Chapter 29 The Magnetic Field

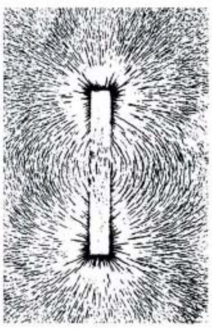
By analogy with electrostatics, why don't we study magnetostatics first? Due to complicated mathematics (lack of magnetic monopole).

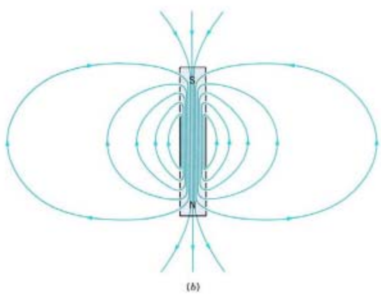
In 1820, Oersted established the link between electricity and magnetism. He found that a compass needle fluctuates during a thunderstorm, particularly when lighting strikes, and later showed that a magnet exerts a force on a current-carrying wire.

Nowadays, magnets are used in meters, motors, loudspeakers, computer memories, chemical analysis, focus electron beams, and a myriad of other applications.

29.1 The Magnetic Field

When iron filings are sprinkled around a bar magnet, they form a characteristic pattern that shows how the influence of the magnet spreads to the surrounding space.

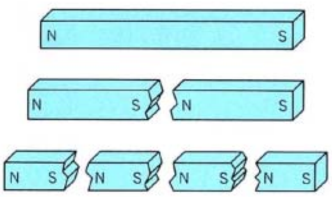




The **magnetic field**, **B**, at a point along the tangent to a field line. The *direction* of **B** is that of the force on the north pole of a bar magnet, or the *direction* in which a compass needle points. The *strength* of the field is proportional to the number of lines passing through a unit area normal to the field (*flux density*).

29.1 The Magnetic Field: monopole?

If one try to isolate the poles by cutting the magnetic, a curious thing happens: One obtains two magnets. No matter how thinly the magnet is sliced, each fragment always have two poles. Even down to the atomic level, no one has found an isolated magnetic pole, called a monopole. Thus magnetic field lines form closed loops.



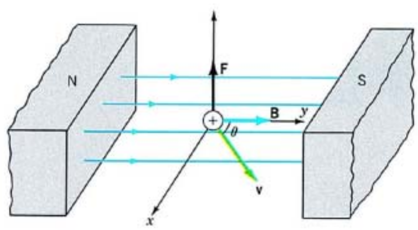
Outside a magnetic the lines emerge from the north pole and enter the south pole; *within* the magnet they are directed from the south pole to the north pole. The ***dots*** represents the tip of an arrow coming toward you. The ***cross*** represents the tail of an arrow moving away.

Definition of the Magnetic Field

When defining of the electric field, the electric field strength can be derived from the following relation: **E**=**F**/q. Since an isolated pole is not available, the definition of the magnetic field is not as simple.

Instead, we examine how an electric charge is affected by a magnetic field.

$$\mathbf{F} = qvB\sin\theta = q\mathbf{v}\times\mathbf{B}$$



Since **F** is always perpendicular to **v**, *a magnetic force does no work on a particle and cannot be used to change its kinetic energy.*

The SI unit of magnetic field is the Tesla (T). 1 T=10^4 G

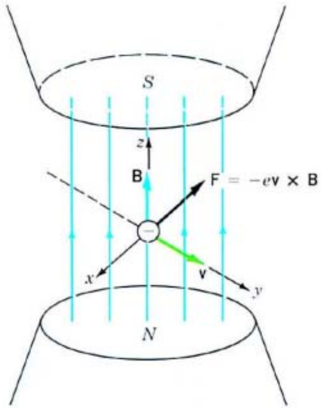
Example 29.1

An electron has a velocity $\mathbf{v}=10^6\mathbf{j}$ m/s in a field $\mathbf{B}=500\mathbf{k}$ G, as shown in Fig.29.6. what is the force on the electron?

Solution:

$$\mathbf{F} = q\mathbf{v}\times\mathbf{B} = -1.6\times10^{-19}\times1.0\times10^{6}\times500(\mathbf{j}\times\mathbf{k})$$
$$= -8.0\times10^{-15}\mathbf{i}\ \ \mathrm{N}$$

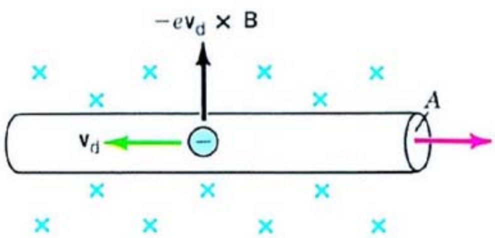
Note that the force is perpendicular to both the velocity and the magnetic field.



29.2 Force on a Current-Carrying Conductor

When a current flows in a magnetic field, the electrons as a whole acquire a slow drift speed, $\mathbf{v}_d$, and experience a magnetic force, which is then transmitted to the wire.

$$F = qvB\sin\theta = (nA\ell e)v_d B$$
$$= (nAev_d)\ell B$$
$$= I\ell B$$
$$\mathbf{F} = I\boldsymbol{\ell}\times\mathbf{B}$$
$$F = I\ell B\sin\theta$$

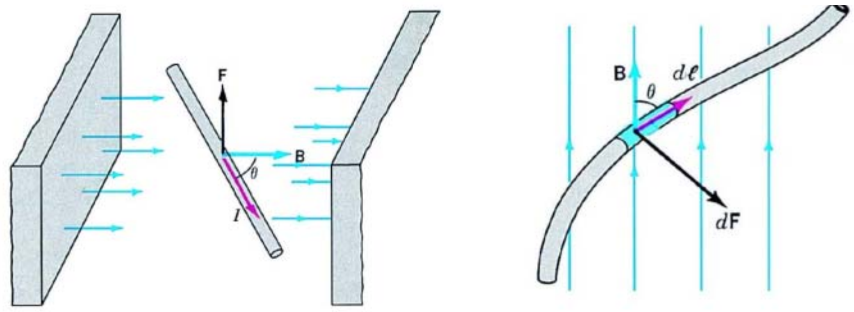


n: is the number of the conductor per unit volume.
ℓ: is defined to be in the direction in which the current is flowing.

29.2 Force on a Current-Carrying Conductor

The force on an infinitesimal current element is

$$d\mathbf{F} = I d\ell \times \mathbf{B}$$



The force on a wire is the vector sum (integral) of the forces on all current elements.

Example 29.3

A straight wire lines along a body diagonal of an imagiary cube of side a=20 cm, and carries a current of 5 A (see Fig. 29.11). Find the force on it due to a uniform field **B**=0.6**j** T.

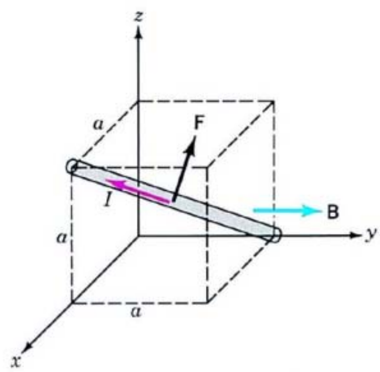
Solution:

$$\ell = a\mathbf{i} - a\mathbf{j} + a\mathbf{k}$$

$$\mathbf{F} = I\ell \times \mathbf{B}$$

$$= IaB(\mathbf{i} - \mathbf{j} + \mathbf{k}) \times \mathbf{j}$$

$$= IaB(\mathbf{k} - \mathbf{i})$$



The force lies in the xz plane and has a magnitude

$$F = \sqrt{2} IaB = 0.85\ N$$

Example 29.4

A wire is bent into a semicircular loop of radius R. It carries a current I, and its plane is perpendicular to a uniform magnetic field B, as shown in Fig. 29.12. find the force on the loop.

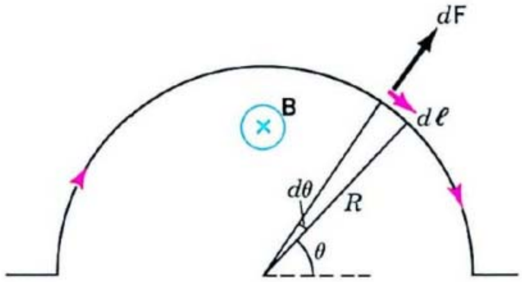
Solution:

$$d\mathbf{F} = I d\ell \times \mathbf{B}$$

$$dF_y = IRB \sin\theta \, d\theta$$

$$F_y = IRB \int_0^{\pi} \sin\theta \, d\theta$$

$$= 2IRB = I(2R)B$$

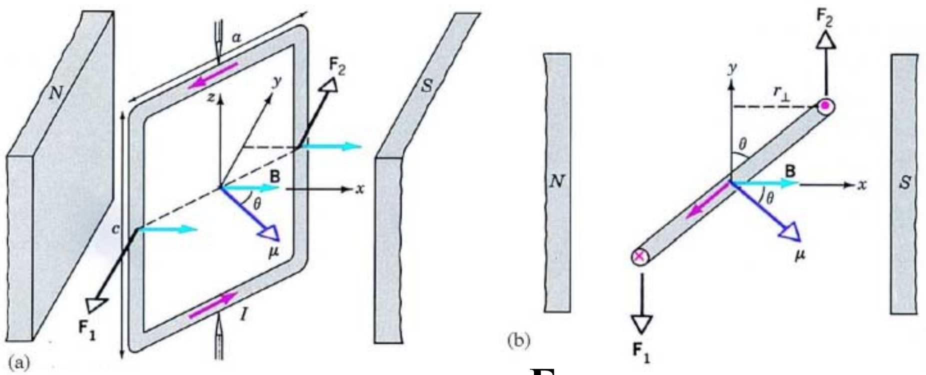


The x-components of the forces on such elements will cancel in pairs.

The net force on any close current-carrying loop is zero.

29.3 Torque on a Current Loop

The net force on a current loop in a uniform magnetic field is zero. However, the loop may experience a net torque that tends to rotate it.



$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

$$\mathbf{F}_1 = I(-c\mathbf{k}) \times B\mathbf{i} = -IcB\mathbf{j}$$

$$\mathbf{F}_2 = I(c\mathbf{k}) \times B\mathbf{i} = IcB\mathbf{j}$$

$$\begin{cases} \tau_1 = \dfrac{1}{2} aIcB \sin\theta \\ \tau_2 = -\dfrac{1}{2} aIcB \sin\theta \end{cases} \Rightarrow \tau = IAB \sin\theta$$

29.3 Torque on a Current Loop (II)

A current loop, like a bar magnet, is a magnetic diople. The magnetic dipole moment of a plane loop of any shape is defined as: $\boldsymbol{\mu} = NIA\hat{\mathbf{n}}$

The SI unit of magnetic moment is A·m^2. The direction of the unit vector **n** is given by a right-hand rule.

The tendency of the torque is to align the magnetic moment along the field --- just the way a compass needle behaves.

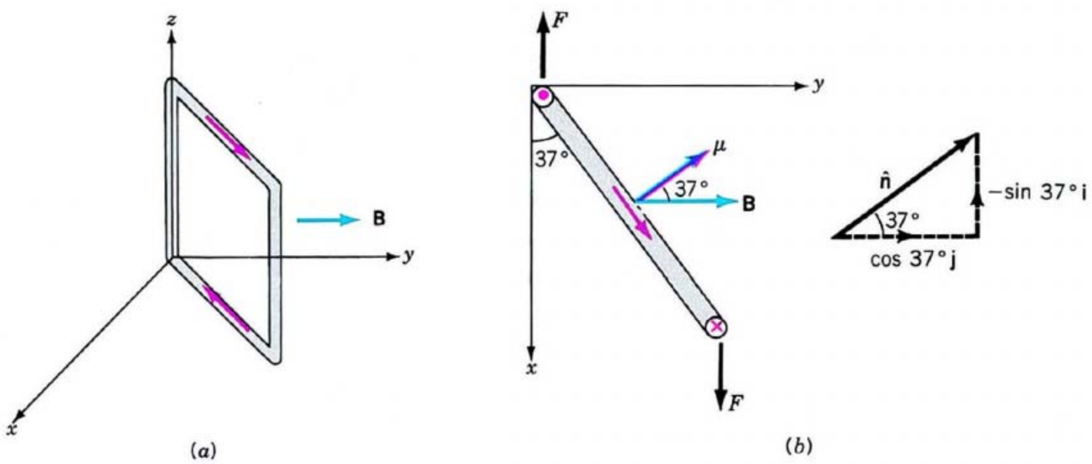
By analogy with electric field, we have

$$\boldsymbol{\tau}_E = \mathbf{p} \times \mathbf{E} \quad U_E = -\mathbf{p} \cdot \mathbf{E}$$

$$\boldsymbol{\tau}_B = \boldsymbol{\mu} \times \mathbf{B} \quad U_B = -\boldsymbol{\mu} \cdot \mathbf{B}$$

Example 29.5

The square loop in Fig. 29.16a has sides of length 20 cm. It has 5 turns and carries a current of 2 A. The normal to the loop is at 37º to a uniform field **B**=0.5**j** T. Find: (a) the magnetic moment; (b) the torque on the loop; (c) the work needed to rotate the loop from its position of minimum energy to the given orientation.

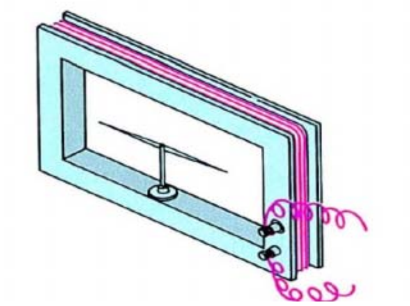


29.4 The Galvanometer

Oersted placed a compass needle below a fine platinum wire that was laid along the north-south axis. When a large current was switched on, he saw the compass needle rotated from its normal alignment.

Soon after Oersted's discovery, it was realized that the deflection of the compass needle could be used to measure current.

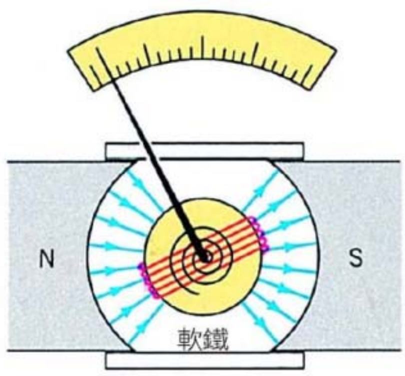
Figure below is a example, using multiple turns to enhance the magnetic effect of current.



Moving-Coil Galvanometer

The current is passed through a coil with many turns of fine wire, suspended in a magnetic field. A coiled spring is used to balance the torque and to restore the system.

The soft iron placed within the loop helps ensure uniform magnetic field.



With this arrangement, the deflection is directly proportional to the current.

$$\tau_B = \mu B \sin\theta = NIAB \sin\theta$$

$$\tau_{sp} = \kappa\phi \, (= \tau_B)$$

$$I = \frac{NAB}{\kappa}\phi, \text{ if } \theta = \frac{\pi}{2}$$

29.5 The Motion of Charged Particles in Magnetic Fields

How does a charged particle move with an initial velocity **v** perpendicular to a uniform magnetic field **B**?

Since **v** and **B** are perpendicular, the particle experiences a force *F=qvB* of constant magnitude directed perpendicular. Under the action of such a force, the particle will move in a circular path at constant speed. From Newton's second law, *F=ma*, we have

$$qvB = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{qB}$$

The radius of the orbit is directly proportional to the linear momentum of the particle and inversely proportional to the magnetic field strength.

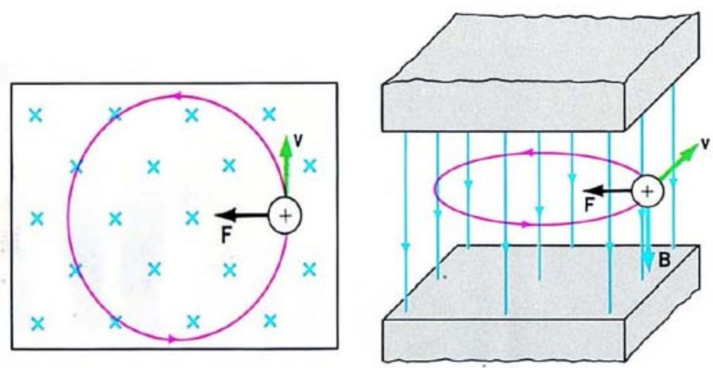
29.5 The Motion of Charged Particles in Magnetic Fields (II)

What are the frequency and the period? Are they independent of the speed of the particle? *Yes*.

The period of the orbit is

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB} = \left(\frac{m}{q}\right)\frac{2\pi}{B}$$

$$f_c = \frac{1}{T} = \frac{qB}{2\pi m} = \left(\frac{q}{m}\right)\frac{B}{2\pi}$$



The frequency is called the cyclotron frequency.

All particles with the same charge-to-mass ratio, *q/m*, have the same period and cyclotron frequency.

Example 29.8

A proton moves in a circle in radius 20 cm perpendicular to a field of magnitude 0.005 T. Find: (a) the magnitude of its momentum, and (b) its kinetic energy in eV.

Solution:

(a) $r = \dfrac{mv}{qB} \Rightarrow p = mv = rqB = 0.2(1.6\times10^{-19}\,\mathrm{C})0.05 = 1.6\times10^{-21}\ \mathrm{kg\cdot m/s}$

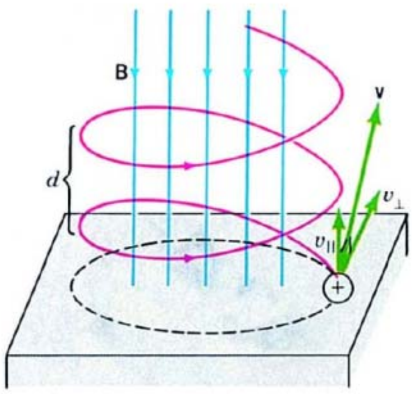
(b) $K = \dfrac{p^2}{2m} = \dfrac{(rqB)^2}{2m}$

$$= \frac{(1.6\times10^{-21})^2}{2\times1.67\times10^{-27}}\ \mathrm{J} = 7.7\times10^{-16}\ \mathrm{J} = 4.8\,\mathrm{keV}$$

Note1: The relation between momentum and kinetic energy.
Note2: The energy unit, electron-volt eV.

Helical Motion

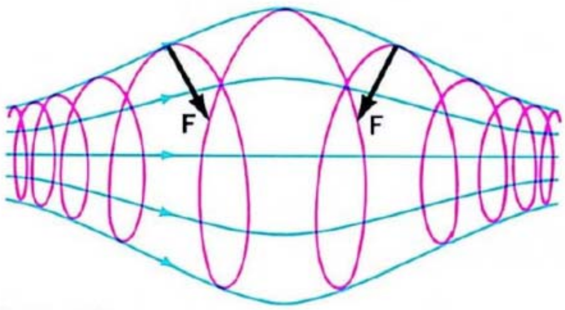
What happens if the charged particle's velocity has not only a perpendicular component $v_\perp$ but also a parallel component $v_{//}$? *Helical Motion*.



The perpendicular component $v_\perp$ gives rise to a force $qv_\perp B$ that produces circular motion, but the parallel component $v_{//}$ is not affected. The result is the superposition of a uniform circular motion normal to the lines and a constant motion along the lines.

Magnetic Bottle/Mirror

What happens if the magnetic field is not uniform? Energy transfer between the perpendicular and parallel components.



In a nonuniform field, the particle experiences a force that points toward the region of week field. As a result, the component of the velocity along the B lines is not constant.

If the particle is moving toward the region of stronger field, as some point it may be stopped and made to reverse the direction of its travel.

29.6 Combined Electric and Magnetic Fields

When a particle is subject to both electric and magnetic fields in the same region, what is the total force on it?

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

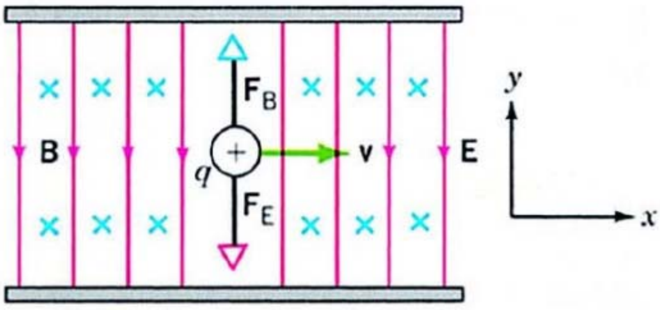
This is called the Lorentz force.

Can we find a condition that a charged particle experiences no net force on it? Yes.

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B}$$

We can use this property to devise a velocity selector.

Velocity Selector

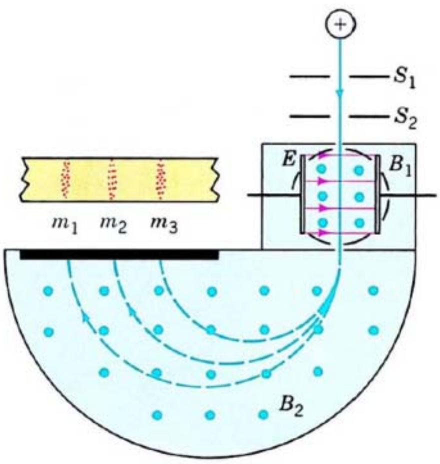


$$\begin{pmatrix} \mathbf{E} = -E\mathbf{j} \\ \mathbf{B} = -B\mathbf{k} \end{pmatrix} \Rightarrow \mathbf{v} = \frac{E}{B}\mathbf{i} = v\mathbf{i}$$

Only those particles with speed v=E/B pass through the crossed fields undeflected. This provides a convenient way of either measuring or selecting the velocities of charged particles.

Mass Spectrometer

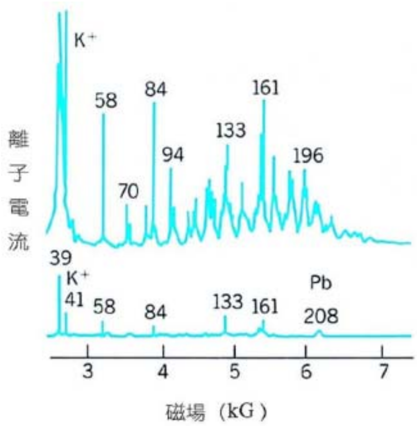
A mass spectrometer is a device that separates charged particles, usually ions, according to their charge-to-mass ratios.



$$v = \frac{E}{B_1}$$

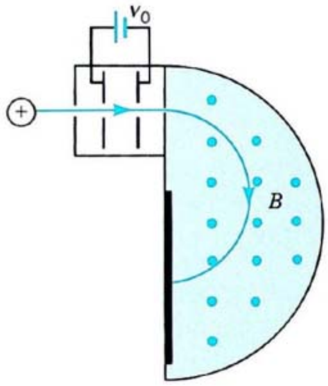
$$r = \frac{mv}{qB_2} = \frac{mE}{qB_1B_2}$$

$$\frac{m}{q} = \frac{B_1B_2}{E}r$$



Example 29.9

In a mass spectrometer shown in Fig. 29.29, two isotopes of an elements with mass m1 and m2 are accelerated from rest by a potential difference V. They then enter a uniform B normal to the magnetic field lines. What is the ratio of the radii of their paths?



Solution:

$$v = \sqrt{\frac{2qV}{m}}$$

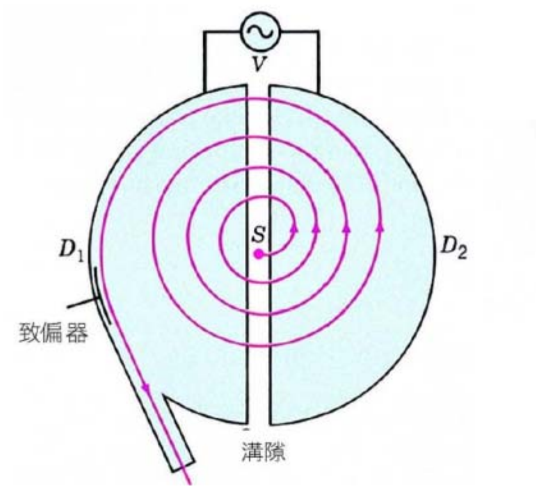
$$r = \frac{mv}{qB} = \sqrt{\frac{2mV}{qB^2}} \quad \text{then } r_1 / r_2 = \sqrt{(m_1 / m_2)}$$

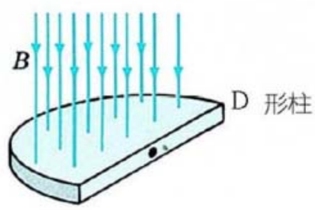
Note1: How particle is accelerated by a potential difference.

29.7 The Cyclotron

The operation of the cyclotron is based on the fact that the orbital period of a particle in a magnetic field is independent of its speed. (non-relativistic limit)

This machine is used to accelerate charged particle to energies up to 200 MeV or even higher. How?





Example 29.10

A cyclotron is used to accelerate protons from rest. It has a radius of 60 cm and a magnetic field of 0.8 T. The potential difference across the dees is 75 kV. Find: (a) the frequency of the alternating potential difference; (b) the maximum kinetic energy; (c) the number of revolutions made by the protons.

Solution:

(a) $$f_c = \frac{qB}{2\pi m} = 12\ \mathrm{MHz}$$

(b) $$K_{\max} = \frac{(q r_{\max} B)^2}{2m} = 1.76\times 10^{-12}\ \mathrm{J} = 11\ \mathrm{MeV}$$

(c) $$\Delta K = 2qV = 150\ \ \mathrm{keV}$$

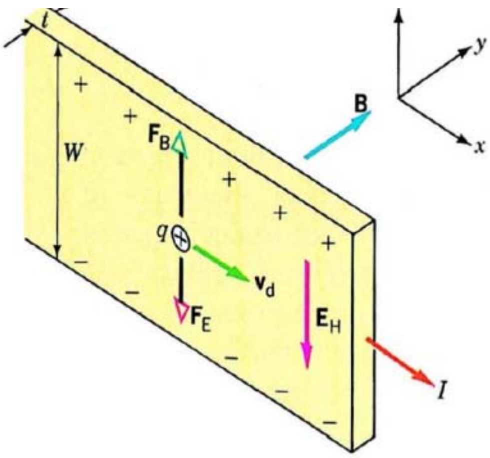
$$K_{\max} / \Delta K = 11000/150 = 73.5\ \ \mathrm{revs.}$$

Hint: How to determine the maximum kinetic energy?

29.8 The Hall Effect

How can we determine the polarity of the charge-carrying particles? The Hall effect.

The charged particles drifting in a current-carrying metal strip, as shown in the Fig. 29.34, experience a magnetic force. This force separates the particles, resulting an electric force to counteract the magnetic force.



The Hall potential difference is

$$\mathrm{V_H} = Ew = v_d B w$$

$$v_d = I / nqwt$$

$$\mathrm{V_H} = \frac{IB}{nqt}$$

Example 29.11

A metal strip of width 1 cm and thickness 2 mm lies normal to a magnetic field. When it carries a current of 10 A, the Hall voltage is 0.4 uV. What is the magnitude of B? Take the carrier charge density to be $5x10^{28}$ m^{-3}.

Solution:

$$B = \frac{nqt\mathrm{V_H}}{I}$$
$$= \frac{(5\times10^{28})(1.6\times10^{-19})(0.002)(4\times10^{-7})}{10}$$
$$= 0.64\ \mathrm{T}$$

Exercises and Problems

Ch.29:

Ex. 8, 21, 37, 43, 46, 58

Prob. 1, 2, 5