Chapter 30 Sources of the Magnetic Field

In the last chapter we studied the force exerted by a magnetic field on current and charges in motion.

We now go on to discuss how the magnetic fields are produced by currents and charges in motion.

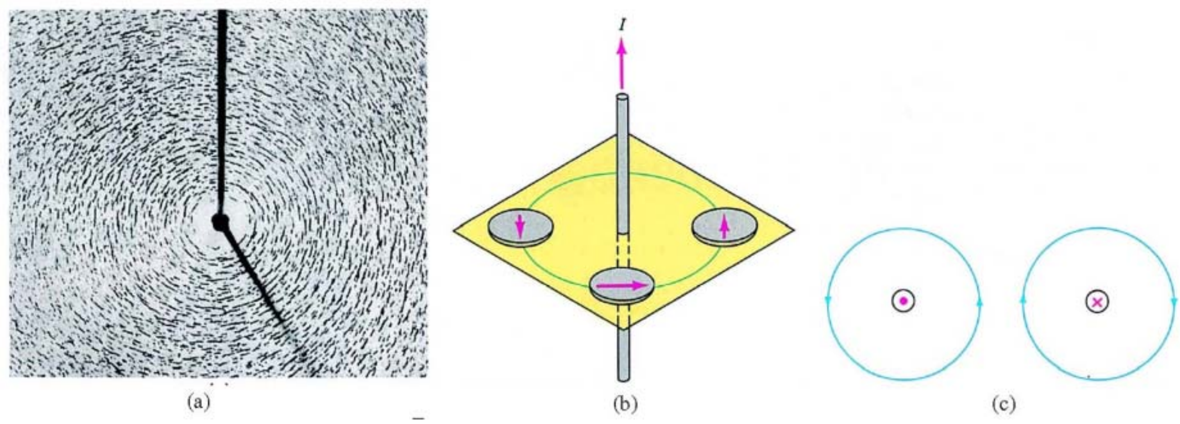
The **Biot-Savart** law is analogous to the expression for the electric field obtained from **Coulomb's** law for the force between point charges.

Ampere's law is analogous to **Gauss's** law in electrostatics. It is useful in determining the magnetic field due to a symmetric current distribution.

30.1 Field due to a Long, Straight Wire

How a current-carry wire produces a magnetic field?

A current in a long, straight wire produces a magnetic field with circular field lines --- as may be verified by sprinkling iron filings on a broad normal to the wire.



30.1 Field due to a Long, Straight Wire (II)

Biot and Sarvart found that the magnetic field is inversely proportional to the distance R from the wire. It was later found that the current is directly proportional to the current.

In SI units, we express these results as

$$B = \frac{\mu_0 I}{2\pi R}$$

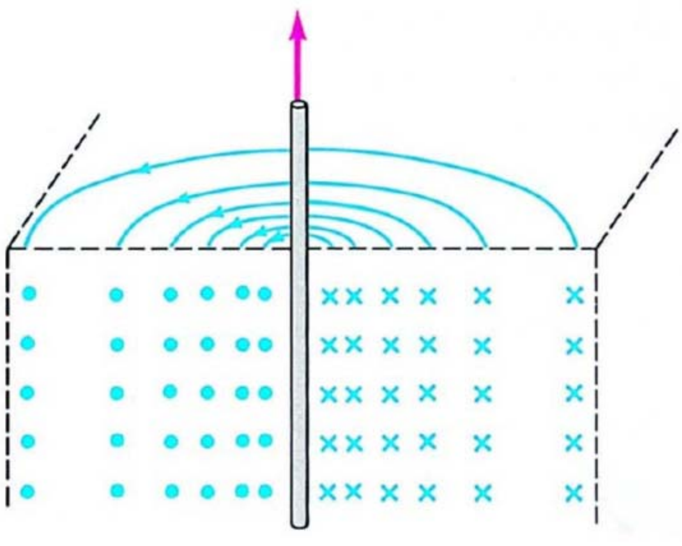
Where μ0, called the permeability constant, is defined to have the value

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m / A}$$

30.1 Field due to a Long, Straight Wire (III)

How do we determine the direction of the field?

The direction of the field is given by the right-hand rule: When the thumb points along the current, the curled fingers indicate the direction of the field.



Example 30.1

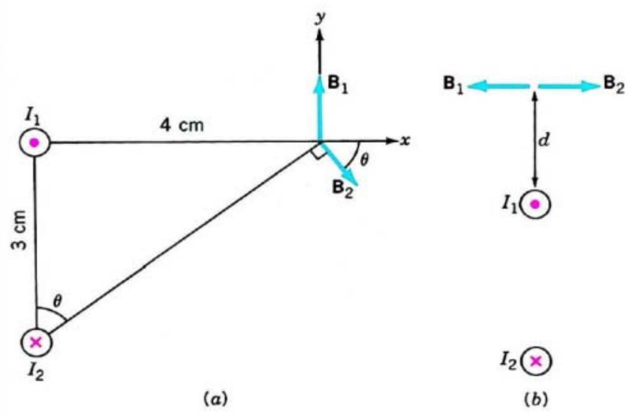
Two long straight, parallel wires are 3 cm apart. They carry currents I_1=3 A and I_2=5 A in opposite directions as shown in Fig. 30.3a. (a) Find the field strength at point P. (b) At what point, besides infinity, is the field strength zero?

Solution:

$$\mathbf{B}_\mathrm{T} = \mathbf{B}_1 + \mathbf{B}_2$$

$$B_1 = \frac{0.2\times 3}{0.04}\,\mathrm{uT} = 15\,\mathrm{uT}$$

$$B_2 = \frac{0.2\times 5}{0.05}\,\mathrm{uT} = 20\,\mathrm{uT}$$



Hint: This is a vector addition. Directions must be taken into consideration.

30.2 Magnetic Force Between Parallel Wires

What is the magnetic force between two current-carrying wires?

Oersted's demonstration that an electric current exerts a force on a compass needle ***did not***, of course, proves that there is an interaction between two currents.

Ampere demonstrated that two current-carrying wires do in fact exert forces on each other.

30.2 Magnetic Force Between Parallel Wires (II)

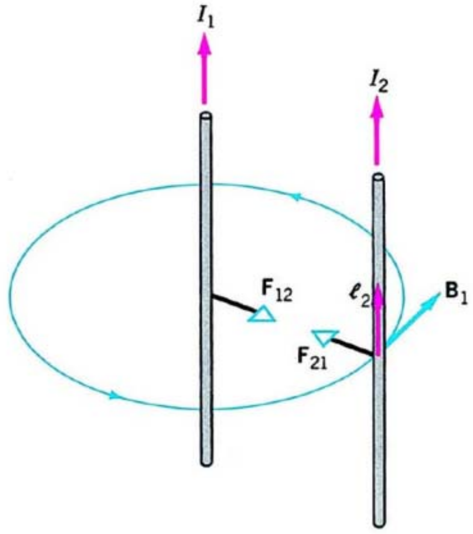
Consider two long, straight wires that carry current I_1 and I_2, as shown in Fig. 30.4.

$$\mathbf{F} = I\boldsymbol{\ell} \times \mathbf{B}$$

$$F_{21} = I_2 \ell_2 B_1 = I_2 \ell_2 \frac{\mu_0 I_1}{2\pi d}$$

The force per unit length on either wire is the same:

$$\frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi d}$$



We see that currents in the same direction attract each other. Conversely, currents in opposite directions repel each other.

30.3 Biot-Sarvart Law for a Current Element

Having determined the magnetic field for a long straight wire, Biot and Sarvart next sought a more general expression for the field due to an infinitesimal length of any current-carry wire.

Laplace pointed out to them that the result for a long wire implies that the field due to a current element should depend on the inverse square of the distance.

long current-carry wire	Long uniform charged line
$B = \frac{2k'I}{R}$	$E = \frac{2k\lambda}{R}$
? $dB = k'\frac{Id\ell}{r^2}\sin\theta$	$d\mathbf{E} = k\frac{\lambda d\ell}{r^2}\hat{\mathbf{r}}$

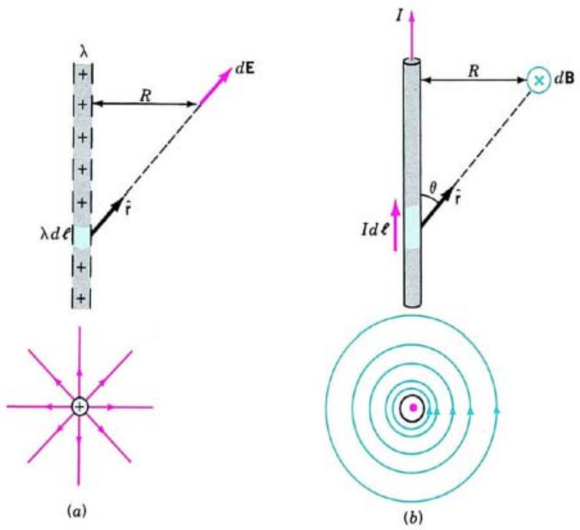
30.3 Biot-Sarvart Law for a Current Element (II)

In SI units and vector notation, the Biot-Sarvart law for the magnetic field due to a current element, shown in Fig. 30.5b. is

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{Id\ell \times \hat{\mathbf{r}}}{r^2}$$

The magnitude of the field is

$$dB = \frac{\mu_0}{4\pi}\frac{Id\ell \sin\theta}{r^2}$$



Example 30.2

Find the field strength at a distance R from an infinite straight wire that carries a current *I*.

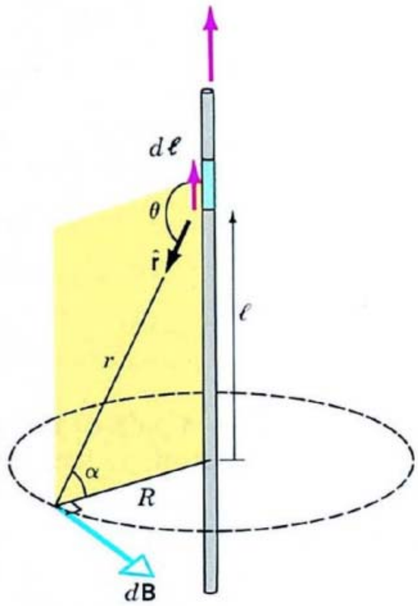
Solution:

$$\left|d\ell \times \hat{\mathbf{r}}\right| = d\ell \sin\theta = d\ell \cos\alpha$$

$$\ell = R\tan\alpha \Rightarrow \mathrm{d}\ell = \mathrm{R}\sec^2\alpha \, d\alpha$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-\pi/2}^{+\pi/2}\frac{\mathrm{R}\sec^2\alpha \cos\alpha \, d\alpha}{(R\sec\alpha)^2}$$

$$= \frac{\mu_0 I}{4\pi R}\int_{-\pi/2}^{+\pi/2}\cos\alpha \, d\alpha = \frac{\mu_0 I}{2\pi R}$$



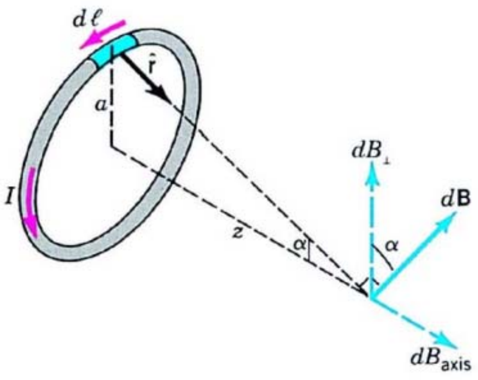
Example 30.3

A circular loop of radius a carries a current *I*. Find the magnetic field along the axis of the loop at a distance z from the center.

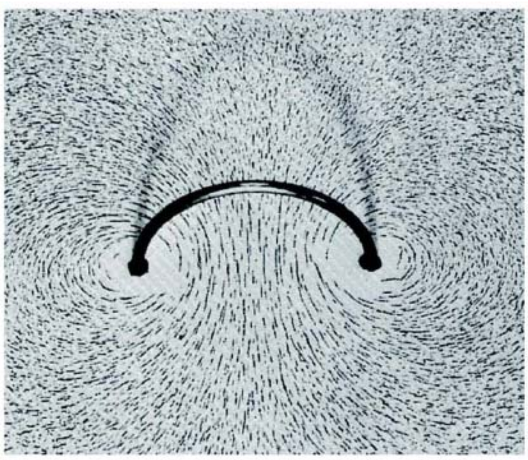
Solution:

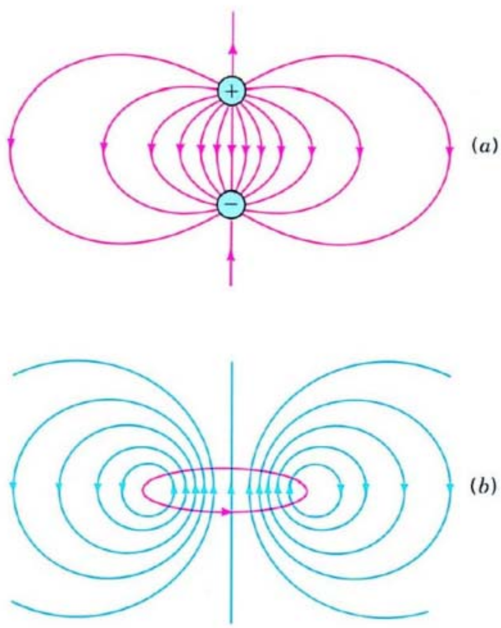
$$dB_{axis} = dB \sin\alpha = \frac{\mu_0 I d\ell}{4\pi r^2}\left(\frac{a}{r}\right)$$

$$B_{axis} = \frac{\mu_0 I a^2}{2r^3}$$

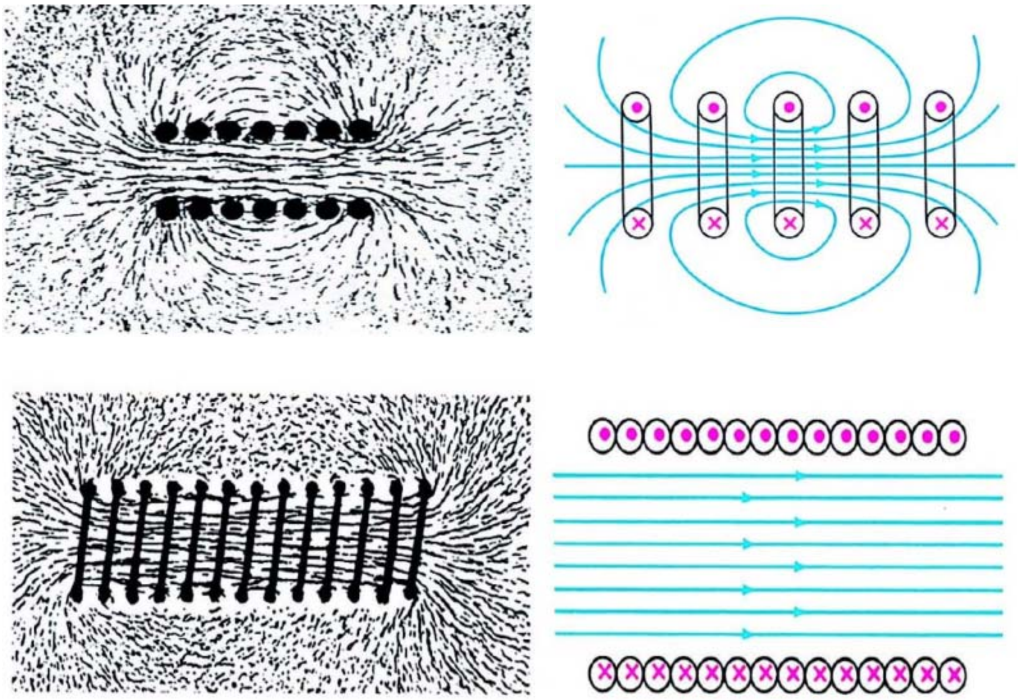


Example 30.3 (II)





The Magnetic Field of Solenoid



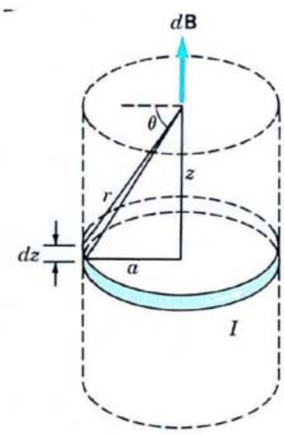
Example 30.4

A solenoid of length *L* and radius *a* has *N* turns of wire and carries a current *I*. Find the field strength at a point along the axis.

Solution:

Sine the solenoid is a series of closely packed loops, we may divided into current loops of width dz, each of which contains ndz turns, where n=N/L is the number of tuns per unit length.

The current within such a loop is (ndz)I.



Example 30.4 (II)

A solenoid of length *L* and radius *a* has *N* turns of wire and carries a current *I*. Find the field strength at a point along the axis.

Solution:

$$z = a\tan\theta \Rightarrow dz = a\sec^2\theta d\theta$$

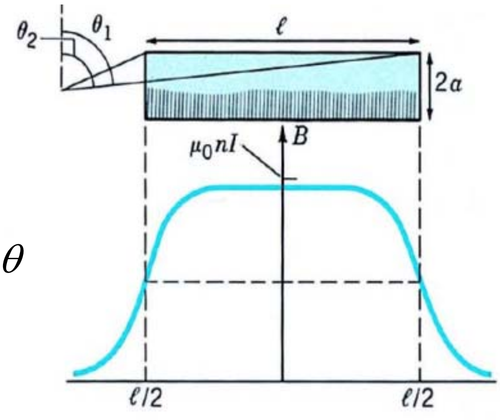
$$nIdz = nIa\sec^2\theta d\theta$$

$$dB_{axis} = \frac{\mu_0 a^2}{2(a^2 + a\tan^2\theta)^{3/2}} nIa\sec^2 d\theta$$

$$= \frac{1}{2}\mu_0 nI\cos\theta d\theta$$

$$B = \int_{\theta_1}^{\theta_2} \frac{1}{2}\mu_0 nI\cos\theta d\theta$$

$$= \frac{1}{2}\mu_0 nI(\sin\theta_2 - \sin\theta_1)$$



(b)

$$B = \mu_0 nI \text{(infinite long solenoid)}$$

30.4 Ampere's Law

Ampere had several objections to the work of Biot and Sarvart. For example, accuracy and assumption.

He pursued his own line of experimental and theoretical research and obtained a different relation, now called Ampere's law, between a current and the magnetic field it produces.

Although Ampere's law can be derived from the Biot-Sarvart expression for d**B**, we will not do so. Instead, we can make it plausible by considering the field due to an infinite straight wire.

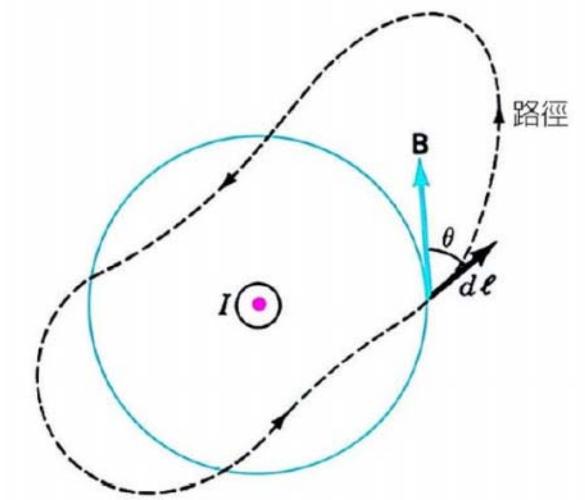
We know that the field lines are concentric circles for a infinite long, straight current-carrying wire.

$$B(2\pi r)=\mu_0 I$$

30.4 Ampere's Law (II)

$B(2\pi r)=\mu_0 I$. We may interpret it as follows: $2\pi r$ is the length of a circular path around the wire, B is the component of the magnetic field tangential to the path, and I is the current through the area bounded by the path.

Ampere generalized this result to *the paths* and wires of *any shape*.



30.4 Ampere's Law (III)

According to Ampere's law the sum (integral) of this product around a closed path is given by

$$\oint \mathbf{B} \cdot d\ell = \mu_0 I$$

Where I is the net current flowing through the surface enclosed by the path.

The sense (clockwise or counterclockwise) in which the integral is to be evaluated is given by a right-hand rule: When the thumb of the right hand points along the current, the curled fingers indicate the positive sense along the path.

Example 30.5

An infinite straight wire of radius R carries a current I. Find the magnetic field at a distance r from the center of the wire for (a) r>R, and (b) r<R. Assume that the current is uniformly distributed across the cross section of the wire.

Solution:

(a)

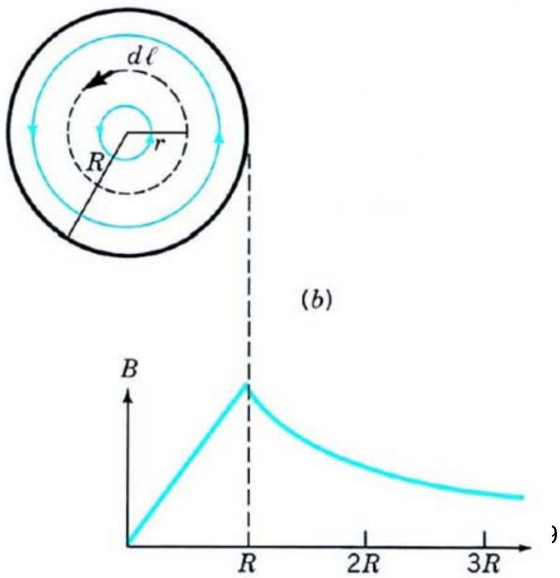
$$\oint \mathbf{B} \cdot d\ell = B 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \quad (r > R)$$

(b)

$$\oint \mathbf{B} \cdot d\ell = B 2\pi r = \mu_0 \frac{\pi r^2}{\pi R^2} I$$

$$B = \frac{\mu_0 I}{2\pi R^2} r \quad (r < R)$$



Example 30.6

An ideal infinite solenoid has n turns per unit length and carries a current I. Find its magnetic field.

Solution:

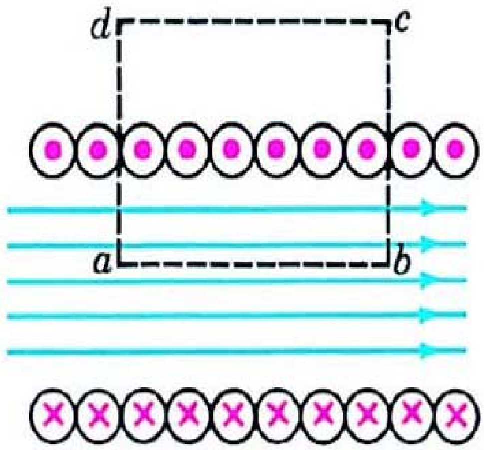
$$\oint \mathbf{B} \cdot d\ell$$

$$= \int_a^b B \cdot d\ell + \cancel{\int_b^c B \cdot d\ell} + \cancel{\int_c^d B \cdot d\ell} + \cancel{\int_d^a B \cdot d\ell}$$

$$= \int_a^b B \cdot d\ell$$

$$BL_{ab} = \mu_0 n L_{ab} I$$

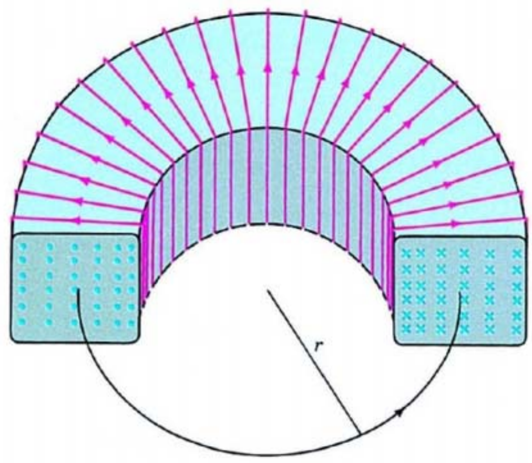
$$B = \mu_0 n I$$



Example 30.7

A toroidal coil (shaped like a doughnut) is tightly wound with N turns and carries a current I. We assume that it has a rectangular cross section, as shown in Fig. 30.18. Find the field strength within the toroid.

Solution:



$$\oint \mathbf{B} \cdot d\ell = B \oint d\ell = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi r}$$

The field is not uniform; it varies as 1/r. The toroidal fields are used in research on fusion power.

Exercises and Problems

Ch.30:

Ex. 9, 13, 28, 29

Prob. 1, 3, 5, 9, 10