Chapter 16 Mechanical Waves

A **wave** is a disturbance that travels, or propagates, without the transport of matter.

Examples: sound/ultrasonic wave, EM waves, and earthquake wave.

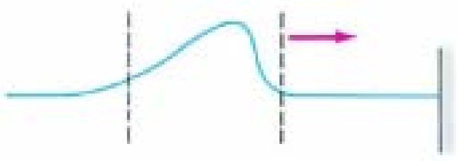
Mechanical waves, such as water waves or sound waves, travel within, or on the surface of, a material with elastic properties.

Electromagnetic waves, such as radio, microwave, and light, can propagate through a vacuum.

Matter waves discovered in elementary particles, can display wavelike behavior.

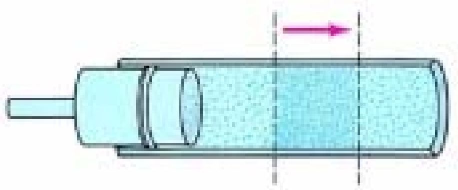
16.1 Wave Characteristics

In a **transverse wave**, shown in Fig. (a), the displacement of a particles is perpendicular to the direction of travel of the wave.



(a)

In a **longitudinal wave**, shown in Fig. (b), the displacement of the particles is along the direction of wave propagation.



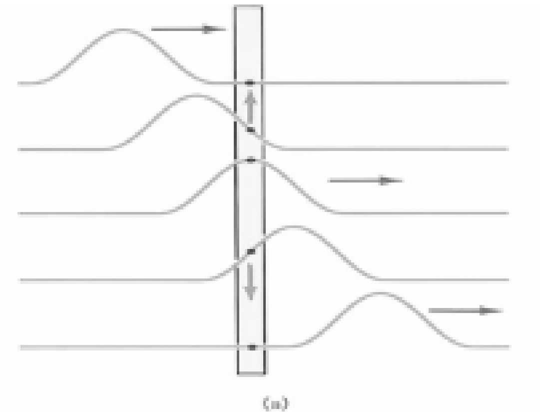
(b)

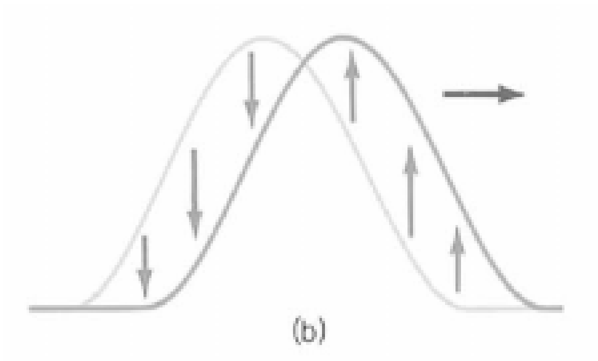
A solid can sustain both kinds of waves, however, an ideal fluid (nonviscous) having no well-defined form can only propagate longitudinal waves.

Seawater, one kind of fluid, can sustain both transverse and longitudinal waves. Why?

Motion of the Medium

The particles of the medium are not carried along with the wave. They undergo small displacements about an equilibrium position, whereas the wave itself can travel a great distance.



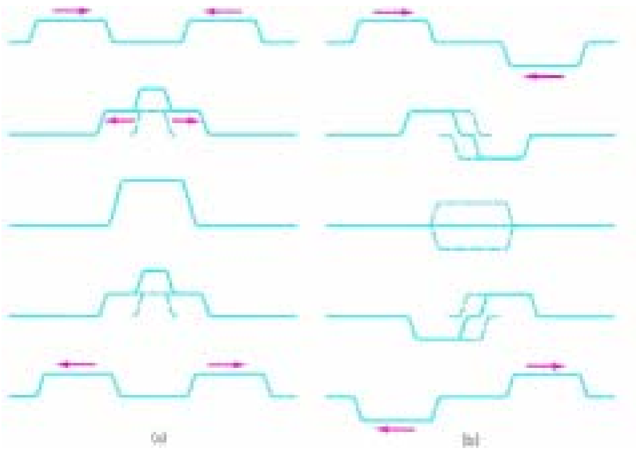


Since the particles of the medium does not travel with the wave, what does? ***Energy and momentum***.

16.2 Superposition of Waves

Principle of linear superposition: The total wave function at any point is the linear sum of the individual wave function; that is,

$$y_T = y_1 + y_2 + y_3 + \cdots + y_N$$



In all our examples we assume that linear superposition is valid.

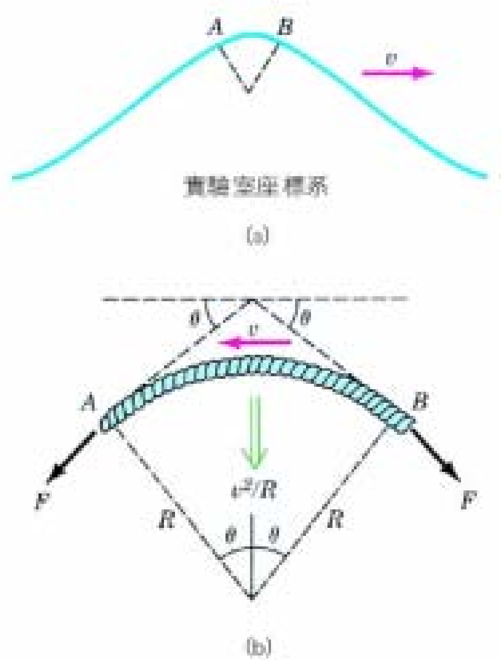
What is the **interference**? *constructive* or *destructive*.

16.3 Speed of a pulse on a string

The speed at which a pulse propagates depends on the properties of the medium.

In the ***pulse*** frame from Newton's second law we have

$$2F\sin\theta = \frac{mv^2}{R}$$

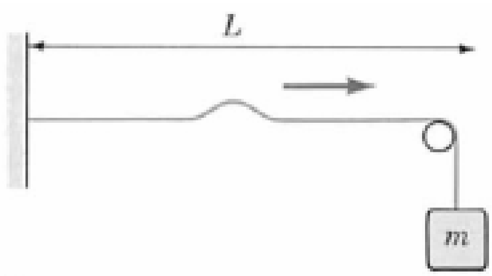


The mass in segment AB is $m=2\mu R\theta$, where μ is the linear mass density*.* In small angle limit, we have

$$v=\sqrt{\frac{F}{\mu}}=\sqrt{\frac{\text{restoring force factor}}{\text{Inertia factor}}}$$

Example 16.1

One end of a string is fixed. It has over a pulley and has a block of mass 2.0 kg attached to the other end. The horizontal part has a length of 1.6 m and a mass of 20 g. What is the speed of a transverse pulse on the string.



Solution:

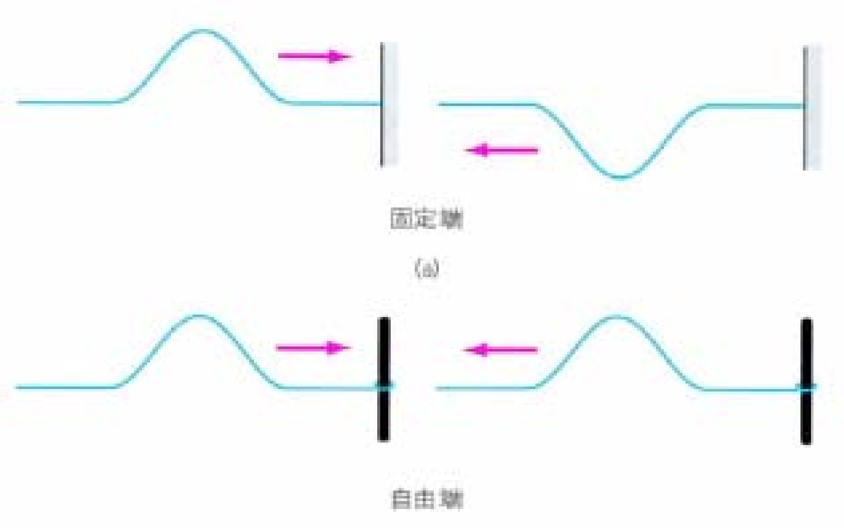
Tension is simply the weight of the block; that is F=19.6 N. The linear mass density is μ=0.0125 kg/m. The speed is

$$v=\sqrt{\frac{F}{\mu}}=\sqrt{\frac{19.6}{0.0125}}=39.6\,\text{m/s}$$

How guitar works? Same concept.

16.4 Reflection and Transmission

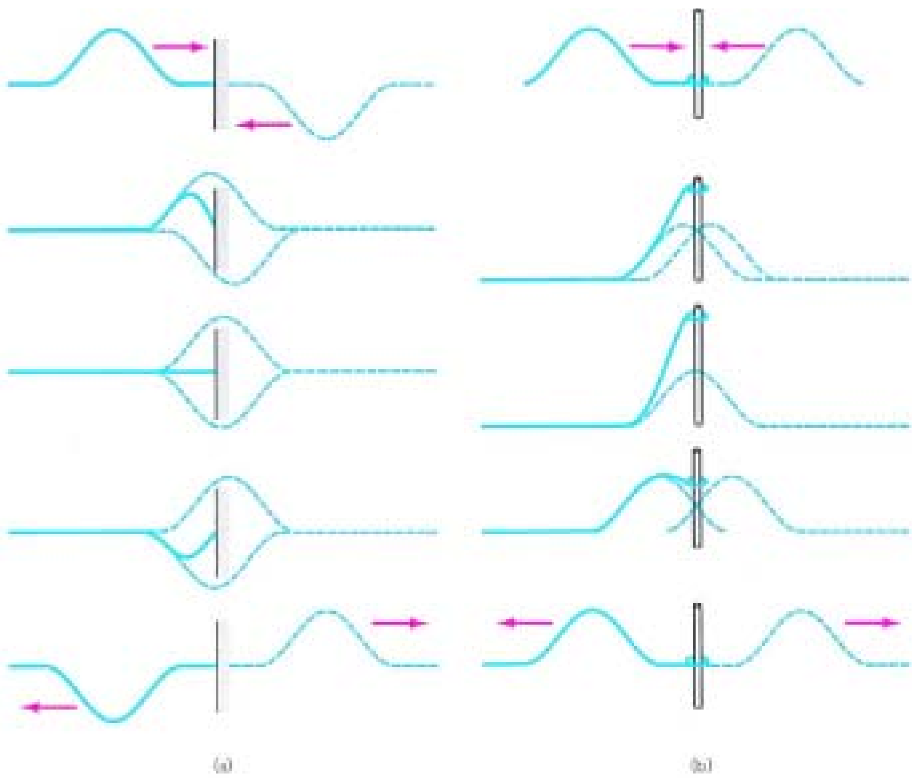
The reflection of a pulse for different end conditions, fixed or free.



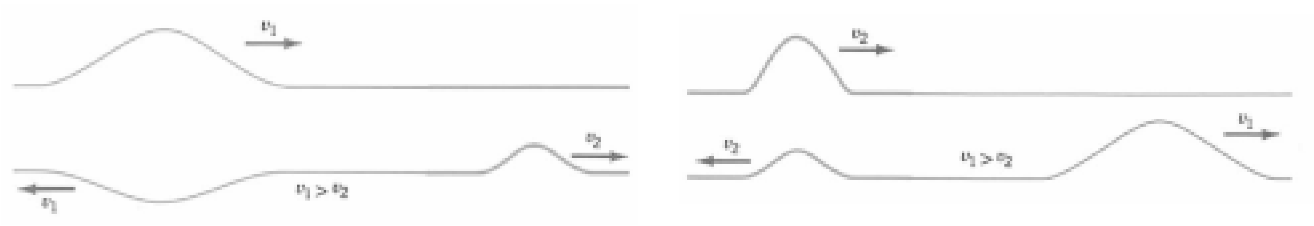
What is the boundary condition of each case?

Difference Perspective:

Superposition of the actual pulse and an imaginary pulse



Between the Extreme Cases of a Fixed End or a Free End



from **light** string to **heavy** string   from **heavy** string to **light** string

This results in **partial** reflection and transmission.

Since the tensions are the same, the relative magnitudes of the wave velocities are determined by **mass densities**.

16.5 Traveling Waves

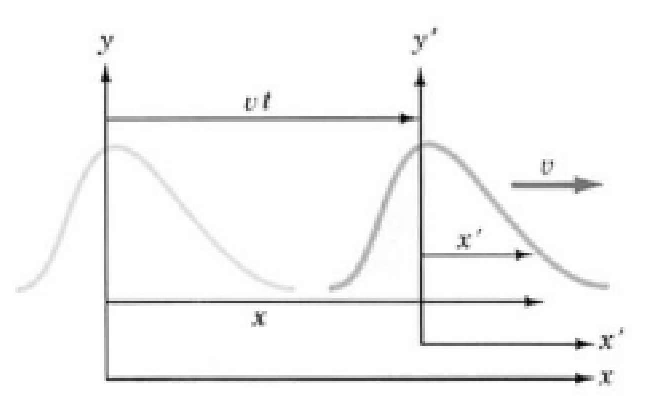
In the *stationary* frame, the pulse has same shape but is moving at a velocity v.

In the *moving* frame the pulse is at rest, so the vertical displacement y' at position x' is given by some function f(x') that describe the shape of the pulse.

$$y'(x') = f(x')$$

$$x' = x - vt$$

$$y(x,t) = f(x - vt)$$



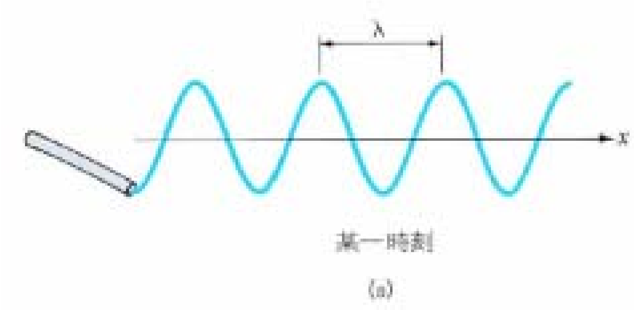
$$y(x,t) = f(x \pm vt)$$

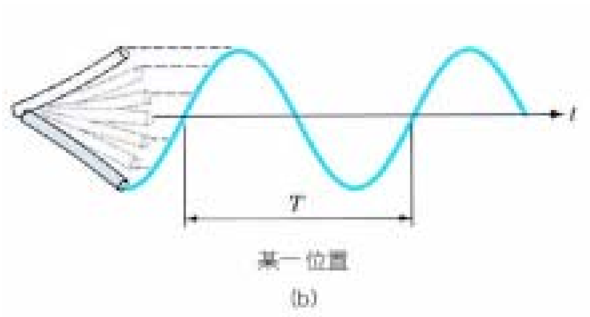
Plus or minus sign represents forward or backward propagation.

16.6 Traveling Harmonic Waves

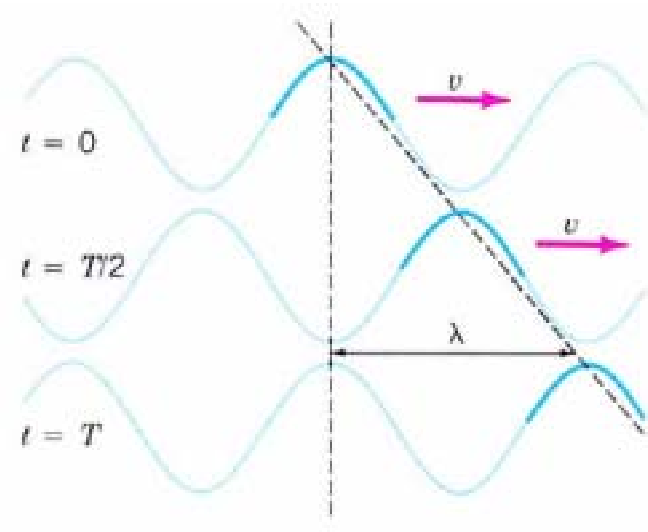
If the source of the waves is a simple harmonic oscillator, the function $f(x \pm vt)$ is sinusoidal and it represents a **traveling harmonic wave**.

The study of harmonic wave is *particularly important* because a disturbance of any shape may be formed *by adding together suitable harmonic components* of different frequencies and amplitudes.





Traveling Harmonic Waves (II)



$$y(x,t) = A\sin(kx \pm \omega t + \phi)$$

A : amplitude

$k = 2\pi/\lambda$: wave number

$\omega = 2\pi/T$: angular frequency

$v = \lambda/T = \omega/k = \lambda f$: velocity

T : period

λ : wavelength

f : frequency

Example 16.3:

The equation of a wave is

$$y(x,t) = 0.05\sin\left[\tfrac{\pi}{2}(10x - 40t) - \tfrac{\pi}{4}\right]\text{m}$$

Find: (a) the wavelength, the frequency, and the wave velocity;
(b) the particle velocity and acceleration at x=0.5 m and t=0.05 s.

Solution:

(a) *A=0.05, k=5π, ω=20π, ϕ=-π/4, λ=2/5, f=10, and T=0.1*

(b)

$$\frac{dy}{dt} = -\pi\cos\left[\tfrac{\pi}{2}(10x - 40t) - \tfrac{\pi}{4}\right] = 2.22\ \text{m/s}$$

$$\frac{d^2y}{dt^2} = -20\pi^2\sin\left[\tfrac{\pi}{2}(10x - 40t) - \tfrac{\pi}{4}\right] = 140\ \text{m/s}^2$$

16.7 Standing Waves

Two harmonic waves of equal frequency and amplitude traveling through a medium in opposite directions form a **standing wave pattern**.

$$y_1(x,t) = A\sin(kx + \omega t + \phi)$$

$$y_2(x,t) = A\sin(kx - \omega t + \phi)$$

$$y = y_1 + y_2 = A\sin(kx + \phi)\cdot\cos(\omega t)$$

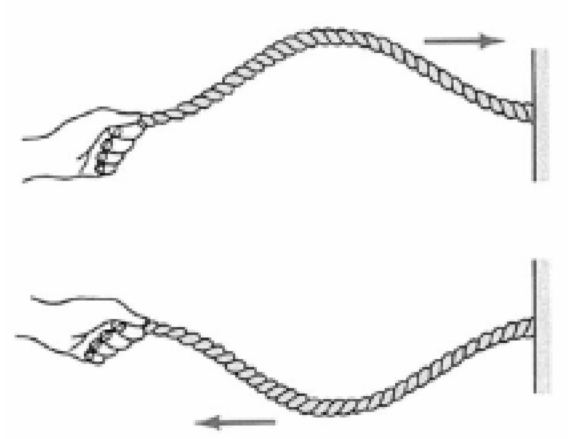
Nodes: the points of permanent zero displacement.

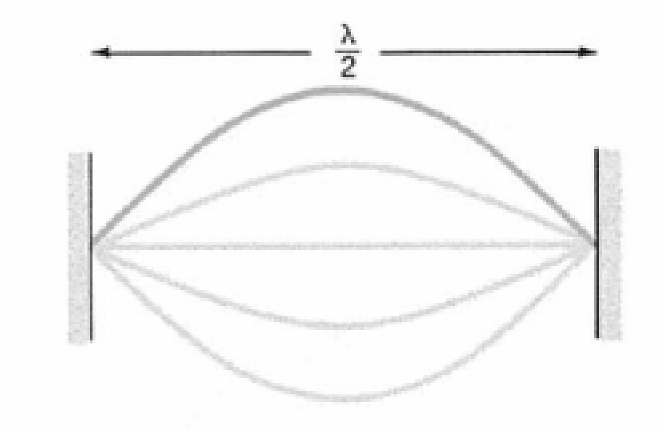
Antinodes: the points of maximum displacement.

16.8 Resonant Standing Waves on a String

How to generate an opposite propagating wave with equal amplitude and frequency? **Reflection**.

Why the resonant frequency is discrete? **Boundary condition**.

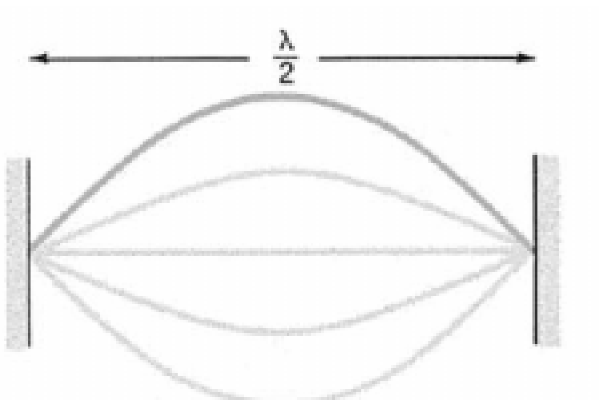




Resonant Standing Waves (II)

What is the minimum frequency allowable on the string?

If the propagating velocity is the same, then the minimum frequency corresponds to the longest wavelength. To satisfy the boundary condition, the maximum wavelength is $\lambda=2L$.

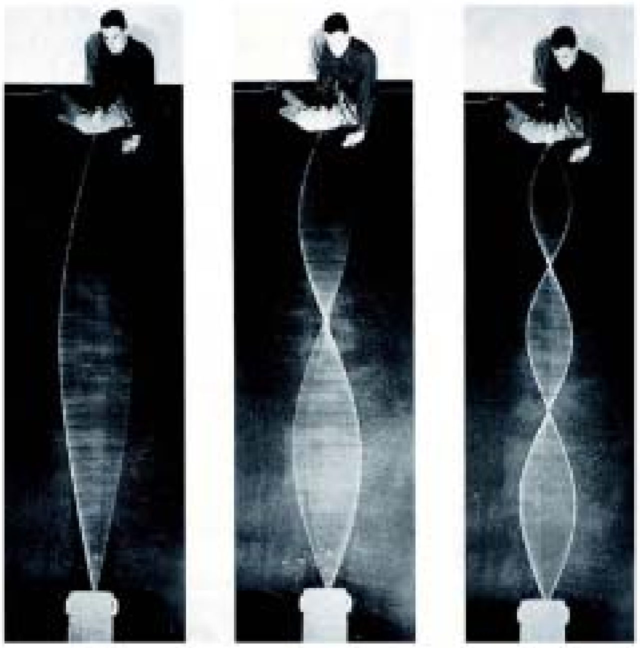


Fundamental frequency (first harmonic)

$$\lambda_1 = 2L$$

$$f_1 = v/\lambda_1 = v/2L$$

Resonant Standing Waves: (III)
Normal modes



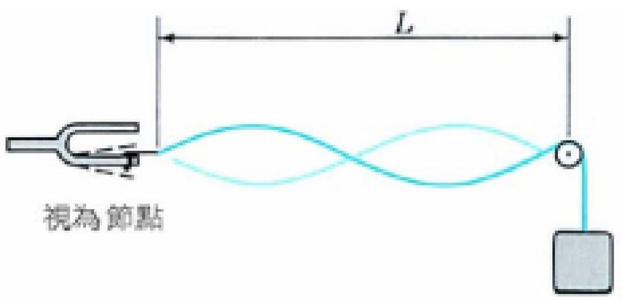
Normal modes

$$\lambda_n = \frac{2L}{n}$$

$$f_n = \frac{nV}{2L}$$

Resonant Standing Waves: (IV)
Resonant frequencies

A simple way to set up a resonant standing is shown below. One prong of a tuning fork is attached to one end of the string. The string hangs over a pulley and a weight determine the tension in it.

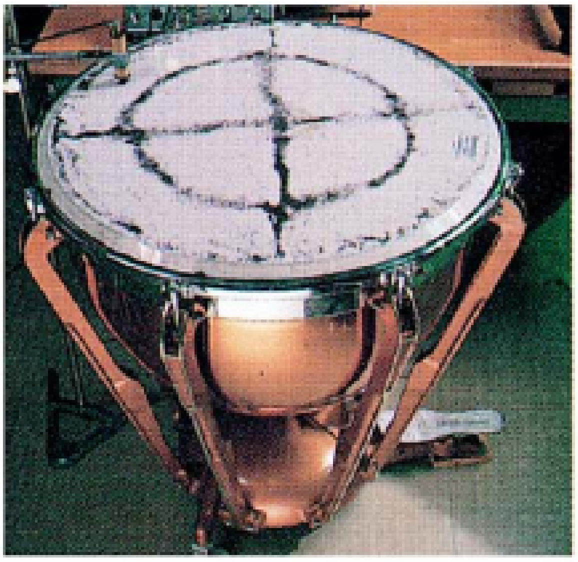


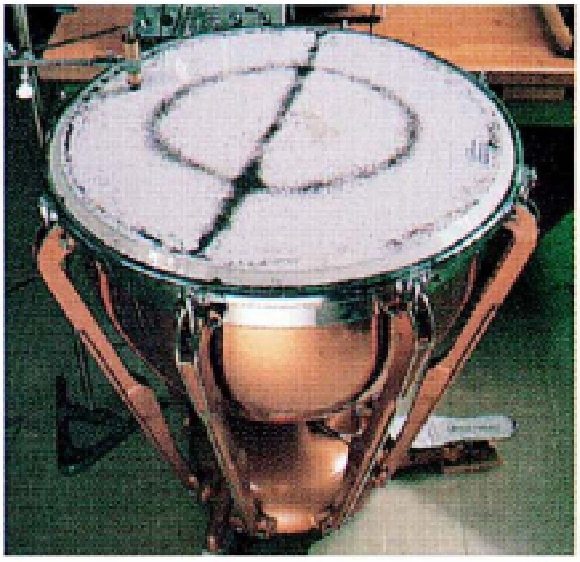
$$f_n = \frac{n}{2L} v = \frac{n}{2L}\sqrt{\frac{F}{\mu}}$$

Example: guitar

Question: Which mode will dominate?

Resonant Standing Waves: (V) 2D normal modes





16.9 The Wave Equation

One can obtain the wave equation by taking the partial derivatives of the wave function for the traveling harmonic wave.

$$y(x,t) = y_o \sin(kx - \omega t + \phi)$$

with respect to t and x:

$$\frac{d^2 y}{dx^2} = -k^2 y_o \sin(kx - \omega t + \phi)$$

$$\frac{d^2 y}{dt^2} = -\omega^2 y_o \sin(kx - \omega t + \phi)$$

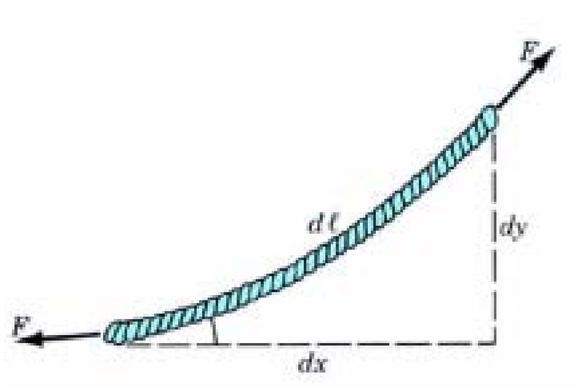
By comparing these derivatives, we see that

$$\frac{d^2 y}{dx^2} = \frac{1}{v^2}\frac{d^2 y}{dt^2}$$

16.10 Energy Transport on a String

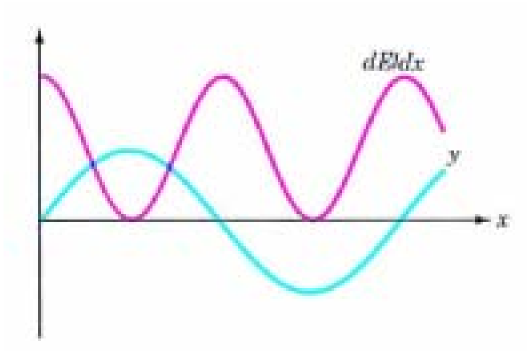
Linear energy density (J/m):

$$\frac{dE}{dx} = \frac{dK}{dx} + \frac{dU}{dx}$$
$$= \mu(\omega y_o)^2 \cos^2(kx - \omega t)$$



Average power (J/s):

$$P_{av} = \frac{dE}{dt}$$
$$= \frac{dE}{dx}\frac{dx}{dt} = \frac{1}{2}\mu(\omega y_o)^2 v$$



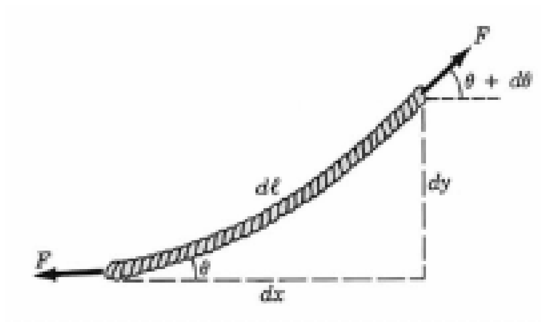
16.11 Velocity of Waves on a String

From Newton's second law we have

$$F[\sin(\theta + \Delta\theta) - \sin(\theta)] = (\mu \Delta x)\frac{\partial^2 y}{\partial t^2}$$

Small angle approximation:

$$\sin\theta \approx \tan\theta = \frac{\partial y}{\partial x}$$
$$\frac{\partial^2 y}{\partial x^2} = (\mu / F)\frac{\partial^2 y}{\partial t^2}$$



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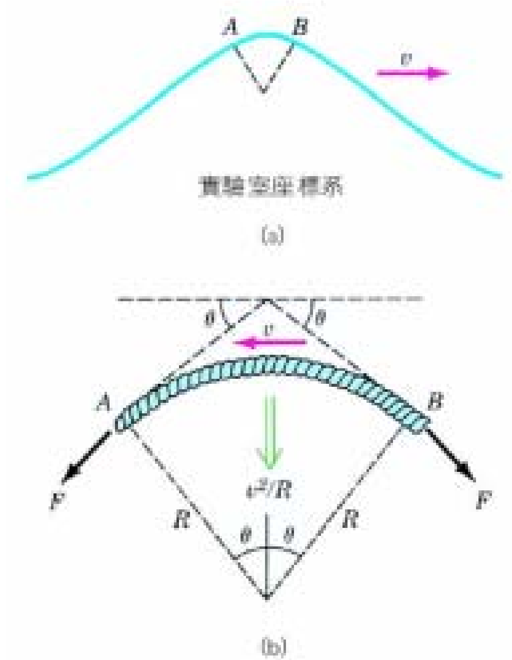

Exercises and Problems

Ch.16:

Ex. 10, 11, 19, 21, 35, 39

Prob. 6, 9, 10, 11