Chapter 23 The Electric Field

Coulomb's law, like Newton's law of gravitation, involves the concept of action at a distance.

It simply states how the particles interact but provides no explanation of the mechanism by which the force is transmitted from one particle to the other,

Even Newton himself is not comfortable with this aspect of his theory.

What is the concept of action at a distance? This leads to the gravitational, electric, and magnetic field.

23.1 The Electric Field

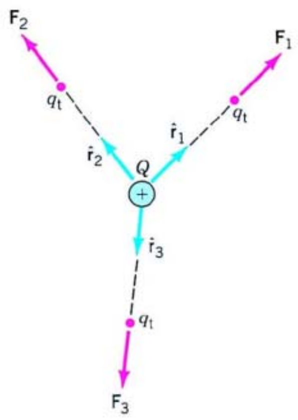
How does one particle sense the presence of the other?

The electric charge creates an electric field in the space around it. A second charged particle does not interact directly with the first; rather, it responds to whatever field it encounters. In this sense, the field acts as an *intermediary* between the particles.

The electric field strength is defined as the force per unit charge placed at that point:

$$\mathbf{E} = \frac{\mathbf{F}}{q_t} = \frac{kq_tQ}{q_t\, r^2}\hat{\mathbf{r}}$$

$$= \frac{kQ}{r^2}\hat{\mathbf{r}}$$



Example 23.1

On a clear day there is an electric field of approximately 100 N/C directed vertically down at the earth's surface. Compare the electrical and gravitational forces on an electron.

Solution:

The magnitude of the electrical force is

$F_e = eE = 1.6 \times 10^{-19} \times 100 = 1.6 \times 10^{-17}$ N. (upward)

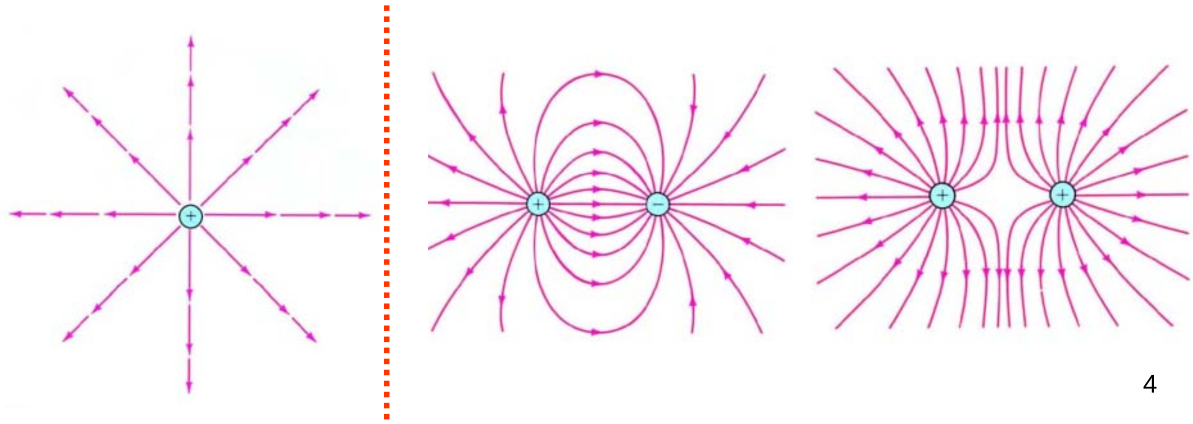
The magnitude of the gravitational force is

$F_g = mg = 9.11 \times 10^{-31} \times 9.8 = 8.9 \times 10^{-30}$ N. (downward)

22.2 Lines of Force

How to express the magnitude and vector properties of the field strength?

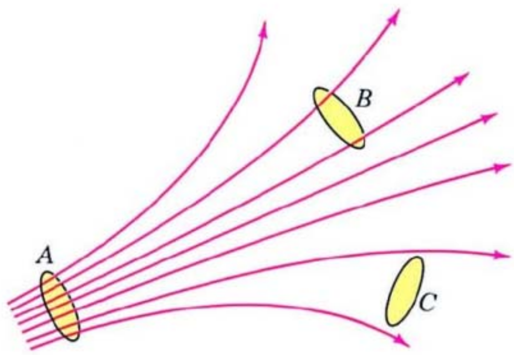
The field strength at any point could be represented by an arrow drawn to scale. However, when several charges are present, the use of arrows of varying length and orientations becomes confusing. Instead we represent the electric field by continuous field lines or lines of force.

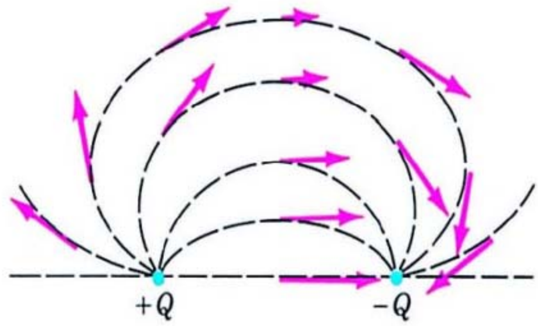


22.2 Lines of Force (II)

How to determine the field strength from the field lines?

The lines are crowed together when the field is strong and spread apart where the field is weaker. The field strength is proportional to the density of the lines.





Example 23.3

Sketch the field lines for two point charges 2Q and –Q.

Solution:

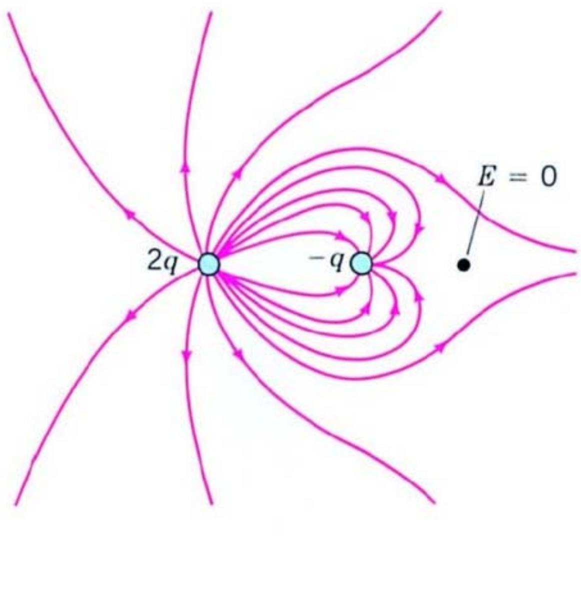
(a)Symmetry

(b)Near field

(c)Far field

(d)Null point

(e)Number of lines

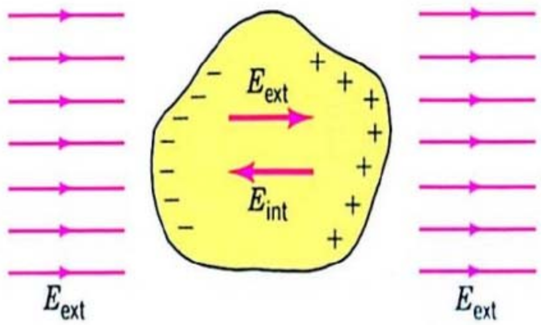


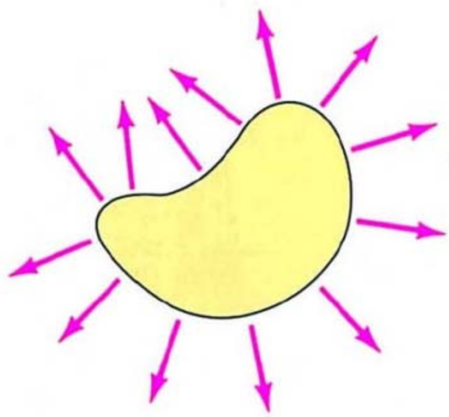
23.3 Electric Field and Conductors

What is the electric field inside a conductor with free electrons?

Under static conditions, the net *macroscopic* field within the material of a *homogeneous* conductor is zero.

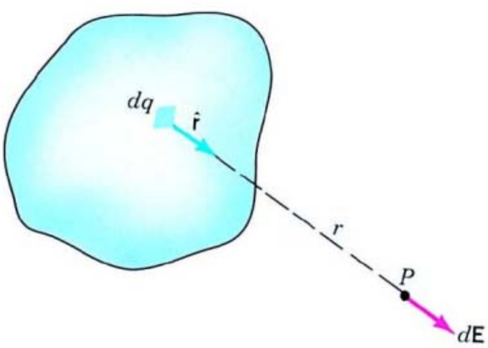
Under static conditions, the electric field at all points on the surface of a conductor is *normal* to the surface.





23.5 Continuous Charge Distribution

In order to find the electric field due to a continuous distribution of charge, one must divide the charge distribution into infinitesimal elements of charge dq which may be considered to be point charges.



$$d\mathbf{E} = \frac{kdq}{r^2}\hat{\mathbf{r}}$$

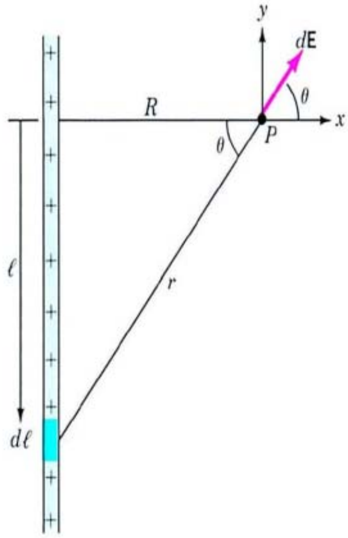
$$\mathbf{E} = k\int \frac{dq}{r^2}\hat{\mathbf{r}}$$

Example 23.7

What is the field strength at a distance R from an infinite line of charge with linear charge density λ C/m.

Solution:

Since the charge carrier is infinite long, the electric field in y-direction completely cancel out. Thus the resultant field is along the x-axis.



$$dE_x = \frac{k\lambda \cos\theta d\ell}{r^2} = \frac{k\lambda R \sec^2\theta \cos\theta \, d\theta}{(R\sec\theta)^2}$$

$$= \frac{k\lambda \cos\theta d\theta}{R}$$

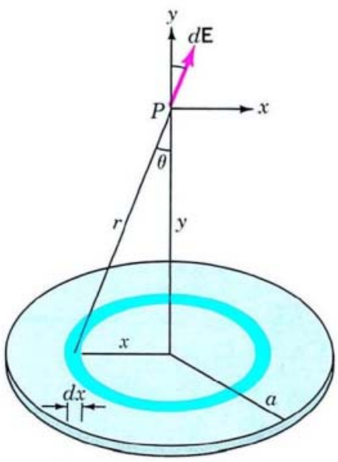
$$E_x = \frac{k\lambda}{R}\int_{-\pi/2}^{\pi/2} \cos\theta d\theta = \frac{2k\lambda}{R}$$

Example 23.8

Non-conducting disk of radius a has a uniform surface charge density σ C/m². What is the field strength at a distance y from the center *along the central axis*.

Solution:

The y-component of the field is



$$dE_y = dE\cos\theta = \frac{kdq}{r^2}\frac{y}{r}$$

$$\text{where } r^2 = \text{x}^2 + \text{y}^2 \text{ and } dq = \sigma(2\pi x dx)$$

$$E_y = \pi k \sigma y \int_0^a \frac{2x dx}{(x^2 + y^2)^{3/2}}$$

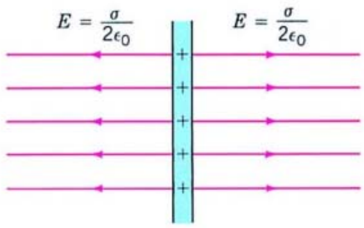
$$= \pi k \sigma y \left[\frac{-2}{(x^2+y^2)^{1/2}} \right]_0^a$$

Example 23.9

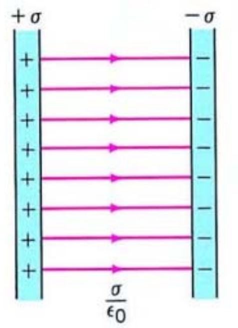
Find the field due to the following: (a) an infinite sheet of charge with surface charge density $+\sigma$; (b) two parallel infinite sheets with charges density $+\sigma$ and $-\sigma$.

Solution:

Using the result of Ex.23.8 with the limit as a → ∞, we obtain.



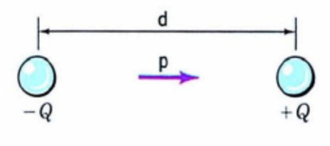
$$E = \pi k\sigma y\left[\frac{-2}{(x^2+y^2)^{1/2}}\right]_0^a = 2\pi k\sigma = \frac{\sigma}{2\varepsilon_o}$$



Is there any easier method to obtain these results? Yes, Gauss's law to be discussed in next chapter.

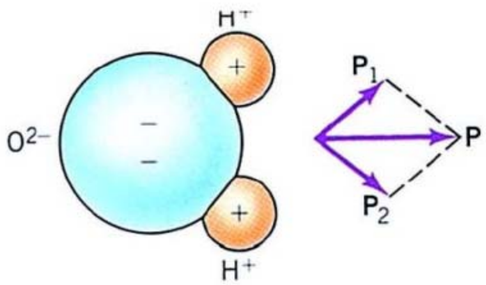
23.6 Dipoles

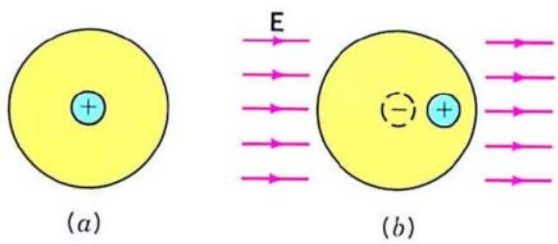
What is dipole? The arrangement of a pair of equal and opposite charges separated by some distance is called an electric dipole.



Permanent dipole: such as molecules of HCl, CO, and H_2O.

Induced dipole: An electric field may also induce a charge separation in an atom or a nonpolar molecule.

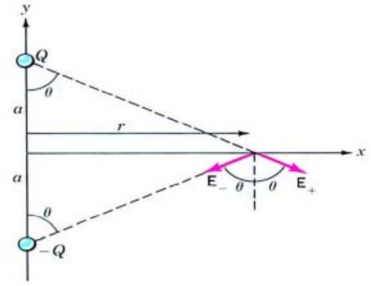




Some *Important* Properties of Dipole

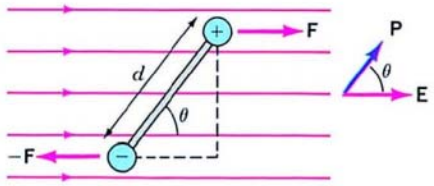
Field due to a dipole:

$$\mathbf{p} = Q\mathbf{d} \; (- \to +)$$



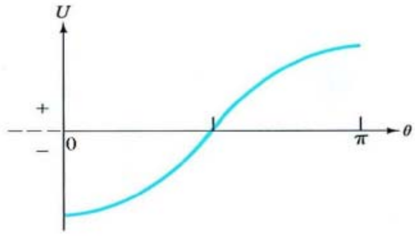
Torque in a uniform field:

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$$



Potential energy:

$$U = -\mathbf{p} \cdot \mathbf{E}$$



Exercises and Problems

Ch.23:

Ex. 35, 49

Prob. 4, 7, 12, 20