

Chapter 25 Electric Potential

Can we apply the concept of **potential**, first introduced in mechanics, to electrostatic system and find the law of **conservation** of energy?

We can define an electrostatic potential energy, analogous to gravitational potential energy, and apply the law of conservation of energy in the analysis of electrical problems.

Potential is a property of a point in space and depends only on the source charges.

It is often easier to analyze a physical situation in terms of **potential**, which is a **scalar**, rather than the **electric field strength**, which is a **vector**.

Can you tell the difference between potential and potential energy?

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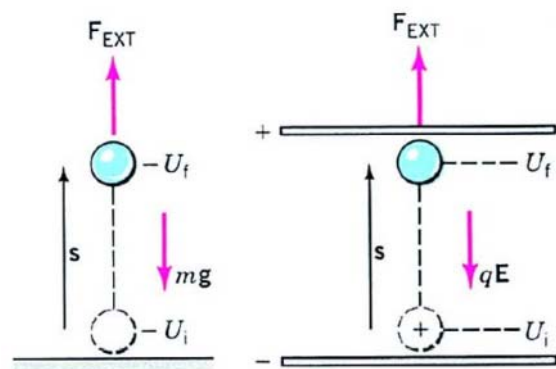
25.1 Potential

The motion of a particle with positive charge q in a uniform electric field is analogous to the motion of a particle of mass m in uniform gravitational field near the earth.

$$W_{EXT} = +\Delta U = U_f - U_i$$

If $W_{EXT} > 0$, work is done by the external agent on the charges.

If $W_{EXT} < 0$, work is done on the external agent by the field.



Potential energy depends not only on the “source” but also on the “test” particle. Thus it will be more convenient if we can define a potential function which is function of “source” only.

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25.1 Potential (II)

When a charge q moves between two points in the electrostatic field, the change in **electric potential**, ΔV , is defined as the change in electrostatic potential energy per unit charge,

$$\Delta V = \frac{\Delta U}{q}$$

The SI unit of electric potential is the volt (V).

$$1V = 1J / C = 1N \cdot m / C$$

The quantity ΔV depends only on the field set up by the source charges, not on the test charge.

$$W_{EXT} = q\Delta V = q(V_f - V_i)$$

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25.1 Potential (III)

We see that only **changes** in potential ΔV , rather than the specific value of V_i and V_f , are significant.

It is convenient to choose the ground connection to earth as the zero of potential.

The potential at a point is the external work need to bring a positive unit charge, at constant speed, from the position of zero potential to the given point.

In an external electric field, both positive and negative charges tend to **decrease** the electrostatic potential energy.

Which side will a charge particle drift if it is in the middle of two conducting plates with potential difference, higher or lower potential side? See next viewgraph.

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25.1 Potential (IV)

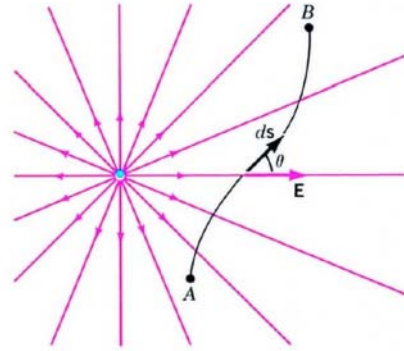
In mechanics, the definition of potential energy in terms of the work done by the conservative force is $\Delta U = -W_c$. The negative sign tells us that positive work by the conservative force leads to a decrease in potential energy.

Therefore, the change in potential energy, associated with an infinitesimal displacement $d\mathbf{s}$, is

$$dU = -\mathbf{F}_c \cdot d\mathbf{s} = -q\mathbf{E} \cdot d\mathbf{s}$$

$$dV = \frac{dU}{q} = -\mathbf{E} \cdot d\mathbf{s}$$

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

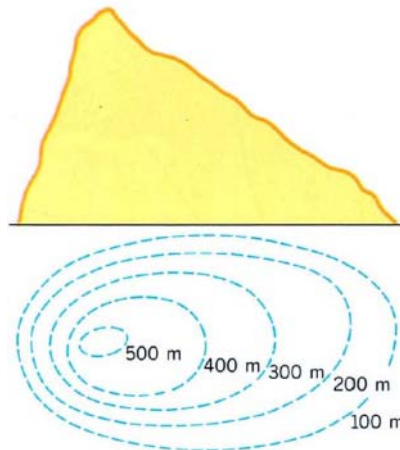
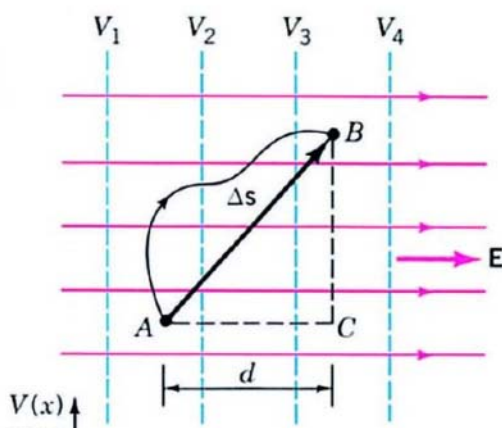


Since the electrostatic field is conservative, the value of this line integral depends only on the end points A and B, not on the path taken.

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25.2 Potential and Potential Energy in a Uniform Field

In a uniform field, the change in potential in moving from A to B is $\Delta V = -\mathbf{E} \cdot d\mathbf{s}$. In a uniform field the potential **decreases** linearly with distance along the field lines.



The electric field lines are perpendicular to the equipotentials and point from higher to lower potentials, that is, “downhill”.

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25.2 Potential and Potential Energy: Motion of Charges

The motion of a charge in an electric field may be discussed in terms of the conservation of energy, $\Delta K + \Delta U = 0$. In terms of potential, the conservation law may be written as

$$\Delta K = -q\Delta V$$

It is convenient to measure the energy of elementary particles, such as electrons and protons, in terms of a non-SI unit called the electronvolt ($1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$).

According to Einstein famous $E=mc^2$, find the energy in terms of eV for an electron of rest mass $9.1 \times 10^{-31} \text{ kg}$, where the speed of light is $3 \times 10^8 \text{ m/s}$.

$$E = 9.1 \times 10^{-31} \times (3 \times 10^8)^2 / 1.6 \times 10^{-19} = 0.511 \text{ MeV}$$

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Example 25.1

A proton, of mass $1.67 \times 10^{-27} \text{ kg}$, enters the region between parallel plates a distance 20 cm apart. There is a uniform electric field of $3 \times 10^5 \text{ V/m}$ between the plates, as shown in Fig. 25.6. If the initial speed of the proton is $5 \times 10^6 \text{ m/s}$, what is its final speed?

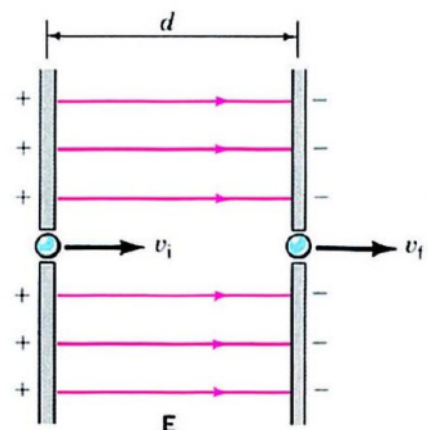
Solution:

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -qV$$

$$v_f = \sqrt{v_i^2 + 2qV/m}$$

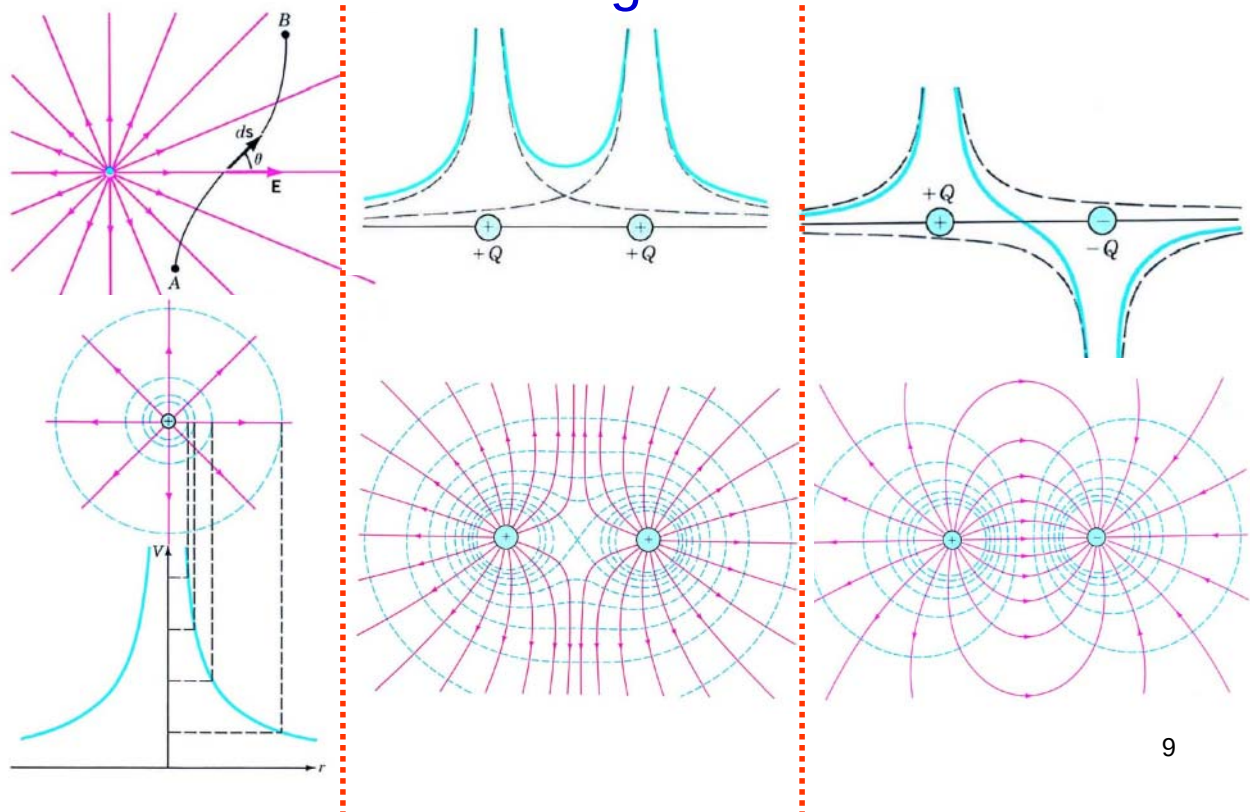
$$= ((5 \times 10^6)^2 - 2 \times 1.6 \times 10^{-19} \times -6 \times 10^4 / 1.67 \times 10^{-27})^{0.5}$$

$$= 6 \times 10^6 \text{ m/s.}$$



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25.3 Potential and Potential Energy of Point Charges



Example 25.2

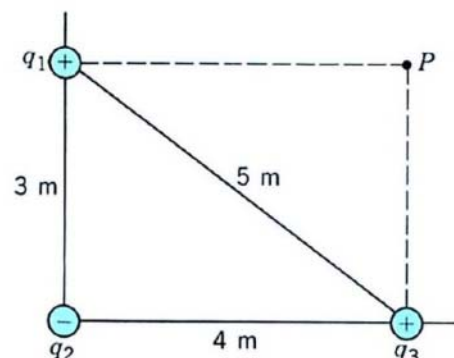
Three point charge, $q_1=1\mu\text{C}$, $q_2=-2\mu\text{C}$, and $q_3=3\mu\text{C}$ are fixed at the positions shown in Fig.25.13a. (a) What is the potential at point P at the corner of the rectangle? (b) What is the total potential energy of q_1 , q_2 , and q_3 ? (c) How much work would be needed to bring a charge $q_4=2.5\mu\text{C}$ from infinity and to place it at P?

Solution:

$$(a) V_p = V_1 + V_2 + V_3 = 7.65 \times 10^3 \text{ V}$$

$$(b) U = U_{12} + U_{13} + U_{23} = -1.41 \times 10^{-2} \text{ J}$$

$$(c) W_{\text{ext}} = q_4 V_p = 0.19 \text{ J}$$



Choosing $V=0$ at $r=\infty$.

Example 25.3

In 1913, Bohr proposed a model of the hydrogen atom in which an electron orbits a stationary proton in a circular path. Find the total mechanical energy of the electron given that the radius of the orbit is 0.53×10^{-10} m.

Solution:

The mechanical energy is the sum of the kinetic and potential energies, $E = K + U$. The centripetal force is provided by the coulomb attraction.

$$U = -\frac{ke^2}{r}$$

$$F = \frac{ke^2}{r^2} = \frac{mv^2}{r} \Rightarrow K = \frac{1}{2}mv^2 = \frac{ke^2}{2r}$$

$$E = U + K = -\frac{1}{2}U = -\frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{2 \times 0.53 \times 10^{-10}} = -2.18 \times 10^{-18} \text{ J} = -13.6 \text{ eV}$$

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25.4 Electric Field Derived from Potential

Can we derive a conservative force from the associated potential energy function? Yes!

$$dU = -\mathbf{F}_c \cdot d\mathbf{s} = -q\mathbf{E} \cdot d\mathbf{s}$$

$$dV = -\mathbf{E} \cdot d\mathbf{s}$$

$$E_s = -\frac{dV}{ds}$$

A derivative in which all variables except one are held constant is called a **partial derivative** and is written with a ∂ instead of d . The electric field is therefore

$$\mathbf{E} = -\frac{\partial V}{\partial x}\mathbf{i} - \frac{\partial V}{\partial y}\mathbf{j} - \frac{\partial V}{\partial z}\mathbf{k}$$

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Example 25.4

The potential due to a point charge is given by $V=kQ/r$.
Find: (a) the radial component of the electric field; (b) the x-component of the electric field.

Solution:

$$(a) \quad V = \frac{kQ}{r} \quad \Rightarrow \quad E = -\frac{dV}{dr} = \frac{kQ}{r^2}$$

$$(b) \quad V = \frac{kQ}{\sqrt{x^2 + y^2 + z^2}} \quad \Rightarrow \quad E_x = -\frac{\partial V}{\partial x} = \frac{kQ}{r^2} \frac{x}{r}$$

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25.5 Continuous Charge Distribution

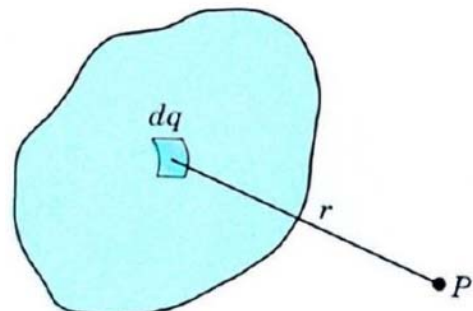
The potential due to continuous charge distribution can be found in two ways.

First approach: direct integration

$$dV = \frac{k dq}{r} \quad \Rightarrow \quad V = \int \frac{k}{r} dq$$

Second approach: use electric field

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$



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Example 25.5

A nonconducting disk of radius a has a uniform surface charge density σ C/m². What is the potential at a point on the axis of the disk at a distance from its center.

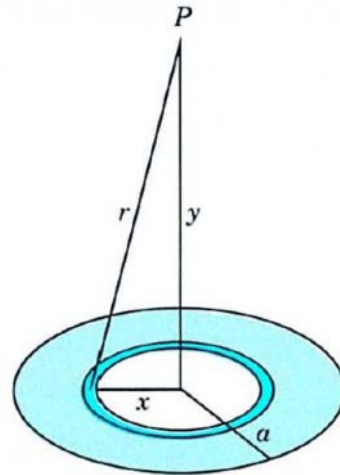
Solution:

$$dV = \frac{k dq}{r}, \quad dq = \sigma(2\pi x dx)$$

$$dV = \frac{k\sigma\pi}{\sqrt{x^2 + y^2}} dx^2$$

$$V = \int_0^a \frac{k\sigma\pi}{\sqrt{x^2 + y^2}} dx^2$$

$$= 2\pi k\sigma \left[(x^2 + y^2)^{0.5} - y \right]_0^a = 2\pi k\sigma \left[(a^2 + y^2)^{0.5} - y \right]$$



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Example 25.6

A shell of radius R has a charge Q uniformly distributed over its surface. Find the potential at a distance $r > R$ from its center.

Solution:

It is more straightforward to use the electric field, which we know from Gauss's law.

$$\mathbf{E} = \frac{kQ}{r^2} \hat{\mathbf{r}} \quad V(r) - V(\infty) = -\int_{\infty}^r \frac{kQ}{r^2} dr = -kQ \left[-\frac{1}{r} \right]_0^r$$

$$V(r) = \frac{kQ}{r}$$

The potential has a fixed value at all points within the conducting sphere equal to the potential at the surface.

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Example 25.7

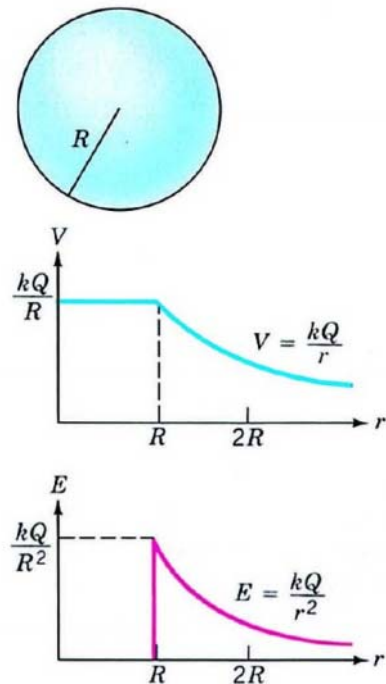
A metal sphere of radius R has a charge Q . Find its **potential energy**.

Solution:

$$dW = Vdq = \frac{kq}{r} dq$$

$$W = \int_0^Q \frac{kq}{r} dq = \frac{kQ^2}{2R}$$

The potential energy $U = \frac{1}{2}QV$ is the work needed to bring the system of charges together.



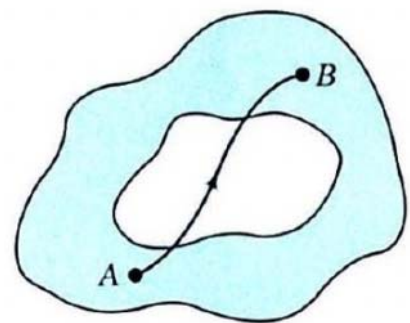
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25.6 Conductors

Within the material of the conductor, the electric field is zero elsewhere.

All points within and on the surface of a conductor in electrostatic equilibrium are at the same potential.

For a displacement $d\mathbf{s}$ along the surface of a conductor, we have $dV = \mathbf{E} \cdot d\mathbf{s} = 0$, which means $\mathbf{E}$ is perpendicular to $d\mathbf{s}$. The field lines are perpendicular to the surface.



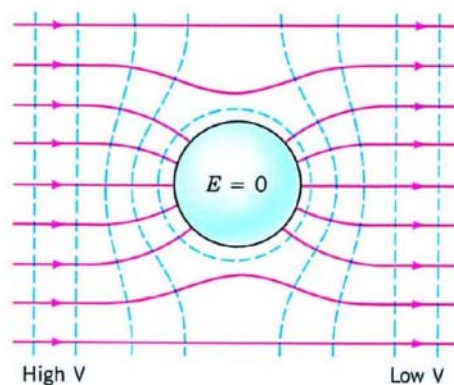
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25.6 Conductors (II) Shielding Effect

Far from the sphere, the field pattern remains the same: The field lines are uniform, and the equipotentials are planes.

Near the sphere, the equipotentials must be spheres and field lines must be radial.

The charges in the sphere **redistribute** themselves so as to ensure that these conditions are met.



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25.6 Conductors (III) Charge Redistribution

Suppose two charged metal spheres with radius R_1 and R_2 are connected by a long wire. Charge will flow from one to the other until their potential are equal. The equality of the potential implies that

$$\frac{Q_1}{R_1} = \frac{Q_2}{R_2}, \text{ since } Q = 4\pi R^2 \sigma$$

$$\sigma_1 R_1 = \sigma_2 R_2$$



We infer that $\sigma \propto 1/R$: The surface charge density on each sphere is inversely proportional to the radius.

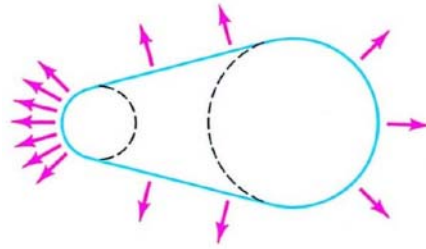
The regions with the smallest radii of curvature have the greatest surface charge densities.

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25.6 Conductors (IV)

Discharge at Sharp Points on a Conductor

$$E = \frac{\sigma}{\epsilon_0} \propto \frac{1}{R}$$



The above equation infer that the field strength is greatest at the sharp points on a conductor.

If the field strength is great enough (about 3×10^6 V/m for dry air) it can cause an electrical discharge in air.

How does the breakdown occur in high voltage transmission line?

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25.6 Conductors (V)

Dust Causing High Voltage Breakdown

The potential at the surface of a charged sphere is $V=kQ/R$ and the field strength is $E=kQ/R^2$. So, for a given breakdown field strength, breakdown voltage is proportional to the radius, $V_B \propto R$.

The potential of a sphere of radius 10 cm may be raised to 3×10^5 V before breakdown. On the other hand, a 0.05 mm dust particle can initiate a discharge at 150 V.

A high voltage system must keep at very clean condition.

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Exercises and Problems

Ch.25:

Ex. 37, 43, 48

Prob. 6, 7, 10, 11