Chapter 8 Conservation of Energy

In the last chapter we introduced the concept of ***kinetic energy***, which is energy that a system possesses *by virtue of its motion*.

In this chapter we introduce ***potential energy***, which is energy that a system possesses *by virtue of the positions of its interacting particles*.

The work-energy theorem then leads to the ***principle of the conservation of mechanical energy***: Under certain conditions, the sum of the kinetic and potential energies in a system stays constant.

Conservation of Energy

The idea that the motion of an object is governed by something that does not change had its origin in the pendulum experiments of Galileo.

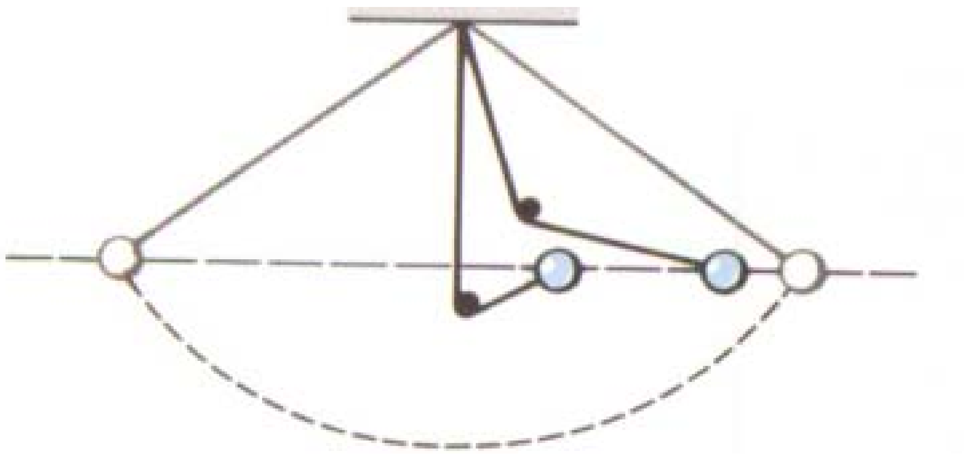
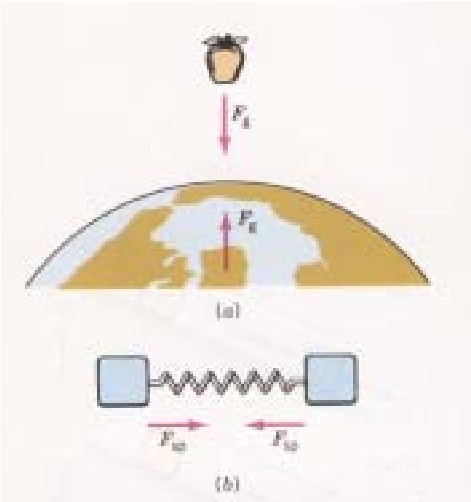


Fig. 8.1 and DVD example

8.1 Potential Energy

Potential energy is energy associated with the relative positions of two or more interacting particles.



Potential energy fits well the idea of energy as the capacity to do work, e.g. gravitational and elastic potential energy.

Potential Energy and External Work

Potential energy can be defined in terms of *work done by an external agent*.

$$W_{ext} = +\Delta U = U_f - U_i$$

The potential energy of a system is the external work needed to bring the particles from the U=0 configuration to the given positions **at constant speed***.

8.2 Conservative Forces

The work done by a conservative force is independent of the path taken.

$$W_{AB}^{(1)} = W_{AB}^{(2)}$$

The work done by a conservative force around any closed path is zero.

$$W_{A\to B}^{(1)} + W_{B\to A}^{(2)} = 0$$

The force of gravity and the force exerted by an ideal spring are called **conservative forces**, whereas the force of friction is a **non-conservative force**.

In other words, if the force can *be expressed in terms only of position, not of velocity or time*, we call this force conservative force.

8.3 Potential Energy and Conservative Forces

Potential energy defined in terms of work done by the associated conservative force.

$$U_B - U_A = -\int_A^B \mathbf{F}_c \cdot d\mathbf{s}$$

*Conservative forces tend to ***minimize*** the potential energy within any system: If allowed to, an apple falls to the ground and a spring returns to its natural length.

Non-conservative force does not imply it is dissipative, for example, magnetic force, and also does not mean it will decrease the potential energy, such as hand force.

Distinction Between Conservative and Non-conservative Forces

The distinction between conservative and non-conservative forces is best stated as follows:

A conservative force may be associated with a scalar potential energy function, whereas a non-conservative force cannot.

$$U_B - U_A = -\int_A^B \mathbf{F}_c \cdot d\mathbf{s}$$

$$\mathbf{F}_c = -\nabla U$$

8.4 Potential Energy Functions

Potential energy is clearly a function of position. We now find this function *for the gravitational force near the surface of the earth* and *for the force exerted by an ideal spring*.

Gravitational Potential Energy (near the earth's surface)

$$U_g(y) = mgy$$

Spring Potential Energy

$$U_{sp}(x) = \frac{1}{2}kx^2$$

Spring Potential Energy

$$U_{sp}(x) = \frac{1}{2}kx^2$$

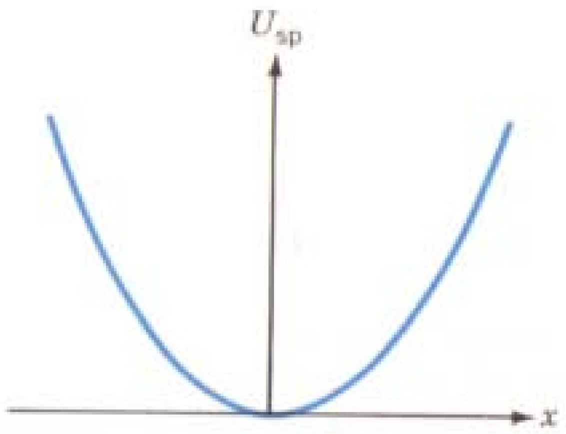


FIGURE 8.6 The potential energy of an ideal spring is a parabolic function of the displacement x from equilibrium.

8.5 Conservation of Mechanical Energy

The mechanical energy *E* is defined as the sum of kinetic energy *K* and potential energy *U*.

$$E = K + U$$

The principle of the conservation of mechanical energy.

$$\Delta E = E_f - E_i = 0$$

This definition offers several advantages.

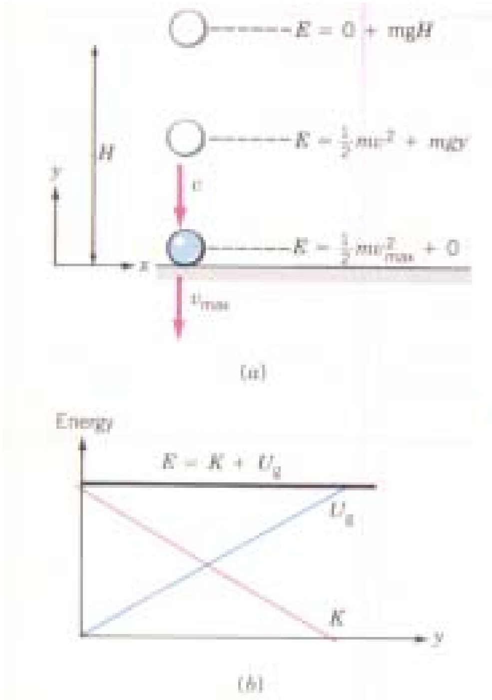
1. Whereas force is vector, work and energy are scalar, which makes them easier to deal with.

2. Only the initial and final states of a system are considered.

3. The concept of energy is useful even when force is unknown.

Conservation of Mechanical Energy: Gravitation

$$E = K + U = \frac{1}{2}mv^2 + mgy$$



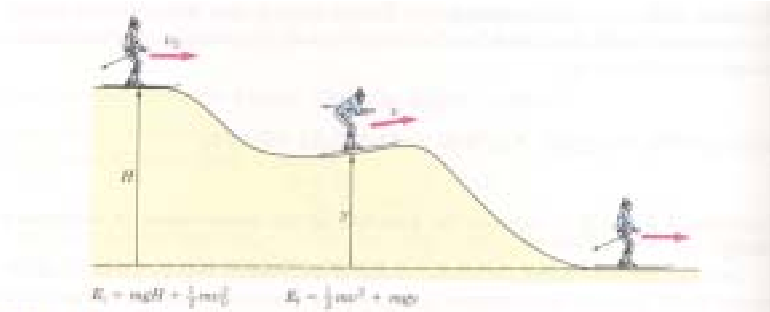
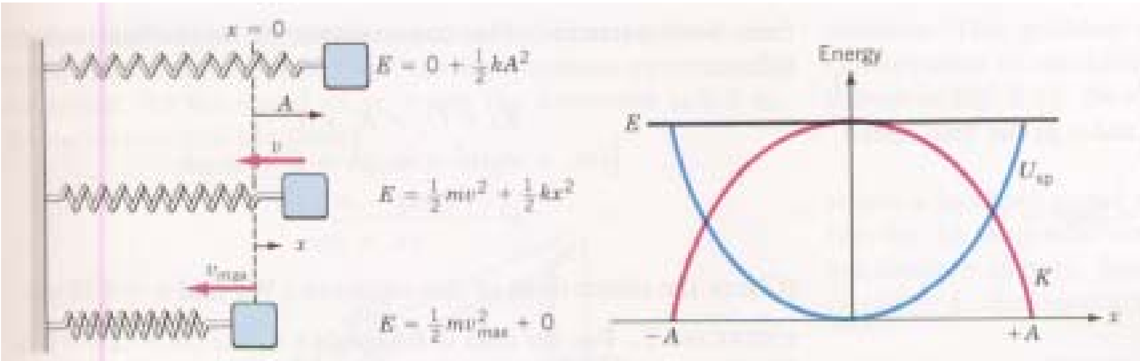


FIGURE 8.8 A skier travels along frictionless slope. The conservation of mechanical energy may be used to find the speed at any point.

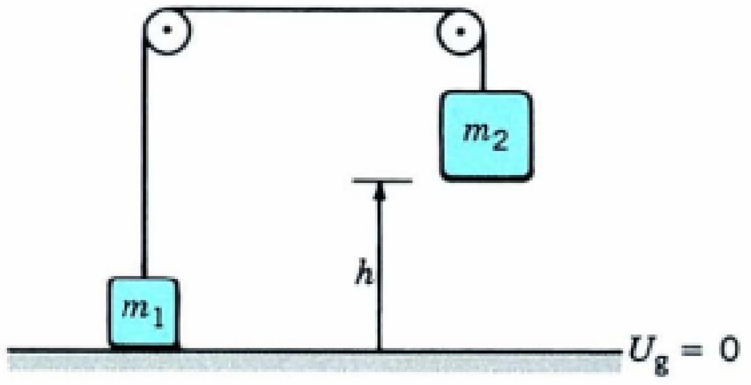
Conservation of Mechanical Energy: Spring

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$



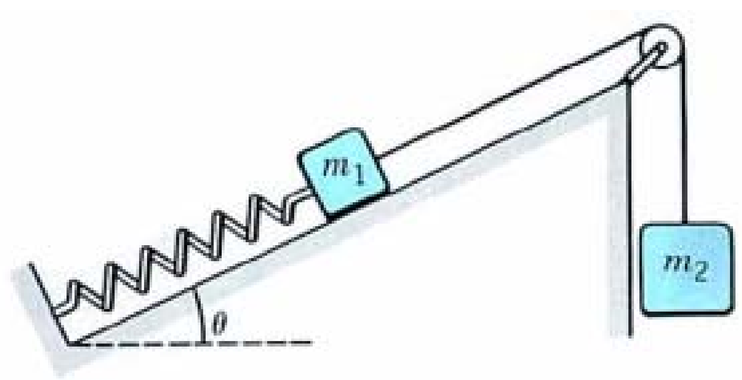
Example 8.4

Two blocks with masses *m*1=3 kg and *m*2=5 kg are connected by a light string that slides over two frictionless pegs as shown in Fig. 8.11. Initially m2 is held 5 m off the floor while m1 is on the floor. The system is then release. At what speed does m2 hit the floor?



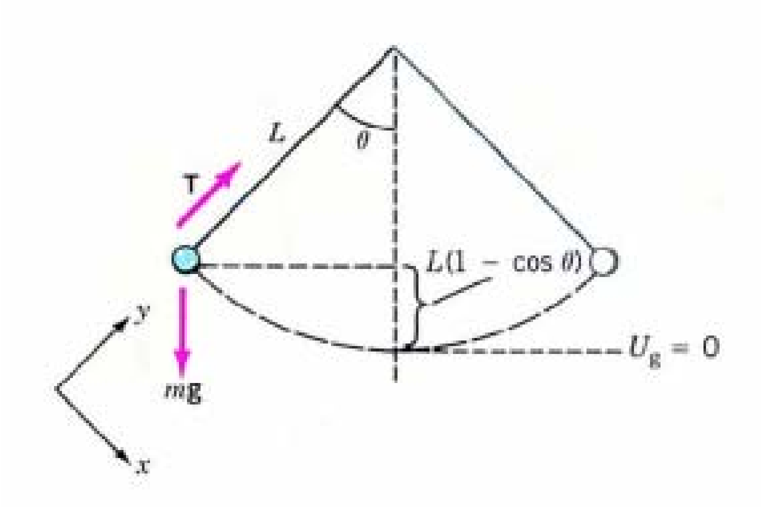
Example 8.6

Two blocks with masses *m*1=2 kg and *m*2=3 kg hang on either side of a pulley as shown in Fig. 8.12. Block *m*1 is on an incline (θ =30°) and is attached to a spring whose constant is 40 N/m. The system is released from rest with spring at its natural length. Find: (a) the maximum extension of the spring; (b) the speed of *m*2 when the extension is 0.5 m. Ignore friction and the pulley mass effect.



Example 8.7

The bob of a simple pendulum of length L=2 m has a mass m=2 kg and a speed $v=1.2$ m/s when the string is at 35° to the vertical. Find the tension in the string at: (a) the lowest point in its swing; (b) the highest point.



8.6 Mechanical Energy and Non-conservative Force

The conservation of mechanical energy may be applied to a system *only when there is no work done by any non-conservative force*.

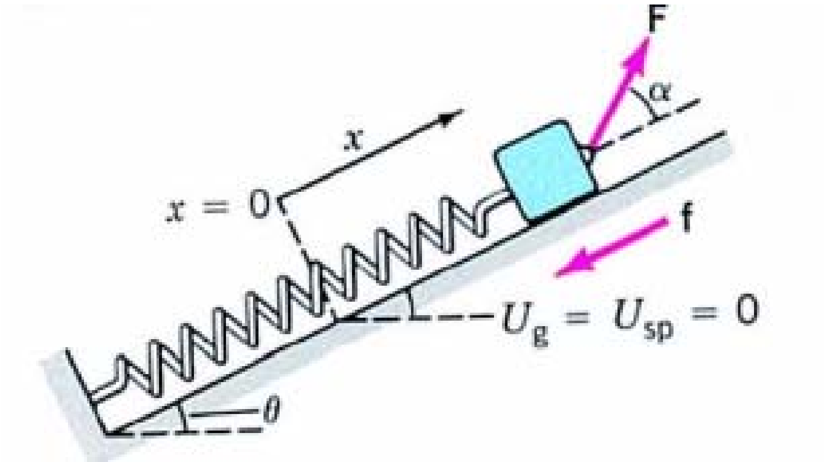
$$\Delta E = E_f - E_i = W_{nc}$$

The above equation is the modified conservation of mechanical energy *when work is done by non-conservative force*.

For example, press or pull a spring and lift a stone by hand.

Example 8.8

A block of mass *m* is attached to a spring and moves on a rough incline as in Fig. 8.15. Initially, the block is at rest with the spring unextended. A force *F* acting at an angle α to the incline pulls the block. Write the modified form of the work-energy theorem.



8.7 Conservative Force and Potential Energy Function

How can we find a conservative force if the associated potential energy function is given?

A conservative force can be derived from a scalar potential energy function.

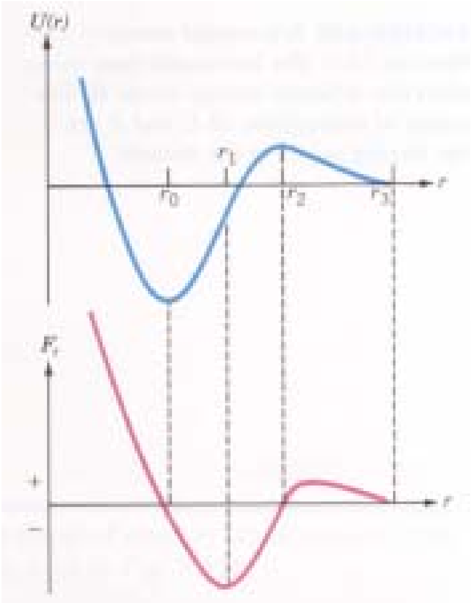
$$\mathbf{F}_c = -\nabla U$$

The negative sign indicates that the force points in the direction of ***decreasing*** potential energy.

Gravity $U_g = mgy;\qquad F_y = -\dfrac{dU_g}{dy} = -mg$

Spring $U_{sp} = \dfrac{1}{2}kx^2;\qquad F_x = -\dfrac{dU_{sp}}{dx} = -kx$

Conservative Force for a Hypothetical Potential Energy Function



(r > r2): Fr > 0, repulsive force.

(r = r2): Fr = 0, unstable equilibrium.

(r2> r > r0): Fr < 0, attractive force.

(r = r0): Fr = 0, stable equilibrium.

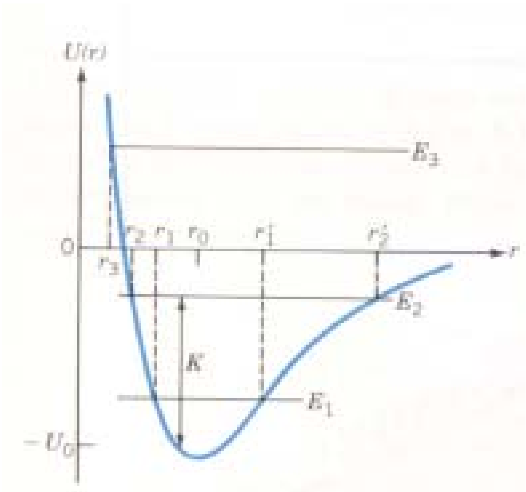
(r0> r): Fr > 0, repulsive force.

Questions

- True/false: The net force acting on a ball being lifted at constant velocity is zero; hence its energy is conserved.

- We use chemical energy in walking up stairs and thereby gain potential energy. Do we regain this energy as we walk down the stairs? If not, where does it go?

8.8 Energy Diagrams



potential well of depth $-U_0$

$$E = K + U$$

(E <0): *bound state*

(E >0): *unbound state*

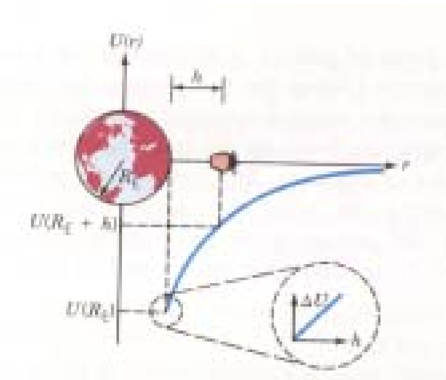
The **binding energy** of a particle in a bound state is the minimum energy requirement to make it an unbound state.

8.9 Gravitational Potential Energy, Escape Speed*

The force of gravity is an example of a **central force**, which means that it is always directed along the line joining the two particles.

$$U_g(r) = -\frac{GmM}{r}$$

$$F(r) = -\frac{dU_g}{dr} = -\frac{GmM}{r^2}$$



This force is also *spherically symmetric* since it depends only on the radial coordinate *r*.

Mechanical Energy

Assuming a system consists of two particles, one mass is very much larger than the other (M>>m) and the kinetic energy of M is negligible. The total mechanical energy is

$$E = \frac{1}{2}mv^2 - \frac{GmM}{r}$$

If particle m is orbiting particle M in a stable circular motion, the centripetal force is provided by the gravitational force.

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

$$E = K + U = -\frac{GmM}{2r} \quad \text{binding energy}$$

Example 8.11

A rocket with a payload of mass *m* is at rest at the surface of the earth. Calculate the work needed to raise the payload to the following states: (a) at rest at a maximum altitude equal to R_E; (b) in circular orbit at an altitude R_E.

Sol:

$$(a)\ \Delta W = E_f - E_i = -\frac{GmM_E}{2R_E} + \frac{GmM_E}{R_E}$$

$$= \frac{GmM_E}{2R_E}$$

$$(b)\ \Delta W = E_f - E_i = (K_f + U_f) + \frac{GmM_E}{R_E} = -\frac{GmM_E}{4R_E} + \frac{GmM_E}{R_E}$$

$$= \frac{3GmM_E}{4R_E}$$

Escape Speed

What is the minimal initial speed if we want to escape from the earth surface, and never return?

Final state $r \to \infty$ with zero speed, $E_f = 0$.

Initial state $r = r_E$, $E_i = \frac{1}{2} m v_{esc}^2 - \frac{GmM}{r_E}$

On setting Ef=Ei, we obtain $v_{esc} = \sqrt{\frac{2GM}{r_E}}$

For a particle at the earth's surface, escape velocity is equal to 11.2 km/s *relative to the center of the earth*.

Example 8.11

A rocket is fired vertically with half the escaped speed. What is its maximum altitude in terms of the radius of the earth R_E? Ignore the earth rotation.

Sol:

$$E_i = \frac{1}{2} m \left(\frac{v_{esc}}{2} \right)^2 - \frac{GmM_E}{R_E} = \frac{GmM_E}{4R_E} - \frac{GmM_E}{R_E} = -\frac{3GmM_E}{4R_E}$$

$$E_i = E_f$$

$$-\frac{3GmM_E}{4R_E} = -\frac{GmM_E}{R_E + h} \Rightarrow h = \frac{R_E}{3}$$

Questions

- Does the escape speed for a rocket depend on the angle of launch?
- Can the rotation of the earth help in the launching of a satellite? If so, explain how.
- Name three forms of energy available on earth whose origin is not ultimately traceable to the sun.

8.10 Generalized Conservation of Energy

Around 1845, several scientists independently concluded that all natural process are subject to an important constrain called the **principle of conservation of energy**:

Energy can change its form, but it can neither be created or destroyed.

The formulation of this principle required the identification of *heat as a form of energy*, which will be discussed in **thermodynamics**.

Exercises and Problems

Ch.8: Ex.12, 13, 14, 26, 37, 38, 51

Prob. 6, 11, 13