## Chapter 14 Solids and Fluids

Matter is usually classified into one of four states or phases: solid, liquid, gas, or plasma.

Shape: A solid has a fixed shape, whereas fluids (liquid and gas) have no fixed shape.
Compressibility: The atoms in a solid or a liquid are quite closely packed, which makes them almost incompressible. On the other hand, atoms or molecules in gas are far apart, thus gases are compressible in general.

The distinction between these states is not always clear-cut.
Such complicated behaviors called phase transition will be discussed later on.

### 14.1 Density

At some time in the third century B.C., Archimedes was asked to find a way of determining whether or not the gold had been mixed with silver, which led him to discover a useful concept, density.

The specific gravity of a substance is the ratio of its density to that of water at $4^{\circ} \mathrm{C}$, which is $1000 \mathrm{~kg} / \mathrm{m}^{3}=1 \mathrm{~g} / \mathrm{cm}^{3}$.
Specific gravity is a dimensionless quantity.

### 14.2 Elastic Moduli

A force applied to an object can change its shape.
The response of a material to a given type of deforming force is characterized by an elastic modulus,

$$
\text { Elastic modulus }=\frac{\text { Stress }}{\text { Strain }}
$$

Stress: force per unit area in general
Strain: fractional change in dimension or volume.
Three elastic moduli will be discussed:
Young's modulus for solids, the shear modulus for solids, and the bulk modulus for solids and fluids.

## Young's Modulus

Young's modulus is a measure of the resistance of a solid to a change in its length when a force is applied perpendicular to a face.

Tensile stress $=\frac{\mathrm{F}_{\mathrm{n}}}{\mathrm{A}} \quad$ Tensile strain $=\frac{\Delta L}{L_{\mathrm{o}}}$

$$
\begin{aligned}
\text { Young's modulus } & =\frac{\text { Tensil stress }}{\text { Tensile strain }} \\
Y & =\frac{F_{n} / A}{\Delta L / L_{\mathrm{o}}}
\end{aligned}
$$

## Young's Modulus (II)



$$
Y=\frac{F_{n} / A}{\Delta L / L_{0}}
$$

## Shear Modulus

The shear modulus of a solid indicates its resistance to a shearing force, which is a force applied tangentially to a surface.

$$
\text { Shear stress }=\frac{\text { Tangential force }}{\text { Area }}=\frac{\mathrm{F}_{\mathrm{t}}}{\mathrm{~A}}
$$

Shear strain $=\frac{\Delta x}{h}$
Shear modulus $=\frac{\text { Shear stress }}{\text { Shear strain }}$

$$
S=\frac{F_{t} / A}{\Delta x / h}
$$

## Shear Modulus (II)



$$
S=\frac{F_{t} / A}{\Delta x / h}
$$

An ideal fluid cannot sustain a shear stress. Although a real fluid cannot sustain a permanent shearing force, there are tangential forces between adjacent layers in relative motion. This produces an internal friction called viscosity.

## Bulk Modulus

The bulk modulus of a solid or a fluid indicates its resistance to a change in volume.
The pressure on the cube is defined as the normal fore per unit area.

$$
\mathrm{P}=\frac{F_{n}}{A}
$$

$$
\text { Bulk }=\frac{\text { Volume stress }}{\text { Volume strain }}, B=-\frac{F_{n} / A}{\Delta V / V}
$$

The negative sign is included to make $B$ a positive number since an increase in pressure leads to an decrease in volume. The inverse of $B$ is called the compressibility, $k=1 / B$.

## Bulk Modulus（II）



$$
B=-\frac{F_{n} / A}{\Delta V / V}
$$

表14．2彈性係數（ $\times 10^{\circ} \mathrm{N} / \mathrm{m}^{2}$ ）

|  | $\mathbf{Y}$ | $\mathbf{S}$ | $\mathbf{B}$ |
| :--- | :---: | :---: | ---: |
| 綪栰 | 100 | 40 | 90 |
| 龬 | 200 | 80 | 140 |
| 鋁 | 70 | 25 | 70 |
| 梗水泥 | 20 |  |  |
| 松木 | 7.6 |  |  |
| 水 |  |  | 2.1 |
| 氷 |  |  | 2.6 |

## 14．3 Pressure in Fluids： <br> Variation of Pressure with Depth in a Liquid

The pressure on a tiny volume element exerted by the surrounding fluid is the same in all direction（equilibrium）．


Pressure is a function only of depth and does not depend on the shape of the container．

## Variation of Pressure with Depth in a Liquid

How the pressure increases with depth?


The dam must be constructed in just the same manner if it had to contain a small body of water of the same depth.

## Pascal's Principle

An external pressure applied to a fluid in an enclosed container is transmitted undiminished to all parts of the fluid and the walls of the container.


$$
\frac{F_{2}}{F_{1}}=\frac{A_{2}}{A_{1}}
$$

Pressure at the two position is the same.

## Measurement of Pressure

A simple way to measure pressure is with a manometer. The absolute pressure $P$ is the sum of the atmospheric pressure Po and the gauge pressure pgh.


$$
P=P_{o}+\rho g h
$$

$$
\begin{aligned}
& 1 \mathrm{~atm}=1.013 \times 10^{5} \mathrm{~Pa}\left(\mathrm{~N} / \mathrm{m}^{2}\right) \\
&=1013 \mathrm{mbar} \\
&=14.7 \mathrm{psi} \\
&=1033.6 \mathrm{~mm} \mathrm{H} \\
& 2
\end{aligned}
$$

### 14.4 Archimede's Principle

When the solid body is immersed in a liquid, it displaces the same volume of liquid. Since the forces exerted by the surrounding liquid are unchanged, we arrive at

## Archimedes' Principle:

Buoyant force $=$ Weight of fluid displaced

$$
F_{B}=\rho_{f} V g
$$

Example 14.1 An iceberg with a density of $920 \mathrm{~kg} / \mathrm{m}^{3}$ floats on an ocean of density $1025 \mathrm{~kg} / \mathrm{m}^{3}$. What fraction of its volume is submerged?
Sol: Suppose the volume of the iceberg is $V_{i}$ and that that of the submerged portion is $\mathrm{V}_{\mathrm{s}}$.
$\rho_{\mathrm{i}} \mathrm{V}_{\mathrm{i}}=\rho_{\mathrm{f}} \mathrm{V}_{\mathrm{s}}$, thus $\mathrm{Vs} / \mathrm{Vi}=920 / 1025=90 \%$

## Example 14.2:

When a 3 kg crown is immersed in water, it has an apparent weight of 26 N . What is the density of the crown?

Solution:
Buoyant force $=3 \times 9.8-26=3.4=1000 \times \mathrm{V} \times \mathrm{g}$
Crown mass $=3 \mathrm{~kg}$
Crown density $=\mathrm{m} / \mathrm{V}=3 / 3.4 \times 1000 \times 9.8=8600 \mathrm{~kg} / \mathrm{m}^{3}$

The metal may be mostly copper.

### 14.5 The Equation of Continuity

The motion of a fluid may be either laminar or turbulent. Laminar flow may be represented by streamlines. For steady flow, streamlines never cross. It is convenient to introduce the concept of a tube of flow.

Turbulent flow commonly observes when the fluid velocity is large or encounters obstacles, generally involving loss of mechanical energy.


## Assumptions for our Present Discussion

1. The fluid is non-viscous: There is no dissipation of energy due to internal friction between adjacent layers in the liquid.
2. The flow is steady: The velocity and pressure at each point are constant in time.
3. The flow is irrotational: A tiny paddle wheel placed in the liquid will not rotate. In rotational flow, for example, in eddies, the fluid has net angular momentum about a given point.

## Equation of Continuity

The velocity of a particle will not be constant along a streamline. The density and the cross-sectional area of a tube of flow will also change. Since fluid does not leave the tube of flow, the mass passing through zone 1 will later move to zone 2.

$$
\text { (Incompressible) } A_{1} v_{1}=A_{2} v_{2}
$$

### 14.6 Bernoulli's Equation

Bernoulli's equation may be derived when the fluid is compressible and nonviscous and the flow is steady and

The net work done on the system
 by $\mathrm{F}_{1}$ and $\mathrm{F}_{2}$ is
$\Delta W=F_{1} \ell_{1}-F_{2} \ell_{2}=P_{1} A_{1} \ell_{1}-P_{2} A_{2} \ell_{2}$ $=\left(P_{1}-P_{2}\right) \Delta V$

The change in potential and kinetic energy are

$$
\begin{aligned}
& \Delta U=\Delta m g\left(y_{2}-y_{1}\right) \\
& \Delta K=\frac{1}{2} \Delta m\left(v_{2}^{2}-v_{1}^{2}\right)
\end{aligned}
$$

### 14.6 Bernoulli's Equation (II)

The changes are bought about by the net work done on the system, $\Delta \mathrm{W}=\Delta \mathrm{U}+\Delta \mathrm{K}$.

$$
\begin{gathered}
\left(P_{1}-P_{2}\right) \Delta V=\Delta m g\left(y_{2}-y_{1}\right)+\frac{1}{2} \Delta m\left(v_{2}^{2}-v_{1}^{2}\right) \\
P_{1}+\rho g y_{1}+\frac{1}{2} \rho v_{1}^{2}=P_{2}+\rho g y_{2}+\frac{1}{2} \rho v_{2}^{2}
\end{gathered}
$$

Bernoulli derived this equation in 1738. It applies to all points along a streamline in a nonviscous, imcompressible fluid. It is a disguised form of the work-energy theorem.

## Example 14.1:

Water emerges from a hole at the bottom of a large tank, as shown below. If the depth of water is $h$, what is the speed at which the water emerges?

Solution:

$$
\begin{aligned}
& P_{1}=P_{2}=P_{o} \\
& v_{1}=0, y_{1}-y_{2}=h \\
& \rho g h=\frac{1}{2} \rho v_{2}^{2}, v_{2}=\sqrt{2 g h}
\end{aligned}
$$



The speed of the emerging fluid is the same as that of a particle that falls freely through the same vertical distance. This rather surprising result is called Torricelli's theorem.

## Exercises and Problems

Ch.14:
Ex. 7, 11
Prob. 2, 3, 4, 7, 8, 9, 10, 11

