Chapter 15 Oscillations

Any motion or event that repeats itself at regular intervals is said to be **periodic**.

Oscillation: In general, an oscillation is a periodic fluctuation in the value of a physical quantity above and below central or equilibrium value.

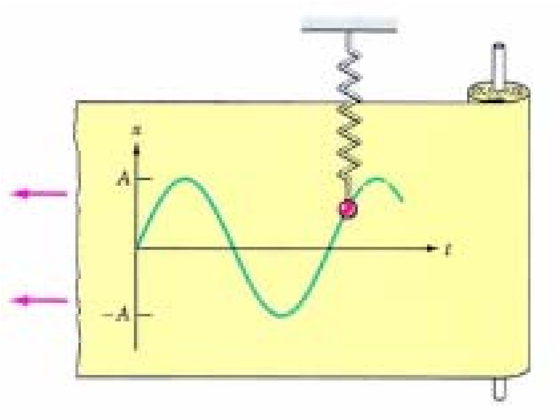
Examples: Mechanical and non-mechanical oscillations.

Galileo probably made the first qualitative observations of oscillations, which led to the property of ***isochronism***.

We confine our attention on **simple harmonic oscillation**.

Damped and **forced** oscillations are treated as supplementary material.

15.1 Simple Harmonic Oscillation



The displacement from equilibrium is given by

$$x(t) = A\sin\omega t$$

where A is the *amplitude* and ω, measured in rad/s, is called the *angular frequency*, rather than angular velocity.

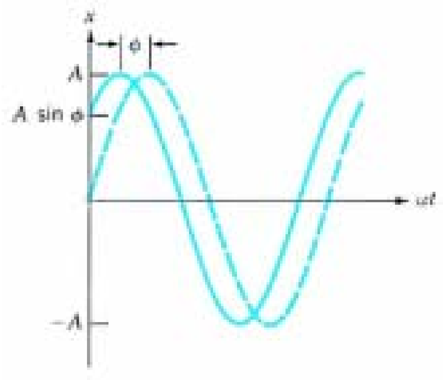
What is the difference between frequency and angular frequency?

Simple Harmonic Oscillation

A complete form of simple harmonic oscillation:

$$x(t) = A\sin(\omega t + \Phi)$$

The argument $\omega t+\phi$ is called the *phase*, while ϕ is called the *phase constant* (or *phase angle*), measured in radians.



$$\omega = \frac{2\pi}{T} = 2\pi f$$

Simple Harmonic Oscillation (III)

$$x(t) = A\sin(\omega t + \Phi)$$

A simple harmonic oscillator has the following characteristics:

1. *Simple*: the amplitude is constant.
2. *Isochronism*: the period is independent of amplitude.
3. *Harmonic*: The time dependence of the fluctuating quantity can be expressed in terms of a sinusoidal function of a single frequency.

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

This differential equation characterizes all types of simple harmonic oscillation.

Example 15.1:

The position of a particle moving along the x-axis is given by x=0.08 sin(12t+0.3) m, where t is in seconds. (a) What are the amplitude and period of the motion? (b) Determine the position, velocity, and acceleration at t=0.6 s.

Solution:

(a) The amplitude A is 0.08 m and the period T is 2π/12=0.52 s.

(b)

$$\begin{cases} x(t) = A\sin(\omega t + \phi) \\ v(t) = A\omega\cos(\omega t + \phi) \\ a(t) = -A\omega^2 \sin(\omega t + \phi) \end{cases}$$

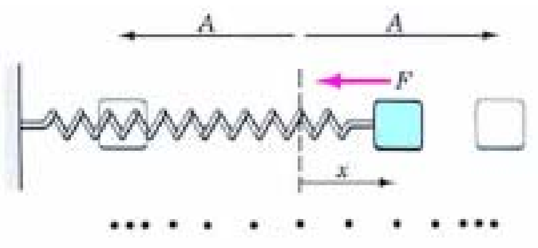
15.2 The Block-Spring System

$$F_{sp} = -kx$$

$$a = \frac{F_{sp}}{m} = -\frac{k}{m}x$$

$$a = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\Rightarrow \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$



This differential equation is merely another way of writing Newton's second law.

$$\omega = \sqrt{\frac{k}{m}}, \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

Example 15.2:

A 2-kg block is attached to a spring for which *k*=200 N/m. It is held at an extension of 5 cm and then release at t=0. Find: (a) the displacement as a function of time; (b) the velocity when x=+A/2; (c) the acceleration when x=+A/2.

Solution:

(a) k=200 N/m, m=2 kg, ω=10 rad/s, T=π/5 s, A= 0.05 m and ϕ=π/2 (obtained from initial condition).

$$x(t) = 0.05\sin(10t + \pi/2)$$

(b) and (c)

$$x(t) = A/2 \Rightarrow \sin(\omega t + \phi) = 1/2, \cos(\omega t + \phi) = \pm 1/\sqrt{2}$$

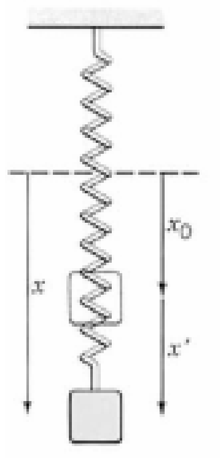
$$v(t) = 0.05 \times 10 \times (\pm 1/\sqrt{2})$$

two velocities, why?

$$a(t) = -0.05 \times 100/2$$

Example 15.4:

Show that a block hanging from a vertical spring, as shwon in Fig. 15.7, executes simple harmonic motion.



Solution:

mg=kx$_o$, F=mg-kx=-k(x-x$_o$)=-kx', where *x'=x-xo is the displacement from the equilibrium position.*

Since the restoring *force* is linearly proportional to the *displacement* from equilibrium, the motion will be simple harmonic,

Gravitation force plays what role in this case?

15.3 Energy in Simple Harmonic Motion

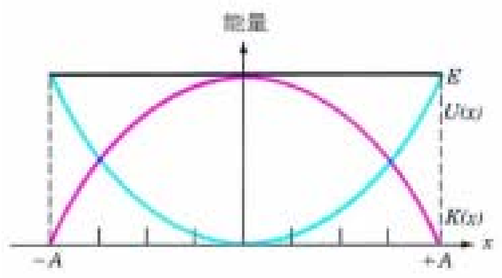
Since the force exerted by an ideal spring is conservative, the energy of the block-spring system is constant.

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2\sin^2(\omega t + \phi)$$
$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\omega^2 A^2\cos^2(\omega t + \phi)$$
$$= \tfrac{1}{2}kA^2\cos^2(\omega t + \phi)$$
$$E = U + K = \tfrac{1}{2}kA^2$$

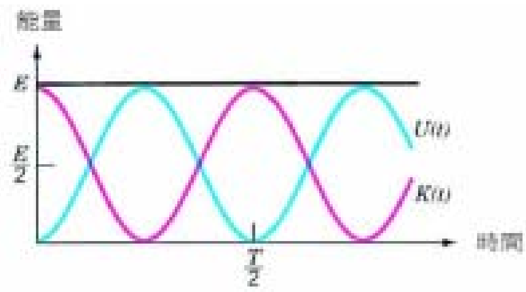
The total energy of any simple harmonic oscillator is constant and proportional to the square of amplitude.

Energy in Simple Harmonic Motion

All SHM is characterized by a parabolic potential well.



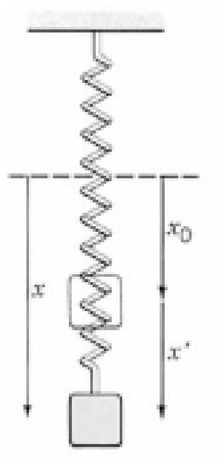
The variation of the kinetic , potential, and total energy as a function of time.



Example 15.4:

vertical block-spring system

Show that a block hanging from a vertical spring, as shwon in Fig. 15.7, executes simple harmonic motion.



Solution:

$$U_{sp} = \tfrac{1}{2}kx^2 = \tfrac{1}{2}k(x_0 + x')^2$$

$$U_g = -mgx = -mg(x_0 + x')$$

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\dot{x}'^2$$

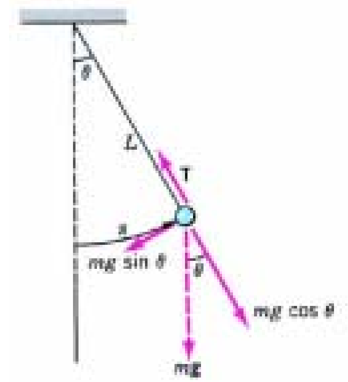
$$\text{since } x_0 = mg/k$$

$$E = U_{sp} + U_g + K$$

$$= \tfrac{1}{2}kx'^2 + \tfrac{1}{2}m\dot{x}'^2$$

15.4 Pendulums

A simple pendulium is an idealized system in which a point mass is suspended at the end of a massless string.



Newton's second law applied along this directions is:

$$-mg\sin\theta = m\frac{d^2 s}{dt^2}$$

Its physical meaning is that the component of the weight acts as a restoring force.

For small angle, sin $\theta \approx \theta$. Substituting this together with s=L θ into above equation to find

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0$$

Pendulums (II)

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0$$

$$\omega = \sqrt{\frac{g}{L}}$$

$$T = 2\pi / \omega = 2\pi\sqrt{\frac{L}{g}}$$

$$\theta = \theta_o \sin(\omega t + \phi)$$

Note here that the angular frequency ω should not be confused with the instantaneous angular velocity d θ /dt.

The Physical Pendulum

The rotational form of Newton's second law,

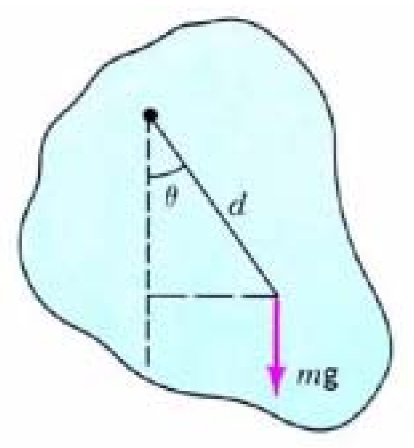
$$-mgd\sin\theta = I\frac{d^2\theta}{dt^2}$$

Small-angle approximation, $\sin\theta \approx \theta$

$$\frac{d^2\theta}{dt^2} + \frac{mgd}{I}\theta = 0$$

$$\omega = \sqrt{\frac{mgd}{I}}$$

$$T = 2\pi / \omega = 2\pi\sqrt{\frac{I}{mgd}}$$



The Torsional Pendulum

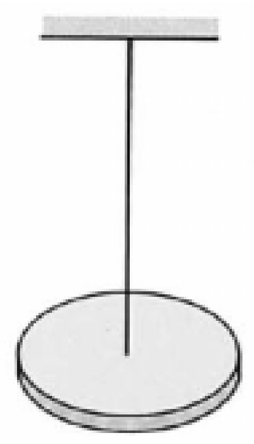
The restoring torque obeys Hooke's law,

$$-\kappa\theta = I\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} + \frac{\kappa}{I}\theta = 0$$

$$\omega = \sqrt{\frac{\kappa}{I}}$$

$$T = 2\pi/\omega = 2\pi\sqrt{\frac{I}{\kappa}}$$



Exercises and Problems

Ch.15:

Ex. 14, 22

Prob. 3, 4, 5, 6, 12, 13, 14