## Chapter 26 Capacitors and Dielectrics

## How to store charge for long periods? Capacitors.

It is a big challenge. Even when a charged body is placed on an insulated stand, the charge tends to leak away. That is why people wants to "condense" charge without losing it.
Kleist invent the Leyden jar and served as a basis of electric researches for the next 50 years.


### 26.1 Capacitance

The magnitude of the charge $Q$ stored on either plate of a capacitor is directly proportional to the potential difference $V$ between the plates. Therefore, we may write

$$
Q=C V
$$

Where C is a constant of proportionality called the capacitance of the capacitor.
The SI unit of a capacitance is the farad (F). 1Farad =1 coulomb/volt


The capacitance of a capacitor depends on the geometry of the plates (their size, shape, and relative positions) and the medium (such as air, paper, or plastic) between them.

### 26.1 Capacitance: parallel-plate capacitor

A common arrangement found in capacitors consists of two plates.

$$
\begin{aligned}
& E=\frac{\sigma}{\varepsilon_{0}}=\frac{Q}{\varepsilon_{0} A} \Rightarrow V=E d=\frac{d Q}{\varepsilon_{0} A} \\
& C=\frac{\varepsilon_{0} A}{d}
\end{aligned}
$$

where $\varepsilon_{0}$ is $8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m}$.


## Example 26.1

A parallel-plate capacitor with a plate separation of 1 mm has a capacitance of 1 F . What is the area of each plate?

Solution:

$$
A=\frac{C d}{\varepsilon_{0}}=\frac{1 \times 10^{-3}}{8.85 \times 10^{-12}}=1.13 \times 10^{8} \mathrm{~m}^{2}
$$

This is approximately $10 \mathrm{~km} \times 10 \mathrm{~km}$ ! Clearly, the farad is very large unit.

## Example 26.3

What is the capacitance of an isolated sphere of radius $R$ ?

Solution:

$$
V=\frac{Q}{4 \pi \varepsilon_{0} R} \Rightarrow C=4 \pi \varepsilon_{0} R
$$

If we assume that earth is a conducting sphere of radius 6370 km , then its capacitance would be 710 uF .

Is earth a good capacitor? No.

## Example 26.4

A spherical capacitor consist of two concentric conducting spheres, as shown in Fig. 26.6. The inner sphere, of radius $R_{1}$, has charge $+Q$, while the outer shell of radius $R_{2}$, has charge $-Q$. Find its capacitance.
Solution:

$$
\begin{aligned}
& E=\frac{Q}{4 \pi \varepsilon_{0} r^{2}} \Rightarrow V=-\int_{R 1}^{R 2} E d r=\frac{Q}{4 \pi \varepsilon_{0}}\left(\frac{1}{R_{2}}-\frac{1}{R_{1}}\right) \\
& C=4 \pi \varepsilon_{0}\left(\frac{R_{1} R_{2}}{R_{2}-R_{1}}\right)
\end{aligned}
$$



The capacitance happens to be negative quantity.

## Example 26.5

A cylindrical capacitor consists of a central conductor of radius a surrounded by a cylindrical shell of radius $b$, as shown in Fig. 26.7. Find the capacitance of a length $L$ assuming that air is between the plates.
Solution:

$$
\begin{aligned}
E_{r} & =\frac{\lambda L}{\varepsilon_{0} 2 \pi r L}=\frac{\lambda}{2 \pi \varepsilon_{0} r} \\
V_{r} & =-\int_{a}^{b} E_{r} d r=-\frac{\lambda}{2 \pi \varepsilon_{0}} \ln \left(\frac{b}{a}\right) \\
& =-\frac{Q}{2 \pi \varepsilon_{0} L} \ln \left(\frac{b}{a}\right) \\
C & =-\frac{2 \pi \varepsilon_{0} L}{\ln (b / a)}
\end{aligned}
$$



Again, we are interested only in the magnitude of the capacitance.

### 26.2 Series and Parallel Combinations

A capacitance is rated according to its capacitance and the maximum potential difference that can be applied without damaging the insulator between the plates.

Series: same charge Q

$$
\left\{\begin{array}{l}
V_{1}=\frac{Q}{C_{1}} \\
V_{2}=\frac{Q}{C_{2}} \quad V=V_{1}+V_{2}=\frac{Q}{C_{1}}+\frac{Q}{C_{2}}=\frac{Q}{C_{e q}} \\
V=\frac{Q}{C_{e q}}=\frac{1}{C_{1}}+\frac{1}{C_{2}}
\end{array}\right.
$$



### 26.2 Series and Parallel Combinations (II)

Parallel: same potential difference $V$
$\left\{\begin{array}{l}Q_{1}=C_{1} V \\ Q_{2}=C_{2} V \\ Q=C_{e q} V\end{array}\right.$
$Q=Q_{1}+Q_{2}$
$C_{e q}=C_{1}+C_{2}$

(a)

(b)

## Example 26.6

For the circuit in Fig.26.10a, find: (a) the equivalent capacitance; (b) the charge and potential difference for each capacitor.
Solution:


### 26.3 Energy Stored in a Capacitor

The energy stored in a capacitor is equal to the work done--for example, by a battery---to charge it.
The work needed to transfer an infinitesimal charge $d q$ from the negative plate to the positive plate is $d W=V d q=q / C d q$.
The total work done to transfer charge $Q$ is

$$
W=\int_{0}^{Q} \frac{q}{C} d q=\frac{Q^{2}}{2 C}=\frac{C V^{2}}{2}
$$

## What kind of the potential energy does this work convert?

Electric potential energy.

## Example 26.7

Two capacitors, $\mathrm{C} 1=5 \mathrm{uF}$ and $\mathrm{C} 2=3 \mathrm{uF}$, are initially in parallel with a 12-V battery, as in Fig. 26.11a. They are disconnected and then reconnected as shown in Fig. 26.11b. Note carefully the numbering on the plates. Find the charges, potential differences, and energies stored (a) in the initial state, and (b) in the final state.

(b)

## Example 26.7 (II)

Solution:
Initial state:
Charge Q1=C1V=60uC, Q2=C2V=36uC
Potential difference $\mathrm{V} 1=\mathrm{V} 2=\mathrm{V}=12 \mathrm{~V}$
Energy stored 1/2Q1V=360uJ, 1/2Q2V=216uJ
final state: equal potential difference Vp
Charge $Q=Q 1-Q 2=24 u C=Q 1 p+Q 2 p=8 u F x V p$
Potential difference $\mathrm{Vp}=3 \mathrm{~V}$, $\mathrm{Q} 1 \mathrm{p}=15 \mathrm{uC}, \mathrm{Q} 2 \mathrm{p}=9 \mathrm{FF}$
Energy stored $1 / 2 \mathrm{Q} 1 \mathrm{pVp}=22.5 \mathrm{JJ}, 1 / 2 \mathrm{Q} 2 \mathrm{pVp}=13.5 \mathrm{JJ}$

### 26.4 Energy Density of the Electric Field

Where is the potential energy stored for a charged parallelplates capacitor?

$$
\begin{aligned}
& U_{E}=\frac{1}{2} Q V=\frac{1}{2} C V^{2}, C=\varepsilon_{0} A / d, V=E d \\
& U_{E}=\frac{1}{2} \frac{1}{2} A \\
& u_{E}=\frac{1}{2}(E d)^{2}=\frac{1}{2} \varepsilon_{0} E^{2}(A d) \quad E=\frac{\sigma}{\epsilon_{0}},
\end{aligned}
$$

$u_{E}$ is a generally valid expression for the energy density of an electric field.

Why don't we use the potential instead of using the electric field?

## Example 26.9

Use Eq. 26.10 to derive the potential energy of a metal sphere of radius $R$ with charge $Q$

Solution:


Apply Gauss's law, the electric field outside the metal sphere is

$$
E=\frac{k Q}{r^{2}} \quad(\mathrm{r}>\mathrm{R})
$$

The energy of a imaginary shell of radius $r$ and infinitesimal thickness dr is

$$
\begin{aligned}
d U_{E} & =\frac{1}{2} \varepsilon_{0} E^{2} d V=\frac{1}{2} \varepsilon_{0}\left(\frac{k Q}{r^{2}}\right)^{2} 4 \pi r^{2} d r=\frac{k Q^{2}}{2} \frac{1}{r^{2}} d r \\
U_{E} & =\int_{R}^{\infty} \frac{k Q^{2}}{2} \frac{1}{r^{2}} d r=\frac{k Q^{2}}{2 R}\left(=\frac{1}{2} Q V\right)
\end{aligned}
$$

### 26.5 Dielectric

When certain nonconducting materials, such as glass, paper, or plastic, are introduced between the plates of a capacitor, its capacitance increases. Such materials calls dielectric.
(i) Battery not connected (constant charge)
(ii) Battery connected (constant voltage)

(a)

(b)

$$
V_{D}=\frac{V_{0}}{\kappa}
$$

$$
C_{D}=\kappa C_{0}
$$

## 26．5 Dielectric（II）

| 物質 <br> 名稱 | 介電 <br> 常數 | 介電強度 <br> $\left(10^{\circ} \mathrm{V} / \mathrm{m}\right)$ |
| :--- | :--- | :---: |
| 空氣 | 1.00059 | 3 |
| 紙 | 3.7 | 16 |
| 玻璃 | $4-6$ | 9 |
| 石蠟 | 2.3 | 11 |
| 橡翏 | $2-3.5$ | 30 |
| 雲母 | 6 | 150 |
| 水 | 80 | - |



What does the dielectric strength mean？

## 26．6 Atomic View of Dielectrics

The dielectric constant of a substance is a measure of the response of its charges to an external electric field．

（a）

（b）

（a）


## Example 26.10

A dielectric slab of thickness t and dielectric constant k is inserted into a parallel plate capacitor with plates of area A , separated by distance d, as shown in the figure. Assume that the battery is disconnected before the slab is inserted. What is the capacitance?


## Exercises and Problems

Ch.26:
Ex. 7, 31, 34, 42
Prob. 2, 3, 7, 12

