Chapter 24 Gauss's Law

In principle, the electrostatic field due to a continuous charge distribution can always be found by using Coulomb's law, but the integration required may be ***complex***.

In this chapter we present an ***alternative approach***, base on the concept of ***lines of force***, which, in some cases, can be much ***simpler***.

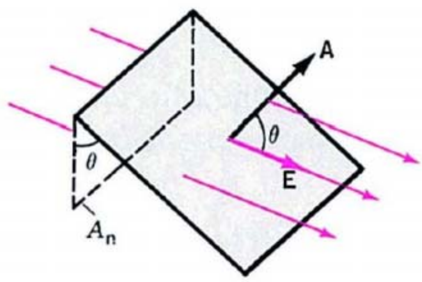
Gauss built on the picture of lines "flowing" through a closed surface by introducing a quantity called ***flux*** and related it to the ***net*** charge enclosed by the surface.

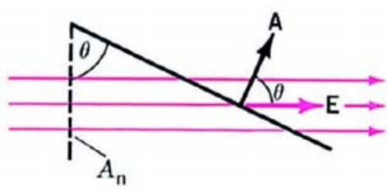
Gauss's law is a general statement about the properties of electric fields; it is ***not restricted*** to electrostatic fields as in Coulomb's law.

24.1 Electric Flux

The electric flux Φ_E through this surface is defined as

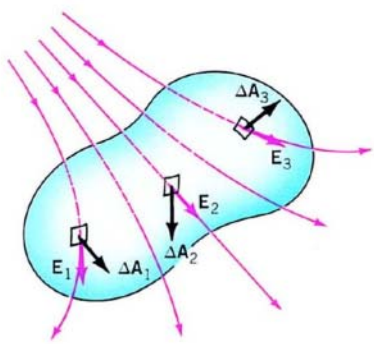
$$\Phi_E = EA\cos\theta$$
$$= \mathbf{E}\cdot\mathbf{A}$$





For a nonuniform electric field

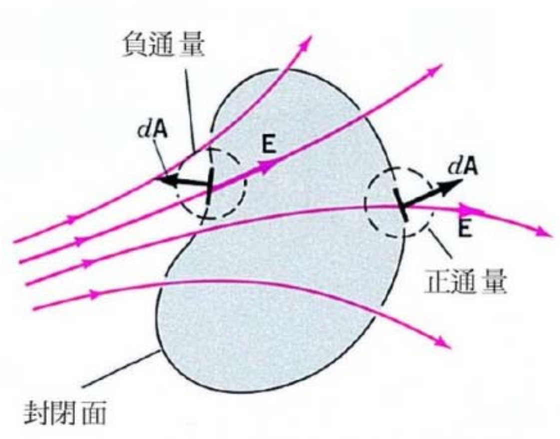
$$\Phi_E = \int \mathbf{E}\cdot\hat{\mathbf{n}}\,da$$



24.1 Electric Flux (II)

Flux leaving a closed surface is positive, whereas flux entering a closed surface is negative.

The net flux through the surface is zero since the number of lines that enter the surface is equal to the number that leave.



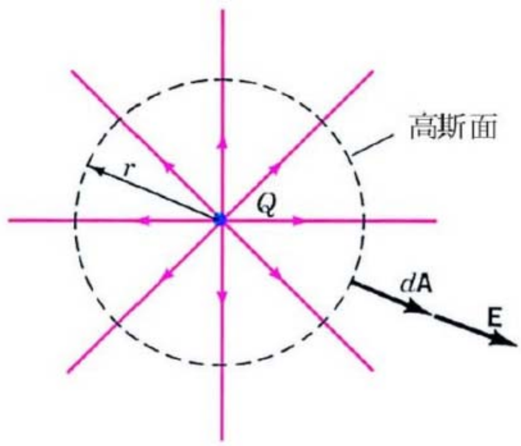
24.2 Gauss's Law

How much is the flux for a spherical Gaussian surface around a point charge?

The total flux through this closed Gaussian surface is

$$\Phi_E = \oint \mathbf{E} \cdot \hat{\mathbf{n}} da = \frac{kQ}{r^2} \cdot 4\pi r^2$$

$$= 4\pi KQ = \frac{Q}{\varepsilon_0}$$



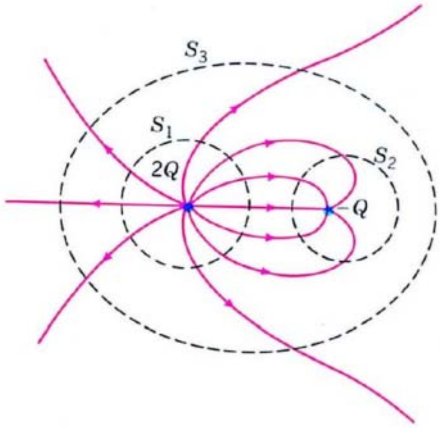
The net flux through a closed surface equals $1/\varepsilon_0$ times the net charge enclosed by the surface.

Can we prove the above statement for arbitrary closed shape?

24.2 Gauss's Law (II)

•The circle on the integral sign indicates that the Gaussian surface must be enclosed.

•The flux through a surface is determined by the ***net*** charge enclosed.



How to apply Gauss's law?

1. Use symmetry.
2. Properly choose a Gaussian surface (E//A or E⊥A).

Example 24.2

A *non-conducting* uniform charged sphere of radius *R* has a total charge *Q* uniformly distributed throughout its volume. Find the field (a) inside, and (b) outside, the sphere.

Solution:

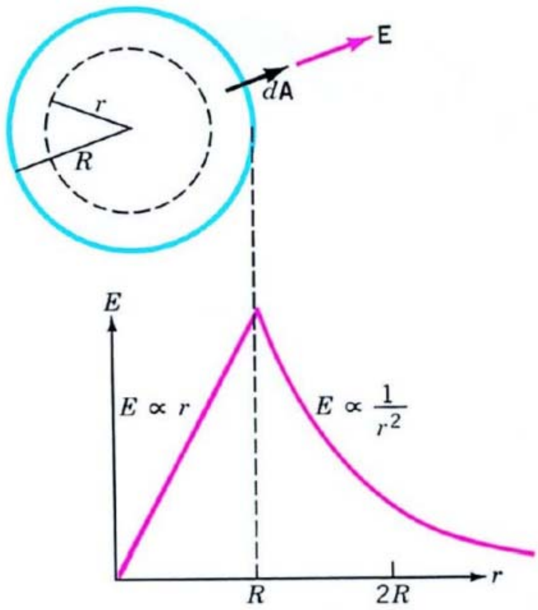
(a) inside

$$E = \frac{\Phi_{in}}{4\pi r^2} = \left(\frac{Q\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}\right)\frac{1}{4\pi\varepsilon_0 r^2}$$

$$= \frac{Q}{4\pi\varepsilon_0 R^3} r = \frac{kQ}{R^3} r$$

(b) outside

$$E = \frac{\Phi}{4\pi r^2} = \frac{kQ}{r^2}$$



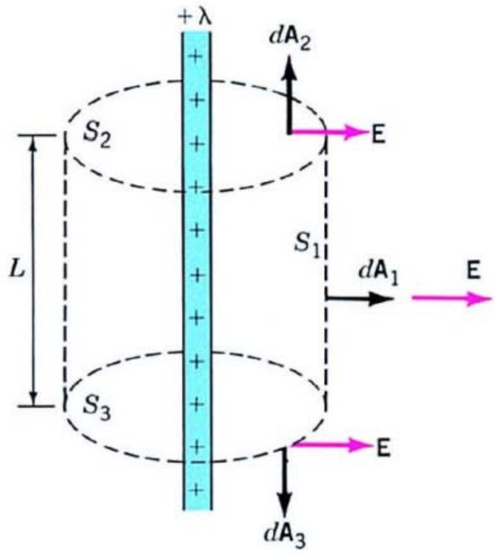
Example 24.3

An *infinite line of charge* has a linear charge density λ C/m. Find the electric field at a distance r from the line.

Solution:

$$E = \frac{\Phi_{enclosed}}{2\pi rL} = \frac{\lambda L}{2\pi\varepsilon_0 rL}$$

$$= \frac{2k\lambda}{r}$$



Choosing an appropriate Gaussian surface is the key.

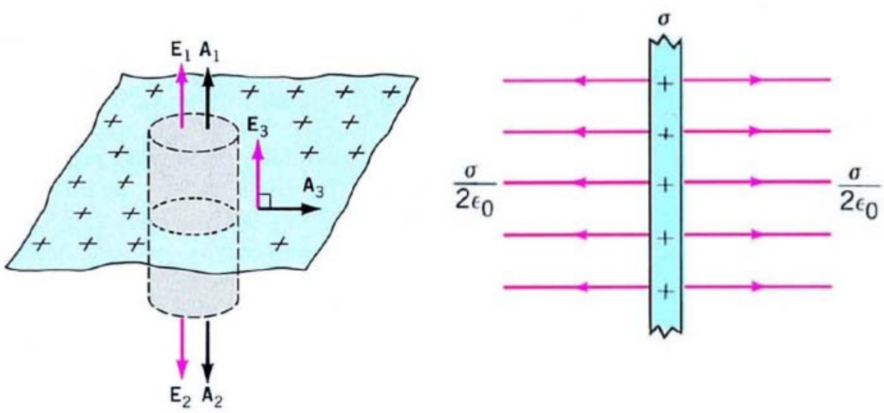
Example 24.4

Find the field due to an infinite flat sheet of charge with a uniform surface charge density σ C/m^2.

Solution:

$$2EA = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

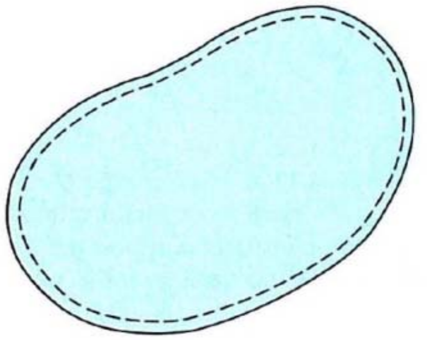


24.3 Conductors

Is the electric field inside a conductor zero or non-zero?

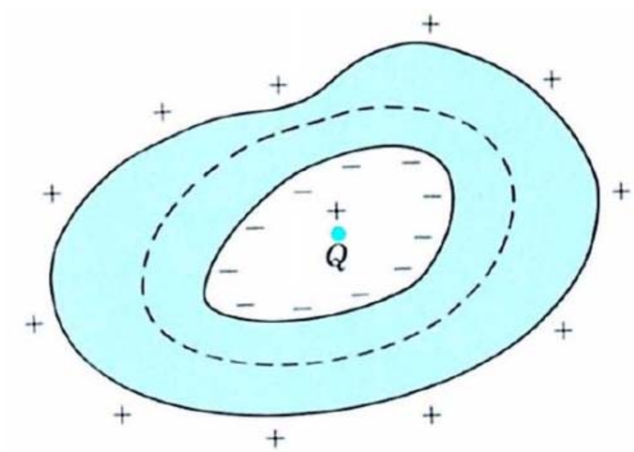
An electric field can be set up temporarily in the body of the conductor. The free electrons will redistribute themselves and within a tiny fraction of a second (~ps) the internal field will vanish.

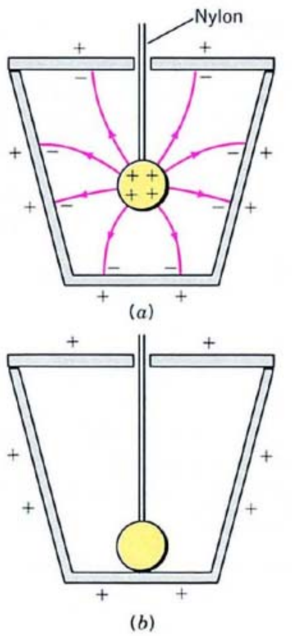
We conclude that any net charge on a conductor must reside on its surface.



24.3 Conductors : Cavity in a Conductor

A point charge Q within a cavity in a conductor induces equal and opposite charges on the surface of the cavity and on the surface of the conductor.





Exercises and Problems

Ch.24:

Ex. 8, 16, 24

Prob. 2, 4, 6, 9, 11, 14