Chapter 9 Linear Momentum

Descartes believed that God had created the world like a perfect and never-changing clockwork mechanism, and asserted that the total "***quantity of motion***", which he defined as ***mass*** *x* ***speed***, would remain constant.

Although Descartes' analysis of specific examples was weak, he had sown the seed of an extremely important idea in physics: There is some physical quantity that does not change within an isolated system. This is called a ***principle of conservation***.

In 1668, scientists began to communicate their result and soon realized that the quantity of motion was conserved if it was re-defined as ***mass*** x ***velocity***.

Linear Momentum and Kinetic Energy

In the meantime, Huygens and Wren independently concluded the quantity mv^2 is conserved in collisions between hard balls.

Later, Newton conducted carefully experiments on collision between all kinds of substances, such as glass, wood, steel, and putty.

Newton found the vector $m\mathbf{v}$ was always conserved, but that the scalar quantity mv^2 was conserved only in special case of collisions between hard spheres.

Force and Momentum

In Newton's own version of his second law, he stated that the "motive force" exerted on a particle is equal to the change in its linear momentum.

In 1752, Euler modified Newton's definition to include the aspect of time explicitly.

A modern statement of Newton's second law is:

The net force acting on a particle is equal to the time rate of change of its linear momentum.

$$\mathbf{F} = \frac{d\mathbf{P}}{dt}$$

9.2 Conservation of Linear Momentum

The initial and final velocities are related according to the principle of the conservation of linear momentum:

$$m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$$

In order to apply the conservation of linear momentum, there must be no net external force acting on the system.

$$\mathbf{F}_{ext} = \frac{d\mathbf{P}}{dt}$$

$$\text{If } \mathbf{F}_{ext} = 0, \quad \text{then } \mathbf{P} = \sum \mathbf{P}_\text{i} = \text{constant}$$

If the net external force acting on a system is zero, the total linear momentum is constant.

Type of Collision

Collisions may be either *elastic* or *inelastic*. Linear momentum is conserved in both cases. A *perfectly elastic* collision is defined as one in which the total kinetic energy of the particles is also conserved.

elastic and *inelastic* $$m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$$

elastic $$\frac{1}{2}m_1\mathbf{u}_1^2 + \frac{1}{2}m_2\mathbf{u}_2^2 = \frac{1}{2}m_1\mathbf{v}_1^2 + \frac{1}{2}m_2\mathbf{v}_1^2$$

Super-elastic collision refers to the possibility that total kinetic energy increases as a result of collision. This is because the collision triggers a system to release extra potential energy.

Example 9.1

A 2000-kg Cadillac limousine moving east at 10 m/s collides with a 1000-kg Honda Prelude moving west at 26 m/s. The collision is completely inelastic and takes place on an icy (frictionless) patch of road. (a) Find their common velocity after the collision. (b) what is the fractional loss in kinetic energy?

Sol:

$$\text{(a) } 2000\cdot 10 - 1000\cdot 26 = 3000\cdot v_{after} \Rightarrow v_{after} = 2\text{ m/s}$$

$$\text{(b) } E_{final} = \frac{1}{2}(2000+1000)v_{after}^2 = 6000\text{ J}$$

$$E_{initial} = \frac{1}{2}(2000)10^2 + \frac{1}{2}(1000)26^2 = 438000\text{ J}$$

$$(E_{final} - E_{initial})/E_{initial} = (6000-438000)/438000 = -0.99$$

Example 9.2

A 3.24-kg Winchester Super X rifle, initially at rest, fires a 11.7-g bullet with a muzzle speed of 800 m/s. (a) What is the recoil velocity of the rifle? (b) What is the ratio of the kinetic energies of the bullet and the rifle?

Sol:

(a) momentum conservation

$$11.7\cdot(800-v)=3240\cdot v \quad \Rightarrow v=2.88\text{ m/s}$$

(b) $$K_b=\frac{1}{2}m_b v_b^2=0.5\cdot 0.0117\cdot 800^2=3744\text{ J}$$

$$K_g=\frac{1}{2}m_g v_g^2=0.5\cdot 3.24\cdot 2.88^2=12.5\text{ J}$$

$$K_g/K_b=12.5/3744=0.33\%$$

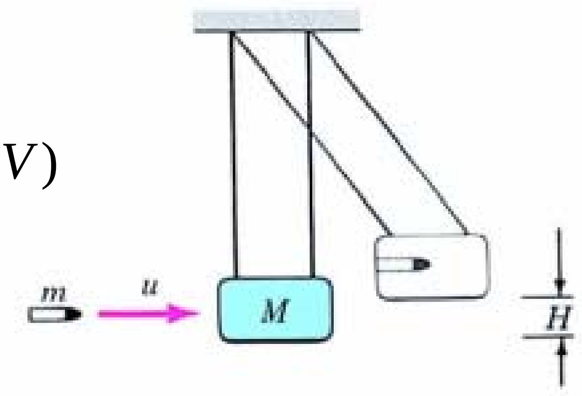
Example 9.5

In 1742, Benjamin Robins devised a simple, yet ingenious, device, called a ballistic pendulum, for measuring the speed of a bullet. Suppose that a bullet , of mass *m*=10 g and speed *u*, is fired into a block of mass *M*= 2 kg suspended as in Fig. 9.8. The bullet embeds in the block and raises it by a height *h*=5 cm. (a) How can one determine *u* from *H*? (b) What is the thermal energy generated?

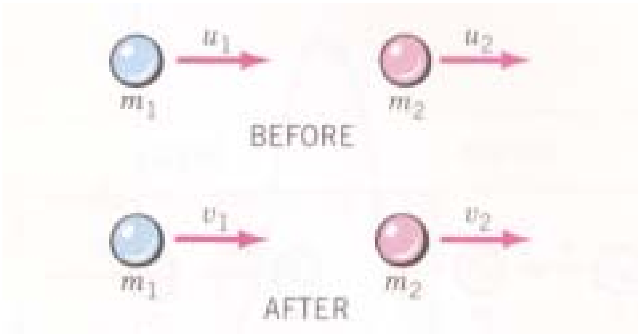
Sol:

Momentum conservation (u,V)

Energy conservation (V, H)



9.3 Elastic Collision in One Dimension



momentum $$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

kinetic energy $$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_1^2$$

elastic $$v_2 - v_1 = -(u_2 - u_1) \qquad (9.10)$$

In a one-dimension elastic collision, the relative velocity is unchanged in magnitude but is reversed in direction.

Elastic Collision in One Dimension

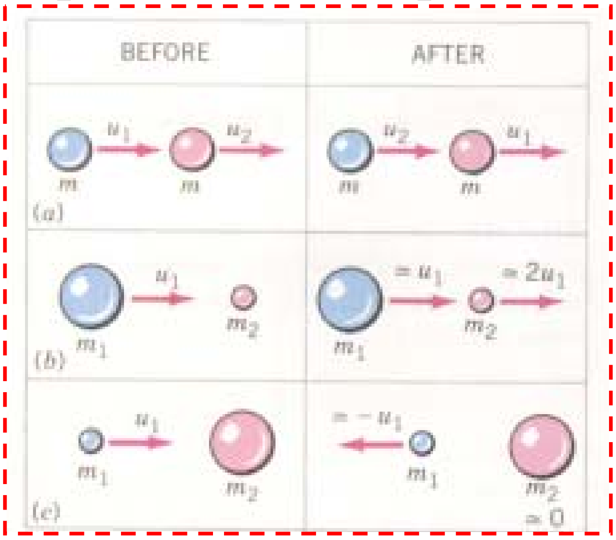
(i) Equal masses: $m_1=m_2=m$

velocities exchange $$v_1 = u_2, \quad v_2 = u_1$$

(ii) Unequal masses: $m_1 \neq m_2$, Target at rest: $u_2=0$

$$v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2}$$

$$v_2 = \frac{2m_1u_1}{m_1 + m_2}$$



9.4 Impulse

The **impulse I** experienced by a particle is defined as the change in its linear momentum:

$$\mathbf{I} = \Delta\mathbf{P} = \mathbf{P}_f - \mathbf{P}_i$$

Impulse is a vector quantity with the same unit as linear momentum (kg·m/s).

$$\mathbf{I} = \Delta\mathbf{P} = \sum_{\Delta t \to 0} \mathbf{F}(t)\Delta t = \int_{t1}^{t2} \mathbf{F}\, dt$$

For the same impulse, prolonging the interacting time will reduce the average applied force.

Examples: jump from a wall and catch a ball.

9.5 Comparison of Linear Momentum with Kinetic Energy

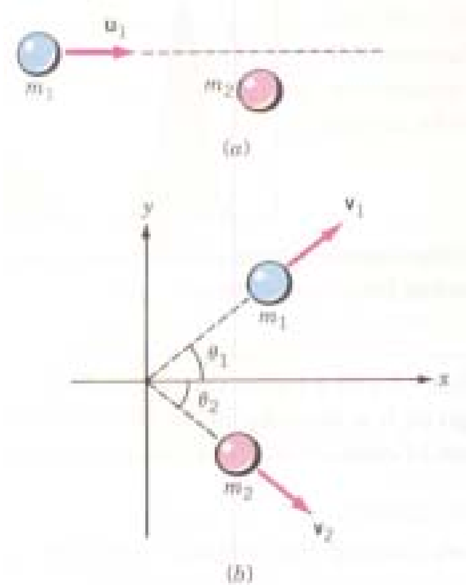
(i) Conservation of linear momentum is a general valid law, whereas the conservation of kinetic energy is true only in the special case of elastic collision.

(ii) Momentum is a vector, whereas the kinetic energy is a scalar.

(iii) Force, momentum, and kinetic energy are all co-related concepts.

$$\mathbf{F} = \frac{d\mathbf{P}}{dt}, \quad K = \int \mathbf{F}\, dx$$

9.6 Elastic Collision in Two Dimension



$$\sum P_x \qquad m_1 u_1 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2$$

$$\sum P_y \qquad 0 = m_1 v_1 \sin\theta_1 - m_2 v_2 \sin\theta_2$$

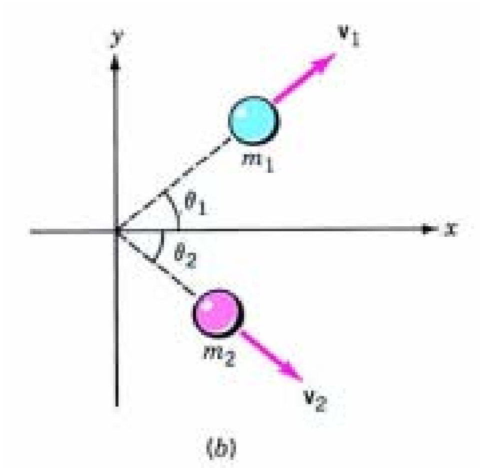
$$\sum K \qquad \tfrac{1}{2} m_1 u_1^2 = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2$$

There are four unknowns v1, v2, θ 1, and θ 2 in three equations. Often, either θ *1 or* θ *2* is measured.

Example 9.9

A proton moving at speed *u*1= 5 km/s makes an elastic collision with another proton initially at rest. Given that θ_1=37°, find v_1, v_2 and θ_2.

Sol:



Momentum conservation

$$\begin{cases} \text{at x-axis} \quad 5 = v_1 \cdot 0.8 + v_2 \cdot \cos\theta_2 \\ \text{at y-axis} \quad v_1 \cdot 0.6 = v_2 \cdot \sin\theta_2 \end{cases}$$

Energy conservation

$$5^2 = v_1^2 + v_2^2$$

Exercises and Problems

Ch.9: Ex.10, 12, 17, 30

Prob. 4, 5, 18, 19