Chapter 7 Work and Energy

In principle, all problems in classical mechanics can be solved by using Netwon's laws.

In practice, we often have little information on the forces involved in a given situation.

Therefore, we introduce **a different approach** to the solution of problems in mechanics that is based on the concepts of **work** and **energy**.

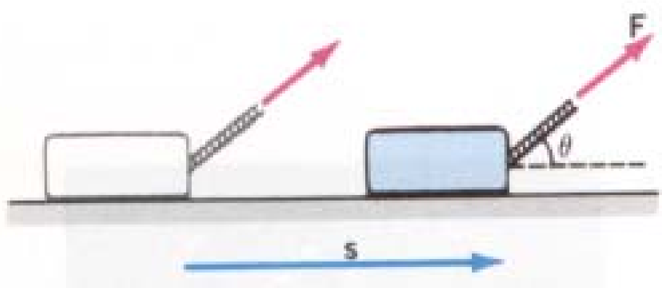
Although energy can change form, it can be neither created nor destroyed. This is called the **principle of conservation of energy**.

7.1 Work Done by a Constant Force

The work **W** done by a constant force **F** when its point of application undergoes a displacement **s** is defined to be

$$W = Fs\cos\theta = \vec{F}\cdot\vec{s}$$

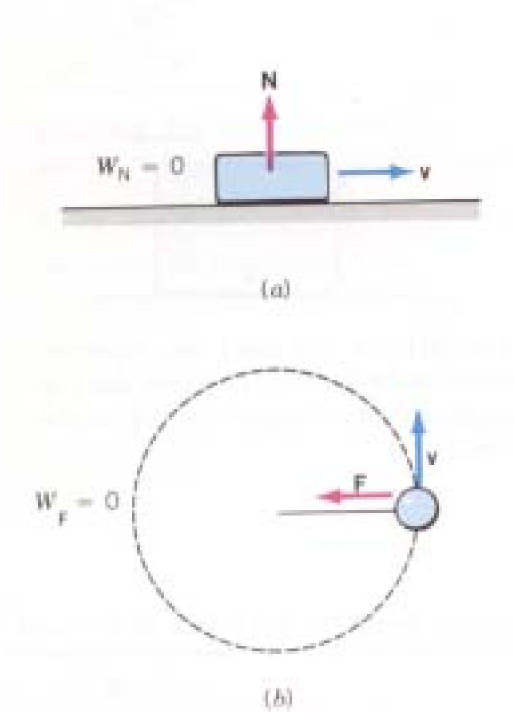
where θ is the angle between **F** and **s**.

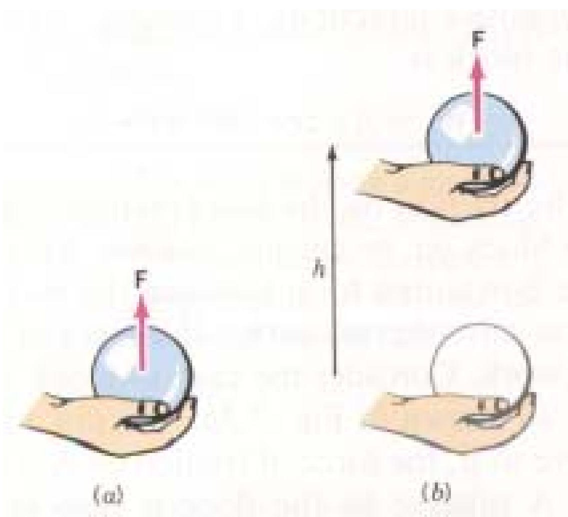


Work is a scalar quantity and its SI unit is joule (J).

$$1\,J = 1\,N\cdot m$$

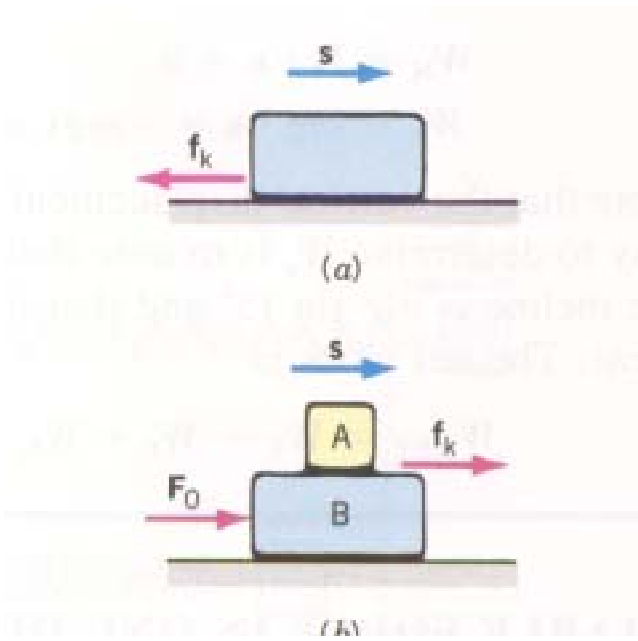
7.1 Work Done by a Constant Force: examples





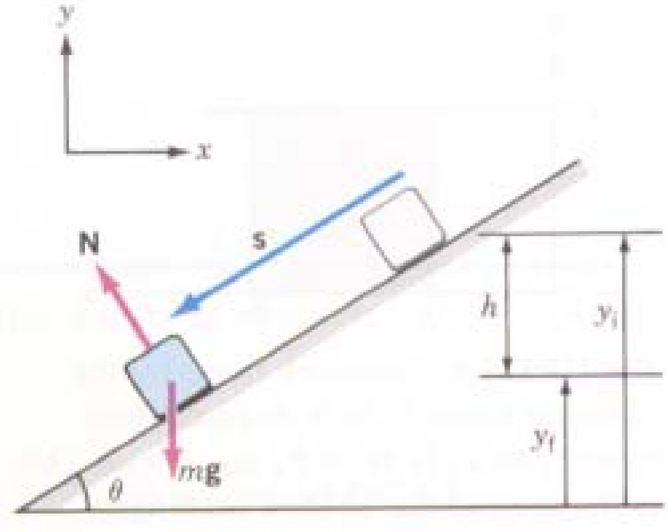
Negative Work and Work Done by Friction

The force of kinetic (sliding) friction always does negative work is a common misconception.



Work Done by Gravity

The work done by the force of gravity depends only on the **initial** and **final** vertical coordinates, *not on the path taken*. This is because the gravitation is a ***conservative force***.



7.2 Work by a Variable Force in One Dimension

Work done by a spring: Hooke's law

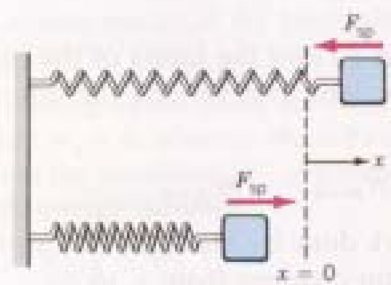


FIGURE 7.9 The force exerted by an ideal spring is given by Hooke's law: $F_{sp} = -kx$, where x is the extension or compression of the spring.

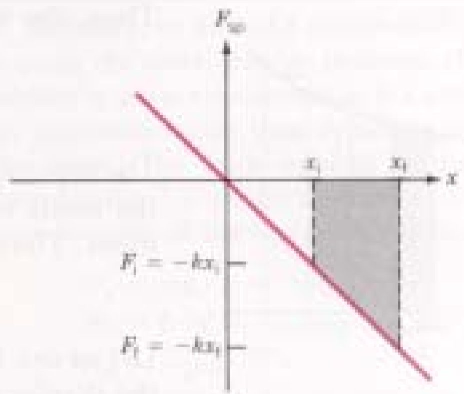
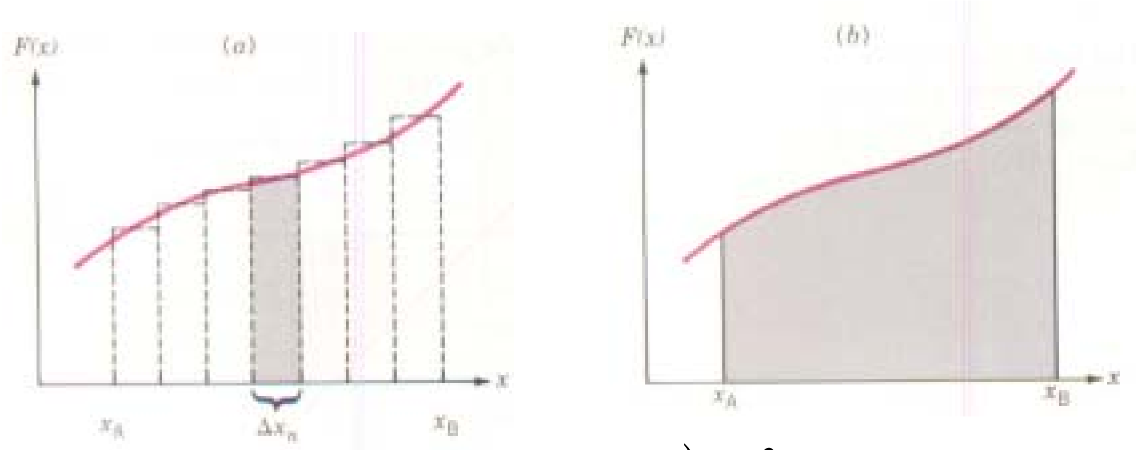


FIGURE 7.10 The work done by the spring when the displacement of its free end changes from x_i to x_f is the area of the trapezoid: $W_{sp} = -\frac{1}{2}k(x_f^2 - x_i^2)$.

Work as an Integral of the F_x versus x Graph



$$W = \lim_{\Delta x_n \to 0}\left(\sum F_n \cdot \Delta x_n\right) = \int F_x \, dx$$

Thus, the work done by a force F_x from an initional point A to final point B is

$$W_{A\to B} = \int_{XA}^{XB} F_x \, dx$$

7.3 Work-Energy Theorem in One Dimension

Work-energy theorem states that the net work done on a particle is equal to the resulting change in its (translational) kinetic energy.

$$W_{net} = \Delta K$$

Whereas **force** and **acceleration** are **vectors**, **work** and **energy** are **scalars**, which make them easier to deal with.

True/false: If the kinetic energy of a body is fixed, the next force on it is zero. Explain your response.

7.4 Power

Mechanical power is defined as the rate at which work is done.

Average Power $$p_{av} = \frac{\Delta W}{\Delta t}$$

Instantaneous Power $$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

The instantaneous **mechanical power** may be written as

$$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

The horsepower (hp), another unit of power, 1 hp=760 W

Tables of Energies and Powers

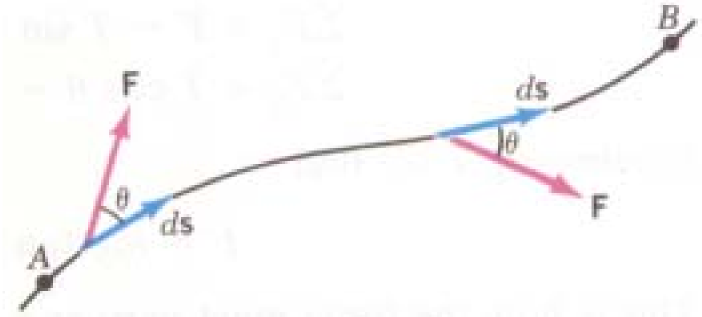
TABLE OF ENERGIES (J)	
Supernova explosion	10^{44}
Annual solar output	1.2×10^{34}
Rotational energy of earth	2×10^{29}
Initial fossil fuels (ca. 1800)	2×10^{23}
Annual global use	2.7×10^{20}
Annual loss tidal friction	10^{20}
Annual U.S. consumption	5×10^{19}
One megaton explosion	4.2×10^{15}
Annual output of power station	10^{16}
Lightning flash	5×10^{9}
One (U.S.) gallon of gasoline	1.3×10^{8}
Daily intake of a person	1.3×10^{7}
Proton in supercollider	3×10^{-7}
Fission of uranium nucleus	3×10^{-11}
Proton in nucleus	10^{-13}
Hydrogen bond	6×10^{-18}
Minimum detectable by eye	3×10^{-18}
Electron in atom	10^{-18}
Kinetic energy of molecule at room temperature	10^{-21}

TABLE OF POWERS (W)	
Quasar (3C273)	4×10^{40}
Solar output	4×10^{26}
Solar power incident on earth	1.74×10^{17}
U.S. consumption	2.7×10^{12}
Photosynthesis (global)	4×10^{12}
Hydroelectric (potential)	3×10^{12}
Tidal friction loss	3×10^{12}
Tidal power in use	7×10^{10}
Geothermal power in use	1.5×10^{9}
Grand Coulee Dam	6.5×10^{9}
Boeing 747	1.4×10^{8}
Aircraft carrier	1.2×10^{8}
Locomotive	3×10^{6}
Powerful laser	10^{6}
Automobile	10^{5}
Radio transmitter	5×10^{4}
Human output (short duration)	2×10^{3}
Person (resting)	75

7.5 Work and Energy in Three Dimension

When the magnitude and direction of a force vary in three dimensions, it can be expressed as a function of the position vector **F**(**r**), or in terms of the coordinates **F**(x, y, z).

$$W_{A\to B} = \int_A^B \vec{F}\, d\vec{s} = \int_A^B F\cos\theta\, ds$$
$$= \int_{XA}^{XB} F_x\, dx + \int_{YA}^{YB} F_y\, dy + \int_{ZA}^{ZB} F_z\, dz$$



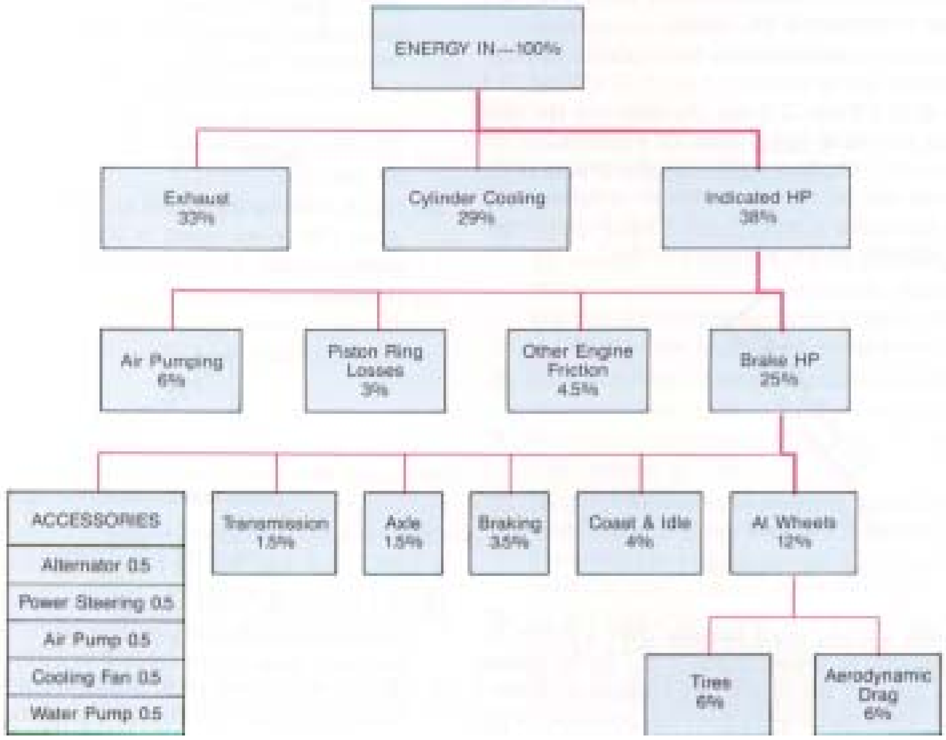
Questions

Do devices such as levers, pulleys, and gears, which allow us to use small forces, also save us work? Explian.

Compare the work done by a person who rises at constant velocity from the ground to the first floor (a) by using the stairs, and (b) by climbing a rope. Does the amount of energy he expends depend on the method?

Can work be done (a) by the force of static friction, or (b) by a centripetal force?

Special Topic: Energy and the Automobile (I)



Special Topic: Energy and the Automobile (II)

These large losses can be reduced in several ways.

- Drive at lower speeds to reduce aerodynamic drag and rolling resistance
- Modify the shape to be streamline to reduce aerodynamic drag
- Close the window to minimize the drag
- Maintain high tires pressures to minimize rolling resistance

Exercises and Problems

Ch.7: Ex.33, 37, 41

Prob. 3, 5, 8, 9