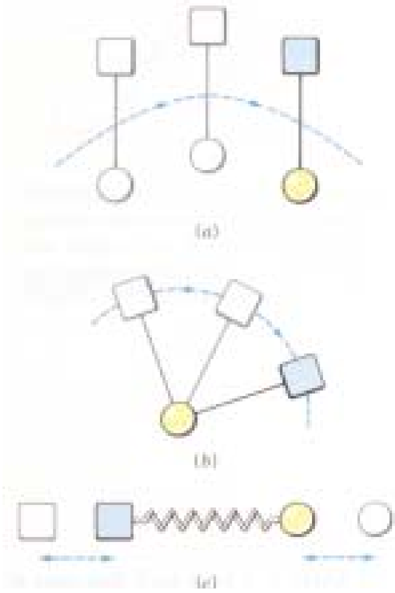
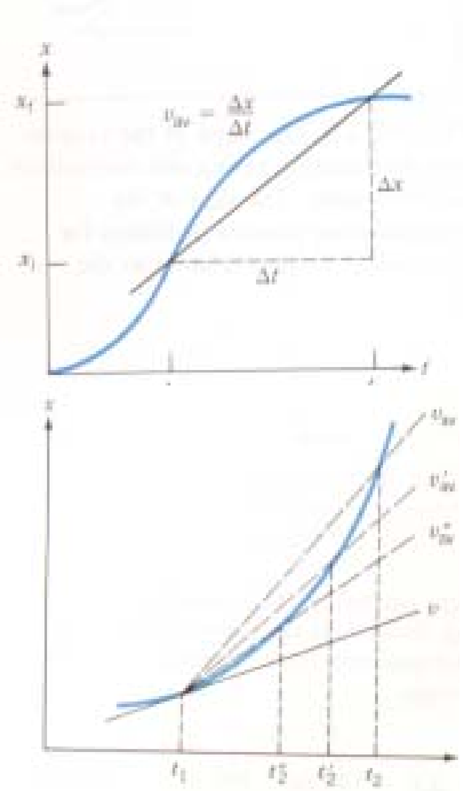
Chapter 3 One-Dimensional Kinematics

3.1 Particle Kinematics

The motion of a body may involve (a) **translation**, (b) **rotation**, (c) **vibration**, or a combination of these.



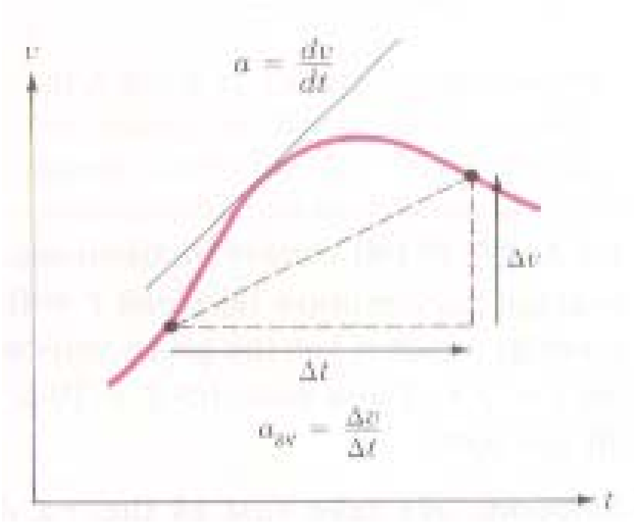
Average and Instantaneous Velocities



$$v_{av} = \frac{\Delta x}{\Delta t}$$

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Average and Instantaneous Accelerations



$$a_{av} = \frac{\Delta v}{\Delta t}$$

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

3.5 The Use of Areas

We now discuss the inverse problems of determining *x* from a graph of *v versus t* graph and *v* from a graph of *a versus t.*

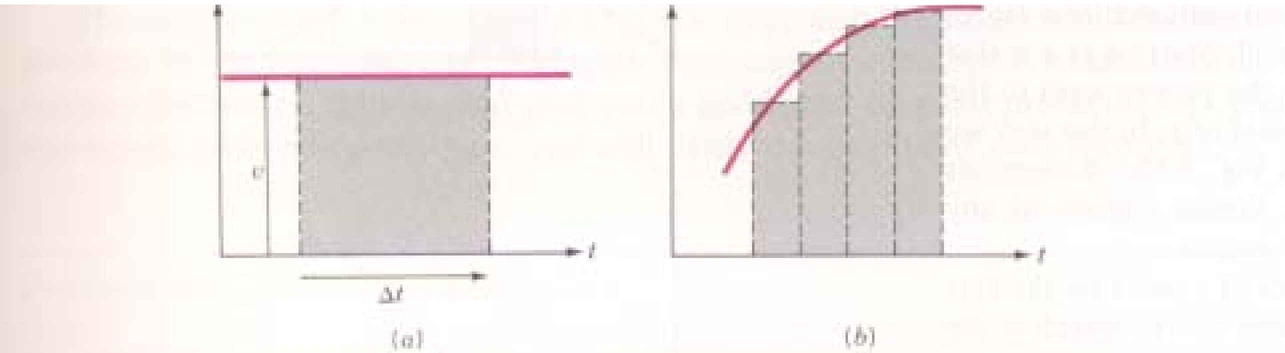
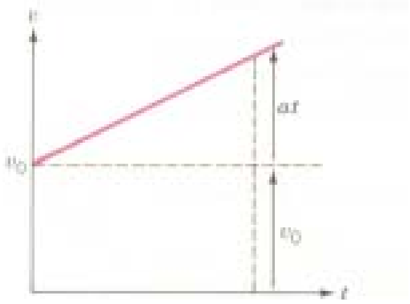
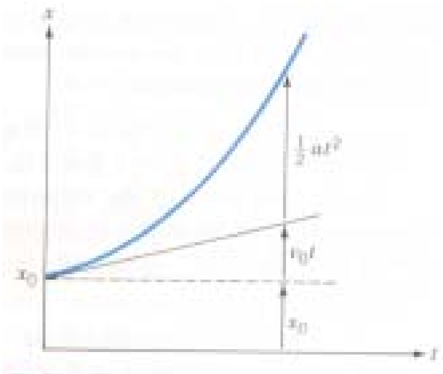


FIGURE 3.13 (*a*) When a particle travels at constant velocity v, the displacement in time interval Δt is $\Delta x = v\Delta t$. This equals the rectangular area under the v versus t graph for this time interval. (*b*) When the velocity is not constant, the actual area may be approximated by the sum of rectangular areas.

3.6 Equations of Kinematics for Constant Acceleration





$$v = v_o + at$$

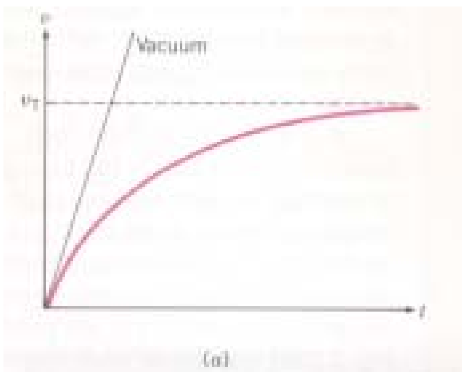
$$x = \int v\,dt$$
$$= x_o + v_o t + \frac{1}{2}at^2$$

$$2a(x - x_0) = 2av_o t + a^2t^2$$
$$= v^2 - v_o^2$$

3.8 Terminal Speed

An object dropped from a great enough height does not accelerate indefinitely. It will ultimately reaches terminal speed, v_T, and then continue to fall at this constant rate.

(b)



In a vertical posture, it takes about 15 s to reach the terminal speed of about 300 km/h (83 m/s); in a spread-eagle posture, v_T is about 200 km/h (55 m/s).

Chapter 4 Inertia and Two-Dimensional Motion

4.3 Projectile Motion

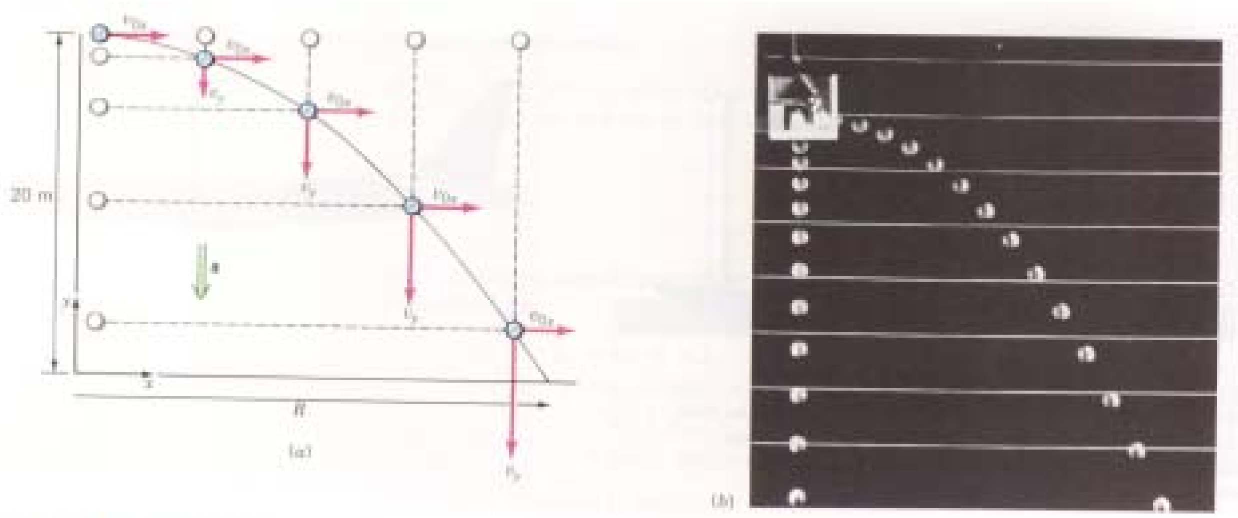


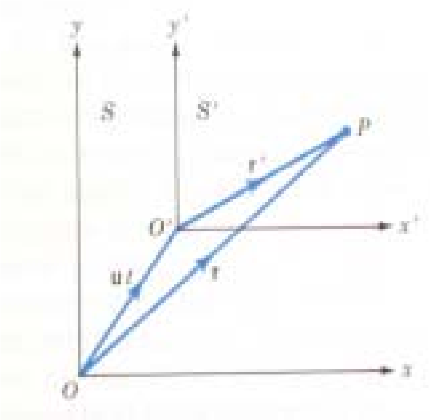
FIGURE 4.8 (a) The horizontal motion of a projectile is at constant velocity while the vertical motion occurs at constant acceleration (provided air resistance is negligible). (b) The vertical component of the motion of a ball projected horizontally is the same as that of a ball that is simply dropped.

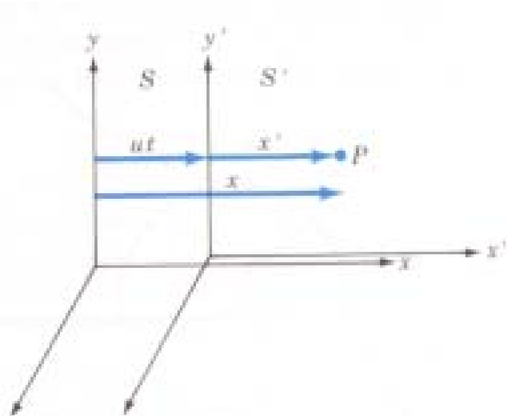
4.5 Inertia Reference Frames & 4.7 The Galilean Transformation

A reference frame in which Newton's first law is valid is called an ***inertial reference frame***.

In an inertial reference frame, a body subject to no net force will either stay at rest or move at constant velocity.

The laws of mechanics have the same form in all inertial frames.





Special Topic: Real Projectiles --- shotput (I)

Q: At what angle should the shot be projected to obtained maximum range?

FIGURE 4.31 When a projectile lands below its initial level, the maximum horizontal range is attained at an angle of projection

$$t_1 = \frac{v_o \sin\theta}{g} \qquad \Delta h = v_o \sin\theta \cdot t_1 - \frac{1}{2} g \cdot t_1^2$$

$$= \frac{v_o^2 \sin^2\theta}{2g}$$

$$h + \Delta h = \frac{1}{2} g \cdot t_2^2 \qquad t_2 = \frac{v_o}{g}\sqrt{\left(\frac{2gh}{v_o^2} + \sin^2\theta\right)}$$

$$L(\theta) = \left(t_1(\theta) + t_2(\theta)\right) \cdot v_o \cos\theta$$

$$= \frac{v_o^2}{g}\left[\sin\theta + \sqrt{\left(\frac{2gh}{v_o^2} + \sin^2\theta\right)}\right]\cos\theta$$

Special Topic: Real Projectiles --- shotput (II)

$$\frac{dL(\theta)}{d\theta} = 0$$

$$= \frac{v_o^2}{g}\left[\cos\theta + \frac{\frac{1}{2} 2\sin\theta\cos\theta}{\sqrt{\left(\frac{2gh}{v_o^2} + \sin^2\theta\right)}}\right]\cos\theta - \frac{v_o^2}{g}\left[\sin\theta + \sqrt{\left(\frac{2gh}{v_o^2} + \sin^2\theta\right)}\right]\sin\theta$$

The angle for maximum range is given by

$$\tan^2\theta_m = \frac{1}{\sqrt{1 + 2gh/v_o^2}}$$

Special Topic: Real Projectiles --- shotput (III)

$$\left[1+\frac{\sin\theta}{\sqrt{\left(\frac{2gh}{v_o^2}+\sin^2\theta\right)}}\right]\cos^2\theta=\left[1+\frac{\sin\theta}{\sqrt{\left(\frac{2gh}{v_o^2}+\sin^2\theta\right)}}\right]\sqrt{\left(\frac{2gh}{v_o^2}+\sin^2\theta\right)}\sin\theta$$

$$\cos^2\theta=\sqrt{\left(\frac{2gh}{v_o^2}+\sin^2\theta\right)}\sin\theta$$

$$\cos^4\theta=\left(\frac{2gh}{v_o^2}+1-\cos^2\theta\right)\sin^2\theta$$

$$\cos^2\theta=\left(\frac{2gh}{v_o^2}+1\right)\sin^2\theta$$

$$\tan^2\theta_m=\frac{1}{\sqrt{1+2gh/v_o^2}}$$

Chapter 5 Particle Dynamics I

We progress from the descriptions of **kinematics** (*how* bodies move) to the explanations of **dynamics** (*why* bodies move).

Newton's second law of motion introduces two concepts, **force** and **mass**, in the same equation.

It appears that one unknown has been defined in terms of another.

It is difficult to avoid this logic flaw in presenting the law of dynamics.

We will try to distinguish clearly between these two concepts.

5.1 Force and Mass

Although every force is a manifestation of one of the fundamental interactions mentioned in Section 1.1, it is sometimes convenient to refer to a force as being either a **contact force** or an **action at a distance**.

Force is clearly a quantity that has both magnitude and direction.

Mass is a measure of its inertia, that is, its resistance to change in velocity.

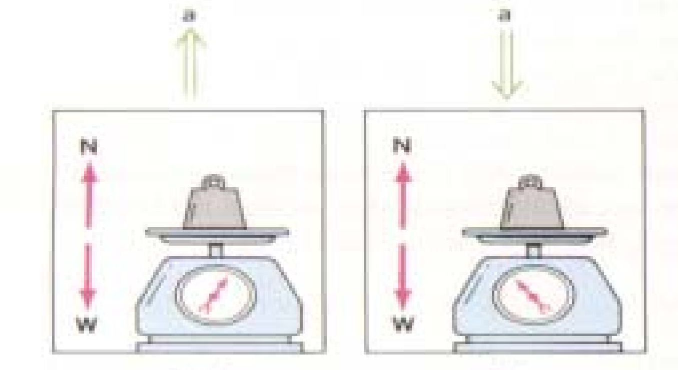
Mass, Weight, and Apparent Weight

The concept of mass and weight are often confused.

Mass: a measure of the inertia of a body (*scalar, intrinsic*).

Weight: the net gravitational force on a body (*vector, varying with gravity*)

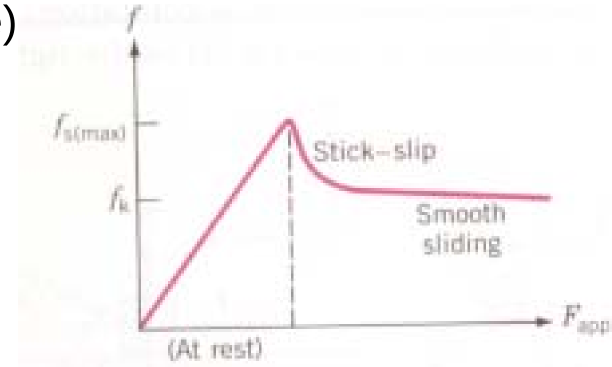
Apparent Weight: the resultant force exerted on a body by a supporting surface (*vector, varying with configuration*).



Chapter 6 Particle Dynamics II

It is natural to assume that friction is caused by roughness of the surfaces in contact.

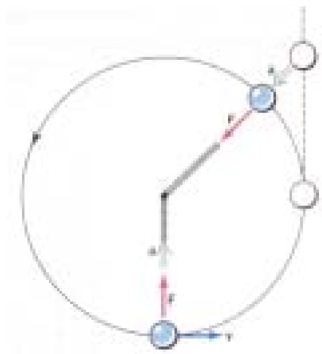
For visibly rough surfaces, above description is **partly true**. As the rough surfaces are sanded, the friction between them **drop**. However, as they are polished further, something strange happens; the friction start to **increase**. (for example, the polished wheels of a locomotive)



6.2 Dynamics of Circular Motion

Centripetal force: This force is always directed radially inward, toward the center. The particle must be subject to a centripetal force of magnitude:

$$\left|\vec{F}\right| = \left|-\frac{mv^2}{r}\hat{r}\right| = \frac{mv^2}{r}$$



Centrifugal force (center-fleeing): This is a misconception. For example: When twirling a stone in a circular motion by a string, the outward force on the hand and inward pull on the stone are equal and opposite forces but they **act on different bodies**.

The Coriolis Force (I)

G.G. Coriolis investigated an additional inertial force that appears if the particle has a velocity relative to a rotating frame.

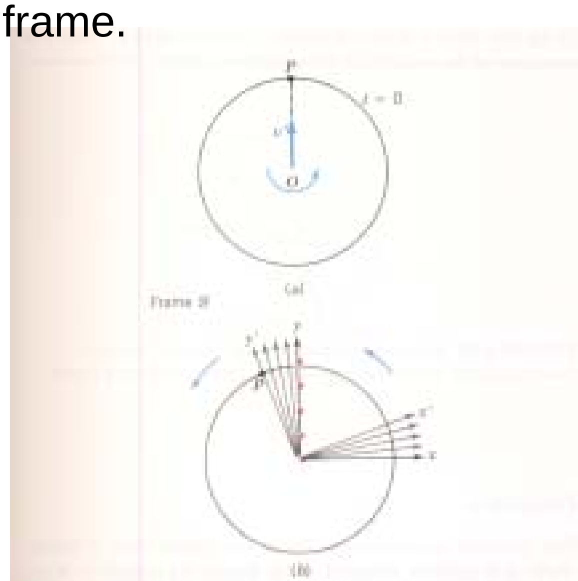


FIGURE 6.23 (a) A ball is thrown with speed v' from the origin of a rotating platform. (b) In the inertial frame (x, y) of the

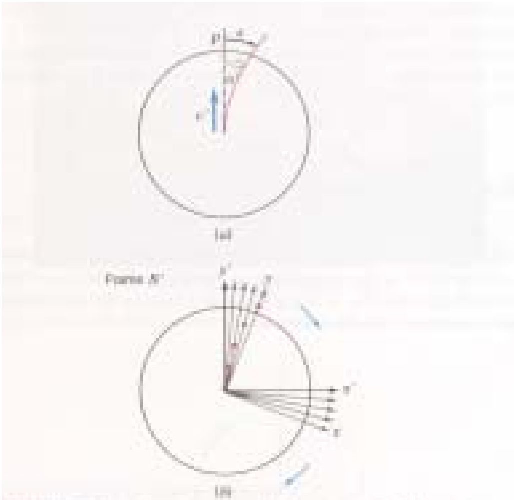


FIGURE 6.24 (a) In the noninertial frame (x', y') of the rotating platform, the path of the ball is curved. This is explained in terms of the fictitious Coriolis force. (b) As the platform rotates in a counterclockwise sense, the Coriolis acceleration a' deflects it toward the right.

The Coriolis Force (II)

Dujuan Typhoon

More examples: Foucault pendulum, The red spot on Jupiter